Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

MATHEMATICS
PAPER 1

4052/01

14 August 2023

Monday

2 hours 15 minutes

2023 SECONDARY FOUR EXPRESS
PRELIMINARY EXAMINATION

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

INSTRUCTIONS TO CANDIDATES:
Write your name, index number and class on the spaces provided above.
Write in dark blue or black pen. You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

Note that all the diagrams in this paper are not drawn to scale.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

For Examiner's Use						Total Marks
Category	Accuracy	Notations	Others	Marks Deducted		
Question No.						90

This question paper consists of **23** printed pages (including this cover page) and **1** blank page.

2

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

3

Answer **all** the questions.

1   Solve $7x = 18 + 3x$.

Answer $x =$ .... [1]

2   **(a)** Calculate $\dfrac{26.18^3}{\sqrt{4.52 - 0.4^2}}$.

Write your answer correct to 5 significant figures.

Answer   .... [1]

(b) Write your answer to **part (a)** in standard form.

Answer   .... [1]

4

3 **(a)** Express 784 as the product of prime factors.

Answer .... [1]

(b) It is given that a and b are prime numbers.

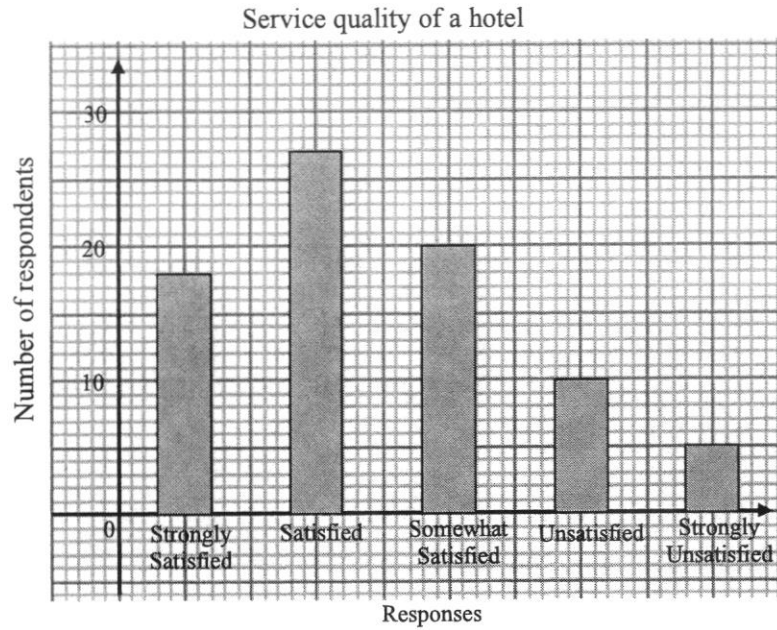Find the smallest values of a and b such that $784 \times \dfrac{a}{b}$ is a perfect cube.

Answer $a =$ .... [1]

$b =$ .... [1]

4 Expand and simplify $(w+5)(1-w)$.

Answer .... [2]

5

5 The bar graph below shows the results of a survey conducted on the service quality of a hotel.

Service quality of a hotel

(a) Find the percentage of respondents who answered 'Strongly Satisfied' and 'Satisfied'.

Answer ....% [1]

(b) Suggest the use of another statistical diagram to represent the results of the survey conducted, that can show the relative size of a part in relation to the whole.

Answer .... [1]

6

6   Find the largest integer that satisfies $2y - 3 < 4$.
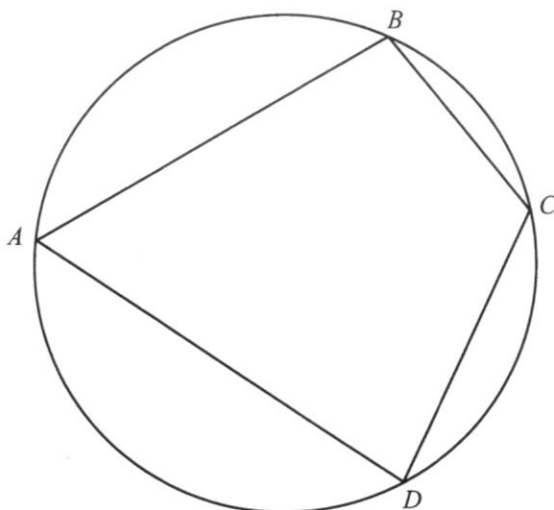
Answer .... [2]

7   P is directly proportional to Q^3.
     When $Q = 2$, $P = 64$.
     When the value of Q is halved, the value of P changes by a factor of m.
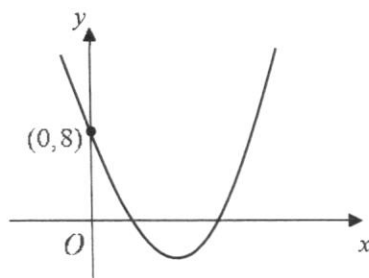   Find the value of m.

Answer $m =$ .... [2]

7

8 The diagram shows a quadrilateral playground *ABCD*.
A circular fence is constructed around the playground such that the vertices, *A*, *B*, *C* and *D* of the playground touch the circumference of the fence.

(a) Construct the perpendicular bisector of *AB*. [1]

(b) Construct the bisector of angle *ADC*. [1]

(c) A sand pit is to be constructed inside the circular fence but outside the quadrilateral playground. The sand pit is nearer to *AD* than *CD* and nearer to *B* than *A*.
Shade the region for the sand pit to be constructed. [1]

8

9 The diagram below shows the graph of $y = 3(x-h)^2 - 4$.

(a) Find the value of h.

Answer $h =$ .... [2]

(b) Explain why the graph of $y = 3(x-h)^2 + 1$ does not cut the x-axis.

Answer

....

....

.... [1]

---

10 A group of six students took a Mathematics quiz and the marks were recorded below.

8   10   9   13   10   9

(a) Calculate the standard deviation.
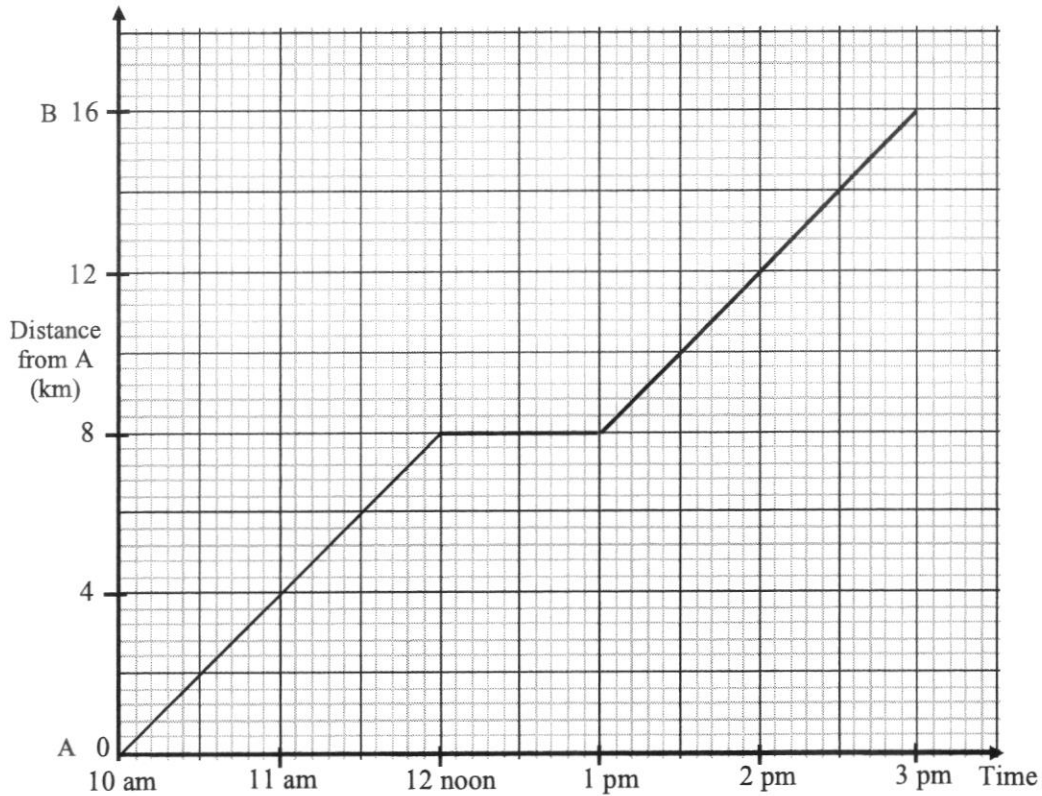
Answer   .... [1]

(b) **Two** other students also took the quiz, and their marks were recorded.
Given that the mean mark obtained by the eight students was 10 and the mode was also 10, find the marks of these two students.

Answer   .... and .... [2]

9

11

The distance-time graph shows the journey Tan took to run from town A to B.

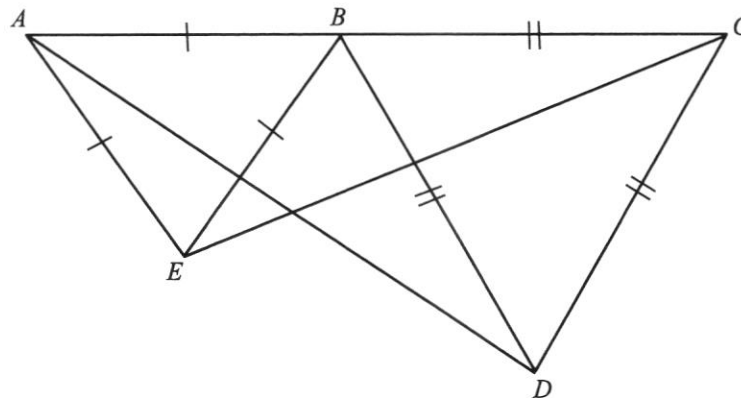**(a)** Find the distance Tan ran in the first two hours.

Answer ....km [1]

(b) Calculate the average speed, in m/s, for the whole journey Tan ran.

Answer ....m/s [2]

12 Simplify $\dfrac{2y^2 + y - 3}{4y^2 - 9}$.

Answer .... [3]

13

In the diagram, *ABC* is a straight line and triangles *ABE* and *BCD* are equilateral triangles.

Show that triangle *ABD* and triangle *EBC* are congruent.
Give a reason for each statement you make.

Answer

....

....

....

.... [3]

11

14 The first three terms in a sequence of numbers, T_1, T_2, T_3, ... are given below.

$$T_1 = 1 - \frac{1}{2}$$
$$T_2 = \frac{1}{2} - \frac{1}{3}$$
$$T_3 = \frac{1}{3} - \frac{1}{4}$$
.
.
.

(a) Write down T_4.

Answer .... [1]

(b) Show that the total sum of T_1, T_2, T_3, ... , T_n in the above sequence is $1 - \dfrac{1}{n+1}$.

Answer

[2]

12

15   A, B and C are points $(-1, 0)$, $(3, 8)$ and $(2, 1)$ respectively.

   (a)   Find the length of AB.

Answer $AB =$ .... units   [2]

   (b)   Find the equation of the line that passes through B and has the same gradient as AC.

Answer   .... [2]

16   (a)   Find the interior angle of a regular 18-sided polygon.

Answer   ....° [2]

   (b)   An n-sided polygon has two of its exterior angles at $45°$ and $75°$.
      If the remaining exterior angles are each $20°$, calculate the value of n.

Answer $n =$ .... [2]

13

17 (a) Simplify $\left(\dfrac{a^{-6}}{b^9}\right)^{\frac{1}{3}}$ and leave your answer in positive index notation.

Answer .... [2]

(b) Given that $2^{4x} \div 2^x = \sqrt[3]{2}$, find x.

Answer $x = $........................... [2]

---

18 (a) Given that $m^2 - 8mn + 16n^2 = 0$, find the value of $\dfrac{m}{n}$.

Answer .... [2]

(b) Factorise completely $3ac - 7c + 18ab - 42b$.

Answer .... [2]

14

19 A florist sells three types of bouquets, Bliss, Love and Commitment.
The number of stalks for each type of flower in each type of bouquet is shown in the table.

		Type of Flower			
		Rose	Lily	Gerbera	Sunflower
Type of Bouquet	Bliss	2	0	7	3
	Love	3	1	5	1
	Commitment	8	2	4	0

(a) Represent the above information in a 3×4 matrix, **F**.

Answer $\mathbf{F} = \begin{pmatrix} & & & \end{pmatrix}$ [1]

(b) The cost of each stalk of Rose, Lily, Gerbera and Sunflower is \$6, \$7.80, \$2.50 and \$3 respectively.

(i) Represent this information in a 4×1 column matrix, **H**.

Answer $\mathbf{H} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$ [1]

(ii) Evaluate the matrix $\mathbf{J} = \mathbf{F}\mathbf{H}$.

Answer $\mathbf{J} =$ [1]

(iii) State what the elements of **J** represent.

Answer

....

....

....

.... [1]

15

20 Box X contains 5 balls numbered 2, 3, 4, 7 and 9.
Box Y contains another 5 balls numbered 1, 5, 6, 8, and 10.
In a game, Ming drew a ball at random from each box, and the sum of both numbers is obtained.

(a) Complete the possibility diagram below.

Box Y

+	1	5	6	8	10
2	3			10	12
3		8			
4			10		
7				15	
9	10	14			19

Box X (label to the left of rows)

[2]

(b) Find the probability that

(i) the sum of both numbers is an odd number,

Answer .... [1]

(ii) the sum is a multiple of **one** of the two numbers drawn.

Answer .... [1]

16

21 The upper part of a solid wooden right circular cone was cut off leaving the frustum as shown in the diagram. The frustum has top radius 4 cm, base radius 8 cm and height 7.5 cm.
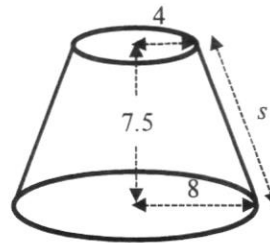
(a) Show that the slant height, s, is 8.5 cm.

Answer

[1]

(b) Find the curved surface area of the frustum.

Answer .... cm^2 [3]

17

22 In triangle *MNR*, point *M* is $(-3, 0)$ and $\sin \angle NMR = \dfrac{5}{13}$.

Q is a point on the negative *x*-axis.
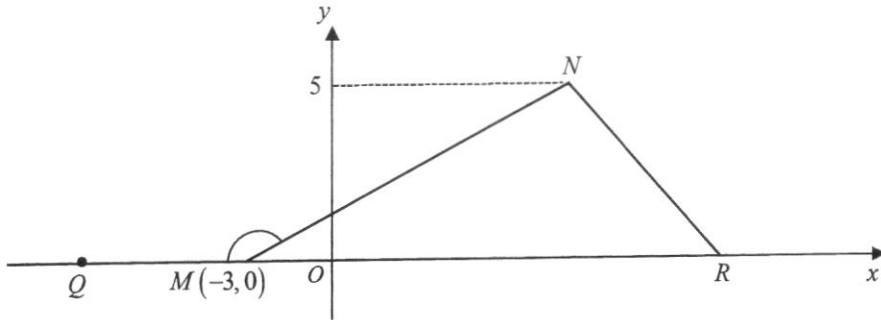
(a) Express the following as a fraction

(i) $\cos \angle NMQ$,

Answer .... [2]

(ii) $\tan \angle NMR$.

Answer .... [1]

(b) The area of triangle *MNR* is 50 square units.

Find the coordinates of *R*.

Answer (..................,..................) [2]

18

23 The diagram below shows a tree AB of height 7 m that stands vertically on a slope inclined at $20°$ with the horizontal PQ.

At a particular time in the morning, the tree casts a shadow, BC, on the slope.
AC is perpendicular to the slope.
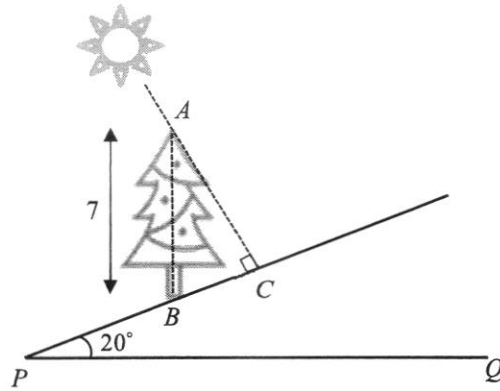
(a) Calculate the length of the shadow, BC.

Answer .... m [2]
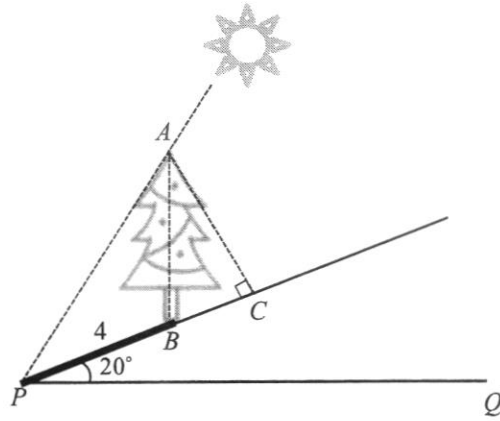
19

After some time, the sun goes into a position as shown below.

(b) If the shadow, *BP*, of the tree on the slope is 4 m, find the angle that the sunray makes with the horizontal *PQ*.

Answer ....° [3]

20

24 (a) $\xi = \{\text{integers } x : 1 \le x \le 12\}$

$P = \{\text{prime numbers}\}$

$Q = \{\text{multiples of 3}\}$

(i) Represent the above information on the Venn diagram shown in the answer space below.
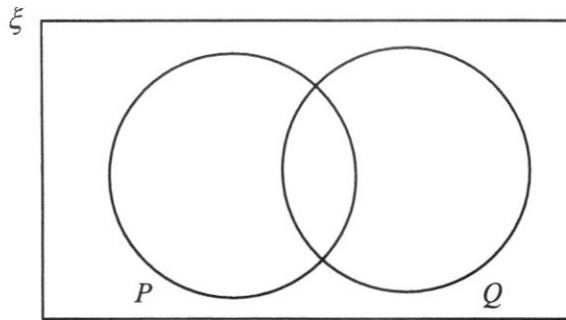
Answer

[2]

(ii) List the elements in $(P' \cap Q') \cup (P \cap Q)$.

Answer .... [1]

(iii) $R = \{x : x \text{ is a multiple of 6}\}$

Use set notation to describe the relationship between Q and R.

Answer .... [1]

(b) On the Venn diagram, shade the region which represents the set $A \cap B'$.

[1]
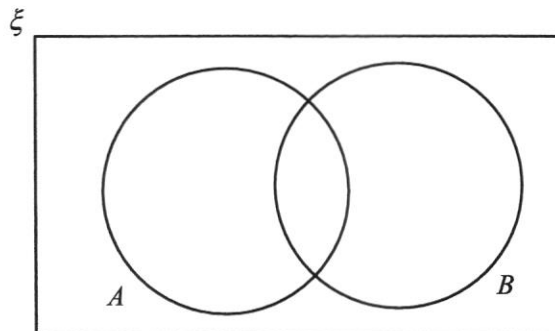
21

25 *OWXY* is a sector of a circle, centre *O*, of radius *r* cm and reflex angle 240°.
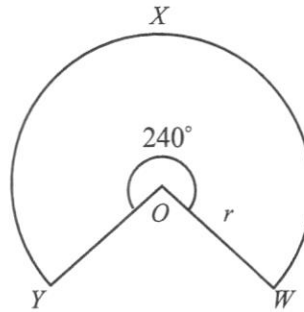
The sector *OWXY* has an area of 150π cm².

(a) Express 240° in terms of π radians.

Answer ....rad [2]

(b) Show that $r = 15$.

Answer

[2]

(c) The radii, *OW* and *OY*, are joined together to form a cone.
Find the base radius of the cone.

Answer .... cm [2]

26
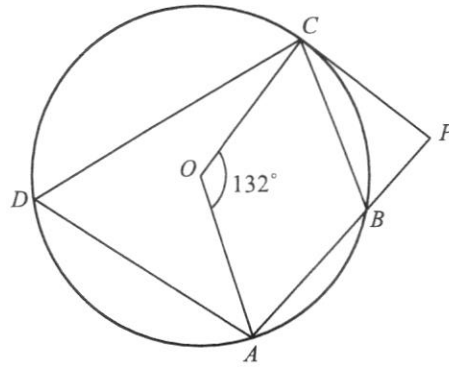
In the diagram above, O is the centre of the circle, such that angle $COA = 132°$.
PC is a tangent to the circle at C and PBA is a straight line.

By giving a reason for each step of your working, find

(a) $\angle CDA$,

Answer ....° [2]

(b) $\angle CBP$.

Answer ....° [2]

23

(c) If the radius of the circle is 3.55 cm, calculate the area of triangle *AOC*.

Answer .... cm^2 [2]

END OF PAPER

1	$x = \dfrac{9}{2}$
2a	8593.4
2b	8.5934×10^3
3a	$784 = 2^4 \times 7^2$
3b	$a = 7$ and $b = 2$
4	$-w^2 - 4w + 5$
5a	56.25%
5b	Pie chart
6	3
7	$m = \dfrac{1}{8}$
8	(a) B3 (b) B1
9a	$h = 2$
9b	**Either one** 1. The minimum point of the graph $y = 3(x-h)^2 + 1$ is $(h, 1)$. or $(2, 1)$.

	2. The equation $(x-h)^2 = -\dfrac{1}{3}$ has no solution for x
10a	1.57
10b	10 and 11
11a	8 km
11b	$\dfrac{8}{9}$ m/sec or 0.889 m/sec
12	$\dfrac{y-1}{2y-3}$
13	1) $AB = EB$ (sides of an equilateral triangle / given) 2) $BD = BC$ (sides of an equilateral triangle / given) 3) $\angle ABD = 180° - 60°$ (adj. $\angle$ on a st. line) $\quad = \angle EBC$ $\quad = 120°$ $\therefore$ triangle ABD is congruent to triangle EBC (SAS)
14a	$T_4 = \dfrac{1}{4} - \dfrac{1}{5}$
14b	$\left(1 - \dfrac{1}{2}\right) + \left(\dfrac{1}{2} - \dfrac{1}{3}\right) + \ldots + \left(\dfrac{1}{n} - \dfrac{1}{n+1}\right) = 1 - \dfrac{1}{n+1}$
15a	8.94 units
15b	$y = \dfrac{1}{3}x + 7$
16a	160°
16b	$n = 14$
17a	$\dfrac{1}{a^2 b^3}$
17b	$x = \dfrac{1}{9}$
18a	$\dfrac{m}{n} = 4$

18b	$(c+6b)(3a-7)$
19a	$F = \begin{pmatrix} 2 & 0 & 7 & 3 \\ 3 & 1 & 5 & 1 \\ 8 & 2 & 4 & 0 \end{pmatrix}$
19bi	$H = \begin{pmatrix} 6 \\ 7.80 \\ 2.50 \\ 3 \end{pmatrix}$
19bii	$\begin{pmatrix} 38.50 \\ 41.30 \\ 73.60 \end{pmatrix}$
19biii	The elements of **J** represent the total cost of the four types of flowers - Rose, Lily, Gerbera and Sunflower in bouquet Bliss, Love and Commitment respectively.
20a	

+	1	5	6	8	10
2	3	7	8	10	12
3	4	8	9	11	13
4	5	9	10	12	14
7	8	12	13	15	17
9	10	14	15	17	19

20bi	$\dfrac{13}{25}$
20bii	$\dfrac{2}{5}$
21a	Show

21b	320 cm^2
22ai	$-\dfrac{12}{13}$
22aii	$\dfrac{5}{12}$
22b	Coordinates of $R = (17,\,0)$
23a	2.39 m
23b	65.8°
24ai	
24aii	$(P' \cap Q') \cup (P \cap Q) = \{1, 3, 4, 8, 10\}$
24aiii	$R \subset Q$
24b	
25a	$\dfrac{4}{3}\pi$ rad

25b	Show
25c	10
26a	$\angle CDA = 132° \div 2$ $= 66°$ ($\angle$ at the centre = twice $\angle$ at circumference)
26b	$\angle CBA = 180° - 66°$ $= 114°$ ($\angle$s in opp. segment) $\angle CBP = 180° - 114°$ (adj. $\angle$s on a st. line) $= 66°$
26c	4.68 cm^2

Name:	Register Number:	Class:

PRESBYTERIAN HIGH SCHOOL

MATHEMATICS
PAPER 2

4052/02

16 August 2023   Wednesday   2 hours 15 minutes

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Qn	1	2	3	4	5	6	7	8	9	*Marks Deducted*	
Marks											

TOTAL MARKS

90

Category	Accuracy		Notations		Others	
Question						

Setter: Mr Tan Lip Sing
Vetter: Mrs Joyce Yeo

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1 **(a)** Rearrange the formula $c = \dfrac{d^2 + 5}{d^2 - 3}$ to make d the subject.

Answer $d =$ .... [3]

(b) Write as a single fraction in its simplest form $\dfrac{3}{(x-2)^2} - \dfrac{1}{2-x}$.

Answer .... [2]

(c) Solve these simultaneous equations.

$$5x + 3y = 14$$
$$3x + 5y = 18$$

You must show your working.

Answer $x = $ ....

$y = $ .... [3]

(d) Solve the equation $\dfrac{2x-1}{5x-6} = \dfrac{1}{2x-3}$.

Answer $x = $ .... or .... [3]

2   **(a)**   Before departing London for Singapore, Peter bought 3000 Singapore dollars
from the bank. The exchange rate between British pounds (£) and Singapore
dollars ($) was £1 = $1.71. He also had to pay the bank an additional commission
fee of 1.5% for the exchange of currency.
Calculate the total amount of pounds, inclusive of commission, he paid the bank.
Give your answer correct to the nearest pound.

Answer £ ………..……………. [2]

(b)   Peter bought a laptop while he was in Singapore. He paid $664.20 inclusive of
the 8% GST (Goods & Services Tax) for the laptop after getting a discount of A%
on the original price. The laptop's original price is $750 before GST.

(i)   Find the GST amount paid for the laptop.

Answer $ ……………………………. [2]

(ii)   Calculate the value of A.

Answer A = ………………………. [2]

(c) Mary invests $20 000 in an endowment plan that offers 4% per year compound interest. How much interest will she receive after 10 years? Give your answer correct to the nearest cent.

Answer $ .... [2]

(d) A map of a province has a scale of 1 : 500 000.

    (i) The length of an expressway on the map is 25 cm.
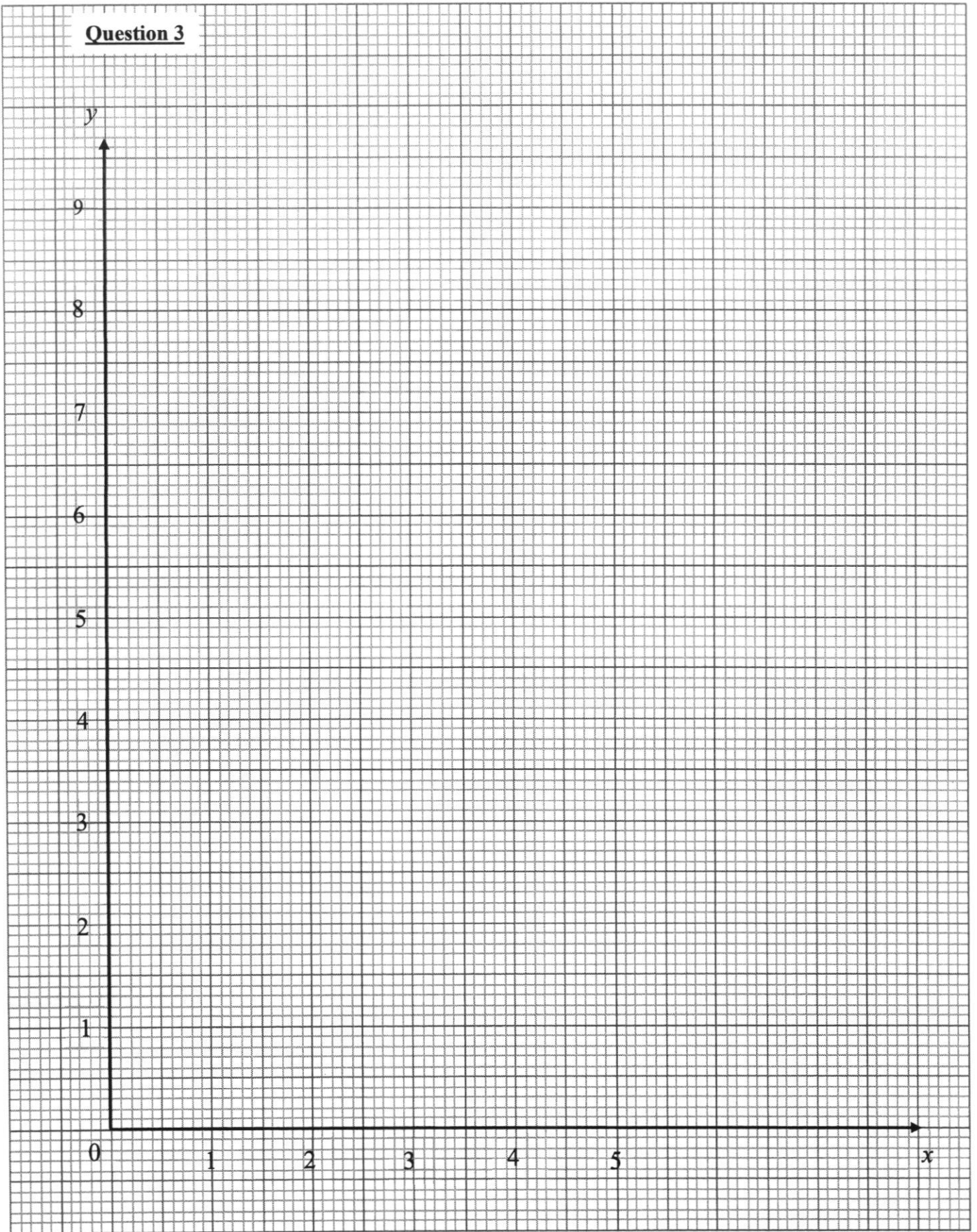   Calculate the actual length, in kilometres, of the expressway.

Answer .... km [1]

    (ii) The area of a reservoir is 180 km².
        Calculate the area, in square centimetres, of the reservoir on the map.

Answer .... cm² [2]

Question 3

3 The variables x and y are connected by the equation $y = \dfrac{x^2}{5} + \dfrac{4}{x}$.

The table below shows some corresponding values of x and y, correct to 2 decimal places.

x	0.5	1	1.5	2	2.5	3	4	5
y	8.05	4.20	3.12	2.80	2.85	3.13	4.20	5.80

(a) On the grid provided, draw the graph of $y = \dfrac{x^2}{5} + \dfrac{4}{x}$ for $0.5 \le x \le 5$.

Plot the points given in the table and join them with a smooth curve. [3]

(b) By drawing a tangent, find the gradient of the curve at $x = 3$.

Answer .... [2]

(c) (i) On the same grid, draw the line $y = 7 - \dfrac{1}{2}x$ for $0 \le x \le 5$. [1]

   (ii) Write down the x-coordinates of the points where this line intersects the curve.

Answer $x =$ .... or .... [2]

(iii) Find the equation, in the form $2x^3 + ax^2 + bx + c = 0$, which is satisfied by the values of x found in **(c)(ii)**.

Answer .... [2]

(d) Use your graph to find the values of x in the range $0 \le x \le 5$ for which

$$0.2x^2 + \frac{4}{x} - 2 = 3.$$

Answer $x = $ .... or .... [2]

4   (a)   The daily average temperature at Town *A* was recorded for 60 days.
            The cumulative frequency curve below shows the distribution of the temperatures.

Daily Average Temperature (°C)
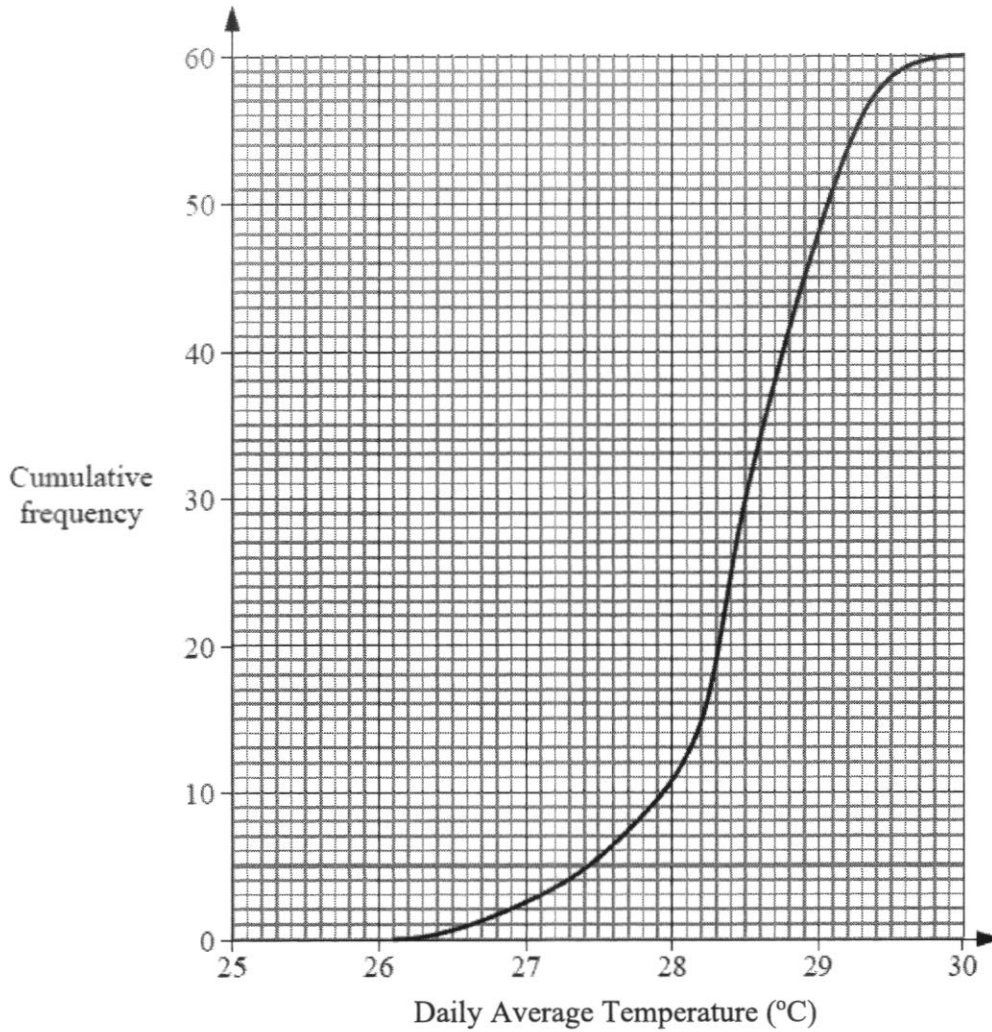
(i)   Use the curve to estimate
(a)   the median temperature,

Answer .... °C [1]

(b)   the interquartile range of the temperatures,

Answer .... °C [2]

(c)   the number of days that Town A had temperatures above $29°\text{C}$.

Answer .... days [1]

(ii) The daily average temperature at Town B was recorded for the same period.
The interquartile range of the temperatures at Town B is $1.5°\text{C}$.
Use this information to comment on one difference between the
temperature at Town A and at Town B.

....

....

....

.... [1]

(b) Box A contains 6 red cards, 4 blue cards and 2 green cards.
Box B contains 3 red cards and 5 blue cards.
A card is drawn at random from Box A and put into Box B.
Next, a card is drawn at random from Box B.

Find, as a fraction in its simplest form, the probability that

(i)   two green cards are drawn,

Answer .... [1]

(ii) neither of the cards is green,

Answer .... [1]

(iii) the two cards are of the same colour.

Answer .... [2]

5 **(a)** The diagram shows a parallelogram *ABCD* with *CD* produced to *E*.

F is the point of intersection of *BE* and *AD*.
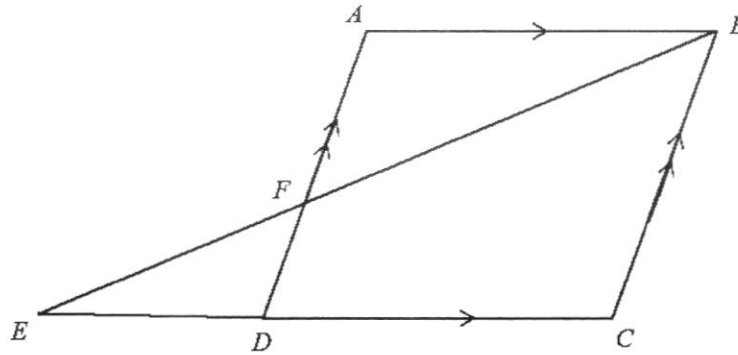
(i) Show that triangle *BAF* and triangle *EDF* are similar.

Give a reason for each statement you make.

....

....

....

....

.... [2]

(ii) State another triangle that is similar to *BAF* and *EDF* .

Answer Triangle .... [1]

(iii) The ratio $ED:DC = 2:3$.
Find the ratio $BC:AF$.

Answer .... : .... [1]

(iv) Given that the area of triangle *EDF* is 9.5 cm², find the area of triangle *BAF* .

Answer .... cm² [2]

(b) The diagram below shows a cone of height 50 cm.

The volume of the liquid in the cone is $\dfrac{3}{4}$ of the volume of the cone.

Calculate the depth, *h* cm, of the liquid.

50

h
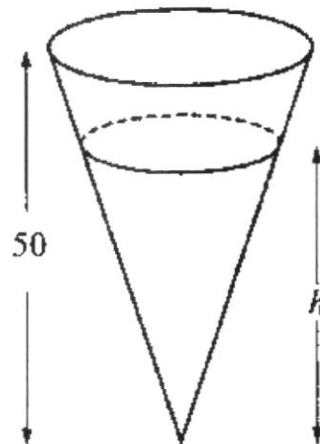
Answer .... cm [2]

6   In the diagram below, P is a point on DC, such that $DC = 2DP$ and X is a point on BP such that $3BX = 2BP$.

It is given that $\overrightarrow{AD} = 4\mathbf{a}$, $\overrightarrow{AB} = 3\mathbf{b}$, and $\overrightarrow{DP} = 2\mathbf{a} + 3\mathbf{b}$.
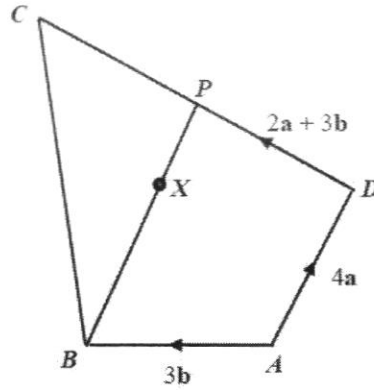
(a)   Express, as simply as possible, in terms of $\mathbf{a}$ and/or $\mathbf{b}$,

(i) $\overrightarrow{BP}$,

Answer .... [1]

(ii) $\overrightarrow{AX}$,

Answer .... [2]

(iii) $\overrightarrow{AC}$.

Answer .... [2]

(b) Explain whether the points A, X and C lie on the same straight line.

....

....

....

....

....

.... [2]

(c) Given that the area of triangle $BCP = 24\,\text{cm}^2$, find the area of triangle CXP.

Answer ....cm^2 [2]

7
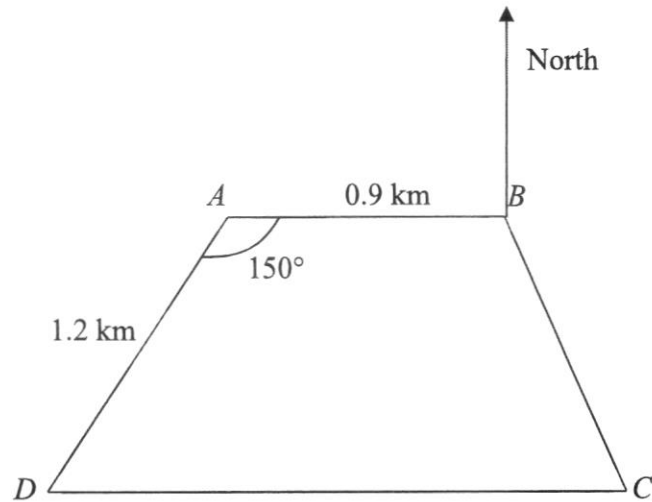
The diagram shows four towns A, B, C and D on a piece of horizontal land. Town A is due west of Town B. $ABCD$ is a trapezium such that $AB = 0.9$ km, $AD = 1.2$ km and angle $BAD = 150°$.

(a) Calculate the distance between Town B and Town D.

Answer .... km [3]

(b) Find angle BDC.

Answer ....° [2]

(c)   Calculate the bearing of *D* from *B*.

Answer ....° [2]

(d)   A tower is standing at Town *B*.
The greatest angle of elevation of the top of the tower, *T*, from the path *CD* is 18°. Find the height of the tower in metres.

Answer .... m [3]

8 James bought some essential oil for $720 at x per litre.

(a) Write an expression, in terms of x, for the number of litres of essential oil he bought.

Answer .... litres [1]

(b) Due to a leakage in the container, 5 litres of essential oil were lost.
James sold the remaining essential oil at $2 per litre more than what he had paid for. Write an expression, in terms of x, for the amount of money he received from the sale of essential oil.

Answer $ .... [1]

(c) Given that James made a profit of $100, write down an equation in x to represent this information and show that it reduces to $x^2 + 22x - 288 = 0$.

[3]

(d) Solve the equation $x^2 + 22x - 288 = 0$.

Answer $x =$ .... or $x =$ .... [3]

(e) Find, to the nearest litre, the amount of essential oil James sold.

Answer .... litres [2]

9   The table below shows the Income Tax Rate in Singapore.

Table 1: Income Tax Rate

Chargeable Income	Rate (%)	Gross Tax Payable ($)
On the first $120,000	-	7,950
On the next $40,000	15	6,000
On the first $160,000	-	13,950
On the next $40,000	18	7,200
On the first $200,000	-	21,150
On the next $40,000	19	7,600
On the first $240,000	-	28,750
On the next $40,000	19.5	7,800

(a)   Henry enjoyed a total tax relief of $15 000 and paid $14 130 of income tax for
      the year of assessment 2022. Calculate his annual income in 2022.
      [Annual income = Chargeable income + tax relief]

Answer $ .... [2]

Henry recently got a pay rise and his income is now $15 500 per month.
He is keen to buy a private condominium which is priced at $1 200 000. To afford this
condominium, he needs to apply for a bank loan of $800 000.

(b) The maximum duration of a housing loan for private properties is up to 35 years
or 65 years of age, whichever is lower.

Given that Henry is 45 years old, find the maximum number of years Henry can
loan from the bank.

Answer .... years [1]

(c) Henry decides to apply for a loan for the maximum duration allowed for his age.
The loan from the bank is subject to a simple interest of 3.5% per annum.

The government introduced the Total Debt Servicing Ratio (TDSR) to prevent
individuals from over-borrowing.

Information about TDSR

- Total Debt Servicing Ratio $= \dfrac{\text{Total monthly debt repayment}}{\text{Monthly income}}$

- Total monthly debt repayment includes repayments for car loans,
personal loans, credit card expenditure, home loans and other loans.

- The maximum TDSR allowed is 55%.

His current monthly debt repayment is shown in the table below:

Type	Amount ($)
Car loan	1000
Credit card Expenditure	1000
Personal loans	1000

By considering the TDSR ratio, will the bank approve his loan request?
Justify your answer and show your calculations clearly.

PRESBYTERIAN HIGH SCHOOL

MATHEMATICS
PAPER 1

14 August 2023

Monday

4052/01

2 hours 15 minutes

MARK SCHEME

2

Answer **all** the questions.

1   Solve $7x = 18 + 3x$.

$7x = 18 + 3x$

$4x = 18$

$x = \dfrac{9}{2}$ --- B1

---

2   **(a)**   Calculate $\dfrac{26.18^3}{\sqrt{4.52 - 0.4^2}}$.

Write your answer correct to 5 significant figures.

8593.4   --- B1

(b)   Write your answer to **part (a)** in standard form.

8.5934×10^3 --- B1

---

3   **(a)**   Express 784 as a product of prime factors.

2	784
2	392
2	196
2	98
7	49
7	7
	1

$784 = 2^4 \times 7^2$ --- B1

(b)   It is given that a and b are prime numbers.

Find the smallest values of a and b such that $784 \times \dfrac{a}{b}$ is a perfect cube.

$784 \times \dfrac{a}{b} = \left(2^4 \times 7^2\right) \times \dfrac{7}{2}$ $\therefore$ $a = 7$ and $b = 2$ --- B1, B1

---

4   Expand and simplify $(w + 5)(1 - w)$.

$(w + 5)(1 - w)$

$= -w^2 + w - 5w + 5$ --- M1: at least 2 correctly expanded terms

$= -w^2 - 4w + 5$ --- A1

---

5   The bar graph below shows the results of a survey conducted on the service quality of a hotel.

Service quality of a hotel
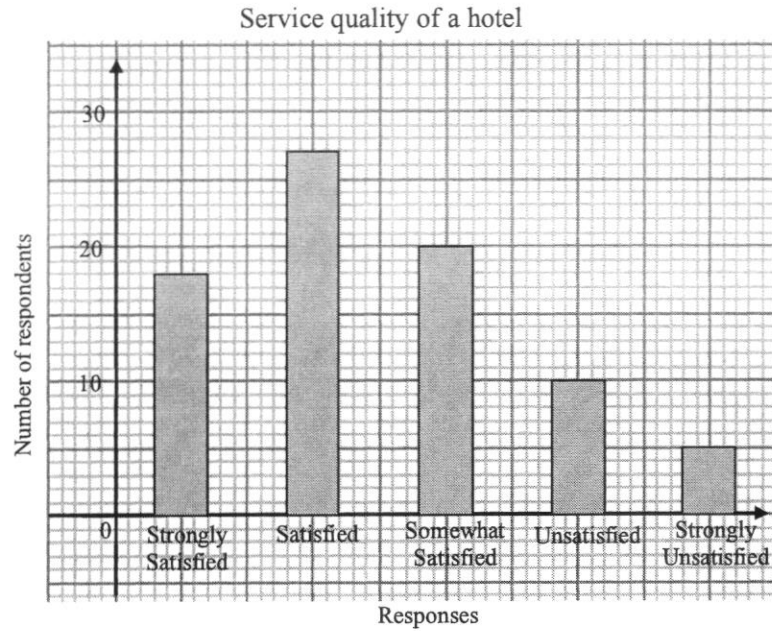
(a)   Find the percentage of respondents who answered 'Strongly Satisfied' and 'Satisfied'.

$$\frac{18+27}{18+27+20+10+5} \times 100\%$$
$$= 56.25\% \quad \text{--- B1}$$

(b)   Suggest the use of another statistical diagram to represent the results of the survey conducted, that can show the relative size of a part in relation to the whole.

Pie Chart --- B1

6   Find the largest integer that satisfies $2y - 3 < 4$.
   $2y - 3 < 4$
   $2y < 7$
   $y < \dfrac{7}{2}$ B1: seen this answer
   The largest integer is 3.   ---- B1

4

7   P is directly proportional to Q^3.

When $Q = 2$, $P = 64$.
When the value of Q is halved, the value of P changes by a factor of m.
Find the value of m.

$P = kQ^3$

When $Q = 2$ and $P = 64$

$64 = k(2)^3$ ---- M1: attempt to find the proportionality constant by substitution

$k = 8$

$P = 8Q^3$

$P_{new} = 8\left(\dfrac{1}{2}Q\right)^3 = 8\left(\dfrac{Q^3}{8}\right) = Q^3$

Hence the factor m is $\dfrac{1}{8}$.   --- A1

---

8   The diagram shows a quadrilateral playground $ABCD$.
     A circular fence is constructed around the playground such that the vertices, A, B, C and D of the
     playground touch the circumference of the fence.

(a) Construct the perpendicular bisector of AB.   [1]

(b) Construct the bisector of angle ADC.   [1]

(c) A sand pit is to be constructed inside the circular fence but outside the quadrilateral
     playground. The sand pit is nearer to AD than CD and nearer to B than A.
     Shade the region for the sand pit to be constructed.   [1]

5

9 The diagram below shows the graph of $y = 3(x-h)^2 - 4$.
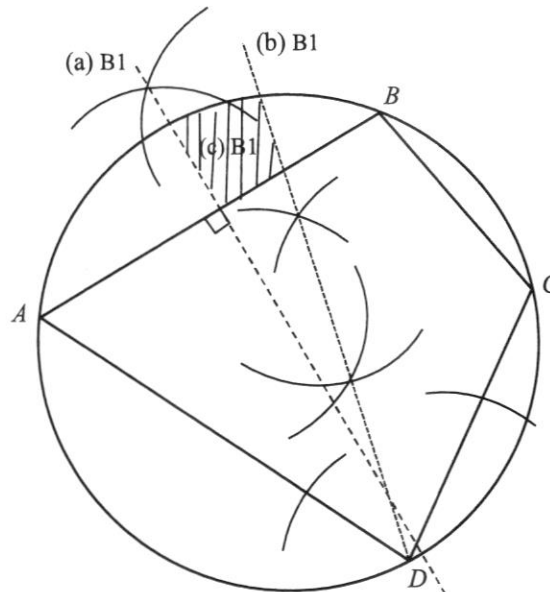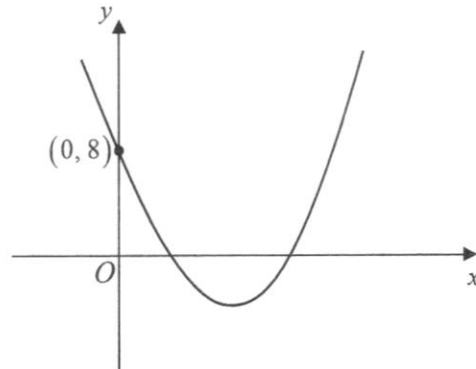
(a) Find the value of h.

Substitute $(0, 8)$:

$8 = 3(0-h)^2 - 4$ --- M1: shows subsitution

$h = 2$ or -2 (reject)

$\therefore h = 2$ --- A1

(b) Explain why the graph of $y = 3(x-h)^2 + 1$ does not cut the x-axis.

Either one

1. The minimum point of the graph $y = 3(x-h)^2 + 1$ is $(h, 1)$ or $(2, 1)$.

2. The equation $(x-h)^2 = -\dfrac{1}{3}$ has no solution for x. [1]

---

10 A group of six students took a Mathematics quiz and the marks were recorded below.

$$8 \quad 10 \quad 9 \quad 13 \quad 10 \quad 9$$

(a) Calculate the standard deviation.

1.57

(b) **Two** other students also took the quiz, and their marks were recorded.
Given that the mean mark obtained by the eight students was 10 and the mode was also 10, find the marks of these two students.

The two marks are 10 and 11. --- B1, B1
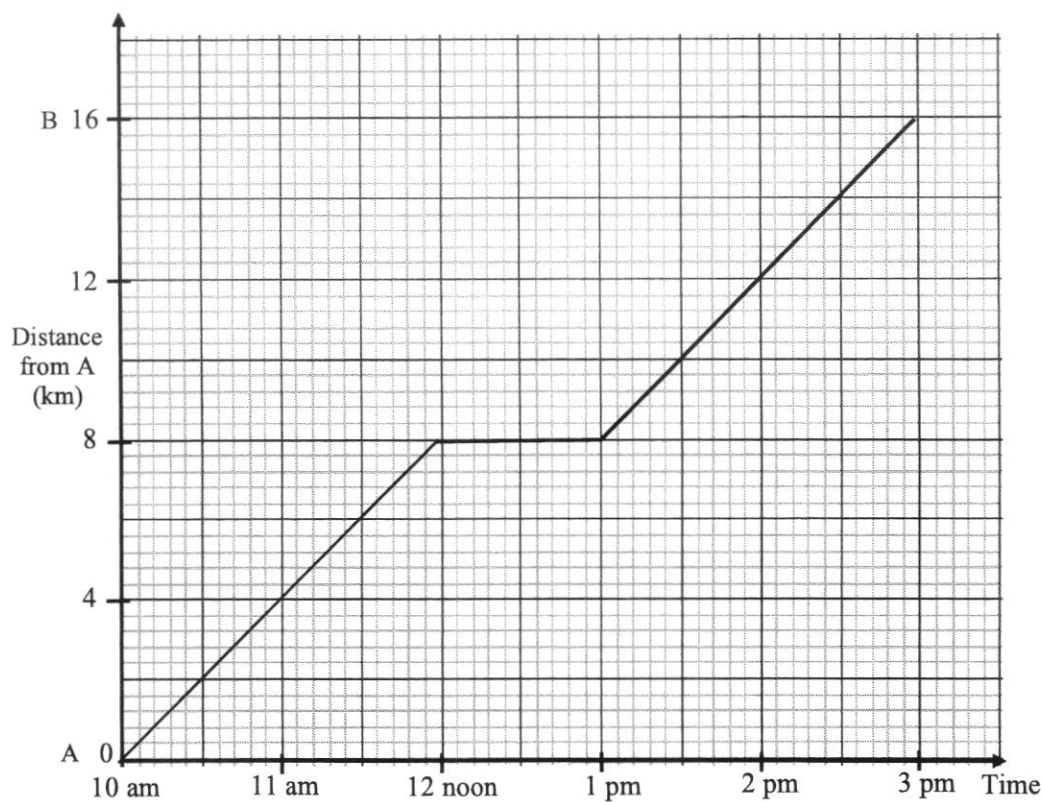
6

11

The distance-time graph shows the journey Tan took to run from town A to B.

(a) Find the distance Tan ran in the first two hours.

8 km --- B1

(b) Calculate the average speed, in m/s, for the whole journey Tan ran.

$$\text{Average speed} = \frac{16000}{5 \times 60 \times 60} \quad \text{--- M1: attempt to convert km to m or h to sec}$$

$$= \frac{8}{9} \text{ m/sec or } 0.889 \text{ m/sec} \quad \text{--- A1}$$

7

12 Simplify $\dfrac{2y^2 + y - 3}{4y^2 - 9}$.

$$\dfrac{2y^2 + y - 3}{4y^2 - 9}$$

$$= \dfrac{(2y+3)(y-1)}{(2y+3)(2y-3)} \quad \text{--- M1, M1: factorise numerator and denominator}$$

$$= \dfrac{y-1}{2y-3} \quad \text{--- A1}$$

13
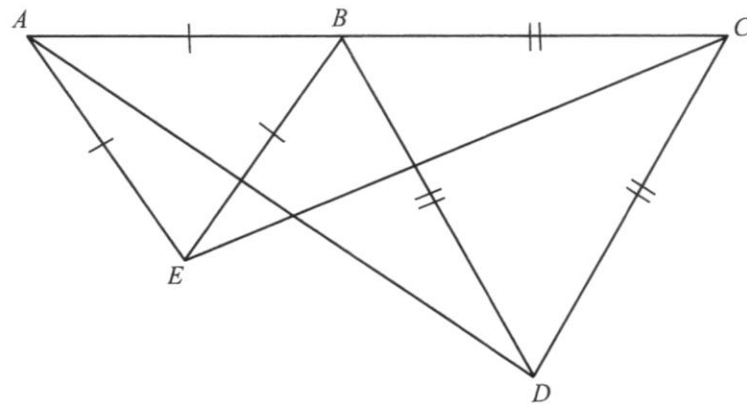
In the diagram, *ABC* is a straight line and triangles *ABE* and *BCD* are equilateral triangles.

Show that triangle *ABD* and triangle *EBC* are congruent.
Give a reason for each statement you make.

Answer

1) $AB = EB$ (sides of an equilateral triangle / given)

2) $BD = BC$ (sides of an equilateral triangle / given) --- B1: at least one statement with reason pr

3) $\angle ABD = 180° - 60°$ (adj. $\angle$ on a st. line)

   $\qquad = \angle EBC \qquad\qquad$ --- B1: show equivalent angles with explanation

   $\qquad = 120°$

$\therefore$ triangle *ABD* is congruent to triangle *EBC* (SAS) --- B1: with name of test   [3]

8

14 The first three terms in a sequence of numbers, T_1, T_2, T_3, ... are given below.

$$T_1 = 1 - \frac{1}{2}$$

$$T_2 = \frac{1}{2} - \frac{1}{3}$$

$$T_3 = \frac{1}{3} - \frac{1}{4}$$

.

.

.

(a) Write down T_4.

$$T_4 = \frac{1}{4} - \frac{1}{5} \quad \text{--- B1}$$

(b) Show that the total sum of T_1, T_2, T_3, ... , T_n in the above sequence is $1 - \frac{1}{n+1}$.

$$\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + ... + \left(\frac{1}{n} - \frac{1}{n+1}\right) \quad \text{--- M1: seen either the formation or } \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$= 1 - \frac{1}{n+1} \quad \text{--- [AG1: shown]}$$

---

15 A, B and C are points $(-1, 0)$, $(3, 8)$ and $(2, 1)$ respectively.

(a) Find the length of AB.

$$\text{Length of } AB = \sqrt{(-1-3)^2 + (0-8)^2} \quad \text{--- M1: correct application of length formula}$$

$$= 8.94 \text{ units } (3\text{s.f.}) \quad \text{--- B1}$$

(b) Find the equation of the line that passes through B and has the same gradient as AC.

$$mAC = \frac{1-0}{2-(-1)} = \frac{1}{3} \quad \text{--- M1}$$

Equation of line passing through B has the **same** gradient $= \frac{1}{3}$

The equation of the line:

$$y = \frac{1}{3}x + c \text{ or } y - 8 = \frac{1}{3}(x-3) \quad \text{(No marks without simplification)}$$

$$y = \frac{1}{3}x + 7 \quad \text{--- A1}$$

9

16 (a) Find the interior angle of a regular 18-sided polygon.

$$\frac{(18-2)\times180°}{18} \quad \text{--- M1}$$

$$= 160° \quad \text{--- A1}$$

(b) An n-sided polygon has two of its exterior angles at $45°$ and $75°$.
If the remaining exterior angles are each $20°$, calculate the value of n.

$$45+75+(n-2)(20)=360 \quad \text{--- M1}$$

$$n=14 \quad \text{--- A1}$$

---

17 (a) Simplify $\left(\dfrac{a^{-6}}{b^9}\right)^{\frac{1}{3}}$ and leave your answer in positive index notation.

$$\left(\frac{a^{-6}}{b^9}\right)^{\frac{1}{3}}$$

$$= \frac{a^{-2}}{b^3} \quad \text{--- M1: applied indices law with at most one error}$$

$$= \frac{1}{a^2 b^3} \quad \text{--- A1}$$

(b) Given that $2^{4x} \div 2^x = \sqrt[3]{2}$, find x.

$$2^{4x} \div 2^x = \sqrt[3]{2}$$

$$2^{4x} \div 2^x = 2^{\frac{1}{3}} \quad \text{--- M1: able to convert to appropriate index form}$$

$$2^{3x} = 2^{\frac{1}{3}}$$

$$3x = \frac{1}{3}$$

$$x = \frac{1}{9} \quad \text{--- A1}$$

10

18 (a) Given that $m^2 - 8mn + 16n^2 = 0$, find the value of $\dfrac{m}{n}$.

Method 1

$m^2 - 8mn + 16n^2 = 0$

$(m - 4n)^2 = 0$ --- M1: attempt to factorise into perfect square

$m - 4n = 0$

$m = 4n$

$\dfrac{m}{n} = 4$ ---- A1

Method 2

$m = \dfrac{-(-8) \pm \sqrt{(-8n)^2 - 4(1)(16n^2)}}{2(1)}$ --- B1

$= \dfrac{8n}{2} = 4n$

$\therefore \dfrac{m}{n} = 4$ --- A1

(b) Factorise completely $3ac - 7c + 18ab - 42b$.

$3ac - 7c + 18ab - 42b$

$= c(3a - 7) + 6b(3a - 7)$ --- M1: identified one common linear factor correctly

$= (c + 6b)(3a - 7)$ --- A1

---

19 A florist sells three types of bouquets, Bliss, Love and Commitment.
The number of stalks for each type of flower in each type of bouquet is shown in the table.

		Type of Flower			
		Rose	Lily	Gerbera	Sunflower
Type of Bouquet	Bliss	2	0	7	3
	Love	3	1	5	1
	Commitment	8	2	4	0

(a) Represent the above information in a 3×4 matrix, **F**.

$$\mathbf{F} = \begin{pmatrix} 2 & 0 & 7 & 3 \\ 3 & 1 & 5 & 1 \\ 8 & 2 & 4 & 0 \end{pmatrix}$$ --- B1

11

(b) The cost of each stalk of Rose, Lily, Gerbera and Sunflower are $6, $7.80, $2.50 and $3 respectively.

(i) Represent this information in a 4×1 column matrix, **H**.

$$\mathbf{H} = \begin{pmatrix} 6 \\ 7.80 \\ 2.50 \\ 3 \end{pmatrix} \quad \text{--- B1}$$

(ii) Evaluate $\mathbf{J} = \mathbf{FH}$.

$$\mathbf{J} = \begin{pmatrix} 2 & 0 & 7 & 3 \\ 3 & 1 & 5 & 1 \\ 8 & 2 & 4 & 0 \end{pmatrix} \begin{pmatrix} 6 \\ 7.80 \\ 2.50 \\ 3 \end{pmatrix} = \begin{pmatrix} 38.50 \\ 41.30 \\ 73.60 \end{pmatrix} \quad \text{--- B1}$$

(iii) State what the elements of **J** represent.

Answer

The elements of **J** represent the total cost of the four types of flowers - Rose, Lily, Gerbera and Sunflower in bouquet Bliss, Love and Commitment respectively. --- B1

---

20 Box X contains 5 balls numbered 2, 3, 4, 7 and 9.
Box Y contains another 5 balls numbered 1, 5, 6, 8, and 10.
In a game, Ming drew a ball at random from each box, and the sum of both numbers is obtained.

(a) Complete the possibility diagram below.

Box Y

+	1	5	6	8	10
2	3	7	8	10	12
3	4	8	9	11	13
4	5	9	10	12	14
7	8	12	13	15	17
9	10	14	15	17	19

Box X (label for rows)

B1: Every 8 correct values
B2: all correct

(b) Find the probability that

(i) the sum of both numbers is an odd number,

$$\frac{13}{25} \quad \text{--- B1}$$

12

(ii) the sum is a multiple of **one** of the two numbers drawn.

$$\frac{10}{25} = \frac{2}{5} \quad \text{--- B1}$$

---

21 The upper part of a solid wooden right circular cone was cut off leaving the frustum as shown in the diagram. The frustum has top radius 4 cm, base radius 8 cm and height 7.5 cm.
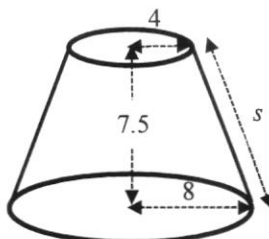
(a) Show that the slant height, s, is 8.5 cm.

$$s = \sqrt{4^2 + 7.5^2} = 8.5 \text{ cm (shown) --- AG1}$$

(b) Find the curved surface area of the frustum.
Curved surface area $= \pi(8)(2 \times 8.5) - \pi(4)(8.5)$

- M1: curved S.A. of the original right circular cone (BIG)
- M1: curved S.A. of the wooden right circular cone (SMALL)

$$= 320.44 \approx 320 \text{ cm}^2 \quad (3\text{s.f.}) \quad \text{--- A1}$$

---

22 In triangle *MNR*, point *M* is $(-3, 0)$ and $\sin \angle NMR = \dfrac{5}{13}$.

Q is a point on the negative x-axis.
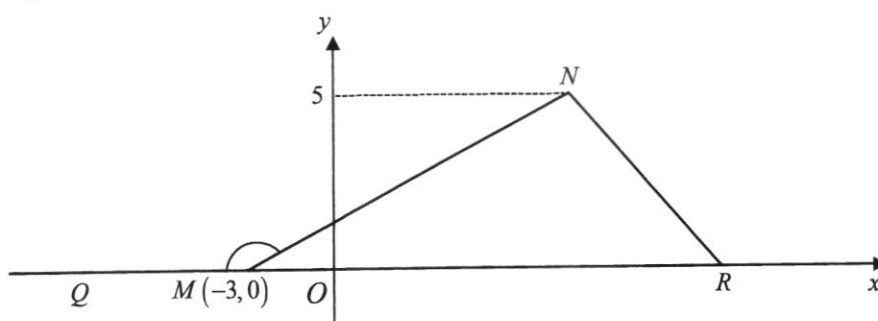
(a) Express the following as a fraction

(i) $\cos \angle NMQ$,

Length of "adjacent" = 12 units --- M1: using Pythagoras' Theorem

$$-\frac{12}{13} \quad \text{--- A1}$$

13

(ii) tan ∠*NMR*.

$$\frac{5}{12}$$

(b) The area of triangle *MNR* is 50 square units.

Find the coordinates of *R*.

Area of triangle $MNR = \frac{1}{2} \times$ base $\times 5 = 50$

Base = 20 units   --- M1

Coordinates of $R = (17,\ 0)$.   --- A1

---

23 The diagram below shows a tree *AB* of height 7 m that stands vertically on a slope inclined at 20° with the horizontal *PQ*.

At a particular time in the morning, the tree casts a shadow, *BC*, on the slope.
AC is perpendicular to the slope.
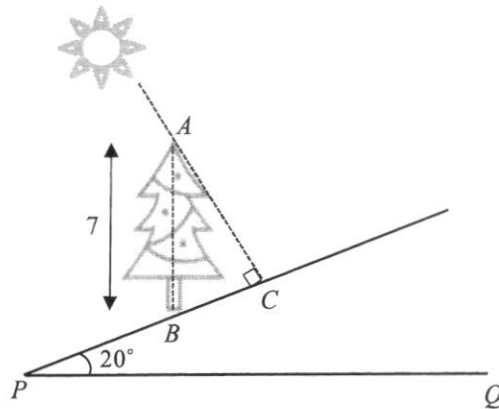
(a) Calculate the length of the shadow, *BC*, on the slope.

$$\sin 20° = \frac{BC}{7}$$ --- M1

$BC = 7 \times \sin 20°$
$\quad = 2.39$ m (3s.f.)   --- A1

Or

$$\cos 70° = \frac{BC}{7}$$ --- M1

$BC = 7 \times \cos 70°$
$\quad = 2.39$ m (3s.f.)   --- A1
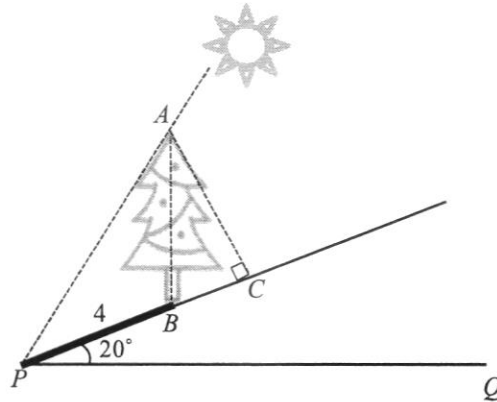
14

After some time, the sun goes into a position as shown below.

(b) If the shadow, BP, of the tree on the slope is 4 m, find the angle that the sun ray makes with the horizontal PQ.
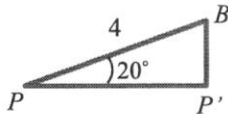
$$\cos 20° = \frac{PP'}{4}$$

$$PP' = 4 \times \cos 20°$$

$$= 3.7587 \text{ m}$$

$$\sin 20° = \frac{BP'}{4}$$

$$BP' = 4 \times \sin 20°$$

$$= 1.3680 \text{ m} \quad \text{--- M1: either length seen}$$

Height of A to the horizontal PQ

$$= (7 + 1.3680) \text{ m}$$

The required angle

$$\tan \angle APP' = \frac{8.3680}{3.7587} \quad \text{--- M1: appropriate use of trigo ratio to find the angle}$$

$$\angle APP' = 65.8° \quad (1 \text{ d.p.}) \quad \text{--- A1}$$

Alternative method

$$\sin 70° = \frac{AC}{7} \quad (\angle ABC = 70°)$$

$$AC = 6.57784 \text{ m}$$

$$PC = 4 + 2.39414 = 6.39414 \text{ m} \quad \text{--- M1: seen length of } PC$$

$$\tan \angle APC = \frac{AC}{PC} = \frac{6.57784}{6.39414}$$

$$\angle APC = \tan^{-1}\left(\frac{6.57784}{6.39414}\right) \quad \text{--- M1}$$

$$= 45.8113°$$

The required angle $= 45.8113° + 20° = 65.8° \quad \text{--- A1}$
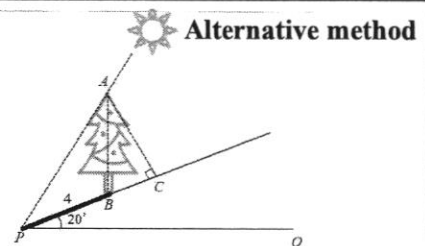
Presbyter

15

24 **(a)** $\xi = \{\text{integers } x : 1 \le x \le 12\}$

$P = \{\text{prime numbers}\}$

$Q = \{\text{multiples of 3}\}$

(i) Represent the above information on the Venn diagram shown in the answer space below.

Answer
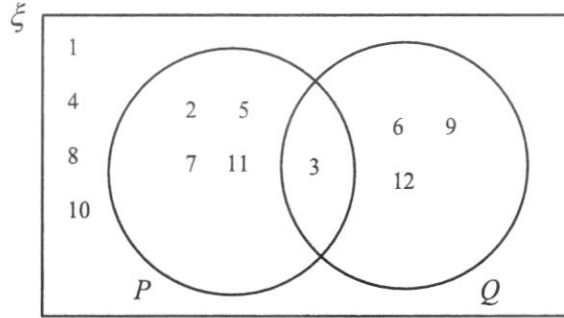
B1: every 6 correct values

B2: correct representation

(ii) List the elements in $(P' \cap Q') \cup (P \cap Q)$.

$(P' \cap Q') \cup (P \cap Q) = \{1, 3, 4, 8, 10\}$ --- B1

(iii) $R = \{x : x \text{ is a multiple of 6}\}$

Use set notation to describe the relationship between Q and R.

$R \subset Q$ --- B1

(b) On the Venn diagram, shade the region which represents the set $A \cap B'$.
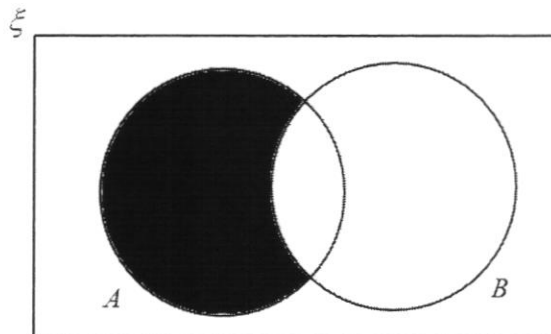
B1: correct shading
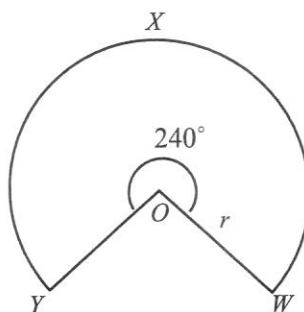
16

25 *OWXY* is a sector of a circle, centre *O*, of radius *r* cm and reflex angle 240°.

The sector *OWXY* has an area of 150π cm².

(a) Express 240° in terms of π radians.

$$240° = 240 \times \frac{\pi}{180} \text{ rad} \quad \text{--- M1}$$

$$= \frac{4}{3}\pi \text{ rad} \quad \text{--- A1}$$

(b) Show that $r = 15$.

Answer

$$\frac{1}{2}(r)^2 \frac{4\pi}{3} = 150\pi \qquad \text{--- M1: applied formula}$$

$$r = 15 \quad \text{--- AG1}$$

(c) The radii, *OW* and *OY*, are joined together to form a cone.

Find the base radius of the cone.

Answer

Method 1:
Arc length = Circumference of circular base

$$15\left(\frac{4}{3}\pi\right) = 2\pi x \quad \text{--- M1}$$

$$x = 10 \quad \text{--- A1}$$

Method 2:

$$\pi x l = 150\pi$$

$$x = \frac{150\pi}{15\pi} \quad \text{--- M1}$$

$$= 10 \quad \text{--- A1}$$

17

26
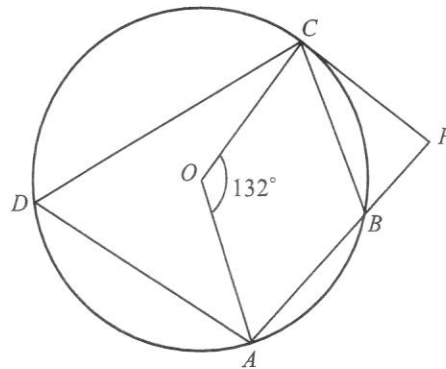
In the diagram above, O is the centre of the circle, such that angle $COA = 132°$.
PC is a tangent to the circle at C and PBA is a straight line.

By giving a reason for each step of your working, find

(a) $\angle CDA$,

$\angle CDA = 132° \div 2$

$= 66° (\angle$ at the centre $=$ twice $\angle$ at circumference) --- B1 reason, B1 answer

(b) $\angle CBP$.

$\angle CBA = 180° - 66°$

$= 114°$ ($\angle$s in opp. segment)

$\angle CBP = 180° - 114°$ (adj. $\angle$s on a st. line)

$= 66°$ --- B1

B1: $\angle$s in opp. segment

(c) If the radius of the circle is 3.55 cm, calculate the area of triangle AOC.

Area of $\triangle AOC = \dfrac{1}{2} \times 3.55 \times 3.55 \times \sin 132°$ --- M1: applied area of triangle formula

$= 4.68 \text{ cm}^2$ (3s.f.) --- A1

PRESBYTERIAN HIGH SCHOOL

MATHEMATICS
PAPER 2

4052/02

16 August 2023

Wednesday

2 hours 15 minutes

2023 SECONDARY FOUR EXPRESS
PRELIMINARY EXAMINATION

MARKING SCHEME

For Examiner's Use										
Qn	1	2	3	4	5	6	7	8	9	10	Marks Deducted
Marks										

Category	Accuracy		Symbols		Others
Question No.					

TOTAL MARKS

90

Setter: Mr Tan Lip Sing
Vetter: Mdm Cynthia Chua

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle ABC} = \frac{1}{2}ab \sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1	(a)	Rearrange the formula $c = \dfrac{d^2 + 5}{d^2 - 3}$ to make d the subject.
		Answer $d = $ .... [3]

		$c = \dfrac{d^2 + 5}{d^2 - 3}$	
		$c(d^2 - 3) = d^2 + 5$	M1
		$cd^2 - 3c = d^2 + 5$	
		$cd^2 - d^2 = 5 + 3c$	
		$d^2(c - 1) = 5 + 3c$	M1
		$d^2 = \dfrac{5 + 3c}{c - 1}$	
		$d = \pm\sqrt{\dfrac{3c + 5}{c - 1}}$	A1

	(b)	Write as a single fraction in its simplest form $\dfrac{3}{(x-2)^2} - \dfrac{1}{2-x}$.
		Answer .... [2]

		$\dfrac{3}{(x-2)^2} - \dfrac{1}{2-x}$	
		$= \dfrac{3}{(x-2)^2} + \dfrac{1}{x-2}$	M1
		$= \dfrac{3 + x - 2}{(x-2)^2}$	
		$= \dfrac{x+1}{(x-2)^2}$	A1

3

(c)	Solve these simultaneous equations.
	$5x + 3y = 14$
	$3x + 5y = 18$
	You must show your working.
	Answer $x =$ .... or $y =$ .... [3]

	$5x + 3y = 14$ ......(1)	
	$3x + 5y = 18$ ......(2)	
	$(1): y = \dfrac{14 - 5x}{3}$ ......(3)	M1
	Substitute (3) into (2):	
	$3x + 5\left(\dfrac{14 - 5x}{3}\right) = 18$	
	$9x + 70 - 25x = 54$	
	$x = 1$	A1
	$y = 3$	A1

(d)	Solve the equation $\dfrac{2x-1}{5x-6} = \dfrac{1}{2x-3}$.
	Answer $x =$ .... or $x =$ .... [3]

	$\dfrac{2x-1}{5x-6} = \dfrac{1}{2x-3}$	
	$(2x-1)(2x-3) = 5x - 6$	M1
	$4x^2 - 6x - 2x + 3 = 5x - 6$	
	$4x^2 - 13x + 9 = 0$	
	$(4x - 9)(x - 1) = 0$	M1 (Factorise)
	$x = 2\dfrac{1}{4}$ or $x = 1$	A1

4

2	(a)	Before departing London for Singapore, Peter bought 3000 Singapore dollars from the bank. The exchange rate between British pounds (£) and Singapore dollars ($) was £1 = $1.71. He also had to pay the bank an additional commission fee of 1.5% for the exchange of currency. Calculate the total amount of pounds, inclusive of commission, he paid the bank. Give your answer correct to the nearest pound. *Answer* £ .... [2]	
		Total amount before commission $= \dfrac{3000}{1.71} = £1754.385965$	M1
		Total amount inclusive of commission $= 1754.385965 \times 1.015$ $\approx £1781$	A1
	(b)	Peter bought a laptop while he was in Singapore. He paid $664.20 inclusive of the 8% GST (Goods & Services Tax) for the laptop after getting a discount of A% on the original price. The laptop's original price is $750 before GST.	
	(i)	Find the GST amount paid for the laptop. *Answer* .... [2]	
		$108\% = \$664.20$ $8\% = \$664.20 \times \dfrac{8}{108} = \$49.20$	M1
		GST amount $= \$49.20$	A1
	(ii)	Calculate the value of A. *Answer* $A =$ ....% [2]	
		Discounted price before GST $= 664.20 - 49.20 = \$615.00$ $A = \dfrac{750-615}{750} \times 100\%$ $A = \dfrac{135}{750} \times 100\% = 18\%$	M1 A1

	(c)		Mary invests $20 000 in an endowment plan that offers 4% per year compound interest. How much interest will she receive after 10 years? Give your answer correct to the nearest cent.	
			Answer $ .... [2	

		Total amount after 10 years $$= 20000\left(1+\frac{4}{100}\right)^{10}$$ $= \$29604.89$ Interest received $= 29604.89 - 20000$ $= \$9604.89$	M1 A1

	(d)		A map of a province has a scale of 1 : 500 000.	
		(i)	The length of an expressway on the map is 25 cm. Calculate the actual length, in kilometres, of the expressway. *Answer* .... km [1]	

		1 cm : 5 km 25 cm : 125 km Actual length = 125 km	B1

		(ii)	The area of a reservoir is 180 km². Calculate the area, in square centimetres, of the reservoir on the map. *Answer* .... cm² [2]	

		(1 cm)² : (5 km)² 1 cm² : 25 km² Area on map $= \frac{180}{25} = 7.2$ cm²	M1 A1

6

3 The variables x and y are connected by the equation $y = \dfrac{x^2}{5} + \dfrac{4}{x}$.

The table below shows some values of x and the corresponding values of y correct to 2 decimal places.

x	0.5	1	1.5	2	2.5	3	4	5
y	8.05	4.20	3.12	2.80	2.85	3.13	4.20	5.80

(a) On the grid provided, draw the graph of $y = \dfrac{x^2}{5} + \dfrac{4}{x}$ for $0.5 \le x \le 5$.

Plot the points given in the table and join them with a smooth curve. [3]

Plot all 8 points correctly.	B2 (6 or 7 points correct – B1)
Join all points with a smooth curve.	B1

(b) By drawing a tangent, find the gradient of the curve at $x = 3$.

Answer .... [2]

Draw the correct tangent line at $x = 3$. $Gradient = \dfrac{6 - 0.95}{7 - 0} \approx 0.721$ (Accept 0.7 to 0.8)	M1 A1

(c) **(i)** On the same grid, draw the line $y = 7 - \dfrac{1}{2}x$ for $0 \le x \le 5$. [1]

Draw correct line $y = 7 - \dfrac{1}{2}x$.	B1

(ii) Write down the x-coordinates of the points where this line intersects the curve.

Answer $x =$ .... or .... [2]

$x = 0.6$ or $x = 4.4$ $(0.55 - 0.65)$ $(4.35 - 4.45)$	B1, B1

7

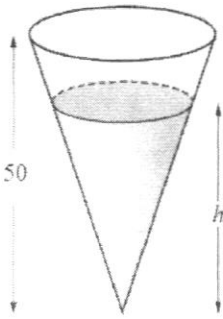|		(iii)	Find the equation, in the form $2x^3 + ax^2 + bx + c = 0$, which is satisfied by the values of x found in **(c)(ii)**.	
			Answer .... [2]	
			$\dfrac{x^2}{5} + \dfrac{4}{x} = 7 - \dfrac{x}{2}$ $2x^3 + 40 = 70x - 5x^2$ $2x^3 + 5x^2 - 70x + 40 = 0$	M1 A1
(d)			Use your graph to find the values of x in the range $0 \le x \le 5$ for which $0.2x^2 + \dfrac{4}{x} - 2 = 3$. *Answer* $x =$ .... or .... [2]	
			$0.2x^2 + \dfrac{4}{x} - 2 = 3$ $\dfrac{x^2}{5} + \dfrac{4}{x} = 5$ Draw the line $y = 5$, from the graph, $x = 0.8$ *or* $x = 4.55$ $(0.75 - 0.85)$ $(4.5 - 4.6)$	 B1, B1
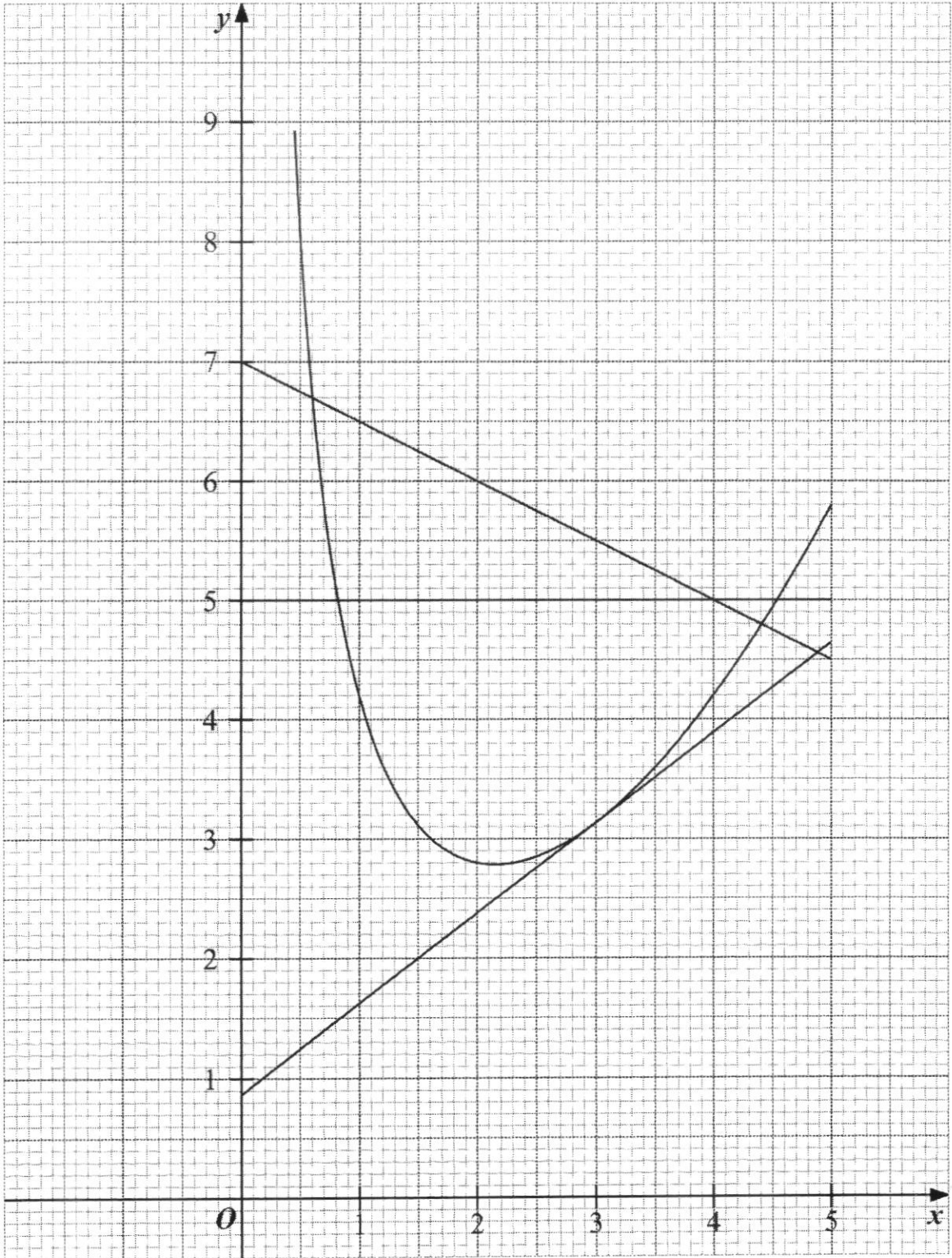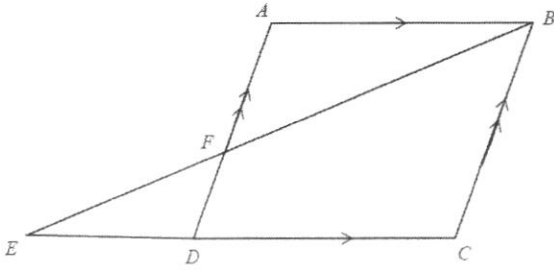
8

| 4 | (a) | The daily average temperature at Town *A* was recorded for 60 days. The cumulative frequency curve below shows the distribution of the temperatures. |

Cumulative frequency

Daily Average Temperature (°C)
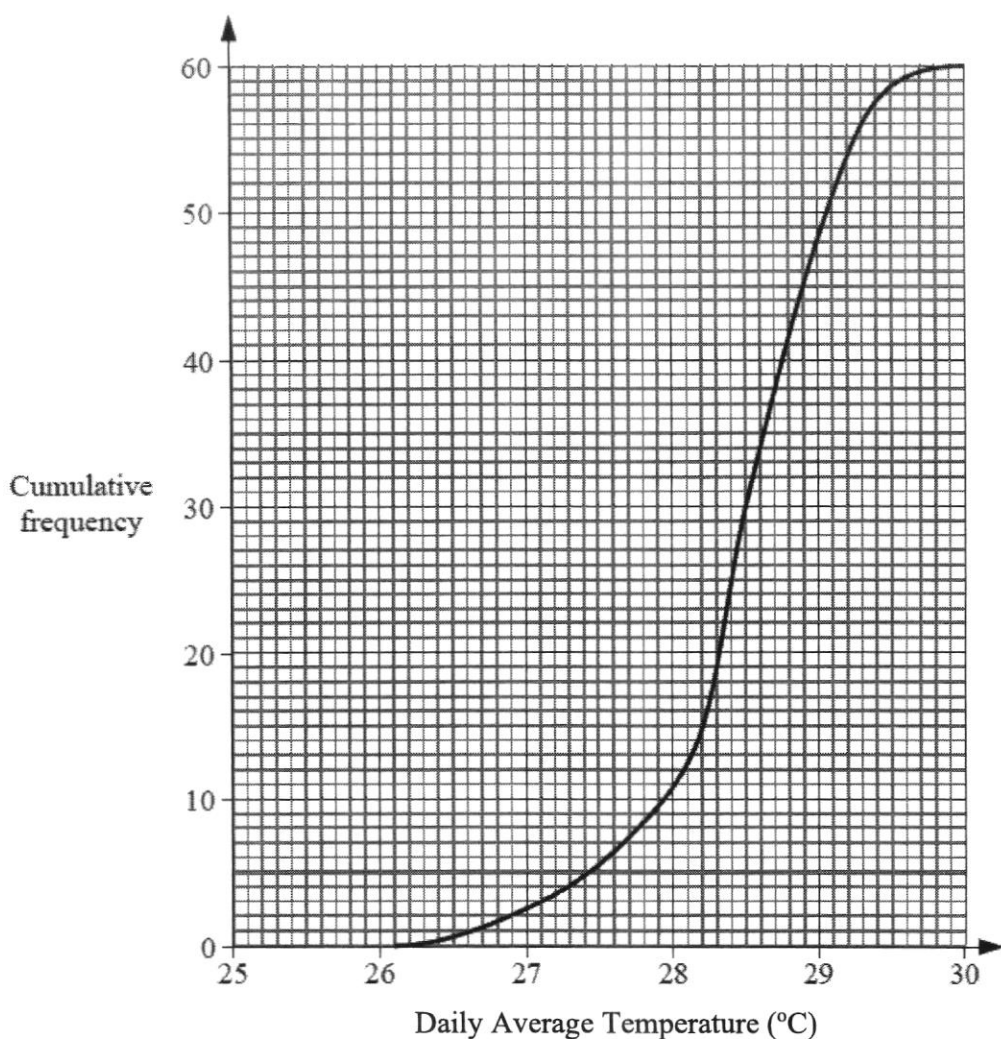
		(i)	Use the curve to estimate (a) the median temperature, *Answer* .... °C [1]
			28.5° B1
			(b) the interquartile range of the temperatures, *Answer* .... °C [2]
			28.9° − 28.2° = 0.7° M1, A1
			(c) the number of days that Town *A* had temperatures above 29°C. *Answer* .... days [1]
			60 − 48 = 12 *days* B1

		(ii)	The daily average temperature at Town B was recorded for the same period. The interquartile range of the temperatures at Town B is 1.5°C. Use this information to comment on one difference between the temperature at Town A and at Town B. .... [1]	
			The temperatures at Town B have a larger spread than the temperatures at Town A. OR The temperatures at Town B were less consistent than the temperatures at Town A.	B1
	(b)		Box A contains 6 red cards, 4 blue cards and 2 green cards. Box B contains 3 red cards and 5 blue cards. A card is drawn at random from Box A and put into Box B. Next, a card is drawn at random from Box B. Find, as a fraction in its simplest form, the probability that	
		(i)	two green cards are drawn, *Answer* .... [1]	
			$P = \dfrac{2}{9} \times \dfrac{1}{9} = \dfrac{1}{54}$	B1
		(ii)	neither of the cards is green, *Answer* .... [1]	
			$P = \dfrac{10}{12} \times 1 = \dfrac{5}{6}$	B1
		(iii)	the two cards are of the same colour. *Answer* .... [2]	
			$P = P(RR) + P(BB) + P(GG)$ $= \dfrac{6}{12} \times \dfrac{4}{9} + \dfrac{4}{12} \times \dfrac{6}{9} + \dfrac{1}{54}$ $= \dfrac{25}{54}$	M1 A1

5	(a)		The diagram shows a parallelogram $ABCD$ with CD produced to E. F is the point of intersection of BE and AD.	
		(i)	Show that triangle BAF and triangle EDF are similar. Give a reason for each statement you make. .... [2]	
			In $\triangle BAF$ and $\triangle EDF$, $\angle BAF = \angle EDF$ (Alternate angles, $AB // ED$) $\angle AFB = \angle DFE$ (Vertically opposite angles) $\therefore \triangle BAF$ and $\triangle EDF$ are similar. (AA Similarity Test)	M1 (at least one correct reason) A1 (with correct reason)
		(ii)	State another triangle that is similar to BAF and EDF. *Answer* Triangle .... [1]	
			Triangle ECB.	B1
		(iii)	The ratio $ED:DC = 2:3$. Find the ratio $BC:AF$. *Answer* .... : .... [1]	
			5 : 3	B1
		(iv)	Given that the area of triangle EDF is 9.5 cm², find the area of triangle BAF. *Answer* .... cm² [2]	
			$$\frac{Area\ of\ \triangle BAF}{Area\ of\ \triangle EDF} = \left(\frac{3}{2}\right)^2$$ $$Area\ of\ \triangle BAF = \frac{9}{4} \times 9.5 = 21.375\ \text{cm}^2$$	M1 A1

(b) The diagram below shows a cone of height 50 cm.

The volume of the liquid in the cone is $\frac{3}{4}$ of the volume of the cone.

Calculate the depth, h cm, of the liquid.

50

h

Answer .... cm [2]

$\dfrac{Volume\ of\ liquid}{Volume\ of\ cone} = \left(\dfrac{h}{50}\right)^3$	M1
$\dfrac{3}{4} = \dfrac{h^3}{125000}$ $h^3 = 93750$ $h \approx 45.4$ cm	A1

6	In the diagram below, P is a point on DC, such that $DC = 2DP$ and X is a point on BP such that $3BX = 2BP$. It is given that $\overrightarrow{AD} = 4\mathbf{a}$, $\overrightarrow{AB} = 3\mathbf{b}$, and $\overrightarrow{DP} = 2\mathbf{a} + 3\mathbf{b}$.	
	(a)	Express, as simply as possible, in terms of $\mathbf{a}$ and/or $\mathbf{b}$,
	(i)	$\overrightarrow{BP}$,
		Answer .... [1]

		$\overrightarrow{BP} = \overrightarrow{BA} + \overrightarrow{AD} + \overrightarrow{DP}$ $= -3\mathbf{b} + 4\mathbf{a} + 2\mathbf{a} + 3\mathbf{b}$ $= 6\mathbf{a}$	B1

	(ii)	$\overrightarrow{AX}$,
		Answer .... [2]

		$\overrightarrow{BX} = \dfrac{2}{3}\overrightarrow{BP} = 4\mathbf{a}$	M1
		$\overrightarrow{AX} = \overrightarrow{AB} + \overrightarrow{BX}$ $= 3\mathbf{b} + 4\mathbf{a}$	A1

	(iii)	$\overrightarrow{AC}$.
		Answer .... [2]

		$\overrightarrow{DC} = 2\overrightarrow{DP} = 4\mathbf{a} + 6\mathbf{b}$	M1
		$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$ $= 4\mathbf{a} + 4\mathbf{a} + 6\mathbf{b}$ $= 8\mathbf{a} + 6\mathbf{b}$ *or* $2(4\mathbf{a} + 3\mathbf{b})$	A1

	(b)	Explain whether the points A, X and C lie on the same straight line. .... [2]	
		$\overrightarrow{AC} = 2(4\mathbf{a} + 3\mathbf{b}) = 2\overrightarrow{AX}$ *AX* is parallel to *AC* and since *A* is a common point, *A, X* and *C* lie on the same straight line.	M1 A1
	(c)	Given that the area of triangle $BCP = 24\,\text{cm}^2$, find the area of triangle CXP. *Answer* ....cm^2 [2]	
		Area of $\Delta CXP = \dfrac{1}{3}$ *Area of* ΔBCP $\qquad = \dfrac{1}{3}(24)$ $\qquad = 8\ cm^2$	M1 A1

15

7	The diagram shows four towns A, B, C and D on a piece of horizontal land. $ABCD$ is a trapezium. $AB = 0.9$ km, $AD = 1.2$ km and angle $BAD = 150°$.	

	(a)	Calculate the distance between Town B and Town D.
		Answer .... km [3]

		$(BD)^2 = (1.2)^2 + (0.9)^2 - 2(1.2)(0.9)\cos 150°$ $\qquad = 4.12061$ $BD = 2.0299 \approx 2.03$ km	M2 (all correct) M1 (2 out of 3 correct) A1

	(b)	Calculate the value of angle BDC.
		Answer ....° [2]

		$\dfrac{\sin \angle BDA}{0.9} = \dfrac{\sin 150°}{2.0299}$ $\sin \angle BDA = 0.22168$ $\angle BDA = 12.808° \approx 12.8°$ Since $ABCD$ is a trapezium, $\angle ADC = 180° - 150° = 30°$ $\angle BDC = 30° - 12.808°$ $\qquad = 17.192° \approx 17.2°$	M1 A1

	(c)	Calculate the bearing of D from B.
		Answer ....° [2]

		$\angle ABD = \angle BDC = 17.192°$ Bearing of D from B $= 270 - 17.192°$ $\approx 252.8°$	M1 A1

16

	(d)	A tower is standing at Town B. The greatest angle of elevation of the top of the tower, T, from the path CD is 18°. Find the height of the tower in metres. *Answer* .... m [3]	
		Let the shortest distance from B to CD be d km. $\sin \angle BDC = \dfrac{d}{2.0299}$ $d = 2.0299 \sin 17.192°$ $\quad = 0.599987 \; km$ Let h be the height of the tower. $\tan 18° = \dfrac{h}{0.599987}$ $h = 0.599987 \tan 18°$ $\quad = 0.194947 \; km$ $\quad \approx 195 \; m$	M1 M1 A1

8	James bought some essential oil for $720 at x per litre.	
(a)	Write an expression, in terms of x, for the number of litres of essential oil he bought. *Answer* .... litres [1]	
	$\dfrac{720}{x}$ litres	B1
(b)	Due to a leakage in the container, 5 litres of essential oil was lost. James sold the remaining essential oil at $2 per litre more than what he had paid for. Write an expression, in terms of x, for the amount of money he received from the sale of essential oil. *Answer* $ .... [1]	
	$\$\left(\dfrac{720}{x}-5\right)(x+2)$	B1
(c)	Given that James made a profit of $100, write down an equation in x to represent this information and show that it reduces to $x^2+22x-288=0$. [3]	
	$\left(\dfrac{720}{x}-5\right)(x+2)-720=100$	M1
	$720+\dfrac{1440}{x}-5x-10-720-100=0$	M1
	$\dfrac{1440}{x}-5x-110=0$	
	$-5x^2-110x+1440=0$	
	$x^2+22x-288=0$	AG1
(d)	Solve the equation $x^2+22x-288=0$. *Answer* $x=$ .... or $x=$ .... [3]	
	$x=\dfrac{-22\pm\sqrt{22^2-4(1)(-288)}}{2(1)}$	M1
	$=\dfrac{-22\pm\sqrt{1636}}{2}$	
	$=9.2237 \quad or \quad -31.2237$	
	$\approx 9.22 \ or \ -31.2$	A1, A1

18

	(e)	Find, to the nearest litre, the amount of essential oil James sold.
		Answer .... litres [2]

		$\dfrac{720}{9.2237} - 5 = 73.09 \approx 73$ litres	M1, A1

9	The table below shows the Income Tax Rate in Singapore.

Table 1: Income Tax Rate

Chargeable Income	Rate (%)	Gross Tax Payable ($)
On the first $120,000	-	7,950
On the next $40,000	15	6,000
On the first $160,000	-	13,950
On the next $40,000	18	7,200
On the first $200,000	-	21,150
On the next $40,000	19	7,600
On the first $240,000	-	28,750
On the next $40,000	19.5	7,800

(a) Henry enjoyed a total tax relief of $15 000 and paid $14 130 of income tax for the year of assessment 2022. Calculate his annual income in 2022.

[Annual income = Chargeable income + tax relief]

Answer $ .... [2]

Let the chargeable income be $(160\,000 + A)$ $14130 = 13950 + (A \times 18\%)$ $A = \$1000$	M1	
Annual income $= \$(160000 + 1000 + 15000)$ $\qquad = \$176\,000$	A1	

20

Henry recently got a pay rise and his income is now $15 500 per month.
He is keen to buy a private condominium which is priced at $1 200 000. To afford this condominium, he needs to apply for a bank loan of $800 000.

(b) The maximum duration of a housing loan for private properties is up to 35 years or 65 years of age, whichever is lower.
Given that Henry is 45 years old, find the maximum number of years Henry can loan from the bank.

Answer .... years [1]

Maximum number is $65 - 45 = 20$ years	B1

(c) Henry decides to apply for a loan for the maximum duration allowed for his age. The loan from the bank is subject to a simple interest of 3.5% per annum.

The government introduced the Total Debt Servicing Ratio (TDSR) to prevent individuals from over-borrowing.

Information about TDSR

- Total Debt Servicing Ratio = $\dfrac{\text{Total monthly debt repayment}}{\text{Monthly income}}$

- Total monthly debt repayment includes repayments for car loans, personal loans, credit card expenditure, home loans and other loans.

- The maximum TDSR allowed is 55%.

His current monthly debt repayment is shown in the table below:

Type	Amount ($)
Car loan	1000
Credit card Expenditure	1000
Personal loans	1000

By considering the TDSR ratio, will the bank approve his loan request? Justify your answer and show your calculations clearly.

.... [7]

		Interest on housing loan for 1 year $= \$800\,000 \times 0.035 = \$28\,000$	M1
		Interest on housing loan for 20 years $= \$28\,000 \times 20 = \$560\,000$	M1
		Total debt (loan + interest) $= \$(800\,000 + 560\,000) = \$1360\,000$	M1
		Monthly debt repayment for housing loan over 20 years $= \$1360\,000 \div (12 \times 20) = \$5666.67$	M1
		Maximum debt allowable per month under TDSR $= \$15\,500 \times 0.55 = \$8525$	M1
		Henry's total monthly debt $= \$(5666.67 + 3000) = \$8666.67$	M1
		Since Henry's monthly debt of \$8666.67 has exceeded the maximum debt allowable per month (\$8525) under the TDSR, the bank will not approve his loan.	A1