Name	Index Number	Class

WOODGROVE SECONDARY SCHOOL

A COMMUNITY OF FUTURE-READY LEARNERS AND THOUGHTFUL LEADERS

O LEVEL PRELIMINARY EXAMINATION 2023

LEVEL & STREAM	**:** SECONDARY 4 EXPRESS/ 5 NORMAL ACADEMIC
SUBJECT (CODE)	**:** ADDITIONAL MATHEMATICS (4049)
PAPER NO	**:** 01
DATE (DAY)	**:** 11 SEPTEMBER 2023 (MONDAY)
DURATION	**:** 2 HOURS 15 MINUTES

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
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Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks in this paper is 90.

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/ 90

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Setter : Ms Nicole Ng

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

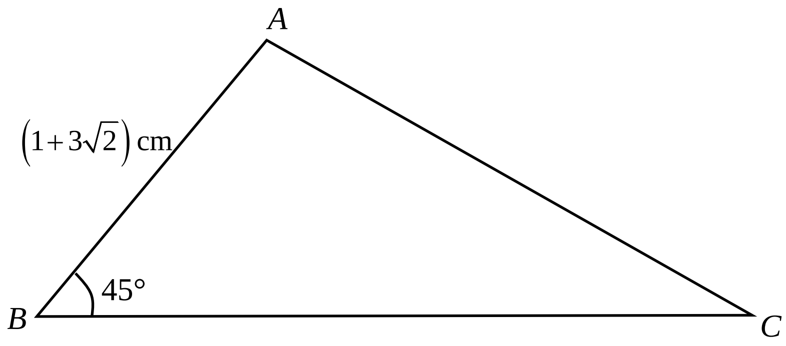
Formulae for Δ *ABC*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Triangle *ABC* is such that the length of side *AB* is $\left(1+3\sqrt{2}\right)$ cm, angle *ABC* is 45° and its area is $\left(7+4\sqrt{2}\right)$ cm^2. Find, without using a calculator, the exact length of *BC*, in cm. Leave your answer in the form of $\left(a+b\sqrt{2}\right)$, where *a* and *b* are integers. [4]



2 Given that $4^x \times 6^{2x+3} = 24^{2+x}$, find the value of 6^x without using a calculator. [4]

3 When a polynomial $\mathrm{f}(x)$ is divided by $(x+1)$ and $(x+2)$, the remainders are 3 and 5 respectively. Find the remainder when $\mathrm{f}(x)$ is divided by $(x+1)(x+2)$. [4]

4 Given that $\int_{-1}^{2} \mathrm{f}(x)\,\mathrm{d}x = \int_{2}^{4} \mathrm{f}(x)\,\mathrm{d}x = 6$, find

(a) $\int_{-1}^{4} 2\mathrm{f}(x)\,\mathrm{d}x + \int_{4}^{2} \mathrm{f}(x)\,\mathrm{d}x$, [2]

(b) the value of k for which $\int_{-1}^{2} \left[\mathrm{f}(x) + kx\right] \mathrm{d}x = 9$. [3]

5 **(a)** Find the $\frac{1}{x}$ term in the expansion of $\left(x^2+\frac{2}{x}\right)^{10}$. [3]

(b) Hence, find the constant term in the expansion of $(1+3x)\left(x^2+\frac{2}{x}\right)^{10}$. [2]

6 A spherical balloon expands at a constant rate of 8 cm^3/s. The balloon is initially empty.

(a) Find the rate of increase of its radius when the radius is 2.5 cm, leaving your answer in terms of π.

[The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.] [3]

(b) When the radius is beyond 5 cm, besides the expansion, air begins to leak out from the balloon at a rate of 2 cm^3/s. Find the rate of change of the radius when it is 8 cm. [2]

7 Given that $\sin x = \frac{5}{13}$ and x is obtuse, find the exact value of the following.

(a) $\sec(-x)$ [3]

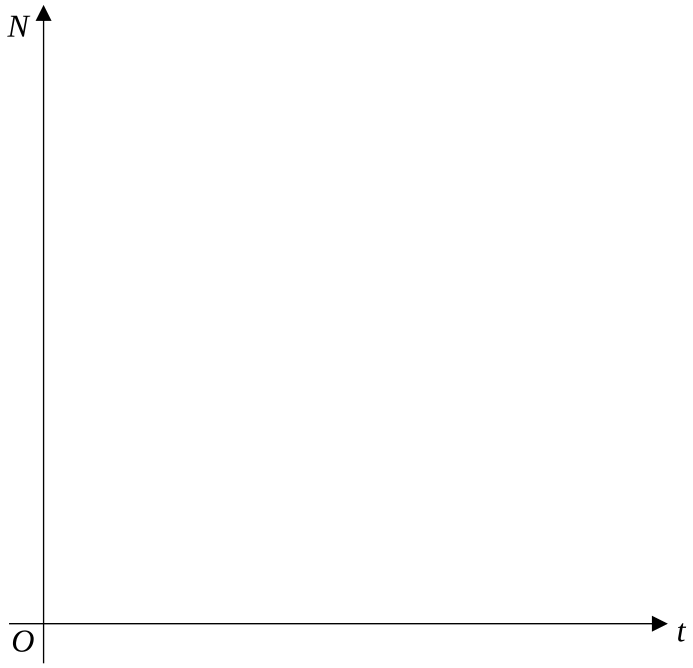
(b) $\cos\frac{x}{2}$ [3]

8 The number of ants, N, in a colony after t days can be modelled by $N = 1200e^{at}$, where a is a constant. There are 10 000 ants after 6 days.

(a) Find the initial number of ants in the colony. [1]

(b) How many ants are there after 15 days? Give your answer correct to 2 significant figures. [3]

(c) Sketch the graph of $N = 1200e^{at}$ for the first 15 days. [2]

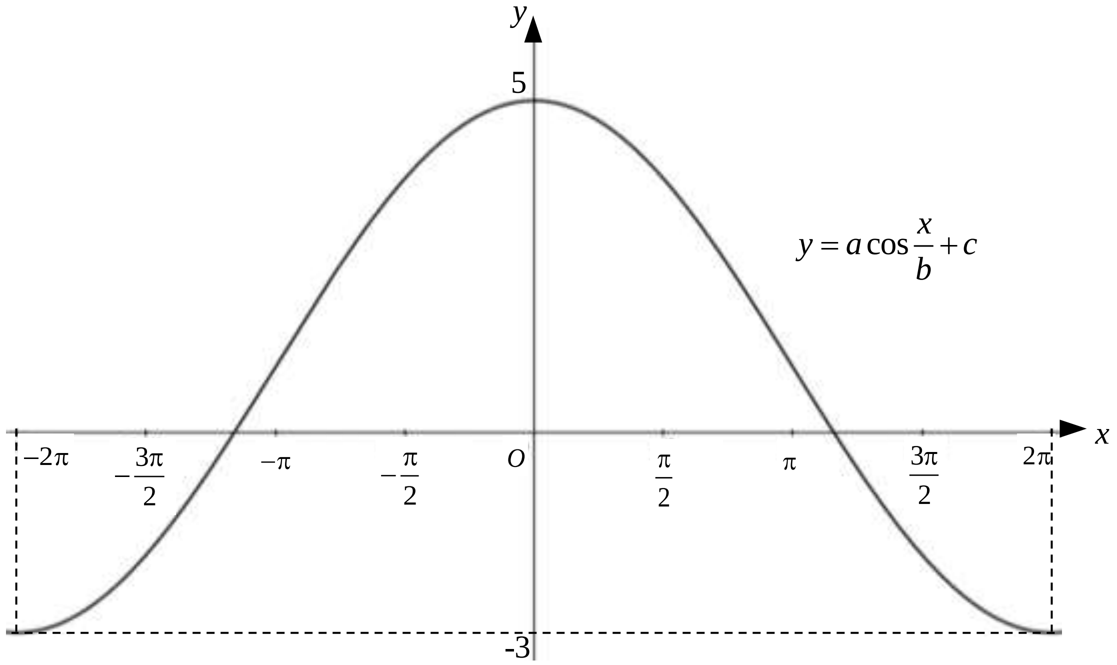


9 **(a)** Find the range of values of m for which the function $y = x^2 - 4mx + 3 - m$ is always positive for all real values of x. [3]

(b) Show that the line $y = 4x + p$ intersects the curve $y = px^2 - 2p - 6$ for all real values of x, where p is positive. [4]

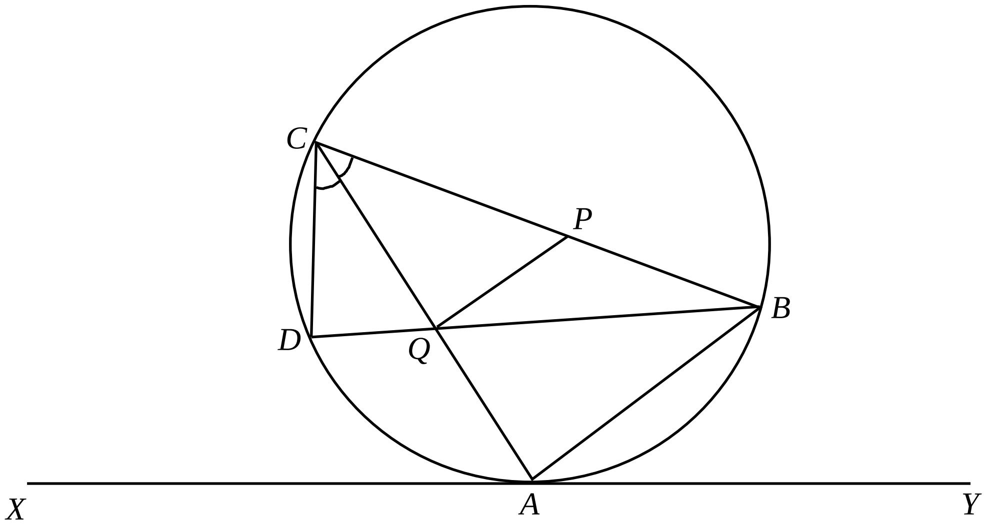
10 **(a)** State the principal value of $\tan^{-1}\left(-\sqrt{3}\right)$ in degrees. [1]

(b) The diagram shows a sketch of the graph $y = a\cos\frac{x}{b} + c$, where a, b and c are integers. Find the values of a, b and c. [3]



(c) Given that $y = 8\cos^2 x - 2\sin^2 x$, express y in the form of $p\cos 2x + q$, stating the value of each of the integers p and q. Explain why y will never reach 10. [4]

11 The diagram below shows a circle with points A, B, C and D at its circumference where XY is a tangent to the circle at point A. P and Q are the midpoints of BC and AC respectively. BQD is a straight line and $\angle QCD = \angle QCP$.



(a) Prove that $\angle BAY = \angle QCD$. [2]

(b) **(i)** Show that ΔQCP is similar to ΔDCQ. [4]

(b) (ii) Show that $2QC \times DQ = AB \times DC$. [2]

12 It is given that $y = \dfrac{2x^2 + 3}{x}$, $x \neq 0$.

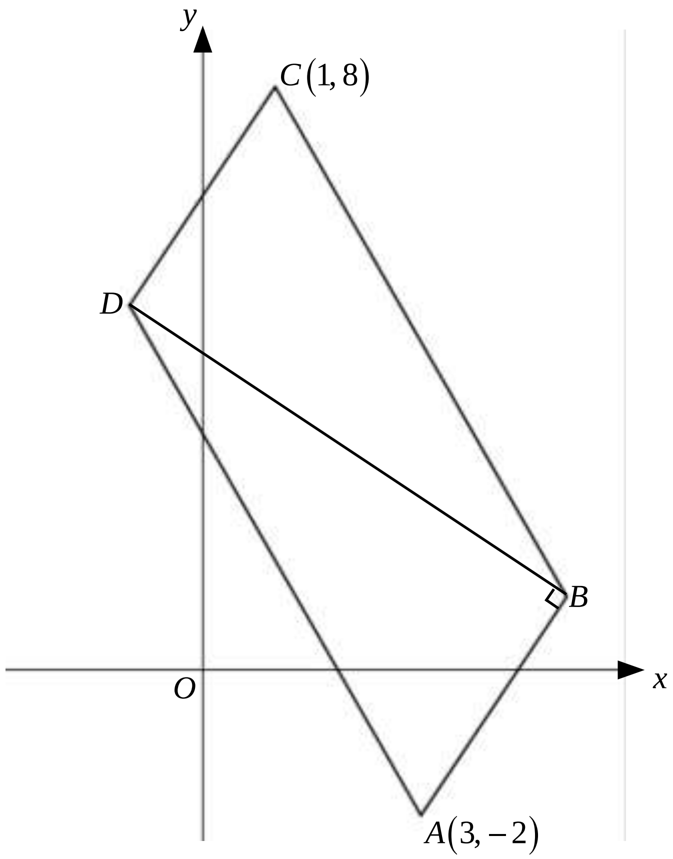
(a) Prove that $x^2 \dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} + x \dfrac{\mathrm{d}y}{\mathrm{d}x} = y$. [4]

(b) Find, in exact values, the x-coordinates of the turning points of y. [2]

(c) Determine the nature of each of the turning points. [2]

13 **Solutions to this question by accurate drawing will not be accepted.**

The parallelogram *ABCD* is such that the points *A* and *C* are $(3, -2)$ and $(1, 8)$ respectively. The line *BD* is parallel to the line $2x + 3y = 4$ and is perpendicular to *AB*.



(a) Show that the equation of *BD* is $2x + 3y = 13$. [4]

(b) Calculate the coordinates of *B*. [4]

(c) Calculate the coordinates of *D*. [2]

14 A particle starts from rest at a fixed point *O* and moves in a straight line such that its velocity v ms^{-1} is given by $v = 4t - \frac{3}{2}t^2$, where t is the time in seconds after leaving *O*.

Calculate

(a) the velocity of the particle when its acceleration is zero, [3]

(b) the time when the particle is instantaneously at rest again, [2]

(c) the total distance travelled by the particle when it returns to *O*. [5]

END OF PAPER

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MARKING SCHEME

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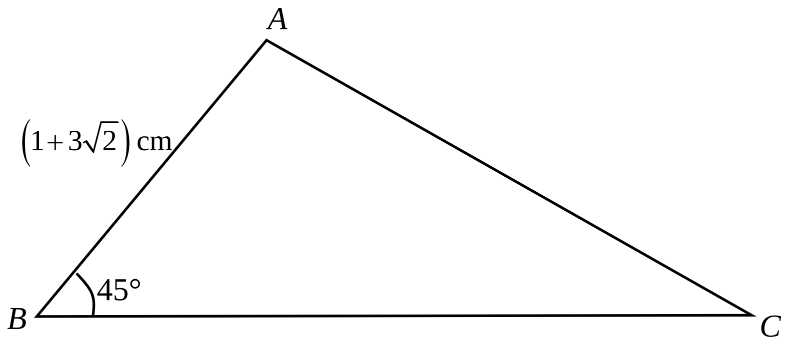


$7+4\sqrt{2}=\frac{1}{2}\left(1+3\sqrt{2}\right)\left(BC\right)\sin 45°$ $\quad M1$

$7+4\sqrt{2}=\frac{1}{2}\left(1+3\sqrt{2}\right)\left(BC\right)\left(\frac{1}{\sqrt{2}}\right)$ $\quad M1$(for sin45°)

$BC=\frac{2\sqrt{2}\left(7+4\sqrt{2}\right)}{1+3\sqrt{2}}\times\frac{1-3\sqrt{2}}{1-3\sqrt{2}}$ $\quad M1$

$=\frac{\left(14\sqrt{2}+16\right)\times\left(1-3\sqrt{2}\right)}{-17}$

$=\frac{14\sqrt{2}-84+16-48\sqrt{2}}{-17}$

$=\frac{-34\sqrt{2}-68}{-17}$

$=\left(4+2\sqrt{2}\right)$cm $\quad A1$

2 Given that $4^x \times 6^{2x+3} = 24^{2+x}$, find the value of 6^x without using a calculator. [4]

$4^x \times 6^{2x+3} = 24^{2+x}$

$2^{2x} \times 2^{2x+3} \times 3^{2x+3} = 8^{2+x} \times 3^{2+x}$

$2^{4x+3} \times 3^{2x+3} = 2^{6+3x} \times 3^{2+x}$ $M1$

$\dfrac{3^{2x+3}}{3^{2+x}} = \dfrac{2^{6+3x}}{2^{4x+3}}$

$3^{x+1} = 2^{-x+3}$ $M1$

$3^x \times 3^1 = 2^{-x} \times 2^3$

$\dfrac{3^x}{2^{-x}} = \dfrac{2^3}{3}$ $M1$

$6^x = \dfrac{8}{3}$ $A1$

3 When a polynomial $f(x)$ is divided by $(x+1)$ and $(x+2)$, the remainders are 3 and 5 respectively. Find the remainder when $f(x)$ is divided by $(x+1)(x+2)$. [4]

$f(x) = (x+1)(x+2)Q(x) + ax + b$

$f(-1) = 3$

$3 = -a + b$.........................(1) $M1$

$f(-2) = 5$

$5 = -2a + b$.......................(2) $M1$

$(1)-(2):$ $M1$

$-2 = a$

$b = 1$

$\therefore$ remainder = $-2x+1$. $A1$

4 Given that $\int_{-1}^{2} \mathrm{f}(x)\,\mathrm{d}x = \int_{2}^{4} \mathrm{f}(x)\,\mathrm{d}x = 6$, find

(a) $\int_{-1}^{4} 2\mathrm{f}(x)\,\mathrm{d}x + \int_{4}^{2} \mathrm{f}(x)\,\mathrm{d}x$, [2]

$\int_{-1}^{4} 2\mathrm{f}(x)\,\mathrm{d}x + \int_{4}^{2} \mathrm{f}(x)\,\mathrm{d}x$

$= 2\left[\int_{-1}^{2} \mathrm{f}(x)\,\mathrm{d}x + \int_{2}^{4} \mathrm{f}(x)\,\mathrm{d}x\right] - \int_{2}^{4} \mathrm{f}(x)\,\mathrm{d}x$ $\quad M1$

$= 2(6+6) - 6$

$= 18$ $\quad A1$

(b) the value of k for which $\int_{-1}^{2} [\mathrm{f}(x) + kx]\,\mathrm{d}x = 9$. [3]

$\int_{-1}^{2} [\mathrm{f}(x) + kx]\,\mathrm{d}x = 9$

$\int_{-1}^{2} \mathrm{f}(x)\,\mathrm{d}x + \int_{-1}^{2} kx\,\mathrm{d}x = 9$

$6 + \left[\frac{kx^2}{2}\right]_{-1}^{2} = 9$ $\quad M1$

$\left[\frac{kx^2}{2}\right]_{-1}^{2} = 3$

$\left[\frac{k(2)^2}{2}\right] - \left[\frac{k(-1)^2}{2}\right] = 3$ $\quad M1$

$2k - \frac{k}{2} = 3$

$k = 2$ $\quad A1$

5 **(a)** Find the $\frac{1}{x}$ term in the expansion of $\left(x^2+\frac{2}{x}\right)^{10}$. [3]

$$T_{r+1}=\binom{10}{r}\left(x^2\right)^{10-r}\left(2x^{-1}\right)^r \qquad M1$$

$$=\binom{10}{r}2^r x^{20-3r}$$

$$20-3r=-1 \qquad M1$$

$$r=7$$

$$T_8=\binom{10}{7}2^7 x^{-1}=\frac{15360}{x} \qquad A1$$

(b) Hence, find the constant term in the expansion of $(1+3x)\left(x^2+\frac{2}{x}\right)^{10}$. [2]

$$(1+3x)\left(x^2+\frac{2}{x}\right)^{10}=(1)(0)+(3x)\left(\frac{15360}{x}\right) \qquad M1$$

$$=46080 \qquad A1$$

6 A spherical balloon expands at a constant rate of 8 cm^3/s. The balloon is initially empty.

(a) Find the rate of increase of its radius when the radius is 2.5 cm, leaving your answer in terms of π.

[The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.] [3]

$$\frac{dV}{dr} = 4\pi r^2 \qquad M1$$

$$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt} \qquad OR \qquad \frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

$$= \frac{1}{4\pi(2.5)^2} \times 8 \quad M1 \qquad\qquad 8 = 4\pi(2.5)^2 \times \frac{dr}{dt} \quad M1$$

$$= \frac{8}{25\pi} \text{ cm/s} \qquad A1$$

(b) When the radius is beyond 5 cm, besides the expansion, air begins to leak out from the balloon at a rate of 2 cm^3/s. Find the rate of change of the radius when it is 8 cm. [2]

$$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt} \qquad OR \qquad \frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$$

$$= \frac{1}{4\pi(8)^2} \times 6 \qquad M1\left(\text{for } \frac{dV}{dt} = 6\right) \qquad 6 = 4\pi(8)^2 \times \frac{dr}{dt}$$

$$= \frac{3}{128\pi} \text{ cm/s} \qquad A1$$

Accept 0.00746 m (3 s.f)

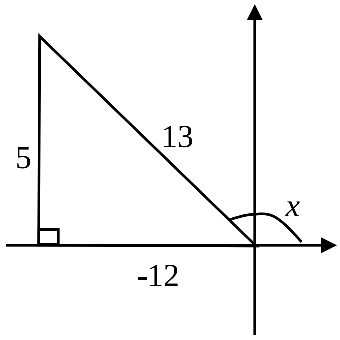
7 Given that $\sin x = \frac{5}{13}$ and x is obtuse, find the exact value of the following.

(a) $\sec(-x)$ [3]

$\sec(-x) = \frac{1}{\cos(-x)}$ $M1$

$= \frac{1}{\cos x}$ $M1$

$= -\frac{13}{12}$ $A1$



(b) $\cos \frac{x}{2}$ [3]

$\cos x = -\frac{12}{13}$

$-\frac{12}{13} = 2\cos^2 \frac{x}{2} - 1$ $M1$

$\cos^2 \frac{x}{2} = \frac{1}{26}$ $M1$

$\cos \frac{x}{2} = \frac{\sqrt{26}}{26} \left(\text{accept } \frac{1}{\sqrt{26}}\right)$ *or* $-\frac{\sqrt{26}}{26} (rej)$ $A1$

8 The number of ants, N, in a colony after t days can be modelled by $N = 1200e^{at}$, where a is a constant. There are 10 000 ants after 6 days.

(a) Find the initial number of ants in the colony. [1]

$N = 1200e^{a(0)} = 1200$ $\quad B1$

(b) How many ants are there after 15 days? Give your answer correct to 2 significant figures. [3]

$10000 = 1200e^{6a}$ $\quad M1$

$e^{6a} = \dfrac{10000}{1200}$

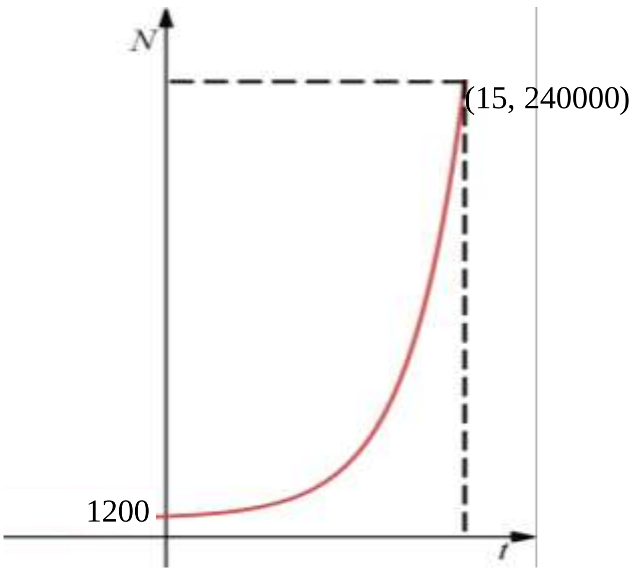
$6a = \ln\dfrac{10000}{1200}$ $\quad M1$

$a = 0.353377$

$N = 1200e^{(0.353377)(15)}$

$= 240000$ $\quad A1$

(c) Sketch the graph of $N = 1200e^{at}$ for the first 15 days. [2]



B1 – for the shape of the graph

B1 – for the y-intercept at 1200 and the point at t = 15 days.

9 **(a)** Find the range of values of m for which the function $y = x^2 - 4mx + 3 - m$ is always positive for all real values of x. [3]

$b^2 - 4ac = (-4m)^2 - 4(1)(3-m)$ $M1$

$= 16m^2 + 4m - 12$

$16m^2 + 4m - 12 < 0$ $M1$

$4m^2 + m - 3 < 0$

$(4m-3)(m+1) < 0$

$-1 < m < \frac{3}{4}$ $A1$

(b) Show that the line $y = 4x + p$ intersects the curve $y = px^2 - 2p - 6$ for all real values of x, where p is positive. [4]

$px^2 - 2p - 6 = 4x + p$ $M1$

$px^2 - 4x - 3p - 6 = 0$

$b^2 - 4ac = (-4)^2 - 4(p)(-3p-6)$ $M1$

$= 12p^2 + 24p + 16$

Method 1

For $12p^2 + 24p + 16$,

$b^2 - 4ac = (24)^2 - 4(12)(16)$

$= -192 < 0$ $M1$

$\therefore 12p^2 + 24p + 16 > 0$

$\therefore$ will intersect. $A1$

Method 2

$b^2 - 4ac = 12(p^2 + 2p) + 16$

$= 12(p+1)^2 - 12(1)^2 + 16$

$= 12(p+1)^2 + 4$ $M1$

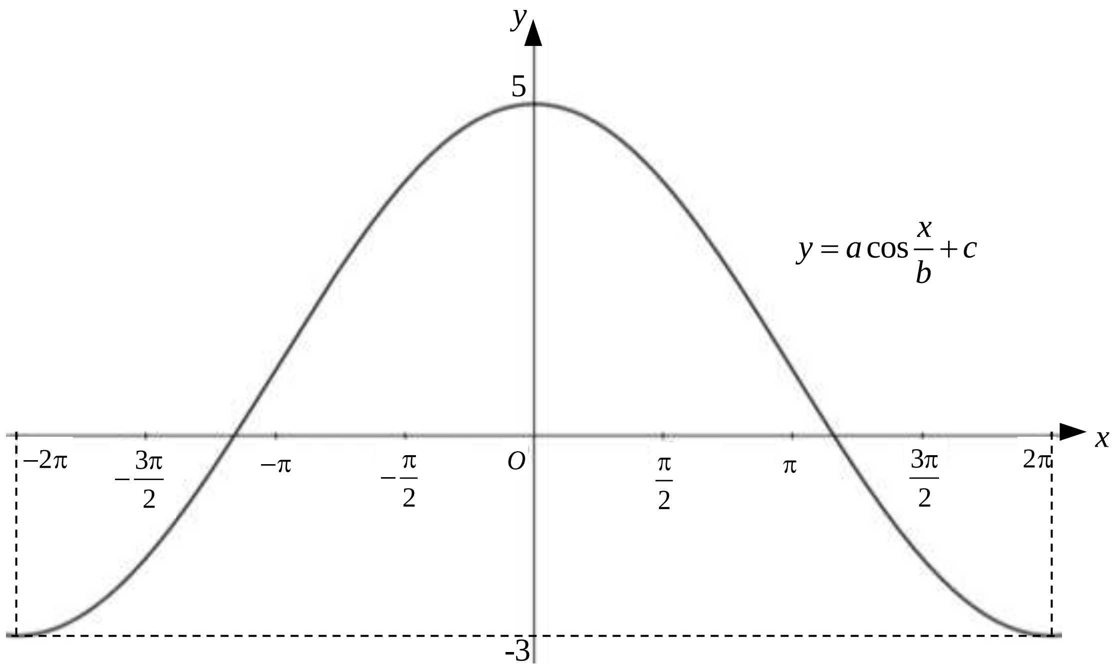
min value = 4>0, $\therefore b^2 - 4ac > 0$

$\therefore$ will intersect. $A1$

10 **(a)** State the principal value of $\tan^{-1}\left(-\sqrt{3}\right)$ in degrees. [1]

$-60°$ $B1$

(b) The diagram shows a sketch of the graph $y = a\cos\dfrac{x}{b} + c$, where a, b and c are integers. Find the values of a, b and c. [3]



$a = 4, b = 2, c = 1$ $B3$

(c) Given that $y = 8\cos^2 x - 2\sin^2 x$, express y in the form of $p\cos 2x + q$, stating the value of each of the integers p and q. Explain why y will never reach 10. [4]

$y = 8\cos^2 x - 2\sin^2 x$

$= 8\cos^2 x - 2\left(1 - \cos^2 x\right)$ $\quad M1$

$= 10\cos^2 x - 2$

$= 5\left(2\cos^2 x - 1 + 1\right) - 2$

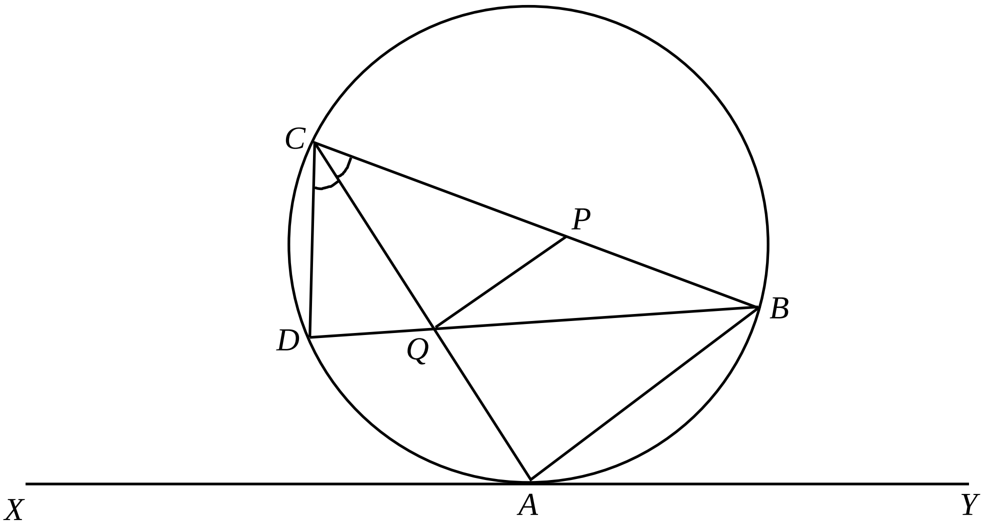
$= 5\left(\cos 2x + 1\right) - 2$ $\quad M1$

$= 5\cos 2x + 3$

$p = 5, q = 3$ $\quad A1$

Max value of $y = 5 + 3 = 8 < 10$ $\quad B1$

11 The diagram below shows a circle with points A, B, C and D at its circumference where XY is a tangent to the circle at point A. P and Q are the midpoints of BC and AC respectively. BQD is a straight line and $\angle QCD = \angle QCP$.



(a) Prove that $\angle BAY = \angle QCD$. [2]

$\angle BAY = \angle QCP$ (angles in alternate segments or tangent chord thm) $M1$
$\angle QCP = \angle QCD$ (given)
$\therefore \angle BAY = \angle QCD$ (shown) $A1$

(b) **(i)** Show that ΔQCP is similar to ΔDCQ. [4]

In ΔQCP and ΔDCQ,
$\angle QCP = \angle DCQ$ (given)
$QP // AB$ (Midpoint Thm) $M1$
$\angle CQP = \angle CAB$ (corresponding angles) $M1$
$\angle CAB = \angle CDQ$ (angles in same segment) $M1$
$\therefore \angle CQP = \angle CDQ$
$\therefore$ ΔQCP and ΔDCQ and similar.(AA test) $A1$

(b) **(ii)** Show that $2QC \times DQ = AB \times DC$. [2]

From **(bi),**

$$\frac{QC}{DC} = \frac{QP}{DQ} \qquad M1$$

$$QC \times DQ = QP \times DC$$

$$QC \times DQ = \frac{1}{2} AB \times DC \text{ (Midpt Thm)}$$

$$\therefore 2QC \times DQ = AB \times DC \qquad A1$$

12 It is given that $y = \dfrac{2x^2 + 3}{x}$, $x \neq 0$.

(a) Prove that $x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} = y$. [4]

$$y = \frac{2x^2+3}{x} = 2x + 3x^{-1}$$

$$\frac{dy}{dx} = 2 - 3x^{-2} \qquad M1$$

$$\frac{d^2y}{dx^2} = 6x^{-3} \qquad M1$$

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = x^2\left(\frac{6}{x^3}\right) + x\left(2 - \frac{3}{x^2}\right) \qquad M1$$

Accept e.c.f

$$= \frac{6}{x} + 2x - \frac{3}{x}$$

$$= \frac{3}{x} + 2x$$

$$= \frac{2x^2+3}{x} = y \qquad A1$$

(b) Find, in exact values, the x-coordinates of the turning points of y. [2]

$\frac{dy}{dx} = 0$

$2 - \frac{3}{x^2} = 0$ $M1$

$x^2 = \frac{3}{2}$

$x = \pm\sqrt{\frac{3}{2}}$ OR $\pm\frac{\sqrt{6}}{2}$ $A1$

(c) Determine the nature of each of the turning points. [2]

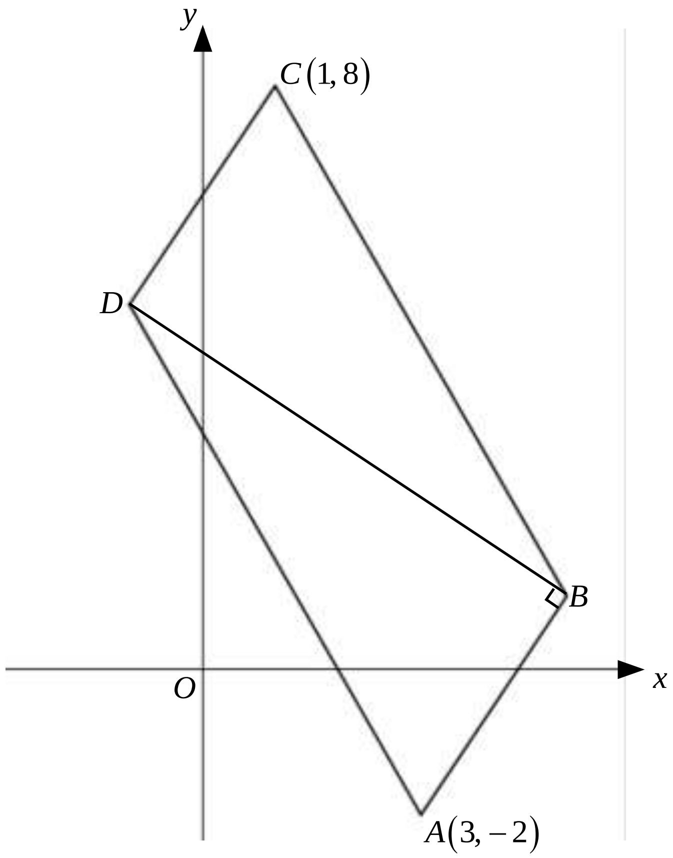
For $x = \frac{\sqrt{6}}{2}$, $\frac{d^2y}{dx^2} > 0, \therefore$ min. $B1$

For $x = -\frac{\sqrt{6}}{2}$, $\frac{d^2y}{dx^2} < 0, \therefore$ max. $B1$

Accept e.c.f (full marks awarded if x values were wrong in previous parts.

13 Solutions to this question by accurate drawing will not be accepted.

The parallelogram *ABCD* is such that the points *A* and *C* are $(3,-2)$ and $(1,8)$ respectively. The line *BD* is parallel to the line $2x+3y=4$ and is perpendicular to *AB*.



(a) Show that the equation of *BD* is $2x+3y=13$. [4]

$m_{BD}=-\frac{2}{3}$ $B1$

Midpoint of $BD=(2,3)$ $M1$

$y=-\frac{2}{3}x+c$ $M1$ *or* $\frac{y-3}{x-2}=-\frac{2}{3}$

$3=-\frac{2}{3}(2)+c$

$c=\frac{13}{3}$

$y=-\frac{2}{3}x+\frac{13}{3}$

$2x+3y=13$ (shown) $A1$

(b) Calculate the coordinates of *B*. [4]

Find the equation of *AB*,

$m_{AB} = \frac{3}{2}$ $B1$

$y = \frac{3}{2}x + c$ *OR* $\frac{y+2}{x-3} = \frac{3}{2}$

$-2 = \frac{3}{2}(3) + c$

$c = -\frac{13}{2}$

$y = \frac{3}{2}x - \frac{13}{2}$ $A1$ *OR* $2y = 3x - 13$

$y = -\frac{2}{3}x + \frac{13}{3}$ ....(1)

$y = \frac{3}{2}x - \frac{13}{2}$ ....(2)

$-\frac{2}{3}x + \frac{13}{3} = \frac{3}{2}x - \frac{13}{2}$ $M1$ Accept e.c.f (if previous eqn of AB is wrong.

$-4x + 26 = 9x - 39$

$13x = 65$

$x = 5$

$B(5,1)$ $A1$

(c) Calculate the coordinates of *D*. [2]

Let *D* be (x, y).

$(2,3) = \left(\frac{x+5}{2}, \frac{y+1}{2}\right)$ $M1$

$\therefore D(-1, 5)$. $A1$

14 A particle starts from rest at a fixed point *O* and moves in a straight line such that its velocity v ms^{-1} is given by $v = 4t - \frac{3}{2}t^2$, where t is the time in seconds after leaving *O*.

Calculate

(a) the velocity of the particle when its acceleration is zero, [3]

$a = \frac{\mathrm{d}v}{\mathrm{d}t} = 4 - 3t$ $M1$

$4 - 3t = 0$

$t = \frac{4}{3}s$ $M1$

$v = 4\left(\frac{4}{3}\right) - \frac{3}{2}\left(\frac{4}{3}\right)^2 = \frac{8}{3}$ m/s $A1$

(b) the time when the particle is instantaneously at rest again, [2]

$4t - \frac{3}{2}t^2 = 0$ $M1$

$t = 0s$(rej) *or* $4 - \frac{3}{2}t = 0$

$t = \frac{8}{3}s$ $A1$

Must rej t = 0 or show evidence like # symbol to show this is the final answer.

(c) the total distance travelled by the particle when it returns to O. [5]

$s = \int v \, dt$

$= \int 4t - \frac{3}{2}t^2 \, dt$ $M1$

$= 2t^2 - \frac{1}{2}t^3 + c$ $M1$

$t = 0, s = 0, c = 0$

$s = 2t^2 - \frac{1}{2}t^3$ $A1$

$t = \frac{8}{3},$

$s = 2\left(\frac{8}{3}\right)^2 - \frac{1}{2}\left(\frac{8}{3}\right)^3 = \frac{128}{27}$ $M1$

Accept e.c.f

Total distance = $\frac{128}{27} \times 2 = \frac{256}{27}$ m OR $9\frac{13}{27}$ m $A1$

Accept 9.48 m (3 s.f)

OR

$s = 2\int_0^{\frac{8}{3}} 4t - \frac{3}{2}t^2 \, dt$ $M1$(for 2)

$M1$(for integrating)

$= 2\left[2t^2 - \frac{1}{2}t^3\right]_0^{\frac{8}{3}}$ $M1$

$= 2\left(\frac{128}{27} - 0\right)$ $M1$

$= \frac{256}{27}$

Total distance = $\frac{256}{27}$ m $A1$

END OF PAPER

Name	Index Number	Class

WOODGROVE SECONDARY SCHOOL

A COMMUNITY OF FUTURE-READY LEARNERS AND THOUGHTFUL LEADERS

O-LEVEL PRELIMINARY EXAMINATIONS 2023

LEVEL & STREAM	:	SECONDARY 4 EXPRESS/ 5 NORMAL ACADEMIC
SUBJECT (CODE)	:	ADDITIONAL MATHEMATICS (4049)
PAPER NO	:	02
DATE (DAY)	:	12 SEPTEMBER 2023 (TUESDAY)
DURATION	:	2 HOURS 15 MINUTES

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The total number of marks in this paper is 90.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

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$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

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$$\sin 2A = 2 \sin A \cos A$$

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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 It is given that $f(x) = 2e^{x}(\sin x - \cos x)$.

(a) Show that $f'(x) = 4e^{x} \sin x$. [3]

(b) Hence evaluate $\int_{0}^{\pi} e^{x} \sin x \, dx$. [4]

2 **(a)** Prove that $\sin 3x = 3\sin x - 4\sin^3 x$. [4]

(b) Hence solve the equation $6\sin x - 8\sin^3 x = 1$ for $0° < x < 120°$. [4]

3 **(a)** Show that $x^4 + 3x^2 - 4 = (x+1)(x-1)(x^2+4)$. [2]

(b) Hence express $\dfrac{3x^2+7}{x^4+3x^2-4}$ in partial fractions. [6]

4 A curve y, is such that $\frac{d^2y}{dx^2} = 2x$ and the point $P(0,-3)$ lies on the curve. The gradient of the curve at P is 5.

(a) Determine if the curve passes through point $Q(3,21)$. [5]

(b) Explain why the curve has no turning point. [2]

(c) Determine whether the curve is an increasing or decreasing function. [2]

5 The points $A(-2,1)$, $B(3,-4)$ and $C(3,1)$ lies on a circle.

(a) Show that the centre of the circle is $\left(\frac{1}{2},-\frac{3}{2}\right)$. [6]

(b) Explain why AB is the diameter of the circle. [1]

(c) Find the equation of the circle. [3]

(d) Show that point $D(2,2)$ lies outside the circle. [2]

6 Solve the following equations.

(a) $3+\log_2(x+4)=2\log_2(3x-4)$. [4]

(b) $2\log_3 y - \log_y 3 = 1.$ [5]

7 **(a)** Factorise $(x-1)^3-8$ completely. [3]

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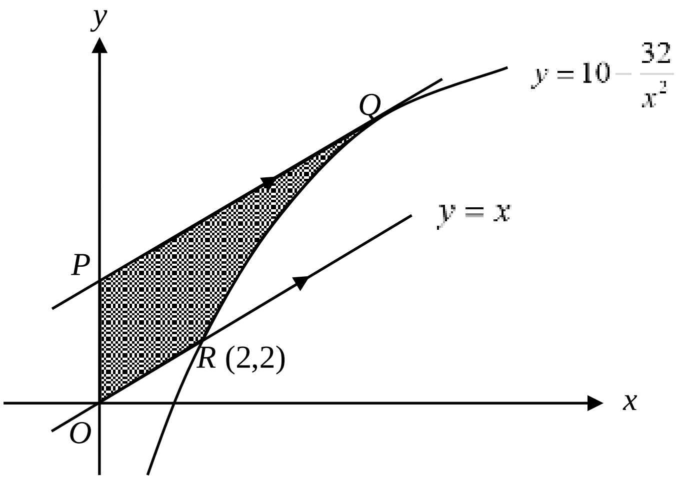
8

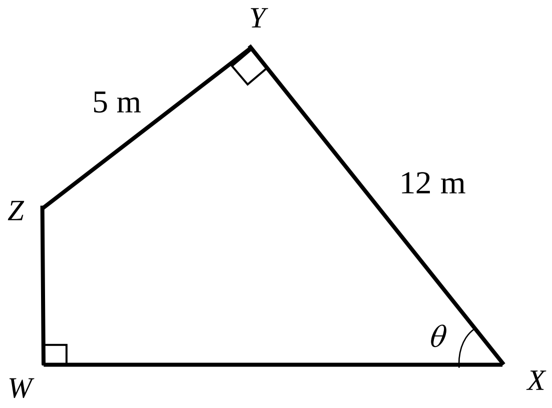


The diagram shows part of the curve $y = 10 - \frac{32}{x^2}$ and two parallel lines *OR* and *PQ*. The equation of *OR* is $y = x$ and the line intersects the curve at point $R(2, 2)$. *PQ* is the tangent to the curve at point *Q*.

(a) Find the coordinates of *Q* and of *P*. [5]

(b) Find the area of the shaded region *OPQR*. [5]

9



In the diagram $WXYZ$ is a quadrilateral with $XY = 12$ m , $YZ = 5$ m and $\angle WXY = \theta$.

(a) Show that the perimeter, P cm, of $WXYZ$ is $17\sin\theta + 7\cos\theta + 17$. [4]

(b) Express P in the form $R\sin(\theta + \alpha) + k$, where $R > 0$, $k > 0$ and $0° < \alpha < 90°$. [3]

(c) Find the maximum value of P and the corresponding value of θ. [2]

10 A cylindrical pipe of surface area S m^2 has a circumference of $\left(a+\dfrac{b}{x^2}\right)$ m and length of x m. Corresponding values of x and S are shown in the table below.

x	0.5	1.0	1.5	2.0
S	23	19	21	24.5

(a) Draw a straight line graph of Sx against x^2. [2]

(b) Use the graph to estimate

(i) the value of each of the constants *a* and *b*, [4]

(ii) the surface area of the pipe with a length of 0.8 m. [3]

(c) By drawing a suitable straight line, find the length of the pipe when its surface area is $5\left(x+\frac{3}{x}\right)$ cm^2. [3]

END OF PAPER

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$$\Delta = \frac{1}{2}bc\sin A$$

1 It is given that $\mathrm{f}(x)=2e^{x}(\sin x-\cos x)$.

(a) Show that $\mathrm{f}'(x)=4e^{x}\sin x$. [4]

$\mathrm{f}(x)=2e^{x}(\sin x-\cos x)$

$\mathrm{f}'(x)=2e^{x}(\sin x-\cos x)+2e^{x}(\cos x+\sin x)$ -------------------- M1

$=2e^{x}\left[(\sin x-\cos x)+(\cos x+\sin x)\right]$ -------------------- M1

$=2e^{x}(2\sin x)$

$=4e^{x}\sin x$ (shown)------------------- A1

(b) Hence evaluate $\int_0^{\pi} e^{x}\sin x\,\mathrm{d}x$. [4]

$\int 4e^{x}\sin x\,\mathrm{d}x=2e^{x}(\sin x-\cos x)+c$-------------------- M1

$4\int e^{x}\sin x\,\mathrm{d}x=2e^{x}(\sin x-\cos x)+c$

$\int e^{x}\sin x\,\mathrm{d}x=\frac{1}{2}e^{x}(\sin x-\cos x)+c$------------------- M1

$\int_0^{\pi} e^{x}\sin x\,\mathrm{d}x=\left[\frac{1}{2}e^{x}(\sin x-\cos x)\right]_0^{\pi}$ ------------------- M1

$=\left[\frac{1}{2}e^{\pi}(\sin\pi-\cos\pi)-\frac{1}{2}e^{0}(\sin 0-\cos 0)\right]$

$=\frac{1}{2}e^{\pi}+\frac{1}{2}$ ------------------- A1

2 **(a)** Prove that $\sin 3x = 3\sin x - 4\sin^3 x$. [3]

$\text{LHS} = \sin 3x$

$= \sin(2x + x)$ ------------------- M1

$= \sin 2x \cos x + \cos 2x \sin x$

$= 2\sin x \cos x \cos x + (1 - 2\sin^2 x)\text{sinx}$ ------------------- M1

$= 2\sin x \cos^2 x + \sin x - 2\sin^3 x$

$= 2\sin x(1 - \sin^2 x) + \sin x - 2\sin^3 x$------------------- M1

$= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$

$= 3\sin x - 4\sin^3 x$

$= \text{RHS}$ ------------------- A1

(b) Hence solve the equation $6\sin x - 8\sin^3 x = 1$ for $0° < x < 120°$. [4]

$6\sin x - 8\sin^3 x = 1$

$2(3\sin x - 4\sin^3 x) = 1$

$2\sin 3x = 1$ ------------------- M1

$\sin 3x = \frac{1}{2}$

$\alpha = 30°$ ------------------- M1

$3x = 30, 180 - 30$

$3x = 30, 150$ ------------------- M1

$x = 10°, 50°$ ------------------- A1

3 **(a)** Show that $x^4 + 3x^2 - 4 = (x+1)(x-1)(x^2+4)$. [2]

LHS = $x^4 + 3x^2 - 4$

$= (x^2-1)(x^2+4)$ -------------------- M1

$= (x+1)(x-1)(x^2+4)$ -------------------- A1

(b) Hence express $\dfrac{3x^2+7}{x^4+3x^2-4}$ in partial fractions. [6]

$$\frac{3x^2+7}{(x+1)(x-1)(x^2+4)} = \frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{Cx+D}{(x^2+4)}$$ -------------------- M1

$3x^2+7 = A(x-1)(x^2+4) + B(x+1)(x^2+4) + (Cx+D)(x+1)(x-1)$

sub $x = 1$

$10 = B(10)$

$B = 1$-------------------- M1

sub $x = -1$

$10 = -10A$

$A = -1$-------------------- M1

sub $x = 0$

$7 = -4A + 4B - D$

$7 = 4 + 4 - D$

$D = 1$-------------------- M1

compare coeff of x^3,

$0 = A + B + C$

$0 = -1 + 1 + C$

$C = 0$-------------------- M1

or when $x = 2$

$19 = -8 + 24 + 6c + 3$

$c = 0$

$$\frac{3x^2+7}{(x+1)(x-1)(x^2+4)} = \frac{-1}{(x+1)} + \frac{1}{(x-1)} + \frac{1}{(x^2+4)}$$

$$= \frac{1}{(x-1)} - \frac{1}{(x+1)} + \frac{1}{(x^2+4)}$$ -------------------- A1

4 A curve y, is such that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2x$ and the point $P(0,-3)$ lies on the curve. The gradient of the curve at P is 5.

(a) Determine if the curve passes through point $Q(3,21)$. [5]

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2x$$

$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 + C$ ------------------ M1

$x = 0, \frac{\mathrm{d}y}{\mathrm{d}x} = 5$ ------------------ M1

$C = 5$

$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 + 5$

$y = \frac{1}{3}x^3 + 5x + D$------------------ M1

$x = 0, y = -3$

$D = -3$

$y = \frac{1}{3}x^3 + 5x - 3$

when $x = 3$ ------------------ M1

$y = 9 + 15 - 3$

$y = 21$

the curve passes through the point (3,21) ------------------ A1

(b) Explain why the curve has no turning point. [2]

$\frac{dy}{dx} = x^2 + 5$

for turning point, $\frac{dy}{dx} = 0$,

$x^2 + 5 = 0$

$x^2 = -5$

$x = \sqrt{-5}$---------------- M1

no Solution, therefore no turning point ---------------- A1

or as $x^2 \geq 0$, $\frac{dy}{dx} \geq 5$---------------- M1

since $\frac{dy}{dx}$ can never be zero,

$\therefore$ the curve has no turning point. ---------------- A1

(c) Determine whether the curve is an increasing or decreasing function. [2]

$\frac{dy}{dx} = x^2 + 5$

$x^2 \geq 0$

$x^2 + 5 > 0$--------------- A1

Therefore the curve is an increasing function for all value of x--------------- A1

5 The points $A(-2,1)$, $B(3,-4)$ and $C(3,1)$ lies on a circle.

(a) Show that the centre of the circle is $\left(\frac{1}{2},-\frac{3}{2}\right)$. [6]

$$A(-2,1), B(3,-4)$$

$$m_{AB}=\frac{-4-1}{3-(-2)}$$

$$=-1 \text{-------------------- M1}$$

$$m_{\perp AB}=1$$

$$midpoint=\left(\frac{-2+3}{2},\frac{1+(-4)}{2}\right)$$

$$=\left(\frac{1}{2},-\frac{3}{2}\right) \text{-------------------- M1}$$

$$y=mx+c$$

$$-\frac{3}{2}=\frac{1}{2}+c$$

$$c=-2$$

$$y=x-2 \text{-------------------- M1}$$

$$B(3,-4), C(3,1)$$

$$m_{BC}=\frac{-4-1}{3-3}$$

$$=\text{undefine} \text{-------------------- M1}$$

$$m_{\perp BC}=0$$

$$\text{midpoint}=\left(3,-\frac{3}{2}\right)$$

$$y=mx+c$$

$$-\frac{3}{2}=0+c$$

$$c=-\frac{3}{2}$$

$$y=-\frac{3}{2} \text{-------------------- M1}$$

$$y=x-2$$

$$y=-\frac{3}{2}$$

$$x=\frac{1}{2}$$

Therefore centre of circle is $\left(\frac{1}{2},-\frac{3}{2}\right)$ -------------------- A1

(b) Explain why AB is the diameter of the circle. [1]

Midpoint of AB is the center of the circle.
Or student show that

$m_{BC} \times m_{AC} = -1$-----------B1

(c) Find the equation of the circle. [3]

$$r^2 = \left(3 - \frac{1}{2}\right)^2 + \left(-4 + \frac{3}{2}\right)^2$$ -------------------- M1

$$= \frac{25}{4} + \frac{25}{4}$$

$$= \frac{25}{2}$$ -------------------- M1

$$\left(x - \frac{1}{2}\right)^2 + \left(y + \frac{3}{2}\right)^2 = \frac{25}{2}$$ -------------------- A1

(d) Show that point $D(2,2)$ lies outside the circle. [2]

Distance of D from centre $= \sqrt{\left(2 - \frac{1}{2}\right)^2 + \left(2 + \frac{3}{2}\right)^2}$ ------------------- M1

$$= \sqrt{\frac{9}{4} + \frac{49}{4}}$$

$= \sqrt{\frac{29}{2}} > \sqrt{\frac{25}{2}}$ -therefore point D lies out side the circle------------------- A1

6 Solve the following equations.

(a) $3+\log_2(x+4)=2\log_2(3x-4)$. [4]

$$3+\log_2(x+4)=2\log_2(3x-4)$$

$$\log_2(3x-4)^2-\log_2(x+4)=3$$

$$\log_2\frac{(3x-4)^2}{(x+4)}=3 \text{ --------------------- M1}$$

$$\frac{(3x-4)^2}{(x+4)}=2^3 \text{ --------------------- M1}$$

$$(3x-4)^2=8(x+4)$$

$$9x^2-24x+16=8x+32$$

$$9x^2-32x-16=0 \text{ --------------------- M1}$$

$$(x-4)(9x+4)=0$$

$$x=4 \quad \text{or } x=-\frac{4}{9} \text{ (Rej) ------------------- A1}$$

(b) $2\log_3 y-\log_y 3=1$. [5]

$$2\log_3 y-\log_y 3=1$$

$$2\log_3 y-\frac{\log_3 3}{\log_3 y}=1 \text{ --------------------- M1}$$

$$\text{let } x=\log_3 y$$

$$2x-\frac{1}{x}=1 \text{ --------------------- M1}$$

$$2x^2-1=x$$

$$2x^2-x-1=0 \text{ --------------------- M1}$$

$$(2x+1)(x-1)=0$$

$$x=-\frac{1}{2} \quad \text{or} \quad x=1$$

$$\log_3 y=-\frac{1}{2} \quad \text{or} \quad \log_3 y=1 \text{ --------------------- M1}$$

$$y=\frac{1}{\sqrt{3}} \quad \text{or} \quad y=3 \text{ --------------------- A1}$$

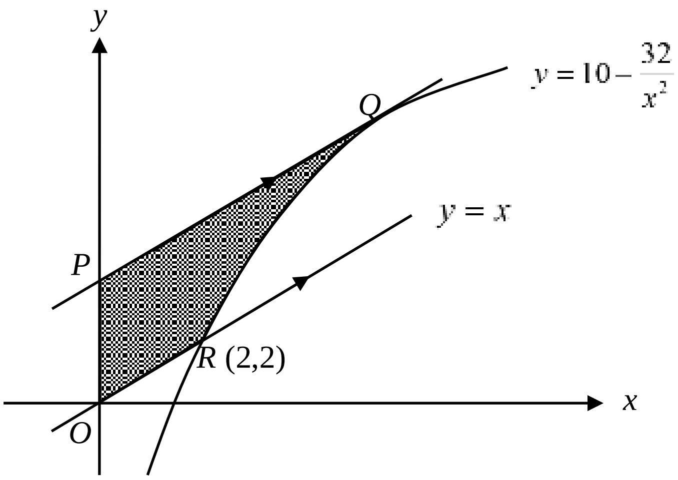
7 **(a)** Factorise $(x-1)^3-8$ completely. [3]

$$(x-1)^3-8=\left[(x-1)-2\right]\left[(x-1)^2+2(x-1)+4\right]\text{-------------------- M1}$$
$$=(x-1-2)(x^2-2x+1+2x-2+4)\text{-------------------- M1}$$
$$=(x-3)(x^2+3)\text{ -------------------- A1}$$

(b) Hence show that $(x-1)^3-8=0$ has only 1 solution. [3]

$$(x-1)^3-8=0$$
$$(x-3)(x^2+3)=0\text{ ------------------ M1}$$
$$x^2+3=0 \quad \text{or} \quad x-3=0$$
$$b^2-4ac=0^2-4(1)(3) \quad \text{or} \quad x=3$$
$$=-12<0 \text{ (no solution)} \quad \text{------------------- M1}$$
$$\therefore\ (x-1)^3-8=0 \text{ has only 1 solution } x=3\text{-----------------------A1}$$

8



The diagram shows part of the curve $y = 10 - \frac{32}{x^2}$ and two parallel lines *OR* and *PQ*. The equation of *OR* is $y = x$ and the line intersects the curve at point $R(2, 2)$. *PQ* is the tangent to the curve at point *Q*.

(a) Find the coordinates of *Q* and of *P*. [5]

$y = 10 - \frac{32}{x^2}$

$y = 10 - 32x^{-2}$

$\frac{dy}{dx} = 64x^{-3}$ -------------------- M1

$64x^{-3} = 1$ -------------------- M1

$x = 4$

$y = 8$

$Q(4,8)$ -------------------- A1

$y = mx + c$

$8 = 4 + c$ -------------------- M1

$c = 4$

$P(0,4)$ -------------------- A1

(b) Find the area of the shaded region *OPQR*. [5]

Area of trapezium $= \frac{1}{2}(4+8)\times 4$

$= 24$ ------------------------------------M1

Area of triangle$= \frac{1}{2}\times 2\times 2$

$= 2$ ------------------------------------M1

Area under the curve$= \int_2^4 10-32x^{-2}\ \mathrm{d}x$

$= \left[10x+32x^{-1}\right]_2^4$ ------------------------------------M1

$= \left[10(4)+\frac{32}{4}\right]-\left[10(2)+\frac{32}{2}\right]$

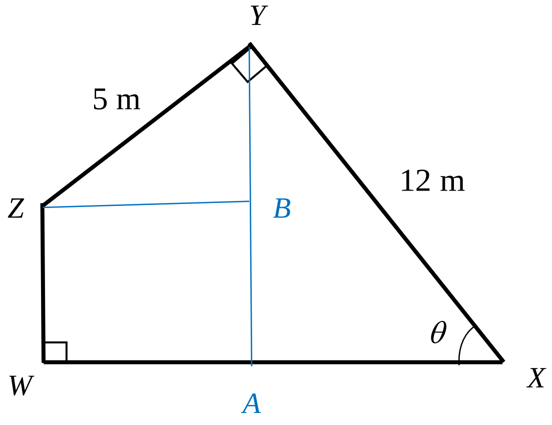
$= 48-36$

$= 12$ ------------------------------------M1

Area of shaded region *OPQR* $= 24-2-12$

$= 10$ units^2 ------------------------------------A1

9



In the diagram $WXYZ$ is a quadrilateral with $XY = 12$ m, $YZ = 5$ m and $\angle WXY = \theta$.

(a) Show that the perimeter, P cm, of $WXYZ$ is $17\sin\theta + 7\cos\theta + 17$. [4]

$\sin\theta = \frac{AY}{12}$ $\cos\theta = \frac{AX}{12}$ $\sin\theta = \frac{BZ}{5}$ $\cos\theta = \frac{BY}{5}$

$AY = 12\sin\theta$ $AX = 12\cos\theta$ $BZ = 5\sin\theta$ $BY = 5\cos\theta$ ----------------------M2

M1 for any pair

$P = 12\cos\theta + 5\sin\theta + 12\sin\theta - 5\cos\theta + 12 + 5$----------------------------------M1

$= 17\sin\theta + 7\cos\theta + 17$ (Shown) -----------------------------A1

(b) Express P in the form $R\sin(\theta + \alpha) + k$, where $R > 0$, $k > 0$ and $0° < \alpha < 90°$. [3]

$17\sin\theta + 7\cos\theta = R\sin(\theta + \alpha)$

$R = \sqrt{17^2 + 7^2}$

$= \sqrt{338}$ -----------------------------M1

$\alpha = \tan^{-1}\frac{7}{17}$

$\alpha = 22.3801°$ -----------------------------M1

$P = 17\sin\theta + 7\cos\theta + 17$

$= 13\sqrt{2}\sin(\theta + 22.4°) + 17$

OR

$=18.4\sin(\theta + 22.4°) + 17$------------------------A1

(c) Find the maximum value of P and the corresponding value of θ. [2]

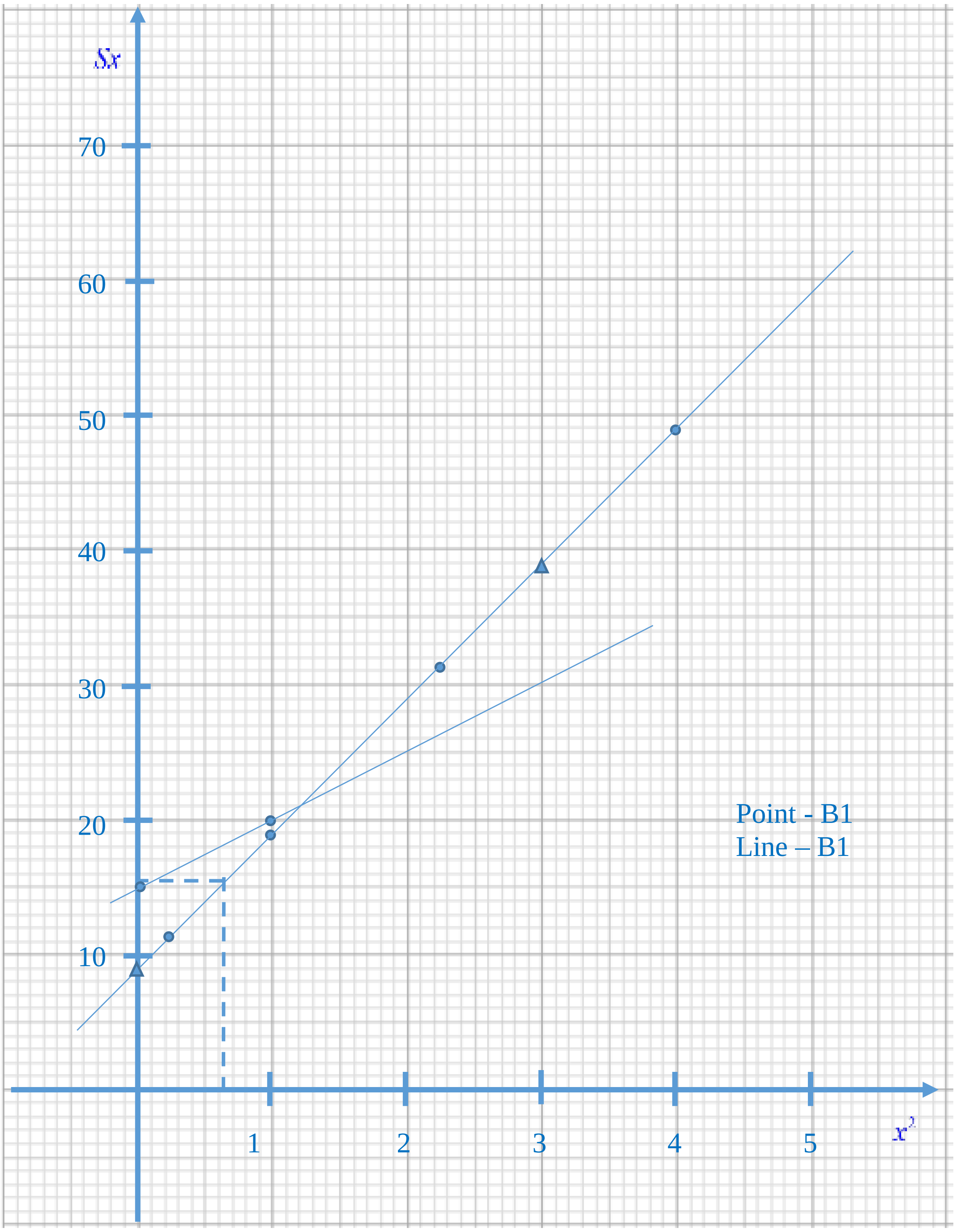
$$\max P = \sqrt{338} + 17$$
$$= 35.38477$$
$$= 35.4 \text{--------------B1}$$
$$\sin(\theta + 22.3801°) = 1$$
$$\theta = 90 - 22.3801$$
$$= 67.6199$$
$$= 67.6°(1\text{dp}) \text{-------------B1}$$

10 A cylindrical pipe of surface area S m^2 has a circumference of $\left(a+\dfrac{b}{x^2}\right)$ m and length of x m. Corresponding values of x and S are shown in the table below.

x	0.5	1.0	1.5	2.0
S	23	19	21	24.5

(a) Draw a straight line graph of Sx against x^2. [2]

x^2	0.25	1.0	2.25	4.0
Sx	11.5	19.0	31.5	49.0



(b) Use the graph to estimate

(i) the value of each of the constants a and b, [4]

$c=9$-----------------------------M1

$m=\dfrac{39-9}{3-0}$

$=10$-----------------------------M1

$S=\left(a+\dfrac{b}{x^2}\right)x$

$S=ax+\dfrac{b}{x}$

$Sx=ax^2+b$-----------------------------M1

a=10

b=9-----------------------A1

(ii) the surface area of the pipe with a length of 0.8 m. [3]

$x^2=0.64$-----------------------------M1

from the graph,

$Sx=15.5$-----------------------------M1

$S=\dfrac{15.5}{0.8}$

$=19.375$

$=19.4\ \text{m}^2$ (3sf)------------------A1

(c) By drawing a suitable straight line, find the length of the pipe when its surface area is $5\left(x+\dfrac{3}{x}\right)$ cm^2. [3]

$S=5\left(x+\dfrac{3}{x}\right)$

$Sx=5x^2+15$-----------------------------M1

From the graph,

$x^2=1.2$-----------------------------M1

$x=\sqrt{1.2}$

$=1.095$

$=1.10$

the length is 1.10 cm-------------------A1

END OF PAPER