ANGLO-CHINESE JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION

Higher 2

/100

CANDIDATE NAME

TUTORIAL/ FORM CLASS

INDEX NUMBER

MATHEMATICS 9758/01

Paper 1

23 August 2022

3 hours

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your index number, class and name on all the work you hand in.
Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

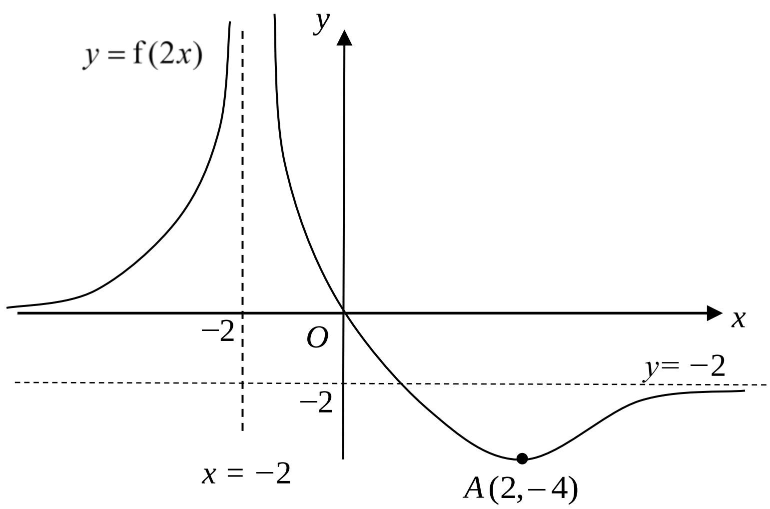
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Question	Marks
1	/3
2	/4
3	/7
4	/6
5	/7
6	/7
7	/10
8	/10
9	/10
10	/10
11	/12
12	/14

This document consists of **30** printed pages and **4** blank pages.

1 The diagram shows the graph of $y = \mathrm{f}(2x)$. The lines $x = -2$ and $y = -2$ are asymptotes to the curve. The minimum point A has coordinates $(2, -4)$ and the curve passes through the origin $(0, 0)$.

Sketch the graph of $y = \dfrac{1}{\mathrm{f}(x)}$, indicating clearly the equations of any asymptotes, axial intercepts and turning points. [3]



2 Find $\int x \tan^{-1}(3x)\,\mathrm{d}x$. [4]

3 With reference to the origin O, the points A, B and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively, where points O, A, B and C are not collinear.

(i) It is given that $|\mathbf{a}| = \sqrt{10}$, $|\mathbf{b}| = 20$, $\mathbf{a} \cdot \mathbf{b} = 40$ and the point N has position vector $\dfrac{5\mathbf{a} + 3\mathbf{b}}{8}$. Find the exact area of triangle ONB. [5]

(ii) Find the angle between the vector $\mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{a}$ and the plane π containing points A, B and C. [2]

4 **(a)** The continuous function $\mathrm{f}(x)$, where $\mathrm{f}(x) > 0$, is strictly decreasing for $x \geq 1$. Sketch the curve $y = \mathrm{f}(x)$ for $k < x < k+1$, where k is an integer and $k \geq 1$.

By comparing the areas of appropriate rectangles and the area under the curve $y = \mathrm{f}(x)$, show that for any integer $k \geq 1$,

$$\mathrm{f}(k+1) < \int_k^{k+1} \mathrm{f}(x)\,\mathrm{d}x < \mathrm{f}(k).$$ [2]

(b) The region under the curve $y = \dfrac{1}{x}$ between $x = 1$ and $x = 10$, is split into 9 vertical strips of equal width. Use the result in part **(a)** to prove

(i) $\displaystyle\int_1^{10} \frac{1}{x}\,\mathrm{d}x < \sum_{k=1}^{9} \frac{1}{k}$, [1]

(ii) $\displaystyle\sum_{k=1}^{9} \frac{1}{k} < 1 + \int_1^{9} \frac{1}{x}\,\mathrm{d}x$. [2]

Hence show that $\ln 10 < 1 + \dfrac{1}{2} + \dfrac{1}{3} + ... + \dfrac{1}{9} < 1 + \ln 9$. [1]

5 **Do not use a calculator in answering this question.**

Two complex numbers are $z_1 = 2\left(\cos\dfrac{\pi}{18} - \mathrm{i}\sin\dfrac{\pi}{18}\right)$ and $z_2 = 2\mathrm{i}$.

(i) Show that $\dfrac{{z_1}^2}{{z_1}^*} + z_2$ is $\sqrt{3} + \mathrm{i}$. [3]

(ii) A third complex number, z_3, is such that $\left(\dfrac{{z_1}^2}{{z_1}^*} + z_2\right) z_3$ is real and $\left|\left(\dfrac{{z_1}^2}{{z_1}^*} + z_2\right) z_3\right| = \dfrac{2}{3}$. Find the possible values of z_3 in the form of $r(\cos\theta + \mathrm{i}\sin\theta)$, where $r > 0$ and $-\pi < \theta \leq \pi$. [4]

6 The function h is defined by $\mathrm{h}: x \mapsto \left|\ln\left(2x\right)\right| + 1,\ x \in \mathbb{R}, 0 < x \le \lambda$.

(i) Find the maximum value of λ for which the inverse function h exist. [1]

(ii) Using $\lambda = \dfrac{1}{2}$,

(a) find h^{-1} and state its domain, [4]

(b) sketch the graph of $y = \mathrm{hh}^{-1}(x)$, [1]

(c) find the solution set for $\mathrm{h}(x) = \mathrm{h}^{-1}(x)$. [1]

7 It is given that $y = \ln(2 + \sin 2x)$.

(i) Show that $\mathrm{e}^{y}\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \mathrm{e}^{y}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = -4\sin 2x$. [2]

(ii) By further differentiation of the above results, find the Maclaurin series for *y*, up to and including the term in x^3. [3]

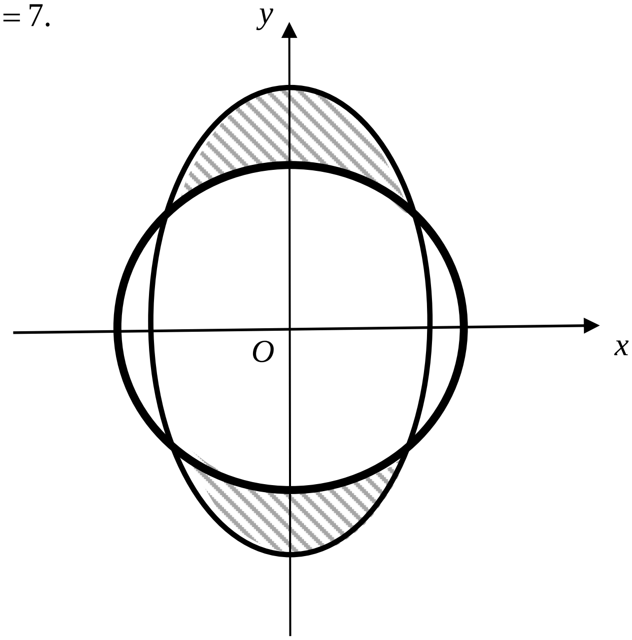
(iii) Verify that the series for $\ln(2 + \sin 2x)$ is the same as the result obtained in part **(ii)**, if the standard series from the List of Formulae (MF26) are used. [3]

(iv) Hence deduce the series expansion for $\dfrac{\ln(2 + \sin 2x)}{\sqrt{1 - x}}$, up to and including the term in x^2. [2]

8 **(i)** Use the substitution $x = k\sin\theta$, to show that

$$\int\sqrt{k^2-x^2}\,\mathrm{d}x = \frac{x}{2}\sqrt{k^2-x^2}+\frac{k^2}{2}\sin^{-1}\left(\frac{x}{k}\right)+c\,.$$ [4]

(ii) The diagram shows the shaded area R, enclosed between two curves, $\frac{x^2}{4}+\frac{y^2}{16}=1$ and $x^2+y^2=7$.



(a) Using the result shown in part **(i)**, find the exact area of region R in the form $A\pi + B\sin^{-1}C$ where A, B and C are exact constants to be determined. [4]

(b) Find the volume generated when region R is rotated through π radians about the y-axis, giving your answer to two decimal places. [2]

9 **(a)** Mr Chan's monthly pay for the first quarter of the year (i.e. first three months) is \$3000. For the second quarter monthly pay, he gets an increment of 60% of his first quarter monthly pay. Subsequently, from the third quarter onwards, he gets an increment of 60% of his **previous increment** every quarter.

Q_n denotes his monthly pay for the n^{th} quarter, where $n \geq 1$.

(i) Prove that $Q_n = 7500(1-0.6^n)$. [3]

(ii) Find to the nearest dollar, his pay for his 17th month of work. [1]

(iii) Find to the nearest dollar, his total salary for the first two years of work. [2]

(iv) Mr Chan decides to quit his job when his increment is less that \$80. How many months will he stay in this job? [1]

(b) Mr Chan's company is planning to implement a loyalty incentive scheme to retain workers with relevant skills and experience. 10 chosen workers will receive a loyalty incentive of \$90 in their pay every month. This monthly loyalty incentive will increase by \$40 every year. The company sets aside a budget of \$200,000 for this scheme.

If the 10 workers continue to work with the company,

(i) what is the maximum number of years for which this scheme can be implemented with this budget? [2]

(ii) what is the highest loyalty incentive amount Mr Chan can hope to receive if he is selected for this scheme? [1]

10

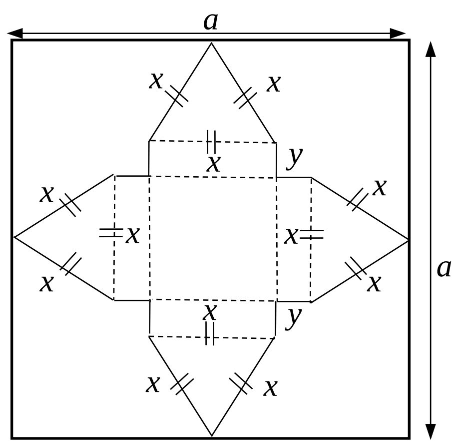


Figure 1

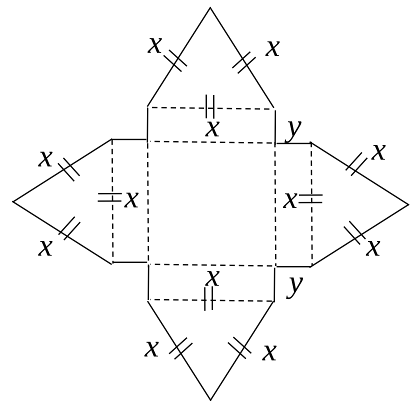


Figure 2

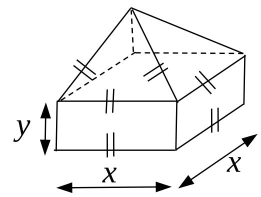


Figure 3

Figure 1 shows a piece of plastic sheet in the shape of a square with sides a metres. Parts of the plastic sheet is cut from each corner to give the shape shown in **Figure 2** which consists of a square, four identical rectangles and four identical equilateral triangles. The sides of the square are x metres each. Each rectangle has a length of x metres and breadth of y metres. Each equilateral triangle has sides of x metres. The remaining plastic sheet shown in **Figure 2** is then folded along the dotted lines to form a container, made up of a cuboid and a square base pyramid, as shown in **Figure 3**. The volume of the container is denoted by V.

(i) Show that $y = \frac{1}{2}\left(a - \left(1+\sqrt{3}\right)x\right)$. [2]

(ii) Use differentiation to find, in terms of a, the value of x that gives a maximum possible value of V, proving that it is a maximum. [5]

[The volume of a square-based pyramid is $\frac{1}{3} \times \text{base area} \times \text{height}$.]

(iii) The container is inverted and is held with its axis vertical and vertex downwards as shown in **Figure 4**. Water is poured into the container at a rate of 0.1 cubic metres per minute. At time t minutes after the start, the depth of water in the container is h metres as shown in the front view diagram of the inverted container in **Figure 5.**

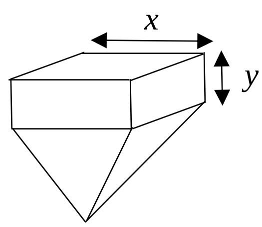


Figure 4

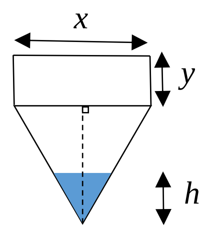


Figure 5

Given that the dimensions of the container, x and y are constants and the water level is still within the pyramid. Find in terms of a, the rate of increase of the water level when $h = 0.05a$ metres. [3]

10 With the proliferation of online social network, sociologists recognised a phenomenon called social diffusion which is the spreading of a piece of information through the population. The members of the population can be classified into two categories namely those who have the information and those who do not.

It is given that N denotes the number of people, in thousands, who have the information in a fixed population size P, in thousands. The rate of diffusion of a piece of information on social media, $\frac{\mathrm{d}N}{\mathrm{d}t}$, where t represents the time taken in hours, can be modelled as being proportional, with a constant of proportionality k, to the product of the number of people who have the information and the number of people who have yet to receive it.

(i) Write down the differential equation involving N, t, k and P. [1]

It is given that $N = 1$ and $\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{P-1}{10}$ when $t = 0$.

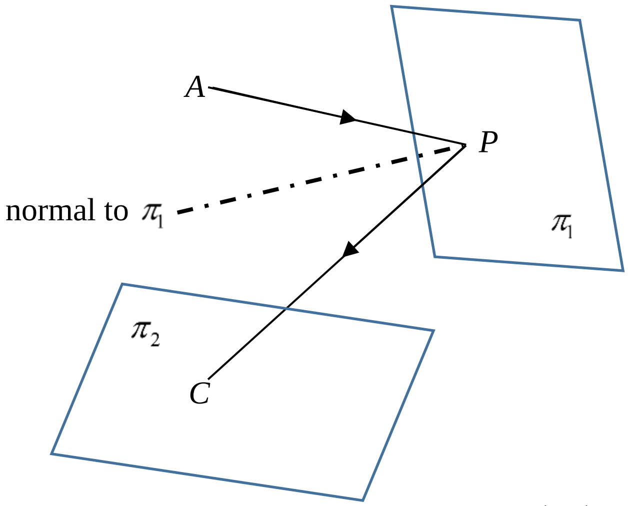
(ii) Show that the general solution of the differential equation in part **(i)** is $N = \frac{AP}{A + \mathrm{e}^{-0.1Pt}}$, where A is an arbitrary constant. [6]

(iii) Given that the population size is 1 million, find the particular solution and hence sketch the graph of N against t. [3]

(iv) With reference to the graph of N against t, explain in context, the long term implication of the model used. [1]

(v) Find the time taken, to the nearest minute, for a piece of information to reach 99% of the population of one million. [1]

11 (In this question you may assume that a laser beam travels in a straight line.)



A laser pointer is used to fire a beam in the direction $\begin{pmatrix} -2 \\ 2 \\ 1 \end{pmatrix}$ from point A with coordinates $(4, -5, 10)$. The beam is reflected at point P off the surface of the mirror π_1 which then, strikes a target plane π_2 at point C as shown in the diagram. It is given that the equation of the plane π_1 is $\mathbf{r} \cdot \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} = 5$.

(i) Show that the coordinates of the point P is $(12, -13, 6)$. [3]

It is given that the angle between AP and the normal to π_1 at P is equal to the angle between PC and the same normal.

(ii) Find the vector $\overrightarrow{AF}$, where F is the foot of perpendicular from A to the normal to π_1 at P. [3]

(iii) Find the vector equation of the line PC. [3]

Express the vector equation of the line PC in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = \mathbf{0}$, where $\mathbf{a}$ and $\mathbf{b}$ are constant vectors. [1]

The equation of the target plane π_2 is $\mathbf{r} \cdot \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = 1$.

(iv) Show that the laser beam AP is parallel to π_2. [1]

(v) Find the shortest distance between the laser beam AP and π_2. [3]

ANGLO-CHINESE JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION

Higher 2

/100

CANDIDATE NAME	

TUTORIAL/ FORM CLASS		INDEX NUMBER	

MATHEMATICS 9758/02

Paper 2

26 August 2022

3 hours

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

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Question	Marks
1	/7
2	/7
3	/7
4	/8
5	/11
6	/7
7	/10
8	/8
9	/9
10	/13
11	/13

This document consists of **24** printed pages and **2** blank pages.

Anglo-Chinese Junior College

Section A: Pure Mathematics [40 marks]

1 The function f is defined by

$$\mathrm{f}(x) = 1 - ax,\ x \in \mathbb{R}, \text{ where } a \text{ is a real constant.}$$

The function g is defined by

$$\mathrm{g}(x) = \begin{cases} x^2 + 3 & \text{for} \quad 0 \le x \le 2, \\ 7 - x & \text{for} \quad 2 < x \le 7. \end{cases}$$

(i) Find the set of possible values of a such that f^{-1} exist. [1]

(ii) If $a = 2$, describe a sequence of transformations that transform the graph of $y = \mathrm{g}(x)$ onto the graph of $y = \mathrm{fg}(x)$. [3]

(iii) The function gg is defined by

$$\mathrm{gg}(x) = \begin{cases} \mathrm{h}_1(x) & \text{for} \quad 0 \le x \le 2, \\ \mathrm{h}_2(x) & \text{for} \quad 2 < x < 5, \\ \mathrm{h}_3(x) & \text{for} \quad 5 \le x \le 7. \end{cases}$$

Find $\mathrm{h}_1(x)$, $\mathrm{h}_2(x)$ and $\mathrm{h}_3(x)$. [3]

2 It is given that $\mathrm{f}(x) = \dfrac{x+a}{b} - \dfrac{a}{x+b}$ where $a > b > 0$.

(i) Sketch the curve with equation $y = \mathrm{f}(x)$ and state the equations of any asymptotes and the points where the curve crosses the axes in terms of a and b. [3]

(ii) Hence or otherwise, solve the inequality $\dfrac{x+a}{b} \ge \dfrac{a}{x+b}$. [1]

(iii) Hence solve the inequality $\dfrac{a-|x|}{b} \ge \dfrac{a}{b-|x|}$. [3]

3 **(i)** Find the roots of the equation $\mathrm{i}z^2 - (5+\mathrm{i})z + 2 - 6\mathrm{i} = 0$, giving your answers in cartesian form $a + b\mathrm{i}$, where $a, b \in \mathbb{R}$. [2]

(ii) Hence find the roots of the equation $-\mathrm{i}w^2 - (1-5\mathrm{i})w + 2 - 6\mathrm{i} = 0$, giving your answers in cartesian form $a + b\mathrm{i}$, where $a, b \in \mathbb{R}$. [2]

(iii) Given that the roots found in part **(i)** are also roots of the equation $P(z)=0$, where $P(z)$ is a polynomial of degree 4 with real coefficients, find $P(z)$. [3]

4 **(i)** It is given that $U_n=\cos\left[(2n+1)\theta\right]$, for $n\geq 0$.

Show that for $n\geq 1$, $U_n+U_{n-1}=2\cos(2n\theta)\cos\theta$. [1]

(ii) Hence show that

$$\sum_{n=1}^{2N}\left[(-1)^{n+1}\cos(2n\theta)\right]=\frac{1}{2}\left(1-\frac{\cos\left[(4N+1)\theta\right]}{\cos\theta}\right).$$ [3]

(iii) Without the use of the graphic calculator, find the value of $\sum_{n=11}^{41}\left[(-1)^{n+1}\cos\frac{n\pi}{3}\right]$, showing your working clearly. [4]

5 A curve C has parametric equations

$$x=\mathrm{e}^{\cos^{-1}2t},\ y=\left(1-4t^2\right)^{\frac{1}{2}},\quad \text{where } -\frac{1}{2}<t<\frac{1}{2}.$$

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. What can be said about the tangent to C at $t=0$? [3]

(ii) Sketch the curve C, stating the coordinates of any axial intercepts. [1]

(iii) Find the equation of the tangent to C at the point $P\left(\mathrm{e}^{\frac{\pi}{3}},\frac{\sqrt{3}}{2}\right)$. [2]

(iv) Find the equation of the normal at the point Q on C with parameter q which is parallel to the y-axis. [2]

(v) Find the area bounded by C, the tangent to C at point P and normal to C at point Q. [3]

Section B: Probability and Statistics [60 marks]

6 Cathy has 14 magnets of which 5 are red, 4 are blue, 3 are orange and 2 are green.

(i) Assuming that the magnets of the same colour are identical, find the number of ways in which Cathy can choose 3 magnets. [2]

(ii) The table below show the amount of money Cathy paid for each type of magnets. Cathy did not pay for the green magnets as they were given to her as free gifts.

Colour of magnet	Red	Blue	Orange	Green
Price paid per magnet	\$3	\$1	\$2	Free

Cathy decided to randomly choose 4 magnets without replacement. Find the probability that she chooses \$7 worth of magnets. [3]

(iii) Cathy decided to label all the magnets such that each magnet will be distinct from the others. If Cathy were to arrange these 14 magnets in a circle on the whiteboard, find the number of different arrangements such that the 2 green magnets are adjacent to each other and the 3 orange magnets are separated from each other. [2]

7 Archer and Betty took part in a competition comprising of at most 3 games. Each game is either won by Archer or Betty. The first person who win 2 games wins the competition. The probability of Archer winning the first game is 0.25. The probability of him winning any subsequent games is p and is independent of any previous games.

(i) Draw a probability tree diagram to represent the above information. [1]

(ii) Find, in terms of p, the probability that Archer will win the competition. [2]

For the rest of the question, use $p = 0.5$.

(iii) Find the probability that Betty won the second game, given that she won the competition. [3]

The number of games won by Archer in a competition is denoted by W. Using $p = 0.5$,

(iv) determine the probability distribution of W, [2]

(v) find $\text{Var}(W)$. [2]

8 Cheddar cheese quality is influenced by starter cultures, milk composition and age. Cheddar takes about two to eighteen months to ripen and develop its texture and flavour. The ages in months (m) and prices in dollars (P) of a random sample of ten 1-kilogram Cheddar cheese are given in the table.

m	2.2	2.8	5.4	6.5	8.8	9.2	10.5	12.4	16.8	17.2
P	25	22	28	32	36	50	72	95	188	240

It is thought that the price after m months can be modelled by one of the formulae

$$P = am + b, \quad \ln P = cm + d,$$

where a, b, c and d are constants.

(i) Explain the meaning of the value of a in the context of the data for the model $P = am + b$. [1]

(ii) Find, correct to 4 decimal places, the value of the product moment correlation coefficient between

(A) m and P

(B) m and $\ln P$ [2]

(iii) Explain which of the two models in part **(ii)** is the better model and find the equation of a suitable regression line for this model. [2]

(iv) Use the equation of the regression line found in **(iii)**, estimate the price of a 1-kilogram Cheddar cheese when it has been aged for 14 months, leaving your answer to the nearest cent. [1]

Explain whether you would expect this value to be reliable. [1]

(v) Re-write your equation from part **(iii)** so that it can be used when the price of the Cheddar cheese, P, is given in dollar **per gram**. [1]

9 In this question you should state clearly all the distributions that you use, together with the values of the appropriate parameters. You should also assume that T and X follow independent normal distributions.

Each KTX train takes T minutes to travel from Seoul Train Station to Pohang Train Station. It is known that T follows the distribution $\mathrm{N}(144, 25)$.

(i) The probability that a randomly selected KTX train takes more than k hours to reach Pohang is 0.5. Without the use of a calculator, explain why $k = 2.4$. [1]

Before 7pm daily, each express bus takes X minutes to travel from Seoul Express Bus Terminal to Pohang Express Bus Terminal. It is known that X follows the distribution

$N(236, 81)$. After 7pm daily, the travel time taken by each express bus will be reduced by 10% as all the express buses will not make a stop at Daegu.

(ii) Find the probability that after 7pm, to travel from Seoul to Pohang, the travel time of a randomly selected KTX train is at most an hour faster than the travel time of a randomly selected express bus. [4]

Kim needs to travel from Seoul to Pohang on Saturday mornings to visit his parents. He prefers to take a KTX train if the tickets are available. On average, 70% of his journeys are by train.

(iii) On a particular Saturday morning, there was a train delay of 1.6 hours due to a train fault. Given that Kim took more than 4 hours to reach Pohang on that morning, find the probability that Kim travels to Pohang by train. [3]

The cost of taking a KTX train and taking an express bus from Seoul to Pohang are \$54 and \$24 respectively.

(iv) Find Kim's expected cost of travelling (one-way) from Seoul to Pohang. [1]

10 A nasi lemak stall holder uses fresh chicken wings as an ingredient for fried chicken wings. Based on his past years records, his mean daily profit was \$535. With the recent lack of fresh chicken supply, the stall holder substituted fresh chicken wings with frozen chicken wings as the ingredient. His wife was hesitant to the change and claimed that the mean daily profit will decrease. To test his wife's claim, the stall holder takes a random sample of 45 days and recorded the daily profits, \$$x$.

(i) State appropriate hypotheses to test the wife's claim and define any symbols that you use. [2]

(ii) State, with a reason, whether it is necessary to assume that his past years records of daily profits are normally distributed for the test to be valid. [1]

(iii) Based on the past years records, it is assumed that the population variance of the daily profit is 2591. If the test shows that there is sufficient evidence that the wife's claim is accepted at 5% level of significance, determine the set of possible values of $\overline{x}$, the mean daily profit in the 45 days. [2]

(iv) The stall holder found that $\overline{x}$ = \$520 and suspects that the population variance of 2591 may be incorrect. Hence he decided to use the sample variance value of 2008 to test his wife's claim. State the conclusion of the test, showing your workings clearly. [3]

(v) State the largest significance level that the stall holder should use so that the conclusion in **(iv)** will be different. Leave your answer in 2 decimal places. [1]

The stall holder now suspects that the mean daily profit does not differ from \$535, even if frozen chicken wings are used instead of fresh chicken wings. To test his claim, the stall holder decides to increase the number of randomly selected days, n, to record his daily profit.

(vi) It is given that $\bar{x} = 526$ and the population variance is assumed to be 2591. Determine the greatest value of n, so that the conclusion of the test shows that there is no reason to reject the null hypothesis at 8% level of significance. [4]

11(a) **(i)** It is known that the probability of a customer using e-payment at a hawker stall is 0.25. A group of customers is chosen at random, find the probability that the 7th chosen customer is the 5th customer using e-payment at the hawker stall. [3]

(ii) A sample of 40 customers were randomly chosen from the hawker stall each day. In a month of 30 days, find the probability that there is at least 15 days with at most 10 customers per sample making e-payment. [3]

(b) The probability that a hawker uses the online delivery platform Foodgowhere is p. A random sample of n hawkers is taken and the random variable X denotes the number of hawkers in the sample that uses Foodgowhere.

(i) Explain what is meant by a random sample in this context. [1]

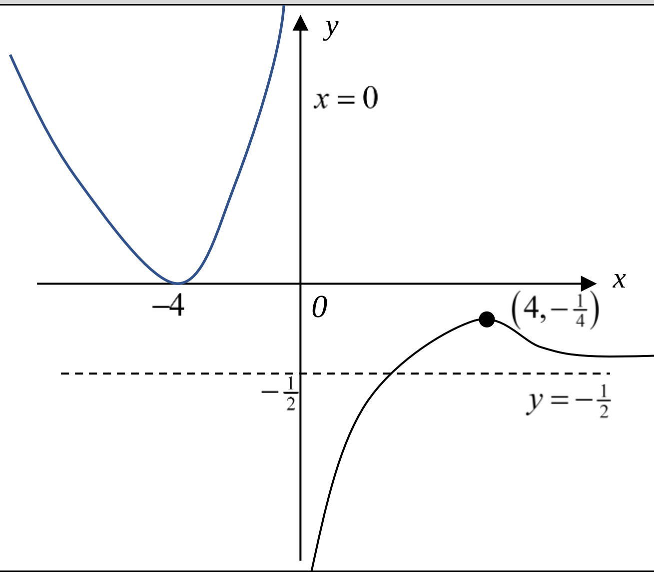
Assuming that X follows a binomial distribution.

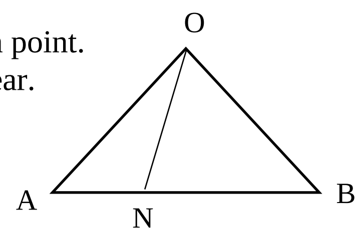
(ii) It is given that $\mathrm{P}(X \le 1) = 0.05303$ and the expected number of hawkers using Foodgowhere is 3.96. Write down two equations satisfied by p and n. Hence find the value of p and n. [3]

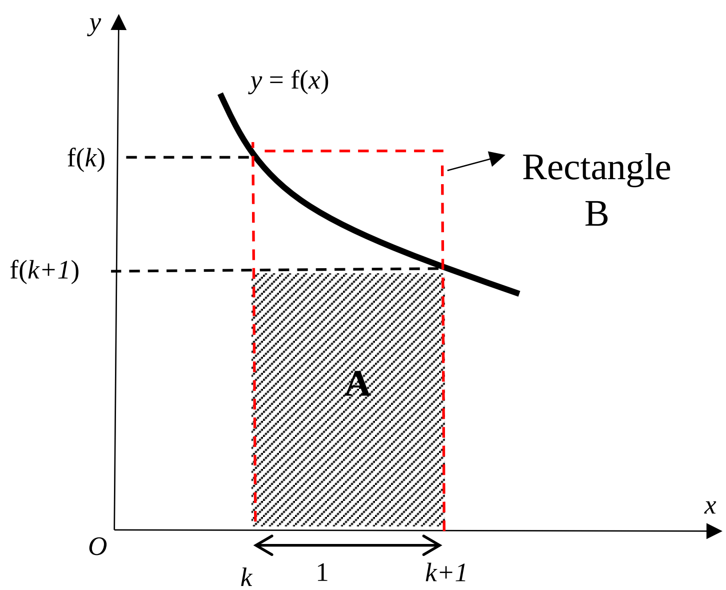
(iii) Given that $n = 15$, find the set of values of p so that that the most likely number of hawkers in the sample who uses Foodgowehere is 5. [3]

Summary of Areas for Improvement			
Knowledge (K)	**Careless Mistakes (C)**	**Read/Interpret Qn wrongly (R)**	**Presentation (P)**

2022 H2 Maths Prelim Paper 1 Markers Report

Qn	Solutions	Comments
1		Many students misread the question and were not aware that the given graph was $y=\mathrm{f}(2x)$. As a result, the x-coordinates remained as −2 and 2, or worse −1 and 1, instead of −4 and 4. It is also good to note that $\left(4,-\frac{1}{4}\right)$ should lie exactly between the x-axis and the asymptote $y=-\frac{1}{2}$.
2	$\int x\tan^{-1}3x\,\mathrm{d}x=\left(\tan^{-1}3x\right)\frac{x^2}{2}-\int\left(\frac{x^2}{2}\frac{3}{1+9x^2}\right)\mathrm{d}x$ $=\frac{x^2}{2}\tan^{-1}3x-\frac{3}{2}\int\frac{x^2}{1+9x^2}\,\mathrm{d}x$ $=\frac{x^2}{2}\tan^{-1}3x-\frac{1}{6}\int\left(1-\frac{1}{1+9x^2}\right)\mathrm{d}x$ $=\frac{x^2}{2}\tan^{-1}3x-\frac{1}{6}\left[x-\frac{1}{3}\tan^{-1}3x\right]+c$ $=\left(\frac{x^2}{2}+\frac{1}{18}\right)\tan^{-1}3x-\frac{1}{6}x+c$	**Common mistakes:** 1) $\frac{\mathrm{d}}{\mathrm{d}x}(\tan^{-1}3x)=\frac{1}{1+(3x)^2}$ 2) Problem with long division $\frac{x^2}{1+9x^2}$ **interpret as** $x^2\,\overline{)9x^2+1}$ **3)** $\int\frac{1}{1+9x^2}=\tan^{-1}3x+C$ **4)** $\int u\mathrm{d}v=uv+\int v\mathrm{d}u$
3 (i)	Area of triangle ONB = $\frac{1}{2}\left\|\overrightarrow{\boldsymbol{ON}}\times\overrightarrow{\boldsymbol{OB}}\right\|$ $=\frac{1}{2}\left\|\left(\frac{5\boldsymbol{a}\times3\boldsymbol{b}}{8}\right)\times\boldsymbol{b}\right\|$ $=\frac{1}{2}\left\|\left(\frac{5\boldsymbol{a}\times\boldsymbol{b}+3\boldsymbol{b}\times\boldsymbol{b}}{8}\right)\right\|$ $=\frac{5}{16}\left\|\boldsymbol{a}\times\boldsymbol{b}\right\|$ $\boldsymbol{a}.\boldsymbol{b}=40=\left\|\boldsymbol{a}\right\|\left\|\boldsymbol{b}\right\|\cos\theta$ where angle $AOB=\theta$ $\cos\theta=\frac{40}{20\sqrt{10}}=\frac{2}{\sqrt{10}}$ Note that : $\sin^2\theta=1-\cos^2\theta$	**Common mistakes & Presentation errors:** 1) wrong notation vector $\underset{\sim}{a}$ written as "a" 2) $\boldsymbol{a}\times\boldsymbol{b}=\left\|\boldsymbol{a}\right\|\left\|\boldsymbol{b}\right\|\sin\theta$ 3) Area of triangle ONB = $\frac{1}{2}(\overrightarrow{\boldsymbol{ON}}\times\overrightarrow{\boldsymbol{OB}})$ 4) Did not include $\left\|\ \right\|$ when finding the area of the triangle. 5) Did not simplify the answer $\frac{5}{16}(20\sqrt{10})\sin[\cos^{-1}(\frac{2}{\sqrt{10}})]$

	$\sin\theta = \dfrac{\sqrt{6}}{\sqrt{10}}$ $\lvert \boldsymbol{a}\times\boldsymbol{b}\rvert = (20\sqrt{10})\times\dfrac{\sqrt{6}}{\sqrt{10}} = 20\sqrt{6}$ Area of triangle ONB = $\dfrac{5}{16}\lvert \boldsymbol{a}\times\boldsymbol{b}\rvert = \dfrac{5}{16}(20\sqrt{6}) = \dfrac{25\sqrt{6}}{4}$	6) Left the answer as 1.53 instead of the exact form
	<u>Method 2</u> $\overrightarrow{NB} = \overrightarrow{OB} - \overrightarrow{ON} = \boldsymbol{b} - \dfrac{5\boldsymbol{a}+3\boldsymbol{b}}{8} = \dfrac{8\boldsymbol{b}-5\boldsymbol{a}-3\boldsymbol{b}}{8}$ $= \dfrac{5\boldsymbol{b}-5\boldsymbol{a}}{8} = \dfrac{5(\boldsymbol{b}-\boldsymbol{a})}{8} = \dfrac{5}{8}\overrightarrow{AB}$ $\overrightarrow{NB} // \overrightarrow{AB}$ with B as common point. Hence A, B and N are collinear. Area triangle ONB = $\dfrac{5}{8}$ area triangle OAB= $(\dfrac{5}{8})\dfrac{1}{2}\lvert \overrightarrow{\boldsymbol{OA}}\times\overrightarrow{\boldsymbol{OB}}\rvert = \dfrac{5}{16}\lvert \boldsymbol{a}\times\boldsymbol{b}\rvert$  $\boldsymbol{a}.\boldsymbol{b} = 40 = \lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert\cos\theta$ where angle AOB = θ $\cos\theta = \dfrac{40}{20\sqrt{10}} = \dfrac{2}{\sqrt{10}}$ $\sin\theta = \dfrac{\sqrt{6}}{\sqrt{10}}$ $\lvert \boldsymbol{a}\times\boldsymbol{b}\rvert = (20\sqrt{10})\times\dfrac{\sqrt{6}}{\sqrt{10}} = 20\sqrt{6}$ Area of triangle ONB = $\dfrac{5}{16}\lvert \boldsymbol{a}\times\boldsymbol{b}\rvert = \dfrac{5}{16}(20\sqrt{6}) = \dfrac{25\sqrt{6}}{4}$	For Method 2 Many students did not explain why A, N and B are collinear.
(ii)	$\overrightarrow{AB}\times\overrightarrow{AC}$ $=(\boldsymbol{b}-\boldsymbol{a})\times(\boldsymbol{c}-\boldsymbol{a})$ $=\boldsymbol{b}\times\boldsymbol{c}-\boldsymbol{b}\times\boldsymbol{a}-\boldsymbol{a}\times\boldsymbol{c}+\boldsymbol{a}\times\boldsymbol{a}$ $=\boldsymbol{b}\times\boldsymbol{c}+\boldsymbol{a}\times\boldsymbol{b}+\boldsymbol{c}\times\boldsymbol{a}+\boldsymbol{0}$ $=\boldsymbol{b}\times\boldsymbol{c}+\boldsymbol{a}\times\boldsymbol{b}+\boldsymbol{c}\times\boldsymbol{a}$ Hence the angle between $\boldsymbol{b}\times\boldsymbol{c}+\boldsymbol{a}\times\boldsymbol{b}+\boldsymbol{c}\times\boldsymbol{a}$ and plane containing points A, B and C is $\dfrac{\pi}{2}$.	Either no attempt or not well done. In this question O is not on the plane π containing A, B and C. Hence π is not parallel to vectors $\boldsymbol{a},\boldsymbol{b}$ and $\boldsymbol{c}$ but students tend to assume that π is parallel to vectors $\boldsymbol{a},\boldsymbol{b}$ and $\boldsymbol{c}$

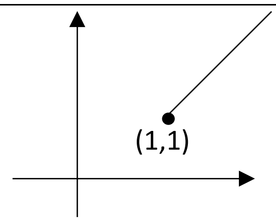
		Hence common errors like $\boldsymbol{a}\times\boldsymbol{b}$, $\boldsymbol{b}\times\boldsymbol{c}$, $\boldsymbol{c}\times\boldsymbol{a}$ are all normals to the plane π containing A, B and C commonly seen. No marks awarded for the answer $\frac{\pi}{2}$ if the method is incorrect.
4 (a)	 Area under curve = $\int_k^{k+1} \mathrm{f}\left(x\right)\mathrm{d}x$ Area of rectangle A= $\mathrm{f}\left(k+1\right)$ x 1 Area of rectangle B= $\mathrm{f}\left(k\right)$ x 1 As seen from diagram: $\mathrm{f}\left(k+1\right) < \int_k^{k+1} \mathrm{f}\left(x\right)\mathrm{d}x < \mathrm{f}\left(k\right)$	Q4 was badly done. The information given in the question was not followed: 1) $\mathrm{f}(x) > 0$ means the graph should not be drawn below the x-axis. 2) <u>curve</u> $y = \mathrm{f}(x)$ means a straight line should not be drawn. 3) for $k < x < k+1$ and $k \geq 1$ means k and $k+1$ should be clearly indicated on the x-axis. There should be only 2 rectangles involved and these should be clearly labelled as *ABCD* and *ABEF* or appropriately shaded.
(b) (i)	$\int_1^{10} \frac{1}{x}\mathrm{d}x = \int_1^{2} \frac{1}{x}\mathrm{d}x + \int_2^{3} \frac{1}{x}\mathrm{d}x + \int_3^{4} \frac{1}{x}\mathrm{d}x + ... + \int_9^{10} \frac{1}{x}\mathrm{d}x$ < f(1) + f(2) + f(3) + ... + f(9) $= \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{9}$ $= \sum_{k=1}^{9} \frac{1}{k}$	Many of the students were totally clueless as to how to use part (a) to do part (b). For the very few who were able to write a few lines of $\int_k^{k+1} \frac{1}{x}\mathrm{d}x < \frac{1}{k}$ as

		$\int_1^2 \frac{1}{x}\,\mathrm{d}x < \frac{1}{1}$, $\int_2^3 \frac{1}{x}\,\mathrm{d}x < \frac{1}{2}$, $\int_3^4 \frac{1}{x}\,\mathrm{d}x < \frac{1}{3}$, etc, they were able to appreciate the upper and lower of limits of an integral and connect all the integrals on LHS to $\int_1^{10} \frac{1}{x}\,\mathrm{d}x$.
(ii)	$\int_1^9 \frac{1}{x}\,\mathrm{d}x = \int_1^2 \frac{1}{x}\,\mathrm{d}x + \int_2^3 \frac{1}{x}\,\mathrm{d}x + \int_3^4 \frac{1}{x}\,\mathrm{d}x + ... + \int_8^9 \frac{1}{x}\,\mathrm{d}x$ > f(2) + f(3) + f(4) + ... + f(9) $= \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{9} = \sum_{k=1}^{9} \frac{1}{k} - 1$ $1 + \int_1^9 \frac{1}{x}\,\mathrm{d}x > \sum_{k=1}^{9} \frac{1}{k}$ $\therefore \sum_{k=1}^{9} \frac{1}{k} < 1 + \int_1^9 \frac{1}{x}\,\mathrm{d}x$	An elegant solution in a script: $1 + \int_1^9 \frac{1}{x}\,\mathrm{d}x$ $= 1 + \int_1^2 \frac{1}{x}\,\mathrm{d}x + \int_2^3 \frac{1}{x}\,\mathrm{d}x + ... + \int_8^9 \frac{1}{x}\,\mathrm{d}x$ $> 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{9}$ $= \sum_{k=1}^{9} \frac{1}{k}$
	$\int_1^{10} \frac{1}{x}\,\mathrm{d}x < \sum_{k=1}^{9} \frac{1}{k} < 1 + \int_1^9 \frac{1}{x}\,\mathrm{d}x$ $[\ln x]_1^{10} < 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{9} < 1 + [\ln x]_1^9$ $\ln 10 < 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{9} < 1 + \ln 9$	Again, majority of the students were not able to combine (b) (i) & (ii) results to write out the inequalities properly. There were bad presentations with no inequalities seen at all but random workings of $\int_1^{10} \frac{1}{x}\,\mathrm{d}x$ and $\int_1^9 \frac{1}{x}\,\mathrm{d}x$ to ln10 and ln9 directly without showing the proper integration process.

5(i)	$z_1 = 2\left(\cos\frac{\pi}{18} - \mathrm{i}\sin\frac{\pi}{18}\right) = 2\mathrm{e}^{-\mathrm{i}\frac{\pi}{18}}$	It is important to note that the properties of modulus and argument can only be applied for $\frac{{z_1}^2}{{z_1}^*}$ and <u>not</u> for $\left(\frac{{z_1}^2}{{z_1}^*} + z_2\right)$.

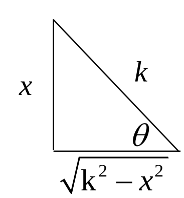
	$\frac{z_1^{\ 2}}{z_1^{\ *}}+z_2$ $=\frac{4e^{-i\frac{\pi}{9}}}{2e^{i\frac{\pi}{18}}}+2i$ $=2e^{-i\frac{\pi}{6}}+2i$ $=2\left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)+2i$ $=2\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)+2i$ $=\sqrt{3}+i$	Conversion of $2e^{-i\frac{\pi}{6}}$ from exponential form to cartesian form would be via the trigonometric form. It was inefficient to solve for a and b in $\lvert a+bi\rvert=2$ and $\arg(a+bi)=-\frac{\pi}{6}$. There was not a need to convert 2i to exponential form $2e^{i\frac{\pi}{2}}$.
(ii)	$\left(\frac{z_1^{\ 2}}{z_1^{\ *}}+z_2\right)z_3$ is real and $\left\lvert\left(\frac{z_1^{\ 2}}{z_1^{\ *}}+z_2\right)z_3\right\rvert=\frac{2}{3}$ $\left(\frac{z_1^{\ 2}}{z_1^{\ *}}+z_2\right)z_3=\frac{2}{3}$ or $-\frac{2}{3}$ $\left(\sqrt{3}+i\right)z_3=\frac{2}{3}$ or $-\frac{2}{3}$ $z_3=\frac{2}{3\left(\sqrt{3}+i\right)}$ or $-\frac{2}{3\left(\sqrt{3}+i\right)}$ $=\frac{2}{3\left(2e^{i\frac{\pi}{6}}\right)}$ or $e^{i\pi}\frac{2}{3\left(2e^{i\frac{\pi}{6}}\right)}$ $=\frac{1}{3}e^{-i\frac{\pi}{6}}$ or $\frac{1}{3}e^{i\frac{5\pi}{6}}$ $=\frac{1}{3}\left(\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right)$ or $\frac{1}{3}\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$	Many students failed to see that z_3 could be found from $\left(\sqrt{3}+i\right)z_3=\pm\frac{2}{3}$ or $\left(2e^{i\frac{\pi}{6}}\right)z_3=\pm\frac{2}{3}$ easily, instead of letting $z_3=a+bi$ and making careless mistakes along the way. Some students thought $\lvert z_3\rvert=\frac{2}{3}$, either reading error or calculation error.
	<u>**Method 2**</u> $\frac{z_1^{\ 2}}{z_1^{\ *}}+z_2=\sqrt{3}+i=2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$	Those students who used method 2 were usually successful in their solutions. Some students did not express the answers in the form

	$\left\|\left(\frac{z_1^{\ 2}}{z_1^{\ *}}+z_2\right)z_3\right\|=\frac{2}{3}$ $\left\|\frac{z_1^{\ 2}}{z_1^{\ *}}+z_2\right\|\left\|z_3\right\|=\frac{2}{3}$ $2\left\|z_3\right\|=\frac{2}{3}$ $\left\|z_3\right\|=\frac{1}{3}$ $\left(\frac{z_1^{\ 2}}{z_1^{\ *}}+z_2\right)z_3$ is real $\Rightarrow \arg\left(\frac{z_1^{\ 2}}{z_1^{\ *}}+z_2\right)z_3=0 \quad \text{or} \quad \pi$ $\Rightarrow \arg\left(\frac{z_1^{\ 2}}{z_1^{\ *}}+z_2\right)+\arg z_3=0 \quad \text{or} \quad \pi$ $\Rightarrow \frac{\pi}{6}+\arg z_3=0 \quad \text{or} \quad \pi$ $\Rightarrow \arg z_3=-\frac{\pi}{6} \quad \text{or} \quad \frac{5\pi}{6}$ $z_3=\frac{1}{3}\left(\cos\left(-\frac{\pi}{6}\right)+\mathrm{i}\sin\left(-\frac{\pi}{6}\right)\right) \quad \text{or} \quad \frac{1}{3}\left(\cos\frac{5\pi}{6}+\mathrm{i}\sin\frac{5\pi}{6}\right)$	$r\left(\cos\theta+\mathrm{i}\sin\theta\right)$, but gave their final answers as $z_3=\frac{1}{3}\left(\cos\frac{\pi}{6}-\mathrm{i}\sin\frac{\pi}{6}\right)$, were not given the accuracy mark.
(6)(i)	Maximum value of λ for which the inverse function h exist.is $\frac{1}{2}$	This is well-done.
(ii) (a)	When $0<x\le\frac{1}{2}$, ln(2x) < 0 $\Rightarrow \left\|\ln(2x)\right\|=-\ln(2x)$ $y=-\ln(2x)+1$ $\ln(2x)=1-y$ $2x=\mathrm{e}^{1-y}$ $x=\frac{1}{2}\mathrm{e}^{1-y}$ $\mathrm{h}^{-1}(x)=\frac{1}{2}\mathrm{e}^{1-x},\ x\ge1$	Most common mistake is writing $\left\|\ln(2x)\right\|$ as $\ln(2x)$ or $\pm\ln(2x)$. Domain is sometimes wrongly written as $(1,\infty)$ or $(\infty,1]$.

(b)		This was poorly done. Many students did not realise that $y = \mathrm{h}\,\mathrm{h}^{-1}(x) = x$ and wasted time finding $\mathrm{h}\,\mathrm{h}^{-1}(x)$. Many wrote $y = \mathrm{h}\,\mathrm{h}^{-1}(x)$ $= \left\|\ln\left(\mathrm{e}^{1-x}\right)\right\| + 1$ $= \|1-x\| + 1$ and sketched a modulus graph instead. $D_{\mathrm{h}^{-1}} = [1, \infty)$ must be considered to further interpret $\|1-x\|$ as $-(1-x)$. Those who correctly wrote $y = \mathrm{h}\,\mathrm{h}^{-1}(x) = x$ often did not consider $D_{\mathrm{h}^{-1}} = [1, \infty)$ in sketching the graph.
(c)	**<u>Method 1</u>** $\mathrm{R_h} = [1, \infty)$ and $\mathrm{R}_{\mathrm{h}^{-1}} = (0, \frac{1}{2}]$ $\mathrm{R_h} \cap \mathrm{R}_{\mathrm{h}^{-1}} = \varnothing$ No solution **<u>Method 2</u>** From GC When h(x) = x $\|\ln(2x)\| + 1 = x$ From GC x = 2.678 > 0.5 (no solution) **<u>Method 3</u>** From GC When h(x) = x $-\ln(2x) + 1 = x$ From GC x = 0.685 > 0.5 (no solution)	This was poorly done. Many do not understand that solution set refers to the values of x such that $\mathrm{h}(x) = \mathrm{h}^{-1}(x)$. Those who wrote the solution as 2.678 or 0.685 failed to consider that solution set is such that $0 < x \le \dfrac{1}{2}$ since $D_{\mathrm{h}} = \left(0, \frac{1}{2}\right]$.

7(i)	$y = \ln(2 + \sin 2x)$ $\mathrm{e}^{y} \dfrac{\mathrm{d}y}{\mathrm{d}x} = 2\cos 2x$	Those who showed by implicit differentiation showed in 2 steps.

	$\mathrm{e}^y \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + \mathrm{e}^y \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = -4\sin 2x$ (shown)	However, a significant number of students did direct differentiation and wasted time by showing the result in at least 7-8 steps.
(ii)	$\mathrm{e}^y \dfrac{\mathrm{d}^3y}{\mathrm{d}x^3} + \mathrm{e}^y \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\dfrac{\mathrm{d}y}{\mathrm{d}x} + 2\mathrm{e}^y \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\dfrac{\mathrm{d}y}{\mathrm{d}x} + \mathrm{e}^y \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^3 = -2\cos 2x$ $\mathrm{e}^y \dfrac{\mathrm{d}^3y}{\mathrm{d}x^3} + 3\mathrm{e}^y \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\dfrac{\mathrm{d}y}{\mathrm{d}x} + \mathrm{e}^y \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^3 = -2\cos 2x$ When $x=0, y=\ln 2, \dfrac{\mathrm{d}y}{\mathrm{d}x}=1, \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=-1, \dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}=-2$ $\therefore\ y = \ln 2 + x + \dfrac{(-1)}{2!}x^2 + \dfrac{(-2)}{3!}x^3 + \ldots$ $= \ln 2 + x - \dfrac{1}{2}x^2 - \dfrac{1}{3}x^3 + \ldots$	Common mistakes: (a) Writing $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^y\right) = \mathrm{e}^y$ (b) Writing $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = 2\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)$ Some students are not able to simplify $\mathrm{e}^{\ln 2}$.
(iii)	$y = \ln\left(2+\sin 2x\right)$ $\approx \ln\left(2 + 2x - \dfrac{(2x)^3}{6}\right)$ $= \ln\left(2\left(1 + x - \dfrac{2}{3}x^3\right)\right)$ $= \ln 2 + \ln\left(1 + x - \dfrac{2}{3}x^3\right)$ $= \ln 2 + \left(x - \dfrac{2}{3}x^3\right) - \dfrac{\left(x-\frac{2}{3}x^3\right)^2}{2} + \dfrac{\left(x-\frac{2}{3}x^3\right)^3}{3} + \ldots$ $\approx \ln 2 + x - \dfrac{2}{3}x^3 - \dfrac{1}{2}x^2 + \dfrac{1}{3}x^3$ $= \ln 2 + x - \dfrac{1}{2}x^2 - \dfrac{1}{3}x^3 + \ldots$ (verified)	A significant number of students do not know how to start on the question. Common mistakes: (a) Writing $= \ln(2+\sin 2x)$ $= \ln\left[1+(1+\sin 2x)\right]$ $= (1+\sin 2x) + \dfrac{(1+\sin 2x)^2}{2}$ $+ \dfrac{(1+\sin 2x)^3}{3} \ldots$ (b) Writing $\sin 2x \approx 2x$, omitting the x^3 term.
(iv)	$\dfrac{\ln(2+\sin 2x)}{\sqrt{1-x}}$	This is not well-done. Many students did not know that they should consider

	$\approx \dfrac{\ln 2 + x - \frac{1}{2}x^2 - \frac{1}{3}x^3}{\sqrt{1-x}}$ $= \left(\ln 2 + x - \frac{1}{2}x^2 - \frac{1}{3}x^3\right)(1-x)^{-\frac{1}{2}}$ $\approx \left(\ln 2 + x - \frac{1}{2}x^2\right)\left(1 + \frac{1}{2}x + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}(-x)^2\right)$ $= \left(\ln 2 + x - \frac{1}{2}x^2\right)\left(1 + \frac{1}{2}x + \frac{3}{8}x^2\right)$ $\approx \ln 2 + x - \frac{1}{2}x^2 + \left(\frac{1}{2}\ln 2\right)x + \frac{1}{2}x^2 + \left(\frac{3}{8}\ln 2\right)x^2$ $= \ln 2 + \left(1 + \frac{1}{2}\ln 2\right)x + \left(\frac{3}{8}\ln 2\right)x^2$	binomial expansion of $(1-x)^{-\frac{1}{2}}$. Some students wrote $\frac{1}{\sqrt{1-x}}$ wrongly as $(1-x)^{\frac{1}{2}}$ or $(1-x)^{-1}$. A significant number of students who considered binomial expansion of $(1-x)^{-\frac{1}{2}}$ made careless mistakes in evaluating the coefficients of x and/or x^2 e.g. $1 - \frac{1}{2}x + \frac{1}{8}x^2$.
8(i)	$x = k\sin\theta$ $\frac{dx}{d\theta} = k\cos\theta$ $\int \sqrt{k^2 - x^2}\,dx$ $= \int \sqrt{k^2 - k^2\sin^2\theta}\,k\cos\theta\,d\theta$ $= \int k^2\cos^2\theta\,d\theta$ $= \frac{k^2}{2}\int \cos 2\theta + 1\,d\theta$ $= \frac{k^2}{2}\left[\frac{1}{2}\sin 2\theta + \theta\right] + c$ $= \frac{k^2}{2}[\sin\theta\cos\theta + \theta] + c$ $= \frac{k^2}{2}\left[\frac{x}{k}\sqrt{1 - \left(\frac{x}{k}\right)^2} + \sin^{-1}\left(\frac{x}{k}\right)\right] + c$ since $\cos\theta = \sqrt{1 - \sin^2\theta}$ $= \frac{x}{2}\sqrt{k^2 - x^2} + \frac{k^2}{2}\sin^{-1}\frac{x}{k} + c$	Generally, well done. Since the result to be proved is given, the working to show that $\frac{k^2}{2}[\sin\theta\cos\theta + \theta] + c$ leads to $\frac{x}{2}\sqrt{k^2 - x^2} + \frac{k^2}{2}\sin^{-1}\frac{x}{k} + c$ should be written out clearly as shown. OR Alternatively, using a right-angled triangle to get expression for $\cos\theta$ given $\sin\theta = \frac{x}{k}$.  $\cos\theta = \frac{\sqrt{k^2 - x^2}}{k}$

(ii) (a)	Point of intersection of $\dfrac{x^2}{4}+\dfrac{y^2}{16}=1$ and $x^2+y^2=7$ Substitute $y^2=7-x^2$ into $4x^2+y^2=16$ $4x^2+7-x^2=16,\ \therefore x=\pm\sqrt{3}$ and $y=\pm 2$	To find the area enclosed between two curves, the points of intersection between the two curves must be found first to get the correct the limits for integration.
	Area $=4\int_0^{\sqrt{3}}\sqrt{16-4x^2}-\sqrt{7-x^2}\,\mathrm{d}x$ $=4\int_0^{\sqrt{3}}2\sqrt{4-x^2}-\sqrt{7-x^2}\,\mathrm{d}x$ $=8\left[\dfrac{x}{2}\sqrt{4-x^2}+\dfrac{4}{2}\sin^{-1}\dfrac{x}{2}\right]_0^{\sqrt{3}}$ $-4\left[\dfrac{x}{2}\sqrt{7-x^2}+\dfrac{7}{2}\sin^{-1}\dfrac{x}{\sqrt{7}}\right]_0^{\sqrt{3}}$ $=8\left[\dfrac{\sqrt{3}}{2}+2\sin^{-1}\dfrac{\sqrt{3}}{2}-0\right]-4\left[\sqrt{3}+\dfrac{7}{2}\sin^{-1}\sqrt{\dfrac{3}{7}}\right]$ $=4\sqrt{3}+16\left(\dfrac{\pi}{3}\right)-4\sqrt{3}-14\sin^{-1}\sqrt{\dfrac{3}{7}}$ $=\dfrac{16\pi}{3}-14\sin^{-1}\sqrt{\dfrac{3}{7}}$ $A=\dfrac{16}{3},\ \ B=-14,\ \ C=\sqrt{\dfrac{3}{7}}$ OR For area taken with respect to y-axis	Many students used the wrong values for the limits of the integral. Note the correct method to use the answer from part (i) to evaluate $\int_0^{\sqrt{3}}\sqrt{16-4x^2}\,\mathrm{d}x$. The coefficient of x^2 must be one before the result can be applied directly as shown. OR Alternatively $\int_0^{\sqrt{3}}\sqrt{16-4x^2}\,\mathrm{d}x$ $=\dfrac{1}{2}\int_0^{\sqrt{3}}2\sqrt{4^2-(2x)^2}\,\mathrm{d}x$ $=\dfrac{1}{2}\left[\dfrac{2x}{2}\sqrt{4^2-x^2}+\dfrac{4^2}{2}\sin^{-1}\dfrac{2x}{4}\right]$ $+c$

<table>
<tr><td></td><td>Area
$$=4\left[\int_2^4 \frac{1}{2}\sqrt{16-y^2}\,\mathrm{d}y-\int_2^{\sqrt{7}}\sqrt{7-y^2}\,\mathrm{d}y\right]$$
$$=2\left[\frac{y}{2}\sqrt{16-y^2}+8\sin^{-1}\frac{y}{4}\right]_2^4-4\left[\frac{y}{2}\sqrt{7-y^2}+\frac{7}{2}\sin^{-1}\frac{y}{\sqrt{7}}\right]_2^{\sqrt{7}}$$
$$=2\left[8\sin^{-1}1-2\sqrt{3}-8\sin^{-1}\frac{1}{2}\right]-4\left[\frac{7}{2}\sin^{-1}1-\sqrt{3}-\frac{7}{2}\sin^{-1}\frac{2}{\sqrt{7}}\right]$$
$$=16\left(\frac{\pi}{2}\right)-4\sqrt{3}-16\left(\frac{\pi}{6}\right)-14\left(\frac{\pi}{2}\right)+4\sqrt{3}+14\sin^{-1}\frac{2}{\sqrt{7}}$$
$$=14\sin^{-1}\frac{2}{\sqrt{7}}-\frac{5\pi}{3}$$
$A=\frac{-5}{3},\ B=14,\ C=\frac{2}{\sqrt{7}}$</td><td></td></tr>
<tr><td>(b)</td><td>$$\text{Volume}=2\pi\left[\int_2^4 \frac{1}{4}\left(16-y^2\right)\mathrm{d}y-\int_2^{\sqrt{7}} 7-y^2\,\mathrm{d}y\right]$$
$$=14.58\ \text{(to 2dp)}$$</td><td>Many students used the wrong expressions for volume.
Since the rotation is about y axis the basic formula should be
$$\text{Volume}=\pi\int_{y_1}^{y_2} x^2\,\mathrm{d}y$$
The limits are y coordinates of the region enclosed and the volume generated using this formula is multiplied by 2 and not 4.</td></tr>
<tr><td>9(a)
(i)</td><td><table><tr><td>Quarter</td><td>Increment</td><td>Monthly pay for the quarter</td></tr><tr><td>1</td><td>-</td><td>3000</td></tr><tr><td>2</td><td>0.6(3000)</td><td>3000+0.6(3000)</td></tr><tr><td>3</td><td>0.6^2(3000)</td><td>3000+0.6(3000)+ 0.6^2(3000)</td></tr><tr><td></td><td></td><td></td></tr></table>
$$Q_n=3000(1+0.6+0.6^2+0.6^3+....0.6^{n-1})$$
$$=3000\frac{1-0.6^n}{1-0.6}=7500(1-0.6^n)$$</td><td>Correct reading and understanding of the question are key here. Many students attempted the question using previously studied standard methods, which will not apply to this question.
Students who took the time to read and write out the monthly pay for at least the first three quarters, manged to proceed well with the question.</td></tr>
</table>

		It is good to label the columns of the table you are forming, as it also clarifies your own thinking. **It is necessary to show the geometric series before summing it using the sum of terms of GP formula.** Since the result to be proved is given, merely working backwards a few steps will not be sufficient to prove the result.
(ii)	17th month is in the 6th quarter $Q_6 = 7500(1-0.6^6) = 7{,}150.08$ His salary for the 17th month of work is \$7,150(to nearest dollar)	Reading the question correctly would have allowed students to use the answer from (i) above, to answer the rest of the question very easily. Many students did this part without understanding what Q_n stands for.
(iii)	Total salary for first two years of work $= 3\sum_{n=1}^{8} 7500(1-0.6^n) = 146816.87$ (from GC) Total salary for two years of work is \$146,817 (to nearest dollar) Alternatively, Total salary for first two years of work $= 3\sum_{n=1}^{8} 7500(1-0.6^n)$ $= 22500\sum_{n=1}^{8}(1-0.6^n)$ $= 22500\left[8-\sum_{1}^{8} 0.6^n\right]$ $= 22500\left[8-\frac{0.6(1-0.6^8)}{1-0.6}\right] = 146816.87$ Total salary for two years of work is \$146,817 (to nearest dollar)	Several students resorted to adding up individual terms and wasting their time, instead of using the GC or the sum of GP formula.
(iv)	Increment in the nth quarter if he stays in the job, $3000\left(0.6^{n-1}\right) \geq 80$ Use GC table: $n \leq 8$ He will stay in his job for 24 months.	The general expression for the increment can be written down easily. Many students used the difference between the monthly pay in two consecutive quarters

<table>
<tr><td></td><td></td><td>to get the expression which is a very long-winded method.

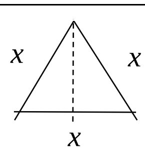
Many also did not use tables from GC to get answer.

When forming the inequality, you should be clear which case you are considering.
Ie if he quits the job then $3000\left(0.6^{n-1}\right) < 80$ will be used, which leads to the answer $n > 8.091$. Therefore, he quits in the 9th quarter. So, he stays on the job for 8 quarters or 24months.</td></tr>
<tr><td>(b)
(i)

(ii)</td><td>Total amount required for 10 workers over n years
$10\times12\times\frac{n}{2}\left[2(90)+(n-1)40\right]\leq200000$
$24n^2+84n-2000\leq0$
$-11.045 < n \leq 7.5449$

Or using GC table
<table><tr><td>n</td><td>Total sum</td></tr><tr><td>7</td><td>176,400</td></tr><tr><td>8</td><td>202,800</td></tr></table>
$n\leq7$
The budget will last for a maximum of 7 years.
Highest incentive amount will be in the 7th year
i.e. 90 +(7-1)40=$330</td><td>Again, reading the question accurately will help.

The loyalty incentive is increasing by a fixed amount each year, meaning it is an arithmetic progression. i.e
90, 90+40, 90+2(40), 90+3(40), ……for *n* years.
Sum of AP can be used to get the total.
Since it is a monthly incentive and it is for 10 persons, the sum must be multiplied by 12 x10 to get the total amount required.</td></tr>
</table>

<table>
<tr><td>**10(i)**</td><td>

By Pythagoras' Theorem,

height of triangle $=\sqrt{x^2-\left(\frac{1}{2}x\right)^2}=\sqrt{\frac{3}{4}x^2}=\frac{\sqrt{3}}{2}x$</td><td>Well done</td></tr>
</table>

	$x+2y+2\left(\frac{\sqrt{3}}{2}x\right)=a$ $y=\frac{1}{2}\left(a-\left(1+\sqrt{3}\right)x\right)$ (shown)	
(ii)	height of pyramid $=\sqrt{\left(\frac{\sqrt{3}}{2}x\right)^2-\left(\frac{1}{2}x\right)^2}=\sqrt{\frac{1}{2}x^2}=\frac{1}{\sqrt{2}}x$ $V=x^2y+\frac{1}{3}x^2\left(\frac{1}{\sqrt{2}}x\right)$ $=\frac{1}{2}ax^2-\frac{1}{2}\left(1+\sqrt{3}\right)x^3+\frac{1}{3\sqrt{2}}x^3$ $=\frac{1}{2}ax^2+\left(\frac{1}{3\sqrt{2}}-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)x^3$ $\frac{\mathrm{d}V}{\mathrm{d}x}=ax+3\left(\frac{1}{3\sqrt{2}}-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)x^2$ $=ax+\left(\frac{1}{\sqrt{2}}-\frac{3}{2}-\frac{3\sqrt{3}}{2}\right)x^2$ When $\frac{\mathrm{d}V}{\mathrm{d}x}=0$ $\Rightarrow ax+\left(\frac{1}{\sqrt{2}}-\frac{3}{2}-\frac{3\sqrt{3}}{2}\right)x^2=0$ $\Rightarrow x\left(a+\left(\frac{1}{\sqrt{2}}-\frac{3}{2}-\frac{3\sqrt{3}}{2}\right)x\right)=0$ $\Rightarrow x=0$ (rejected since $x>0$) or $x=\frac{a}{-\frac{1}{\sqrt{2}}+\frac{3}{2}+\frac{3\sqrt{3}}{2}}=0.295a$ Using second derivative test,	Most students could find the height of pyramid. For V, most students found the volume of the pyramid only. Students need to add the volume of the cuboid: x^2y. Most students went on to differentiate V but did not attempt to solve $\frac{\mathrm{d}V}{\mathrm{d}x}=0$. Some have forgotten to prove maximum. For the first derivative test, students must indicate the actual values in the table. For the second derivative test, students must substitute the x value found and get a negative value for $\frac{\mathrm{d}^2V}{\mathrm{d}x^2}$.

$$\frac{\mathrm{d}^2V}{\mathrm{d}x^2} = a + 2\left(\frac{1}{\sqrt{2}} - \frac{3}{2} - \frac{3\sqrt{3}}{2}\right)x$$

$$= a + 2\left(\frac{1}{\sqrt{2}} - \frac{3}{2} - \frac{3\sqrt{3}}{2}\right)\left(\frac{a}{-\frac{1}{\sqrt{2}} + \frac{3}{2} + \frac{3\sqrt{3}}{2}}\right)$$

$$= a - 2a$$

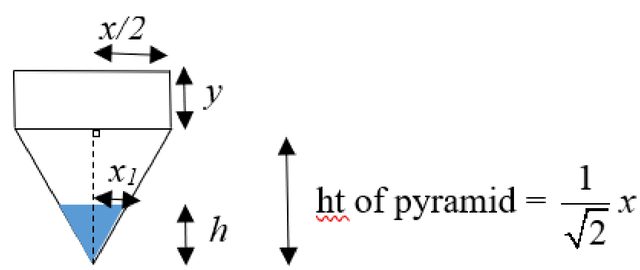
$$= -a < 0$$

Maximum V.

OR Using first derivative test,

x	$0.290a$	$\frac{a}{-\frac{1}{\sqrt{2}} + \frac{3}{2} + \frac{3\sqrt{3}}{2}}$	$0.3a$
$\frac{\mathrm{d}V}{\mathrm{d}x}$	$0.00482a^2$ > 0	0	$-0.00519a^2$ < 0
Slope	/	-	\

(iii)



By similar triangles,

$$\frac{x_1}{\frac{x}{2}} = \frac{h}{\frac{1}{\sqrt{2}}x}$$

$$x_1 = \frac{\sqrt{2}}{2}h$$

Many students cannot distinguish between height of the pyramid, $\frac{1}{\sqrt{2}}x$ in (ii), and the height of water, h in this case.

Many students used the volume, V in (ii) instead of finding the volume of water, V_1 in this case.

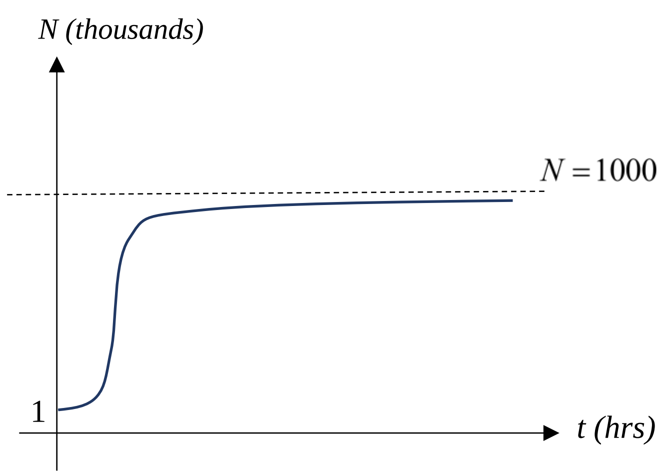
Students should note the following:

They can then express x_1 in terms of h and hence V_1 in terms of h in order to find $\frac{dV_1}{dh}$ before applying the chain rule.

	Since it is a square base pyramid, its base area is $(2x_1)^2$. Volume of water, V_1 $=\frac{1}{3}(2x_1)^2 h=\frac{1}{3}\left(2\left(\frac{\sqrt{2}}{2}h\right)\right)^2 h=\frac{2}{3}h^3$ $\frac{\mathrm{d}V_1}{\mathrm{d}h}=2h^2$ $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}V_1}{\mathrm{d}t}\cdot\frac{\mathrm{d}h}{\mathrm{d}V_1}$ $=(0.1)\left(\frac{1}{2h^2}\right)$ $=(0.1)\left(\frac{1}{2(0.05a)^2}\right)$ $=\frac{20}{a^2}$ The depth of the water is increasing at a rate of $\frac{20}{a^2}$ m/mins.	To find $\frac{dV_1}{dh}$, students must attempt to express V_1 in terms of h first. It is WRONG to differentiate V_1 when it is in terms of x_1 and h.

11 (i)	$\frac{\mathrm{d}N}{\mathrm{d}t}=kN(P-N)$	Well done
11 (ii)	Using $N=1$ and $\frac{\mathrm{d}N}{\mathrm{d}t}=\frac{P-1}{10}$ when $t=0$, $\frac{P-1}{10}=k(1)(P-1)$ $\therefore k=\frac{1}{10}$	Many did not use the conditions given to find k.

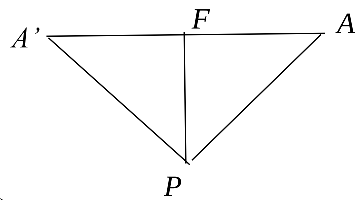
	$\frac{\mathrm{d}N}{\mathrm{d}t}=\frac{N}{10}(P-N)$ $\int \frac{1}{N(P-N)}\,\mathrm{d}N=\int \frac{1}{10}\,\mathrm{d}t$ $\int \frac{\frac{1}{P}}{N}+\frac{\frac{1}{P}}{(P-N)}\,\mathrm{d}N=\int \frac{1}{10}\,\mathrm{d}t$ $\frac{1}{P}\ln(N)-\frac{1}{P}\ln(P-N)=\frac{t}{10}+C$ (Note: $N>0$ and $P>N$) $\frac{1}{P}\ln\left(\frac{N}{P-N}\right)=\frac{t}{10}+C$ $\ln\left(\frac{N}{P-N}\right)=0.1Pt+PC$ $\frac{N}{P-N}=\mathrm{e}^{0.1Pt+PC}$ $\frac{N}{P-N}=A\mathrm{e}^{0.1Pt}$ whereby $A=\mathrm{e}^{PC}$ $N=A\mathrm{e}^{0.1Pt}(P-N)$ $N=AP\mathrm{e}^{0.1Pt}-AN\mathrm{e}^{0.1Pt}$ $N+AN\mathrm{e}^{0.1Pt}=AP\mathrm{e}^{0.1Pt}$ $N\left(1+A\mathrm{e}^{0.1Pt}\right)=AP\mathrm{e}^{0.1Pt}$ $N=\frac{AP\mathrm{e}^{0.1Pt}}{1+A\mathrm{e}^{0.1Pt}}$ $N=\frac{AP\mathrm{e}^{0.1Pt}}{\mathrm{e}^{0.1Pt}\left(\mathrm{e}^{-0.1Pt}+A\right)}$ $N=\frac{AP}{A+\mathrm{e}^{-0.1Pt}}$ (Shown)	Students separated the variables correctly, but they should show this step before they start integrating: $\int \frac{1}{N(P-N)}\,\mathrm{d}N=\int \frac{1}{10}\,\mathrm{d}t$ The integration techniques used were correct, but many careless mistakes were seen. Students should introduce the constant A and attempt to make N the subject. Working MUST be shown to illustrate how students can proceed from this $N=\frac{AP\mathrm{e}^{0.1Pt}}{1+A\mathrm{e}^{0.1Pt}}$ to the expression given.

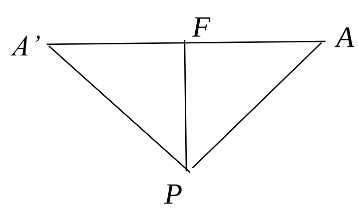
11 (iii)	Since the population size is 1 million, $\therefore P = 1000$. Using $N = 1$ when $t = 0$, $1 = \dfrac{1000A}{A + e^0}$ $A + 1 = 1000A$ $999A = 1$ $A = \dfrac{1}{999}$ $N = \dfrac{\frac{1000}{999}}{\frac{1}{999} + e^{-100t}}$ 	Many students attempted to find *A*, but they did not write down the value of *P* in the expression for *N*. For the sketch, the horizontal asymptote and the *x*-intercept MUST be shown. The curve can only be drawn in the first quadrant as *t*, *N* > 0. Students must learn how to get the curvature (near the origin) from their GC.
11 (iv)	In the long term, the number of people who will receive a piece of information will <u>tend towards</u> the population size of 1 million.	Students must explain in CONTEXT and use key words such as tends to or approaches 1 million. It is *incorrect* to say the entire population *will get* the piece of information.

11 (v)	When $N = 0.99 \times 1000 = 990,$ using GC to solve $990 = \dfrac{\frac{1000}{999}}{\frac{1}{999} + \mathrm{e}^{-100t}}$ $t = 0.1150187 \text{ hr} = 6.901122 \text{ min} \approx 7 \text{ min (nearest minute)}$ It takes about 7 minutes for a piece of information to reach 99% of the population of one million.	Well done for those with the correct A and P stated in the earlier parts. However, students must READ the question carefully to correct their answer to the nearest minute.

12(i)	$\boldsymbol{l}_{AP} : \boldsymbol{r} = \begin{pmatrix} 4 \\ -5 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$ ……..(1) $\pi_1 : \boldsymbol{r} . \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 2 \end{pmatrix} . \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} = 5$ …….(2) Subst (1) into (2) $\begin{pmatrix} 4-2\lambda \\ -5+2\lambda \\ 10+\lambda \end{pmatrix} . \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} = 5$ $-5 + 2\lambda + 30 + 3\lambda = 5$ $\lambda = -4$ $\overrightarrow{OP} = \begin{pmatrix} 4+8 \\ -5-8 \\ 10-4 \end{pmatrix} = \begin{pmatrix} 12 \\ -13 \\ 6 \end{pmatrix}$ $P(12, -13, 6)$	**Most students can score at least 2 marks** **Common mistakes:** 1) Some just verify (12,-13,6) falls on π_1 by substituting $\begin{pmatrix} 12 \\ -13 \\ 6 \end{pmatrix}$ into equation of π_1 i.e $\begin{pmatrix} 12 \\ -13 \\ 6 \end{pmatrix} . \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} = 0 - 13 + 18 = 5$ 2) Some wrote $P = \begin{pmatrix} 12 \\ -13 \\ 6 \end{pmatrix}$ = (12,-13,6) 3) Some did not conclude that coordinates of P is (12,-13,6) 4) Many write vector $\boldsymbol{a}$ as "a" and not "$\underset{\sim}{a}$"
(ii)	**Method 1** Equation of normal to π_1 $L_N : \boldsymbol{r} = \begin{pmatrix} 12 \\ -13 \\ 6 \end{pmatrix} + \alpha \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 12 \\ -13+\alpha \\ 6+3\alpha \end{pmatrix}$	**Not well done especially students who used method 2.** **Common mistakes:** 1)Many assume that

	Let F be the foot of perpendicular from A to normal to plane $\overline{OF} = \begin{pmatrix} 12 \\ -13+\alpha \\ 6+3\alpha \end{pmatrix}$ ....(1) $\overrightarrow{AF} = \begin{pmatrix} 12 \\ -13+\alpha \\ 6+3\alpha \end{pmatrix} - \begin{pmatrix} 4 \\ -5 \\ 10 \end{pmatrix} = \begin{pmatrix} 8 \\ -8+\alpha \\ -4+3\alpha \end{pmatrix}$ $\overrightarrow{AF} \perp L_N$ $\overrightarrow{AF} \bullet \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} = 0$ $\begin{pmatrix} 8 \\ -8+\alpha \\ -4+3\alpha \end{pmatrix} \bullet \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} = 0$ $-8+\alpha-12+9\alpha=0$ $\alpha=2$ $\overrightarrow{AF} = \begin{pmatrix} 8 \\ -8+\alpha \\ -4+3\alpha \end{pmatrix} = \begin{pmatrix} 8 \\ -6 \\ 2 \end{pmatrix}$	$\boldsymbol{r} = \begin{pmatrix} 4 \\ -5 \\ 10 \end{pmatrix} + \alpha \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$ is vector equation of line AF and found the position vector of the foot of perpendicular from A to plane π_1. 2) Some assume that $\overline{OF} = \begin{pmatrix} 4 \\ -5+\alpha \\ 10+3\alpha \end{pmatrix}$

(ii)	**Method 2** Let A' be the mirror image of A in the normal to the plane and F be the foot of perpendicular from A to the normal  $\overrightarrow{AP}=\begin{pmatrix}12\\-13\\6\end{pmatrix}-\begin{pmatrix}4\\-5\\10\end{pmatrix}=\begin{pmatrix}8\\-8\\-4\end{pmatrix}$ $\overrightarrow{FP}=\left[\overrightarrow{AP}\bullet\frac{1}{\sqrt{1+9}}\begin{pmatrix}0\\1\\3\end{pmatrix}\right]\frac{1}{\sqrt{1+9}}\begin{pmatrix}0\\1\\3\end{pmatrix}$ $=\frac{1}{10}\left[\begin{pmatrix}8\\-8\\4\end{pmatrix}\bullet\begin{pmatrix}0\\1\\3\end{pmatrix}\right]\begin{pmatrix}0\\1\\3\end{pmatrix}=\frac{1}{10}(-8-12)\begin{pmatrix}0\\1\\3\end{pmatrix}=\begin{pmatrix}0\\-2\\-6\end{pmatrix}$ $\overrightarrow{AF}=\overrightarrow{AP}+\overrightarrow{PF}=\begin{pmatrix}8\\-8\\-4\end{pmatrix}+\begin{pmatrix}0\\2\\6\end{pmatrix}=\begin{pmatrix}8\\-6\\2\end{pmatrix}$	Students who used method 2 (vector component) to find $\overrightarrow{FP}$ did badly. Common mistakes are 1) $\overrightarrow{FP}=\left\|\overrightarrow{AP}.\hat{\boldsymbol{u}}\right\|\hat{\boldsymbol{u}}$ 2) $\overrightarrow{FP}=(\overrightarrow{PA}.\hat{\boldsymbol{u}})\hat{\boldsymbol{u}}$ 3) $\overrightarrow{AF}=(\overrightarrow{AP}\times\hat{\boldsymbol{u}})\hat{\boldsymbol{u}}$ where $\hat{\boldsymbol{u}}=\frac{1}{\sqrt{1+9}}\begin{pmatrix}0\\1\\3\end{pmatrix}$

(iii)	Let L_N be the normal to π_1 at P **Method 1** From (1) in (ii) $\overrightarrow{OF}=\begin{pmatrix}12\\-13+2\\6+6\end{pmatrix}=\begin{pmatrix}12\\-11\\12\end{pmatrix}$ Let A' be the mirror image of A in L_N $\dfrac{\overrightarrow{OA}+\overrightarrow{OA'}}{2}=\overrightarrow{OF}$ $\overrightarrow{OA'}=2\overrightarrow{OF}-\overrightarrow{OA}=2\begin{pmatrix}12\\-11\\12\end{pmatrix}-\begin{pmatrix}4\\-5\\10\end{pmatrix}=\begin{pmatrix}20\\-17\\14\end{pmatrix}$ $\overrightarrow{PA'}=\begin{pmatrix}20\\-17\\14\end{pmatrix}-\begin{pmatrix}12\\-13\\6\end{pmatrix}=\begin{pmatrix}8\\-4\\8\end{pmatrix}=4\begin{pmatrix}2\\-1\\2\end{pmatrix}$ *A'* falls on line *PC* Hence equation of l_{PC} **:** $\boldsymbol{r}=\begin{pmatrix}12\\-13\\6\end{pmatrix}+\beta\begin{pmatrix}2\\-1\\2\end{pmatrix}$ $\mathbf{l_{PC}}$ **:** $\left(\boldsymbol{r}-\begin{pmatrix}12\\-13\\6\end{pmatrix}\right)\times\begin{pmatrix}2\\-1\\2\end{pmatrix}=\mathbf{0}$	**Not well Done** Many assume that C is the mirror image of A in L_N. i.e $\dfrac{\overrightarrow{OA}+\overrightarrow{OC}}{2}=\overrightarrow{OF}$ commonly seen which is incorrect. The mirror image A' falls on the line PC . Hence although they manage to obtain the equation of line PC because P, A' and C are collinear, the working is incorrect. Expressing the equation in the form $(\mathbf{r}-\mathbf{a})\times\mathbf{b}=\mathbf{0}$ is badly done
(iii)	**Method 2** Let *A'* be the mirror image of A in the normal to the plane and F be the foot of perpendicular from A to the normal  $\overrightarrow{AA'}=2\overrightarrow{AF}=\begin{pmatrix}16\\-12\\4\end{pmatrix}$	Students are careless regarding the directions of the vectors 1)**Method 2** $\overrightarrow{FA'}=\overrightarrow{FA}$

(iii)	$\overrightarrow{PA'} = \overrightarrow{PA} + \overrightarrow{AA'} = \begin{pmatrix} -8 \\ 8 \\ 4 \end{pmatrix} + \begin{pmatrix} 16 \\ 12 \\ 4 \end{pmatrix} = \begin{pmatrix} 8 \\ -4 \\ 8 \end{pmatrix} = 4\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ *A'* falls on line *PC* Hence equation of l_{PC} $: \boldsymbol{r} = \begin{pmatrix} 12 \\ -13 \\ 6 \end{pmatrix} + \beta\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ $\mathbf{l_{PC}} : \left(\boldsymbol{r} - \begin{pmatrix} 12 \\ -13 \\ 6 \end{pmatrix}\right) \times \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} = \mathbf{0}$ **Method 3** $\overrightarrow{PF} = \dfrac{\overrightarrow{PA'} + \overrightarrow{PA}}{2}$ $\overrightarrow{PA'} = 2\overrightarrow{PF} - \overrightarrow{PA} = 2\begin{pmatrix} 0 \\ 2 \\ 6 \end{pmatrix} - \begin{pmatrix} -8 \\ 8 \\ 4 \end{pmatrix} = \begin{pmatrix} 8 \\ -4 \\ 8 \end{pmatrix} = 4\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ *A*' falls on line *PC* Hence equation of l_{PC} $: \boldsymbol{r} = \begin{pmatrix} 12 \\ -13 \\ 6 \end{pmatrix} + \beta\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ $\mathbf{l_{PC}} : \left(\boldsymbol{r} - \begin{pmatrix} 12 \\ -13 \\ 6 \end{pmatrix}\right) \times \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} = \mathbf{0}$	2) **Method 3 (Ratio theorem)** $\overrightarrow{FP} = \dfrac{\overrightarrow{PA'} + \overrightarrow{PA}}{2}$ instead of $\overrightarrow{PF} = \dfrac{\overrightarrow{PA'} + \overrightarrow{PA}}{2}$

(iv)	**<u>Method 1</u>** $\boldsymbol{l_{AP}}:\boldsymbol{r}=\begin{pmatrix}4\\-5\\10\end{pmatrix}+\lambda\begin{pmatrix}-2\\2\\1\end{pmatrix}$ $\pi_2:\boldsymbol{r}.\begin{pmatrix}2\\1\\2\end{pmatrix}=1$ $\begin{pmatrix}-2\\2\\1\end{pmatrix}.\begin{pmatrix}2\\1\\2\end{pmatrix}=-4+2+2=0$ $\therefore \boldsymbol{n}\perp\pi_2$ and $\boldsymbol{n}\perp l_{AP}$ $l_{AP}\ //\ \pi_2$	This is a "show" question but many just wrote $\begin{pmatrix}-2\\2\\1\end{pmatrix}.\begin{pmatrix}2\\1\\2\end{pmatrix}=0$ and did not show the scalar product. Although no marks were deducted, students should explain why when the scalar product is zero, this implies $l_{AP}\ //\ \pi_2$.
	<u>Method 2</u> $\boldsymbol{l_{AP}}:\boldsymbol{r}=\begin{pmatrix}4\\-5\\10\end{pmatrix}+\lambda\begin{pmatrix}-2\\2\\1\end{pmatrix}$…………(1) $\pi_2:\boldsymbol{r}.\begin{pmatrix}2\\1\\2\end{pmatrix}=1$…………(2) Substitute (1) into (2) $\begin{pmatrix}4-2\lambda\\-5+2\lambda\\10+\lambda\end{pmatrix}.\begin{pmatrix}2\\1\\2\end{pmatrix}=1$ LHS= $8-4\lambda-5+2\lambda+20+2\lambda=23\neq1$ No solution None of the points on the line passing through *AP* falls on π_2. Hence $l_{AP}\ //\ \pi_2$	Insufficient working like method 1 was also seen for method 2.
(v)	**Method 1** Pick a point *Q* (0,1,0) on π_2. $A(4,-5,10)$ is a point on l_{AP}. $\overrightarrow{QA}=\begin{pmatrix}4\\-5\\10\end{pmatrix}-\begin{pmatrix}0\\1\\0\end{pmatrix}=\begin{pmatrix}4\\-6\\10\end{pmatrix}$ Shortest distance between the ray *AP* and π_2	Common mistake Shortest distance = $\dfrac{\left\lvert\begin{pmatrix}4\\-6\\10\end{pmatrix}\times\begin{pmatrix}2\\1\\2\end{pmatrix}\right\rvert}{\sqrt{2^2+1^2+2^2}}$

	$= \dfrac{\left\lvert \begin{pmatrix} 4 \\ -6 \\ 10 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \right\rvert}{\sqrt{2^2+1^2+2^2}} = \dfrac{\lvert 8-6+20 \rvert}{3} = \dfrac{22}{3}$	
	Method 2 Let N be the foot of perpendicular from A to plane π_2 $\boldsymbol{l_{AN}} : \boldsymbol{r} = \begin{pmatrix} 4 \\ -5 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4+2\lambda \\ -5+\lambda \\ 10+2\lambda \end{pmatrix}$ $\begin{pmatrix} 4+2\lambda \\ -5+\lambda \\ 10+2\lambda \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = 1$ $8-4\lambda-5+\lambda+20+4\lambda=1$ $\lambda = -\dfrac{22}{9}$ $\overrightarrow{ON} = \begin{pmatrix} 4-\frac{44}{9} \\ -5-\frac{22}{9} \\ 10-\frac{44}{9} \end{pmatrix} = \begin{pmatrix} -\frac{8}{9} \\ -\frac{67}{9} \\ \frac{46}{9} \end{pmatrix}$ $\overrightarrow{AN} = \begin{pmatrix} 4+\frac{8}{9} \\ -5+\frac{67}{9} \\ 10-\frac{46}{9} \end{pmatrix} = \dfrac{1}{9}\begin{pmatrix} 44 \\ 22 \\ 44 \end{pmatrix}$ $\left\lvert \overrightarrow{AN} \right\rvert = \dfrac{1}{9}\sqrt{44^2+22^2+44^2} = \dfrac{22}{3}$	**Not well Done** Some thought that the shortest distance is $\left\lvert \overrightarrow{ON} \right\rvert$

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Qn	Solutions	Comments
1(i)	$a \in \mathbb{R}/\{0\}$	Many students were able to quote the condition for an inverse function to exist but failed to realise that when $a=0$, the line becomes a horizontal straight line, which is not one-one.
(ii)	y = fg(x)= 1 – 2g(x) Reflection of the graph of y = g(x) in the x-axis followed by scaling by scale factor 2 parallel to the y-axis followed by translation 1 unit in the positive y-direction. **OR** Translation of the graph of $y=\mathrm{g}(x)$ by $\frac{1}{2}$ unit in the negative y-direction followed by reflection in the x – axis followed by scaling by scale factor 2 parallel to the y-axis. **OR** Translation of the graph of $y=\mathrm{g}(x)$ by $\frac{1}{2}$ unit in the negative y-direction followed by scaling by scale factor 2 parallel to the y-axis followed by reflection in the x – axis.	Better performing students realized that instead of substituting the rule of g(x) into the composite function directly, they could just describe a series of transformation to get from y = g(x) to y = 1 – 2g(x). Students should avoid using words such as ‘flip’ or ‘transform’ to describe any transformations. Instead, to use ‘reflect’, ‘translate’ and ‘scale’. Do also note that scale factor is a positive numerical value and do not contain x in it.
(iii)	$\mathrm{gg}(\boldsymbol{x})=\begin{cases}7-(\boldsymbol{x}^2+3) & \text{for} \quad 0\le \boldsymbol{x}\le 2\\ 7-(7-\boldsymbol{x}) & \text{for} \quad 2<\boldsymbol{x}<5\\ (7-\boldsymbol{x})^2+3 & \text{for} \quad 5\le \boldsymbol{x}\le 7\end{cases}$ $\therefore \mathrm{gg}(\boldsymbol{x})=\begin{cases}4-\boldsymbol{x}^2 & \text{for} \quad 0\le \boldsymbol{x}\le 2\\ \boldsymbol{x} & \text{for} \quad 2<\boldsymbol{x}<5\\ \boldsymbol{x}^2-14\boldsymbol{x}+52 & \text{for} \quad 5\le \boldsymbol{x}\le 7\end{cases}$	This part of the question tests students on the condition for a composite function to exist. For gg to exist, the range of g $\subseteq$ domain of g. For the answer for $5\le \boldsymbol{x}\le 7$, since the range of g is $[0,2]$ (refer to the function g given in the qns whose rule is $2<\boldsymbol{x}\le 7$), the rule $y=7-x$ will be put into ‘x’ of the rule $y=x^2+3$, since its domain is $[0,2]$. For students who put $y=7-x$ into the ‘x’ of $y=7-x$, you may wish to note that range of g = $[0,2]\subsetneq$ domain of g = $(2,7]$.
2(i)	$y=\frac{x+a}{b}-\frac{a}{x+b}$ Equation of asymptotes are : $y=\frac{x+a}{b}$ and $x=-b$	Some students faced difficulty finding the equation of the oblique asymptote. An easier way to see it would be to rewrite the equation of curve as $y=\frac{x}{b}+\frac{a}{b}-\frac{a}{x+b}$,

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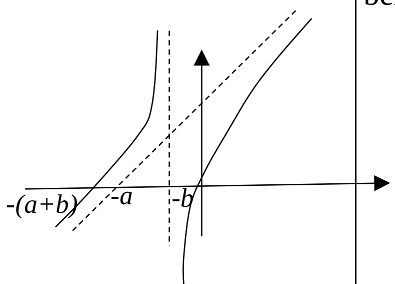
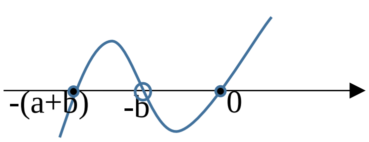
Qn	Solutions	Comments
	When $y=0$, $\dfrac{x+a}{b}=\dfrac{a}{x+b}$ $(x+a)(x+b)=ab$ $x^2+x(a+b)=0$ $x(x+(a+b))=0$ $x=0$ or $x=-(a+b)$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{b}+\dfrac{a}{(x+b)^2}$ $=\dfrac{(x+b)^2+ab}{b(x+b)^2}>0$ since $a>b>0$, using G.C: 	which resembles the form of $y=mx+c-\dfrac{a}{x+b}$. As the question asks for the equations of asymptotes, just writing $-b$ or $\dfrac{x+a}{b}$ will lead to no mark being awarded.
(i)	**Method 1:** From graph $\therefore x\geq 0$ *or* $-(a+b)\leq x<-b$	Some careless mistakes such as putting b instead of $-b$ or missing out the equal signs at 0 or $-a-b$ were commonly seen. There were also some presentation issues, such as leaving the answer as $0\leq x$. Students may wish to note that a standalone ‘,’ is not found in the mathematical notation list provided in the 9758 syllabus. Instead, the use of ‘or’ is more appropriate for the final answer as either this region $x\geq 0$ or this region $-(a+b)\leq x<-b$ will make the inequality true.
	Method 2: $\dfrac{x+a}{b}\geq\dfrac{a}{x+b}$ $\dfrac{x+a}{b}-\dfrac{a}{x+b}\geq 0$ $\dfrac{x^2+ax+bx+ab-ab}{b(x+b)}\geq 0$ $\dfrac{x^2+ax+bx}{b(x+b)}\geq 0$ But $b>0$, $\therefore \dfrac{x^2+ax+bx}{(x+b)}\geq 0$ $\therefore x(x+a+b)(x+b)\leq 0,\quad x\neq -b$  $\therefore x\geq 0$ *or* $-(a+b)\leq x<-b$	

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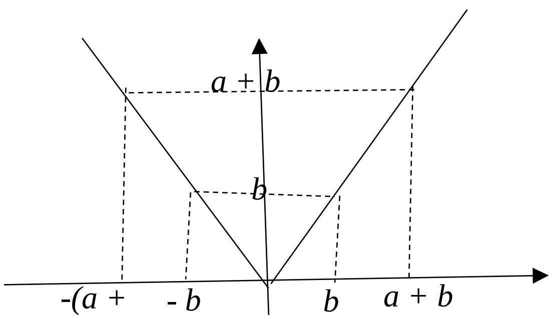
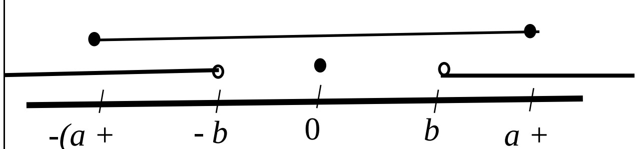
Qn	Solutions	Comments
(ii)	For $\dfrac{a-\lvert x\rvert}{b} \geq \dfrac{a}{b-\lvert x\rvert}$ Replace x in (1) with $-\lvert x\rvert$ $\therefore -\lvert x\rvert \geq 0$ *or* $-(a+b) \leq -\lvert x\rvert < -b$ $\therefore \lvert x\rvert \leq 0$ or $b < \lvert x\rvert \leq (a+b)$	Some common mistakes seen here include: • rejecting $-\lvert x\rvert \geq 0$ immediately, • missing out the equal sign for $a+b$ and $-a-b$ and • poor application of solving inequality with modulus sign. For example, a significant number of students have $-a-b \leq x \leq a+b$ as their answers.
	Method 1:  $\therefore x=0$ or $-(a+b) \leq x < -b$ or $b < x \leq (a+b)$	
	Method 2 $\therefore \lvert x\rvert \leq 0$ or $b < \lvert x\rvert \leq (a+b)$ $\therefore x=0$ or $\lvert x\rvert > b$ and $\lvert x\rvert \leq (a+b)$  $\therefore x=0$ or $-(a+b) \leq x < -b$ or $b < x \leq (a+b)$	
	Method 3 $\dfrac{a-\lvert x\rvert}{b} \geq \dfrac{a}{b-\lvert x\rvert} \quad \Rightarrow \quad \dfrac{a-\lvert x\rvert}{b} - \dfrac{a}{b-\lvert x\rvert} \geq 0$	

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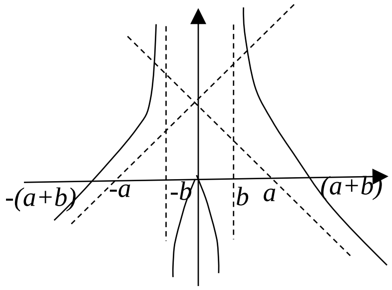
Qn	**Solutions**	**Comments**
	Sketch $y=\frac{a-\lvert x\rvert}{b}-\frac{a}{b-\lvert x\rvert}=\mathrm{f}(-\lvert x\rvert)$  $\therefore x=0$ or $-(a+b)\le x<-b$ or $b<x\le(a+b)$	
3(i)	$\mathrm{i}z^2-(5+\mathrm{i})z+2-6\mathrm{i}=0$ $z=\frac{5+\mathrm{i}\pm\sqrt{[-(5+\mathrm{i})]^2-4(\mathrm{i})(2-6\mathrm{i})}}{2\mathrm{i}}$ $=\frac{5+\mathrm{i}\pm\sqrt{2\mathrm{i}}}{2\mathrm{i}}$ $=\frac{5+\mathrm{i}\pm(1+\mathrm{i})}{2\mathrm{i}}$ $=\frac{6+2\mathrm{i}}{2\mathrm{i}}$ or $\frac{4}{2\mathrm{i}}$ $=1-3\mathrm{i}$ or $-2\mathrm{i}$	There were various methods seen to solve this question, such as completing the square, letting $z=a+b\mathrm{i}$ and proceeding to compare real and imaginary coefficients, etc, but the most efficient way would be to solve it using the quadratic formula and then use the GC to evaluate. For students who convert $a+b\mathrm{i}$ and $a-b\mathrm{i}$ to factors first and compare coefficients with the original question, do note that this method is incorrect as the 2 roots of the quadratic equation do not occur in conjugate pairs, since not all the coefficients are real.
(a) (ii)	$-\mathrm{i}w^2-(1-5\mathrm{i})w+2-6\mathrm{i}=0$ Since $w=\mathrm{i}z$, $w=\mathrm{i}(1-3\mathrm{i})$ or $\mathrm{i}(-2\mathrm{i})$ $=3+\mathrm{i}$ or 2	As this is a hence question, students would have to use the answers from part (i) to make a replacement and then solve for *w*. Solving this part as a fresh new question will warrant zero mark.
(a) (iii)	Since $P(z)$ is a polynomial of degree 4 with real coefficient, hence $1+3\mathrm{i}$ and $2\mathrm{i}$ are also the roots.	The concept tested here is whether students understand that a polynomial with real coefficients would have complex roots in conjugate pairs. Since the 2 roots in part (i) are the roots of $P(z)$, their conjugates would also be roots.

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Qn	Solutions	Comments
	$P(z)=(z+2\mathrm{i})(z-2\mathrm{i})(z-1-3\mathrm{i})(z-1+3\mathrm{i})$ $=(z^2+4)\left((z-1)^2+9\right)$ $=(z^2+4)(z^2-2z+10)$ $=z^4-2z^3+14z^2-8z+40$	Students would also need to be mindful of what they wrote. $P(z)$ is a polynomial in z, not x!
4	$U_n+U_{n-1}=\cos(2n+1)\theta+\cos\left(2(n-1)+1\right)\theta$ $=\cos(2n+1)\theta+\cos(2n-1)\theta$ $=2\cos\dfrac{(2n+1)\theta+(2n-1)\theta}{2}\cos\dfrac{(2n+1)\theta-(2n-1)\theta}{2}$ $=2\cos 2n\theta\cos\theta$	Must show the steps on how the MF26 formulas are applied after second line.
	$\displaystyle\sum_{n=1}^{2N}(-1)^{n+1}\cos 2n\theta=\sum_{n=1}^{2N}(-1)^{n+1}(U_n+U_{n-1})$ $\displaystyle=\sum_{n=1}^{2N}(-1)^{n+1}\frac{1}{2\cos\theta}(U_n+U_{n-1})$ $\displaystyle=\frac{1}{2\cos\theta}\sum_{n=1}^{2N}(-1)^{n+1}(U_n+U_{n-1})$ $\displaystyle=\frac{1}{2\cos\theta}[U_1+U_0$ $-U_2-U_1$ $+U_3+U_2$ $-U_4-U_3$ $\vdots$ $+U_{2N-1}+U_{2N-2}$ $-U_{2N}-U_{2N-1}]$ $\displaystyle=\frac{1}{2\cos\theta}[U_0-U_{2N}]$ $\displaystyle=\frac{1}{2\cos\theta}[\cos\theta-\cos(4N+1)\theta]$ $\displaystyle=\frac{1}{2}\left(1-\frac{\cos(4N+1)\theta}{\cos\theta}\right)$	Need to re-arrange to get this. Some left out $\dfrac{1}{2\cos\theta}$. While doing the Method of Difference, some students mis-read the expression of U_n+U_{n-1} . Instead of starting with U_1+U_0, some started with U_1+U_2 and resulted in cancellation of wrong terms.

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Qn	Solutions	Comments
	$2n\theta=\frac{n\pi}{3}\Rightarrow\theta=\frac{\pi}{6}$ $\sum_{n=11}^{41}(-1)^{n+1}\cos\frac{n\pi}{3}$ $=\sum_{n=1}^{40}(-1)^{n+1}\cos\frac{n\pi}{3}+(-1)^{41+1}\cos\frac{41\pi}{3}-\sum_{n=1}^{10}(-1)^{n+1}\cos\frac{n\pi}{3}$ $=\frac{1}{2}\left(1-\frac{\cos\left(4(20)+1\right)\frac{\pi}{6}}{\cos\frac{\pi}{6}}\right)+\cos\frac{41\pi}{3}-\frac{1}{2}\left(1-\frac{\cos\left(4(5)+1\right)\frac{\pi}{6}}{\cos\frac{\pi}{6}}\right)$ $=\frac{1}{2}\left(1-\frac{\cos(81)\frac{\pi}{6}}{\cos\frac{\pi}{6}}\right)+\cos\frac{41\pi}{3}-\frac{1}{2}\left(1-\frac{\cos(21)\frac{\pi}{6}}{\cos\frac{\pi}{6}}\right)$ $=\frac{1}{2}+\frac{1}{2}-\frac{1}{2}=\frac{1}{2}$	Most students are able to apply $\sum_{n=11}^{41}=\sum_{n=1}^{41}-\sum_{n=1}^{10}$ Many did not realise that the upper limit in (ii) is 2N, so 41 cannot be = 2N. Hence the need to split to sum from n=1 to n=40 then add 41st term separately. To apply $\sum_{n=1}^{40}$ some didn't realise they 2N=40 to get N = 20 and not n=40, thus $\sum_{n=1}^{40}(-1)^{n+1}\cos\frac{n\pi}{3}=\frac{1}{2}\left(1-\frac{\cos\left(4(20)+1\right)\frac{\pi}{6}}{\cos\frac{\pi}{6}}\right)$
5(i)	$x=e^{\cos^{-1}2t}$, $y=\left(1-4t^2\right)^{\frac{1}{2}}$ $\frac{dx}{dt}=\frac{-2}{\sqrt{1-4t^2}}e^{\cos^{-1}2t}$, $\frac{dy}{dt}=\frac{1}{2}(-8t)\left(1-4t^2\right)^{-\frac{1}{2}}=-4t\left(1-4t^2\right)^{-\frac{1}{2}}$ $\frac{dy}{dx}=\frac{-4t\left(1-4t^2\right)^{-\frac{1}{2}}}{-2\left(1-4t^2\right)^{-\frac{1}{2}}e^{\cos^{-1}2t}}$ $=\frac{2t}{e^{\cos^{-1}2t}}$ As $t=0, \frac{dy}{dx}=\frac{2t}{e^{\cos^{-1}2t}}=0$, The tangent to C at $t=0$ is **parallel to the *x*-axis**.	Some students forgot about the – 2 at the numerator of $\frac{dx}{dt}=\frac{-2}{\sqrt{1-4t^2}}e^{\cos^{-1}2t}$. While applying chain rule, some forgot to write down $e^{\cos^{-1}2t}$ in the $\frac{dx}{dt}=\frac{-2}{\sqrt{1-4t^2}}e^{\cos^{-1}2t}$ Some were able to say that $\frac{dy}{dx}=0$ but gave wrong conclusion like tangent is a stationary point or // to *y*-axis

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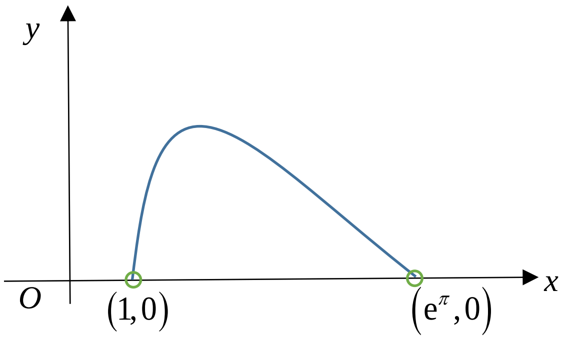
Qn	Solutions	Comments
(ii)		Many students didn't realise that $\frac{-1}{2}<t<\frac{1}{2}$ will result in the open-circles for the end-points on the graph. Only a minority didn't set GC Windows for T-values.
(iii)	Equation of tangent at $P\left(e^{\frac{\pi}{3}},\frac{\sqrt{3}}{2}\right)$, $t=\frac{1}{4}$, $\frac{dy}{dx}=\frac{1}{2}e^{-\frac{\pi}{3}}$ $y-\frac{\sqrt{3}}{2}=\frac{1}{2}e^{-\frac{\pi}{3}}\left(x-e^{\frac{\pi}{3}}\right)$ $y-\frac{\sqrt{3}}{2}=\frac{1}{2}e^{-\frac{\pi}{3}}\left(x-e^{\frac{\pi}{3}}\right)$ $y=\frac{1}{2}e^{-\frac{\pi}{3}}x+\frac{\sqrt{3}}{2}-\frac{1}{2}$	Some didn't use the coordinate of P to solve for the t-value at that point. Some didn't sub t = 0.25 into dy/dx to get the gradient and left the final equation with parameter t.
(iv)	Equation of normal at $Q\left(e^{\cos^{-1}2q},\left(1-4q^2\right)^{\frac{1}{2}}\right)$, $\frac{dy}{dx}=\frac{2t}{e^{\cos^{-1}2t}}=0$, $t=q=0$, Sub $t=q=0$ into $x=e^{\cos^{-1}2(0)}=e^{\frac{\pi}{2}}$ so $Q\left(e^{\frac{\pi}{2}},1\right)$ $\therefore x=e^{\frac{\pi}{2}}$ (Note: the normal is // y-axis)	Some didn't realise that for the normal to be // to y-axis, gradient of the tangent = 0 $\Rightarrow \frac{dy}{dx}=\frac{2t}{e^{\cos^{-1}2t}}=0 \quad \Rightarrow t=0$ Some didn't realise that the normal to be // to y-axis has the x = h type of equation

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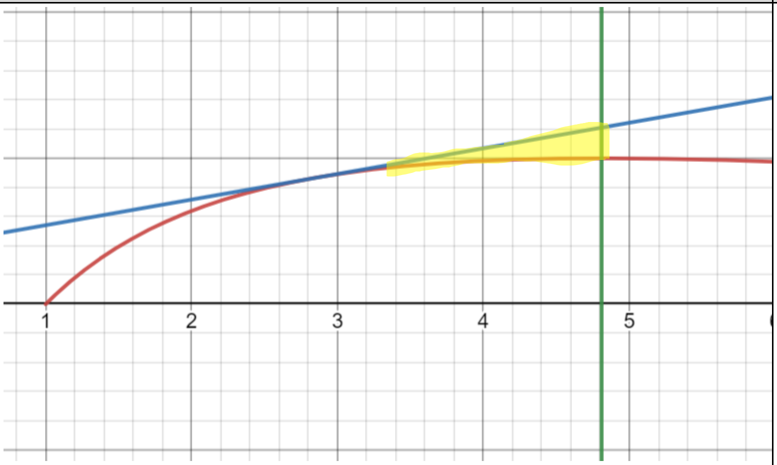
Qn	Solutions	Comments
(v)	 Required area $= \int_{e^{\frac{\pi}{3}}}^{e^{\frac{\pi}{2}}} \left(\frac{1}{2}e^{-\frac{\pi}{3}}x + \frac{\sqrt{3}}{2} - \frac{1}{2}\right) dx - \int_{e^{\frac{\pi}{3}}}^{e^{\frac{\pi}{2}}} (y)\,dx$ $= \int_{e^{\frac{\pi}{3}}}^{e^{\frac{\pi}{2}}} \left(\frac{1}{2}e^{-\frac{\pi}{3}}x + \frac{\sqrt{3}}{2} - \frac{1}{2}\right) dx - \int_{\frac{1}{4}}^{0} \left(1-4t^2\right)^{\frac{1}{2}} \frac{-2}{\sqrt{1-4t^2}} e^{\cos^{-1}2t}\,dt$ $= \int_{e^{\frac{\pi}{3}}}^{e^{\frac{\pi}{2}}} \left(\frac{1}{2}e^{-\frac{\pi}{3}}x + \frac{\sqrt{3}}{2} - \frac{1}{2}\right) dx - \int_{\frac{1}{4}}^{0} -2e^{\cos^{-1}2t}\,dt$ $= 2.03543 - 1.88372$ $= 0.15171$ $= 0.152$	Some were unable to tell where the required area is, therefore using the wrong equation for the upper curve. For the parametric part, some students forgot about the $\frac{dx}{dt}$ part of the integration in $\int_{\frac{1}{4}}^{0} \left(1-4t^2\right)^{\frac{1}{2}} \frac{-2}{\sqrt{1-4t^2}} e^{\cos^{-1}2t}\,dt$.
6 (i)	**Case 1:** All three magnets are different colour. ${}^4C_3 = 4$ ways ${}^4C_3$: Out of 4 colours, select one 3 colors. **Case 2:** Two identical and one different colour. ${}^4C_2 \times {}^2C_1 = 12$ ways ${}^4C_2$: Out of 4 colours choose 2 colours. ${}^2C_1$: Out of the 2 chosen colours, select one colour to be the repeated colour. Alternatively, ${}^4C_1 \times {}^3C_1 = 12$ ways ${}^4C_1$: Out of 4 colours choose 1 colour to be the repeated colour.	Many students are able to identify the 3 categories of cases. However quite a number of students have difficulty finding the correct number of ways for each case. The most common mistake is not realising that the magnets of the same colour are identical and hence not distinct. For example to find the number of ways to choose 3 magnets of different colour, common mistakes includes writing ${}^{14}C_3$, ${}^5C_1 \times {}^4C_1 \times {}^3C_1$, etc.

Qn	Solutions	Comments
	${}^{3}C_1$: Out of the 3 remaining unchosen colours, select one color to be the magnet with a different colour. **Case 3:** All three magnets identical (same colour). All red or all blue or all orange. (3 ways) Total number of ways $=4+12+3=19$ ways	
6(ii)	Method 1: $\text{P}(1 \text{ Red, } 2 \text{ Blue, } 1 \text{ Orange})=\left(\frac{5}{14}\right)\left(\frac{4}{13}\right)\left(\frac{3}{12}\right)\left(\frac{3}{11}\right)\left(\frac{4!}{2!}\right)=\frac{2160}{24024}=\frac{90}{1001}$ $\text{P}(2 \text{ Red, } 1 \text{ Blue, } 1 \text{ Green})=\left(\frac{5}{14}\right)\left(\frac{4}{13}\right)\left(\frac{4}{12}\right)\left(\frac{2}{11}\right)\left(\frac{4!}{2!}\right)=\frac{1920}{24024}=\frac{80}{1001}$ $\text{P}(1 \text{ Red, } 2 \text{ Orange, } 1 \text{ Green})=\left(\frac{5}{14}\right)\left(\frac{3}{13}\right)\left(\frac{2}{12}\right)\left(\frac{2}{11}\right)\left(\frac{4!}{2!}\right)=\frac{720}{24024}=\frac{30}{1001}$ $\text{P}(3 \text{ Orange, } 1 \text{ Blue})=\left(\frac{3}{14}\right)\left(\frac{2}{13}\right)\left(\frac{1}{12}\right)\left(\frac{4}{11}\right)\left(\frac{4!}{3!}\right)=\frac{96}{24024}=\frac{4}{1001}$ $\text{P}(\$7 \text{ worth of magnets})=\frac{90+80+30+4}{1001}=\frac{204}{1001}$ Method 2: $\text{P}(1 \text{ Red, } 2 \text{ Blue, } 1 \text{ Orange})=\frac{{}^{5}C_1\times{}^{4}C_2\times{}^{3}C_1}{{}^{14}C_4}=\frac{90}{1001}$ $\text{P}(2 \text{ Red, } 1 \text{ Blue, } 1 \text{ Green})=\frac{{}^{5}C_2\times{}^{4}C_1\times{}^{2}C_1}{{}^{14}C_4}=\frac{80}{1001}$ $\text{P}(1 \text{ Red, } 2 \text{ Orange, } 1 \text{ Green})=\frac{{}^{5}C_1\times{}^{3}C_2\times{}^{2}C_1}{{}^{14}C_4}=\frac{30}{1001}$ $\text{P}(3 \text{ Orange, } 1 \text{ Blue})=\frac{{}^{3}C_3\times{}^{4}C_1}{{}^{14}C_4}=\frac{4}{1001}$ $\text{P}(\$7 \text{ worth of magnets})=\frac{90+80+30+4}{1001}=\frac{204}{1001}$	Most students are able to identify at least 3 out of the 4 cases for \$7 worth of magnets. For method 1, the most common mistake is missing out on either $\left(\frac{4!}{2!}\right)$ or $\left(\frac{4!}{3!}\right)$. Some students also did not realise that the magnets were chosen <u>without</u> replacement. Method 2 is not a commonly used method among students. Some students who used method 2 have problems finding the correct numerator for each case. For example for the case of 1 Red, 2 Blue and 1 Orange, some made the mistake of writing ${}^{5}C_1\times\left({}^{4}C_1\times{}^{3}C_1\right)\times{}^{3}C_1$.

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Qn	Solutions	Comments
6 (iii)	No. of ways $=(10-1)!\ \times 2!\ \times {}^{10}C_3 \times 3!$ $=522547200$ $(10-1)!$ refers to arranging 5 red, 4 blue and one group of 2 adjacent green in a circle. $2!$ refers to permutating among the 2 adjacent green magnets. ${}^{10}C_3$ refers to choosing 3 slots among the 10 slots for the 3 orange magnets. 3! Refers to permutating among the 3 orange magnets. <u>Complement method (not recommended for this question):</u> Total no. of arrangements such that 2 green magnets are adjacent $=(13-1)!\ \times 2! = 958003200$ No. of arrangements such that 2 green magnets are adjacent and only 2 orange magnets are together $=(10-1)!\ \times 2! \times \left[\left({}^{10}C_2 \times 2!\right) \times \left({}^{3}C_2 \times 2!\right)\right]$ $=391910400$ $(10-1)!\ \times 2!$ refers to arranging 1 group of 2 green magnets, 4 red and 5 blue magnets in a circle. 2! refers to the group of 2 adjacent orange magnets swopping positions. $\left({}^{10}C_2 \times 2!\right)$ refers to choosing 3 slots among the 10 slots for the one group of 2 adjacent orange magnets and another separate orange magnet. 2! refers to the group of 2 adjacent orange magnets and the separate orange magnet swopping positions. $\left({}^{3}C_2 \times 2!\right)$ refers to choosing 2 orange magnets from the 3 orange magnets to be together. 2! refers to the group of 2 adjacent orange magnets swopping positions. No. of arrangements such that 2 green magnets are adjacent and 3 orange magnets are together $=(11-1)!\ \times 2! \times 3!$ $=43545600$ Required no. of ways $=958003200-391910400-43545600$ $=522547200$	This question is manageable for many students. Common mistakes includes missing out on either 2! or 3!. Another common mistake is students writing $(9-1)!\ \times 2!\ \times {}^{9}C_4 \times 4!$ because they first arrange the 5 red and 4 blue marbles in a circle and then choose 4 slots to insert the 3 orange magnets and 1 group of 2 green magnets. However, this method is incorrect as it is missing out on cases whereby the green magnets could be adjacent to the orange magnets. Some students tried the complement method but with only a few being successful.

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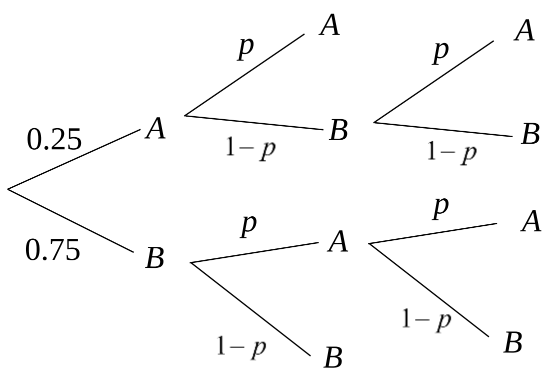
Qn	Solutions	Comments
7 (i)	Let A be the event where Archer wins a game. Let B be the event where Betty wins a game. 	Note that the competition ends once any player wins two consecutive games.
7 (ii)	$\mathrm{P}(\text{Archer wins the competition})$ $= 0.25p + 0.25(1-p)p + 0.75p^2$ $= 0.25p + 0.25p - 0.25p^2 + 0.75p^2$ $= 0.5p + 0.5p^2$ $= 0.5p(1+p)$	There are only 3 possible outcomes for Archer to win the competition, i.e. WW, WLW, LWW (W: Win L:Lose)
7 (iii)	$\mathrm{P}(\text{Betty won 2nd game} \mid \text{Betty won the competition})$ $= \dfrac{\mathrm{P}(\text{Betty lost 1st and won 2nd \& 3rd game}) + \mathrm{P}(\text{Betty won 1st \& 2nd game})}{\mathrm{P}(\text{Betty won the competition})}$ $= \dfrac{(0.25)(1-p)(1-p) + (0.75)(1-p)}{1 - \mathrm{P}(\text{Archer won the competition})}$ $= \dfrac{(0.25)(1-p)(1-p) + (0.75)(1-p)}{1 - 0.5p(p+1)}$ Since $p = 0.5$, $\mathrm{P}(\text{Betty won 2nd game} \mid \text{Betty won the competition})$ $= \dfrac{(0.25)(1-0.5)(1-0.5) + (0.75)(1-0.5)}{1 - (0.25)(0.5+1)}$ $= \dfrac{(0.25)(0.5)(0.5) + (0.75)(0.5)}{1 - 0.375}$ $= 0.7$	"given that" suggests this is a conditional probability question. Note that the event described in the denominator is the complement of the event in **(ii)**. Hence, the calculation is straightforward.

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<table>
<tr><th>Qn</th><th>Solutions</th><th>Comments</th></tr>
<tr><td></td><td>

<u>Alternative method to find P(Betty won the competition)</u>

$\mathrm{P(WW)+P(WLW)+P(LWW)}$
$=0.75(1-p)+0.75p(1-p)+0.25(1-p)^2$
$=0.75(0.5)+0.75(0.5)(0.5)+0.25(0.5)^2$
$=0.625$

</td><td></td></tr>
<tr><td>7 (iv)</td><td>

w	0	1	2
$\mathrm{P}(W=w)$	$\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)=\frac{3}{8}$	$\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{1}{4}$	$\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)=\frac{3}{8}$

</td><td>Common mistakes seen were omitting the case where $w=0$ or including the case where $w=3$, which is clearly a misinterpretation of the question.</td></tr>
<tr><td>7 (v)</td><td>

$\mathrm{E}(W)=(0)\left(\frac{3}{8}\right)+(1)\left(\frac{1}{4}\right)+(2)\left(\frac{3}{8}\right)=1$

$\mathrm{E}(W^2)=(0)^2\left(\frac{3}{8}\right)+(1)^2\left(\frac{1}{4}\right)+(2)^2\left(\frac{3}{8}\right)=\frac{7}{4}=1.75$

$\mathrm{Var}(W)=\mathrm{E}(W^2)-\left[\mathrm{E}(W)\right]^2=1.75-1^2=0.75$

</td><td>Working must be shown when finding $\mathrm{E}(W^2)$.

Writing $\mathrm{E}(X)$ and $\mathrm{E}(X^2)$ are not appropriate here. Students should write the notations according to context.</td></tr>
<tr><td>Q8 (i)</td><td>a represents for every additional month that a 1-kilogram Cheddar cheese is aged, the price increases by $a.</td><td>Students should quote $a instead of a units and mention that it is a <u>fixed increase</u> in the price for each additional month.

Phrasings like “rate of change…with respect to…”, “rate of increase”, ‘change in price” etc. are vague and should be avoided.</td></tr>
<tr><td>(ii)</td><td>(A) 0.9085
(B) 0.9751</td><td>Question clearly stated the accuracy required for the answers (i.e. 4 d.p). Giving your answers to 3 s.f. are not acceptable.</td></tr>
</table>

Qn	Solutions	Comments
(iii)	(B) is a better model since the <u>r value is closer to 1</u> as compared to model (A). Thus the data points in model (B) are more clustered along a straight line. Equation of a suitable straight line: $\ln P = 2.5641 + 0.15806\, m$ $\ln P = 2.56 + 0.158\, m$ (3 s.f) [Final answer]	It is not sufficient to claim that r value of (B) is greater than that of (A). The relativity of these values to 1 is more critical. It is not convenient to justify (B) is a better model by describing the behaviour of P as m increases as the scatter diagrams are not readily available. Coefficients of regression line should be given to 3 s.f. for final answer unless otherwise specified.
(iv)	When $m = 14$, $\ln P = 4.776945$ $P = \$\ 118.74$ (to the nearest cent) $m = 14$ is within the sample data range of m ($2.2 \leq m \leq 17.2$), where the linear relationship still holds, thus the use of this model to predict the value of P is appropriate and reliable.	Use accuracy of 5 s.f. for the coefficients of regression line when performing estimations. It is a serious misconception to say that the estimate of P is reliable because the estimate is in the data range of P. In addition, claiming that "the estimate is reliable because it is in the data range" is ambiguous (Is it referring to m or P?)
(v)	$\ln 1000P = 2.56 + 0.158\, m$ (3 s.f)	Note that the notation P, is **unchanged** in the question except that its unit is now given as dollars per gram. To convert to dollars per kilogram, simply multiply P by 1000.

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Q9 (i)	$T \sim \mathrm{N}(144\,,25)$ Since $\mathrm{P}(T > 60k) = 0.5$, $60k$ is the mean of the distribution, 144 min. $\therefore k = \dfrac{144}{60} = 2.4$	
(ii)	$T \sim \mathrm{N}(144\,,25)$, $X \sim \mathrm{N}(\ 236\,,81)$ $0.9X - T \sim \mathrm{N}\left(\ 0.9\,(236) - (144)\ ,\ \ 0.9^2\,(81) + (25)\ \right)$ $0.9X - T \sim \mathrm{N}(\ 68.4\ ,\ \ 90.61\ )$ $\mathrm{P}(0.9X - T \le 60) = 0.18877 = 0.189$ (to 3 s.f)	**Note:** ➢use $0.9X$ or another variable to denote X after 7pm. ➢ $0.9X \sim \mathrm{N}\left(\ 212.4\ ,\ 0.9^2(81)\right)$ ➢ Since train is faster, it takes a shorter time, hence $\lvert 0.9X - T\rvert$ is NOT necessary. ➢ Qn requires $0.9X - T \le 60$, **not** $T - 0.9X \le 60$, **not** $0.9X - 0.9T \le 60$, **not** $0.9X - T \le 1$. ➢ "$\le$" is required, not "$<$". ➢ $\mathrm{Var}\,(0.9X - T)$ $= \mathrm{Var}\,(0.9X)\ +\ \mathrm{Var}\,(T)$ (Plus! Not minus)
(iii)	Required conditional probability $= \dfrac{\text{P(he takes train and took more than 4hrs, i.e } T > 2.4\text{hrs)}}{\text{P(he takes either train or express bus and took more than 4 hrs)}}$ $= \dfrac{0.7\,\mathrm{P}\left(T > (4-1.6)\times 60\ \right)}{0.7\ \mathrm{P}\left(T > (4-1.6)\times 60\ \right)\ +0.3\ \mathrm{P}(X > 4\times 60)}$ $= \dfrac{0.7(0.5)}{0.7\ (\ 0.5)\ +0.3\,\mathrm{P}(X > 240\,)}$ (from (i)) $= \dfrac{0.7(0.5)}{0.7\ (\ 0.5)\ +0.3(0.32836)}$ $= 0.780365 = 0.780$ (to 3 s.f)	This question is a conditional probability since it is **given** that the time taken is more than 4 hours. In G.C, when normalcdf gives 0.49999, the probability is actually 0.5 because the x value is the mean of the distribution. Eg. $\mathrm{P}(T > 144) = 0.5$ since 144 is the mean. G.C could not give exact value due to rounding off error. 0.7 or 0.3 was commonly missing.
(iv)	Expected cost = 0.7 (54) + 0.3 (24) = $45	Note that (iii) and (iv) are not linked.

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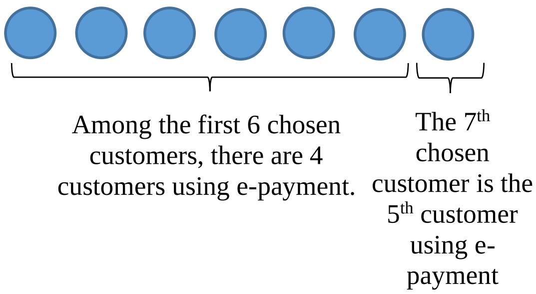
10 (i)	To test $H_0: \mu = 535$ Against $H_1: \mu < 535$ where μ represents the population mean daily profit of the nasi lemak stall holder.	Note that x, $\bar{x}$, μ_O or μ_1 are **not** acceptable.
(ii)	It is not necessary to assume that the daily profits are normally distributed for the test to be valid since $n = 45$ is large, Central Limit Theorem applies. [Note: By Central Limit Theorem, **sample means** of the daily profits of sample size 45 will follow a normal distribution approximately.]	Sample size in this case is 45, **not** the number of days in past years. Central Limit Theorem applies, $\bar{X}$ follows a normal distribution approximately, not X.
(iii)	Since H_0 is rejected at 5% level of significance, $z = \dfrac{\bar{x} - 535}{\sqrt{2591/45}} < -1.645$ $\bar{x} < -1.645\left(\sqrt{2591/45}\right) + 535$ $\{ \bar{x} : \bar{x} < 522.52 \}$ $\therefore \{ \bar{x} : 0 < \bar{x} < 523 \}$ (to 3 s.f)	**Different types of "Variances" (In order of importance)** 1) Population variance : σ^2 ➢ Most useful but usually not available. ➢ $\sigma^2 = 2591$ 2) Unbiased estimate of population variance : s^2 ➢ Quite useful. ➢ Good "substitute" when population variance is not available. ➢ Formulae found in MF26. ➢ $s^2 = \dfrac{n}{n-1} \times$ sample variance 3) Sample variance: No symbol ➢ Most useless. ➢ Cannot be used directly. ➢ Always convert to s^2.
(iv)	Unbiased estimate for population variance is $s^2 = \dfrac{45}{45-1} \times$ sample variance $= \dfrac{45}{45-1} \times 2008 = 2053.6364$ Under H_0, $\bar{X} \sim N\left(535, \dfrac{2053.6364}{45}\right)$ approx. by Central Limit Theorem since $n = 45$ is large Value of test statistic, $z = \dfrac{520 - 535}{\sqrt{2053.6364/45}} = -2.22$ p - value $= 0.013195 < 0.05$ $\therefore$ Reject H_0. There is sufficient evidence at the 5% level of significance to conclude that the population mean daily profit is less than \$535.	

(v)	If H_0 is not rejected, $p-\text{value} > \frac{\alpha}{100}$ $0.013195 > \frac{\alpha}{100}$ $\alpha < 1.3195$ The largest level of significance is 1.31%. (to 2 d.p) (1.32% is also acceptable.)	Level of significance refers to α %, not $\frac{\alpha}{100}$.
(vi)	To test $H_0: \mu = 535$ Against $H_1: \mu \neq 535$ at 8% level of significance Under H_0, $\bar{X} \sim \text{N}\left(535, \frac{2591}{n}\right)$ approx. by Central Limit Theorem since n is large. Value of test statistic, $z = \frac{526-535}{\sqrt{2591/n}}$ In order not to reject H_0, $-1.750686 < \frac{526-535}{\sqrt{2591/n}} < 1.750686$ $-9.901465 < \sqrt{n} < 9.901465$ $0 < n < 98.039016$ Greatest value of n is 98. [Note: In inequalities, before squaring both sides, it is necessary to check whether the inequality "make sense". For example, if $\frac{-9}{\sqrt{2591/n}} > 1.750686$, this doesn't make sense for any real values of n since a negative value (LHS) cannot be larger than a positive value (RHS). It is INCORRECT to square both sides to obtain n. Likewise, if $\frac{-9}{\sqrt{2591/n}} < 1.750686$, this inequality is true for any real values of n, do not square both sides to obtain a range of values of n blindly.]	Note that this is a 2-tail test where $H_1: \mu \neq 535$ at 8% level of significance. Some common incorrect critical values: 1) 1.4050716, due to 1 tail test. 2) 0.1004337, due to invNorm center 0.08. InvNorm center is referring to the area in the middle. In this case, it should be InvNorm center 0.92 instead. Alternatively, invNorm left 0.04 or invNorm right 0.04. **DO NOT WRITE:** $\frac{526-535}{\sqrt{2591/n}} \sim N(0,1)$ The correct way is $\frac{\bar{X}-535}{\sqrt{2591/n}} \sim N(0,1)$ Errors were commonly seen when solving the inequalities algebraically.

11 (a) (i)	 Method 1: Let W be the random variable the number of customers out of first 6 chosen customers who uses e-payment at a hawker stall. $W \sim \mathrm{B}(6, 0.25)$ Required Probability $= \mathrm{P}(W = 4) \times 0.25$ $= 0.00824$ (3.s.f) Method 2: Required Probability $= (0.25)^4 (0.75)^2 \left(\dfrac{6!}{4!2!}\right) \times 0.25$ $= 0.00824$ (3.s.f)	There are many successful attempts for this question for those who uses method 1. For method 2, the most common mistake is the missing term $\left(\dfrac{6!}{4!2!}\right)$. $\left(\dfrac{6!}{4!2!}\right)$ is necessary to account for the various permutations of the first 6 customers whereby 4 of them uses e-payments and 2 of them do not.
(a) (ii)	Let C be the random variable the number of customers out of 40 customers who uses e-payment at a hawker stall. $C \sim \mathrm{B}(40, 0.25)$ $\mathrm{P}(C \le 10) = 0.583904$ Let A be the random variable the number of customers out of 40 customers who uses e-payment at a hawker stall. $A \sim \mathrm{B}(30, 0.583904)$ $\mathrm{P}(A \ge 15) = 1 - \mathrm{P}(A \le 14)$ $= 1 - 0.1322406$ $= 0.867759$ $= 0.868$ (3s.f)	This part is successfully attempted by most students. It is heartening to observe that many student defined the random variables and distributions used. Some students misinterpreted or misread the question and ended up writing things such as $\mathrm{P}(C < 10)$, $\mathrm{P}(C \ge 10)$, $\mathrm{P}(A > 15)$ $\mathrm{P}(A \le 15)$, etc Note that $\mathrm{P}(A \ge 15) \ne 1 - \mathrm{P}(A \le 15)$ $\mathrm{P}(A \ge 15) \ne 1 - \mathrm{P}(A < 14)$

(b) (i)	A random sample in this context means that each hawker has an <u>equal chance of being selected</u> for the sample and the <u>selection</u> of one hawker is <u>independent of any other hawkers</u>.	This part is not well explained by many students. Note that each hawker has an equal chance of being selected for the sample. Quite a number of students thought that only hawkers who uses the Foodgowhere has an equal chance of being selected for the sample. Another common mistake is writing that the probability of a hawker being selected for the sample is independent of other hawkers. Some students did not answer the question but instead wrote about the conditions necessary for a binomial distributions to be applicable.
(b) (ii)	$X \sim \mathrm{B}(n, p)$ Since $\mathrm{E}(X) = 3.96$, $\therefore np = 3.96$ --------------(1) Since $\mathrm{P}(X \le 1) = 0.05303$, $\mathrm{P}(X = 0) + \mathrm{P}(X = 1) = 0.05303$ ${}^{n}C_0 p^0 (1-p)^n + {}^{n}C_1 p^1 (1-p)^{n-1} = 0.05303$ $(1-p)^n + np(1-p)^{n-1} = 0.05303$ --------------(2) Sub. (1) into (2): $(1-p)^{3.96/p} + (3.96)(1-p)^{3.96/p - 1} = 0.05303$ Using GC, $p = 0.3600375 = 0.360$ (3s.f) $\therefore n = \dfrac{3.96}{0.3600375} = 11$ (nearest integer)	This part is well attempted by most students. Most students are able to write out equation (1). When writing out equation (2), some left out $\mathrm{P}(X = 0)$ for the condition $\mathrm{P}(X \le 1) = 0.05303$. Some students did not know how to solve the 2 equations using GC. Note *n* is an integer and hence answer for *n* should not be in 3s.f.

(b) (iii)	$X \sim \mathrm{B}(15, p)$ Since mode $= 5 \Rightarrow$ $\mathrm{P}(X=4) < \mathrm{P}(X=5)$ and $\mathrm{P}(X=5) > \mathrm{P}(X=6)$ Considering $\mathrm{P}(X=4) < \mathrm{P}(X=5)$, $^{15}C_4 p^4 (1-p)^{11} < {}^{15}C_5 p^5 (1-p)^{10}$ $(1365)(p^4)(1-p)^{11} < (3003)(p^5)(1-p)^{10}$ $(p^4)(1-p)^{10}\left[1365(1-p) - 3003p\right] < 0$ Since $(1-p) > 0$ and $p > 0$, $1365(1-p) - 3003p < 0$ $1 - p - 2.2p < 0$ $p > \frac{5}{16}$ Considering $\mathrm{P}(X=5) > \mathrm{P}(X=6)$, $^{15}C_5 p^5 (1-p)^{10} > {}^{15}C_6 p^6 (1-p)^9$ $(3003)(p^5)(1-p)^{10} > (5005)(p^6)(1-p)^9$ $(p^5)(1-p)^9\left[3003(1-p) - 5005p\right] > 0$ Since $(1-p) > 0$ and $p > 0$, $3003(1-p) - 5005p > 0$ $1 - p - \frac{5}{3}p > 0$ $p < \frac{3}{8}$ Combining both results, $\therefore \frac{5}{16} < p < \frac{3}{8}$	Most students are aware that the modal value is 5 and many were able to state the 2 conditions needed for a modal value of 5. Some students only stated one condition while some did not know that we need to combine the results from the 2 conditions. Some attempted to use the GC to solve but ended up only stating one specific value of p whereby the modal value is 5.