TANJONG KATONG GIRLS' SCHOOL
PRELIMINARY EXAMINATION
SECONDARY FOUR EXPRESS

CANDIDATE NAME					
CLASS		E		INDEX NUMBER	

ADDITIONAL MATHEMATICS 4049/01

PAPER 1

14 August 2023

2 hour 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 90.

For Examiner's use

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$
$$\sec^2 A=1+\tan^2 A$$
$$\operatorname{cosec}^2 A=1+\cot^2 A$$
$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$
$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$
$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$
$$\sin 2A=2\sin A\cos A$$
$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$
$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formula for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$
$$a^2=b^2+c^2-2bc\cos A$$
$$\Delta=\tfrac{1}{2}bc\sin A$$

1 A right circular cylinder has a volume of $\left(6+2\sqrt{3}\right)\pi$ cm^3 and a base radius of $\left(1+\sqrt{3}\right)$ cm. Find, **without using a calculator**, the height of the cylinder, in cm, in the form $\left(a+b\sqrt{3}\right)$, where a and b are integers. [3]

2 The curve $y = \dfrac{5}{x} + 3$ and the line $x - 2y - 3 = 0$ intersect at the points P and Q.

Find the coordinates of P and of Q. [3]

3 Express $3-3x-2x^2$ in the form $a(x+b)^2+c$ and hence state the coordinates of the turning point of the curve $y=3-3x-2x^2$. [4]

4 Integrate $\frac{8}{2x-1}+\frac{4}{x^3}+1$ with respect to x. [3]

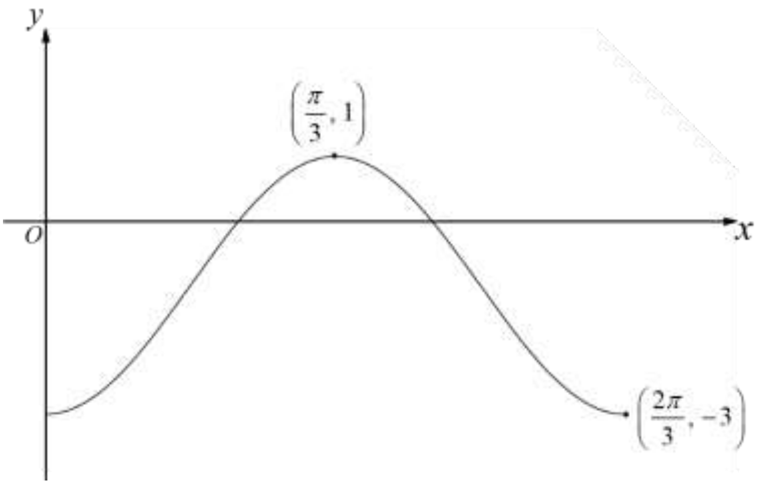
5 Find the value of A, B, C, and D for which $\dfrac{4x^3+7x^2-13x-2}{\left(x^2+3\right)\left(x-1\right)}=A+\dfrac{Bx+C}{x^2+3}+\dfrac{D}{x-1}$. [6]

6 A polynomial, P, is $x^3 - x^2 - x + k$, where k is a constant.

(a) Find the value of k given that P leaves a remainder of 3 when divided by $x - 2$. [2]

(b) In the case where $k = -2$, the quadratic expression $x^2 + ax + 1$ is a factor of P. Find the value of the constant a. [3]

7



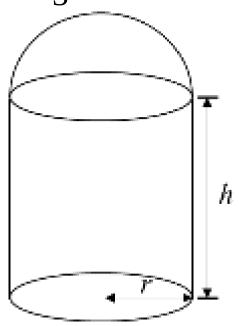
The diagram shows the curve $y = a\cos bx + c$ for $0 \le x \le \dfrac{2\pi}{3}$ radians, where a, b and c are integers. The curve has a maximum point at $\left(\dfrac{\pi}{3}, 1\right)$ and one of its minimum point at $\left(\dfrac{2\pi}{3}, -3\right)$.

(a) Find b. [2]

(b) Explain the effect of c on the curve, and show that $c = -1$. [2]

(c) State the amplitude, and write down the equation of the curve. [2]

8 [The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$, and its surface area is $4\pi r^2$.]

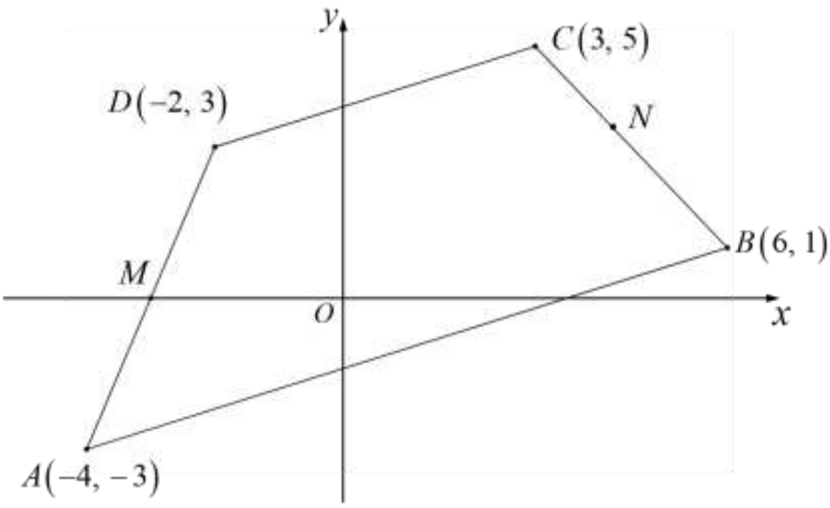


The diagram shows a bubble tea cup of capacity 360π cm^3. It consists of a cylindrical body of base radius r cm and height h cm, and a hemispheric cap. Let S cm^2 be the total surface area of the cup.

(a) Show that $S = \pi\left(\frac{5}{3}r^2 + \frac{720}{r}\right)$. [4]

(b) Given that r can vary, find the stationary value of S and determine its nature. [5]

9



The diagram shows a quadrilateral *ABCD* with vertices $A(-4, -3)$, $B(6, 1)$, $C(3, 5)$ and $D(-2, 3)$. *M* which lies on the *x*-axis is the midpoint of the side *AD*.

(a) Explain why *ABCD* is a trapezium. [3]

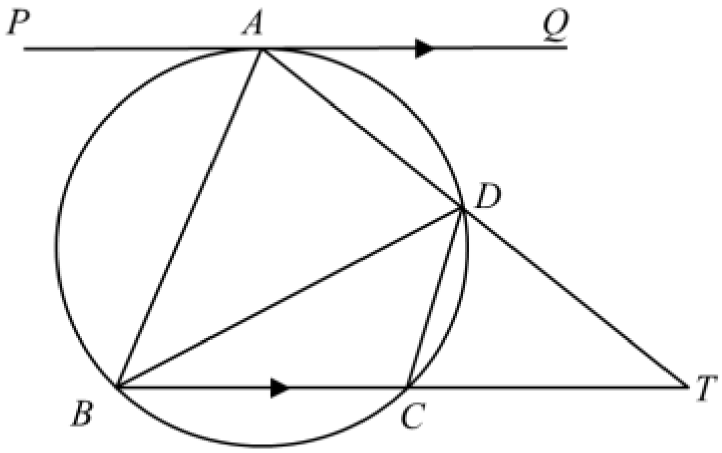
(b) Find the coordinates of *M*. [2]

(c) A point N is on the side BC. Given that the area of the quadrilateral $MNCD$ is $\frac{65}{4}$ units^2, find the coordinates of N. [5]

10 **(a)** Prove the identity $\dfrac{\tan A - \cot A}{\tan A + \cot A} = 1 - 2\cos^2 A$. [4]

(b) Hence solve the equation $\dfrac{\tan 2\theta - \cot 2\theta}{\tan 2\theta + \cot 2\theta} = \dfrac{1}{2}$ for $0 < \theta < \pi$, giving your answers in terms of π. [4]

11



In the diagram, PAQ is the tangent to the circle at A. The line BCT is parallel to PAQ and ADT is a straight line.

(a) Prove that angle $ADB =$ angle CDT. [4]

(b) Prove that triangles *TCD* and *BAD* are similar. [4]

12 **(a)** Show that the solution of the equation $3^{2x+4} \times 5^{x} = 3^{3x} \times 25^{x}$ is $x = \frac{\lg 81}{\lg 15}$. [4]

(b) Express the equation $\log_2 x + \log_4 (x+6) = 2$ as a cubic equation in x. [4]

13 The volume of liquid in a container, V m^3, is given by $V = 0.05\left[(3x+2)^3 - 8\right]$, where x is the height of the liquid in metres.

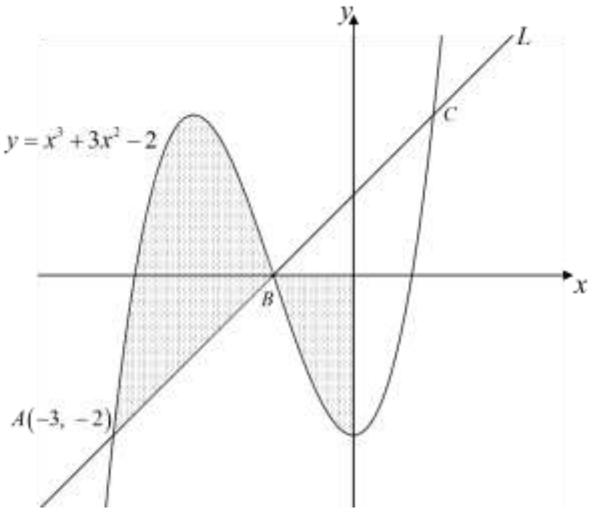
(a) Find an expression for $\frac{\mathrm{d}V}{\mathrm{d}x}$. [2]

The liquid enters the container at a constant rate of 0.081 m^3/s.

(b) Find the value of x when $V = 0.95$. [2]

(c) Hence find $\frac{\mathrm{d}x}{\mathrm{d}t}$ when $V = 0.95$, and explain the significance of your answer. [4]

14



The diagram shows part of the curve $y = x^3 + 3x^2 - 2$. The straight line L cuts the curve at $A(-3, -2)$, the x-axis at B, and intersects the curve again at C.

(a) The gradient of the tangent to the curve at $B = -3$, use it to find the coordinates of B. [3]

(b) Hence find the area of the shaded region. [6]

BLANK PAGE

Qn	Answers
1	$3-\sqrt{3}$
2	The coordinates of P and Q are $(-1, -2)$ and $\left(10, \frac{7}{2}\right)$.
3	$-2\left(x+\frac{3}{4}\right)^2+\frac{33}{8}$ maximum point at $\left(-\frac{3}{4}, \frac{33}{8}\right)$.
4	$4\ln(2x-1)-\frac{2}{x^2}+x+c$
5	$A=4,\ B=12,\ C=-13,\ D=-1$
6	$k=1$ $a=1$
7	$b=3$ $c=-1$ Amplitude $=2$ $y=-2\cos 3x-1$
8	$r=6$
9	$(-3,\ 0)$ $\left(\frac{9}{2}, 3\right)$
10	$\theta=\frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$
12	$x^3+6x^2-16=0$
13	$\frac{dV}{dx}=0.45(3x+2)^2$ $x=\frac{1}{3}$ $\frac{dx}{dt}=0.02$ m/s When the volume of the liquid in the tub is 0.95m^3, the height of liquid in the container is increasing at 0.02 m/s.
14	$y=x+1$ $\frac{21}{4}\text{units}^2$

TANJONG KATONG GIRLS' SCHOOL
PRELIMINARY EXAMINATION
SECONDARY FOUR EXPRESS

CANDIDATE NAME

CLASS	E	

INDEX NUMBER

ADDITIONAL MATHEMATICS — 4049/02

PAPER 2

16 August 2023

2 hour 15 minutes

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 90.

For Examiner's use

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

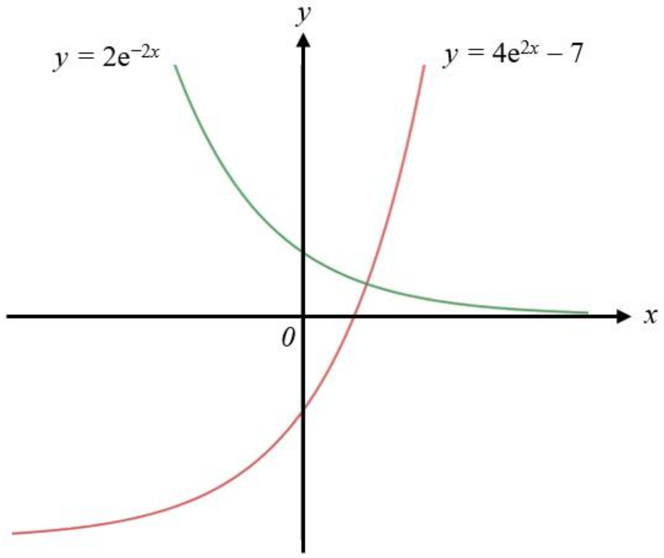
Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formula for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \tfrac{1}{2} bc \sin A$$

1 The diagram below shows the curve $y = 4e^{2x} - 7$ and the curve $y = 2e^{-2x}$.



(a) Using the diagram above, determine with explanation, the number of solutions for the equation $4e^{2x} - 2e^{-2x} = 7$. [1]

(b) Solve the equation $4e^{2x} - 7 = 2e^{-2x}$. [4]

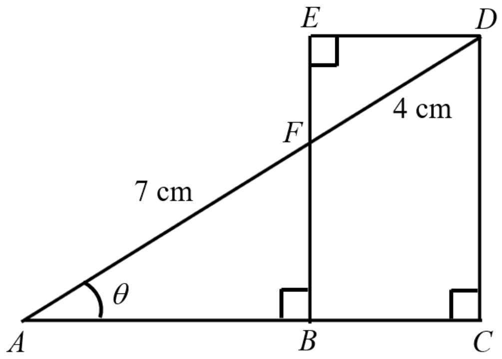
2 **(a)** Show that 1 is a solution of the equation $x^3 - 5x^2 = 2 - 6x$. Hence solve the equation $x^3 - 5x^2 = 2 - 6x$ completely, expressing non-integer roots in in surd form. [4]

(b) Hence solve the equation $(x-1)^3-5(x-1)^2+6x-8=0$ completely.
Express non-integer roots in surd form. [2]

3 **(a)** Given that $y = \dfrac{4x}{\sqrt{3-2x}}$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{12-4x}{\sqrt{(3-2x)^3}}$. [4]

(b) Hence find the value of $\int_0^1 \frac{3-x}{\sqrt{(3-2x)^3}}\,dx$. [4]

4



In the diagram, *BE* intersects *AD* at *F*. *AF* = 7 cm and *FD* = 4 cm. $\angle ABF$, $\angle ACD$ and $\angle DEF$ are right angles. *BCDF* is a trapezium and θ is an acute angle.

(a) Show that the perimeter of *BCDF*, *P* cm, is given by
$P = 4 + 18\sin\theta + 4\cos\theta$. [2]

(b) Express *P* in the form $P = a + R\sin(\theta + \alpha)$, where a is a constant, $R > 0$ and $0° < \alpha < 90°$. [2]

(c) Find the maximum exact value of P and the corresponding value of θ. [3]

(d) Find the value of θ when $P = 11$. [2]

(e) Without evaluating θ, determine whether the perimeter of *BCDF* can have a

a value of 28 cm. Justify your answer. [1]

5 **(a)** Find the set of values of x for which the curve $y = -x^2 + 2x + 4$ lies below another curve $y = x^2 - 2x - 2$ and represent this set on a number line. [4]

(b) The line $y = 2x + k$ is a normal to the curve $y = -x^2 + 2x + 4$ at the point A.

(i) Find the x-coordinate of A. [3]

(ii) Find the value of the constant k. [2]

6 **(a)** **(i)** Write down, and simplify, the first 3 terms in the expansion of $(2-x)^7$ in ascending powers of x. [2]

(ii) Find the coefficient of x^2 in the expansion of $(1-8x+24x^2)(2-x)^7$. [3]

(b) **(i)** Using the general term, find the term in $\frac{1}{x^7}$ in the binomial expansion of $\left(2x - \frac{1}{x^2}\right)^{17}$. [3]

(ii) Explain why there is no term in $\frac{1}{x^2}$ in the expansion of $\left(2x - \frac{1}{x^2}\right)^{17}$. [2]

7 **(a)** Designer bag X was first released in 1950. The price, \$$V$, of the bag is related to t, the number of years since 1950, by the formula $V = ae^{kt}$, where a and k are constants. The table below gives the value of bag X in 1984, 2002, 2008, and 2012.

Year	1984	2002	2008	2012
t (years)	34	52	58	62
V (\$)	1150	2850	4000	4900

(i) On graph paper, plot ln V against t and draw a straight line graph.
Use a scale of 2 cm to 0.5 on the vertical ln V-axis, starting from ln V = 5.0.
Use a scale of 2 cm to 10 years on the t-axis, starting from t = 0. [2]

(ii) Use your graph to estimate the value of a and of k. [3]

(iii) Estimate the year that the value of the bag will hit $7000. [2]

(b) The variables x and y are related by the equation $y = \dfrac{a}{x-b}$ where a and b are constants. Express the equation in a form suitable for drawing a straight line graph, and explain how the values of a and b may be obtained from the graph. [4]

8 A motorist, travelling at a constant velocity of V m/s, passed a fixed point X and saw few vehicles ahead. He stepped on the accelerator and his subsequent velocity, v m/s, is given by $v = 60\mathrm{e}^{\frac{t}{6}}$, where t is the time in seconds after passing X. As he passed a point Y, his velocity has increased to twice his velocity at X.

(a) Find the time taken to travel from X to Y. [3]

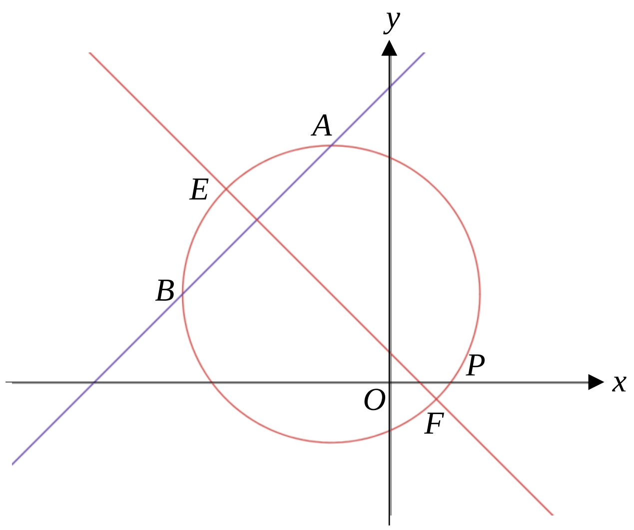
(b) Find the acceleration of the motorist as he passes Y. [2]

(c) Find the distance XY. [3]

9 **(a)** A circle, C_1, has the equation $x^2 + y^2 + 2x - 6y = 10$.

Find the coordinates of the centre and the exact radius of the circle. [3]

(b)



A, B, E and F are 4 points on another circle, C_2.

EF is the perpendicular bisector of chord AB. EF cuts AB at the point $\left(-\frac{9}{2}, \frac{11}{2}\right)$.

The coordinates of point A are (–2, 8).

(i) Find the coordinates of *B*. [3]

(ii) Explain why angle *EAF* = 90°. [1]

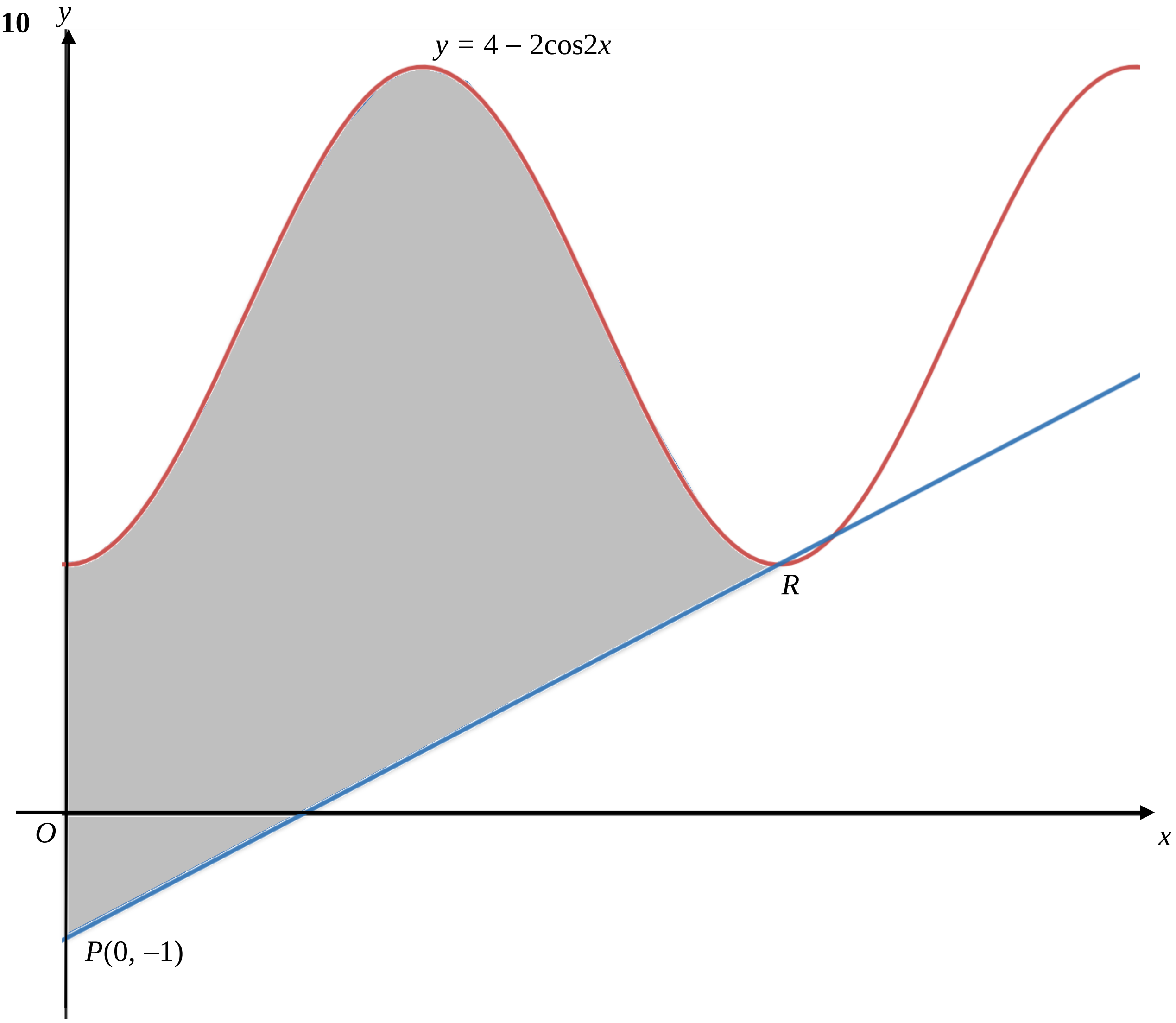
The equation of the perpendicular bisector of chord *AB* is $y+x=1$.
A line $2y=x+8$ also passes through the centre of the circle.

(iii) Find the centre of the circle. [2]

P(2, 0) is another point on the circle.

(iv) Find the equation of the tangent to the circle at *P*. [3]

10



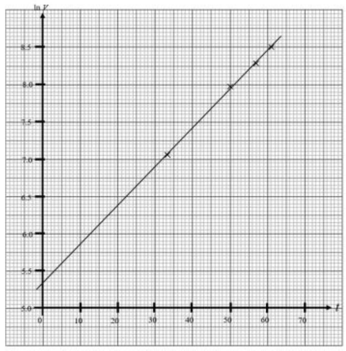
The diagram shows the curve $y = 4 - 2\cos 2x$ for $0 \le x \le \frac{3}{2}\pi$ radians and a straight line passing through points P and R. The coordinates of P are $(0, -1)$ and R is a minimum point of the curve.

Find the area of the region bounded by the curve $y = 4 - 2\cos 2x$, the line segment PR and the y-axis. [11]

Continuation of working space for question **10.**

BLANK PAGE

Question No.	**Answer**
1(a)	1
1(b)	0.347
2(a)	$1,\ 2\pm\sqrt{2}$
2(b)	$2,\ \ 3+\sqrt{2}$ or $3-\sqrt{2}$
3(a)	Proof
3(b)	1
4(a)	Proof
4(b)	$P=4+2\sqrt{85}\sin\left(\theta+12.5°\right)$
4(c)	$4+2\sqrt{85}$, $77.5°$
4(d)	$9.8°$
4(e)	*P* cannot a value of 18 cm.
5(a)	$\left\{x : x\in R,\ \ x<-1\ \text{ or }\ x>3\right\}$ x -1 $\quad$ 3
5(b)(i)	$\frac{5}{4}$
5(b)(ii)	$\frac{39}{16}$
6a(i)	$128-448x+672x^2+...$
6a(ii)	7328
6b(i)	$\frac{12446720}{x^7}$
6b(ii)	$r=\frac{19}{3}$ is not a non-negative integer in range $0\le r\le 17$. $\therefore$ there is no term in $\frac{1}{x^2}$.

Question No.	**Answer**
7(a)(i)	
7(a)(ii)	$a=200.3$, $k=0.0518$
7(a)(iii)	2018
7(b)	$yx=a+by$ Plot yx against y to obtain a straight line graph with a as the intercept on the vertical axis (yx-intercept) and b as the gradient of the line. Thus, a and b may be obtained.
8(a)	4.16 s
8(b)	20 m/s^2
8(c)	360 m
9(a)	$\left(-1,\ 3\right)$, $\quad 2\sqrt{5}$ units
9(b)(i)	$(-7,3)$
9b(iii)	$(-2,\ 3)$
9b(ii)	Since EF is $\perp$ bisector of chord AB, $\therefore$ EF passes through centre of the circle (symmetrical property of circle). $\therefore$ EF is a diameter of the circle. Hence angle EAF = 90° ($\angle$ in a semicircle)
9b(iv)	$3y=4x-8$
10	11.0 square units