Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS **4049/01**
Paper 1

18 August 2023 Friday 2 hours 15 min

2023 SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC) PRELIMINARY EXAMINATIONS

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

INSTRUCTIONS TO CANDIDATES
Write your name, index number and class in the spaces provided above.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided below the questions.
Give non-exact numerical answers correct to 3 significant figures or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use														
Qn	1	2	3	4	5	6	7	8	9	10	11	12	13	*Marks Deducted*
Marks														

Category	Accuracy	Units	Symbols	Others
Question No.				

TOTAL MARKS
90

Setter: Mr Gregory Quek
Vetter: Mr Tan Lip Sing

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The line $y = 2x + 15$ intersects the curve $y = x^2 + 6x + 3$ at points A and B.
Find the value of p for which the distance AB can be expressed as $p\sqrt{5}$. [5]

2 A curve is such that $\dfrac{d^2y}{dx^2} = 12e^{2x} + e^{-x}$. The curve intersects the y-axis at $P(0, 5)$ and the tangent to the curve at P is parallel to $y = 4x + 3$. Find the equation of the curve. [6]

3 A function is defined by $f(x) = x^2 + 2kx + 2k + 3$ for all real values of *x*, where *k* is a constant.

(a) Find the discriminant of $f(x)$ in terms of *k*. [2]

(b) Show that the discriminant of $f(x)$ in **part (a)** can be expressed in the form $4(k - a)^2 - b$, where *a* and *b* are integers. [2]

(c) Find the range of values of *k* for which $f(x) = 0$ has no real roots. [3]

4 It is given that $f(x) = 2x^3 - 5x^2 - 4x + 12$.

(a) Show that $2x+3$ is a factor of $f(x)$. [2]

(b) Factorise $f(x)$ completely. [2]

(c) Hence find the roots of the equation $2\left(2^{3y}\right) - 5\left(2^{2y}\right) - 4\left(2^{y}\right) + 12 = 0$. [3]

5 **(a)** Using long division, show that $\dfrac{x^3-2x^2+5x-10}{x^2+5}=x-2.$ [2]

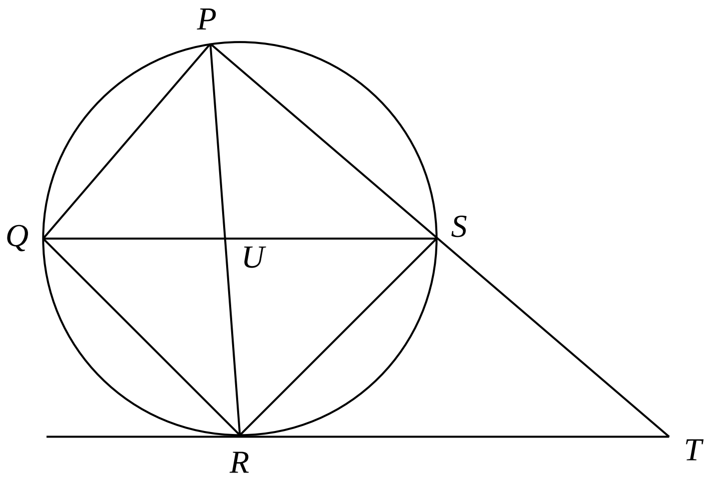
(b) Hence, by first expressing the denominator as a product of two factors,

express $\dfrac{2x^2+1}{x^3-2x^2+5x-10}$ in partial fractions. [5]

6 **(a)** Find the first 3 terms, in ascending powers of x, of the binomial expansion of $\left(2+\frac{ax}{4}\right)^8$, where a is a non-zero constant. Give each term in its simplest form. [2]

(b) Given that the coefficient of x^2 is –320 in the expansion of $(3-x)^2\left(2+\frac{ax}{4}\right)^8$, find the possible value(s) of a. [4]

7



The diagram shows a quadrilateral *PQRS* whose vertices lie on the circumference of a circle.

The diagonals *PR* and *QS* intersect at *U*. The tangent at *R* meets *PS* produced at *T*.

If *QR* = *RS*, prove that

(a) *QS* // *RT*, [3]

(b) triangle *PQR* is similar to triangle *QUR*. [3]

8 **(a)** The equation of a curve is $y = \ln\left(xe^{-3x}\right)$.

The normal to the curve at the point P has a gradient of $\frac{1}{2}$. Find the coordinates of P. [4]

(b) The normal to the curve at P meets the x-axis at Q.
Find the area of triangle OQP, where O is the origin. [3]

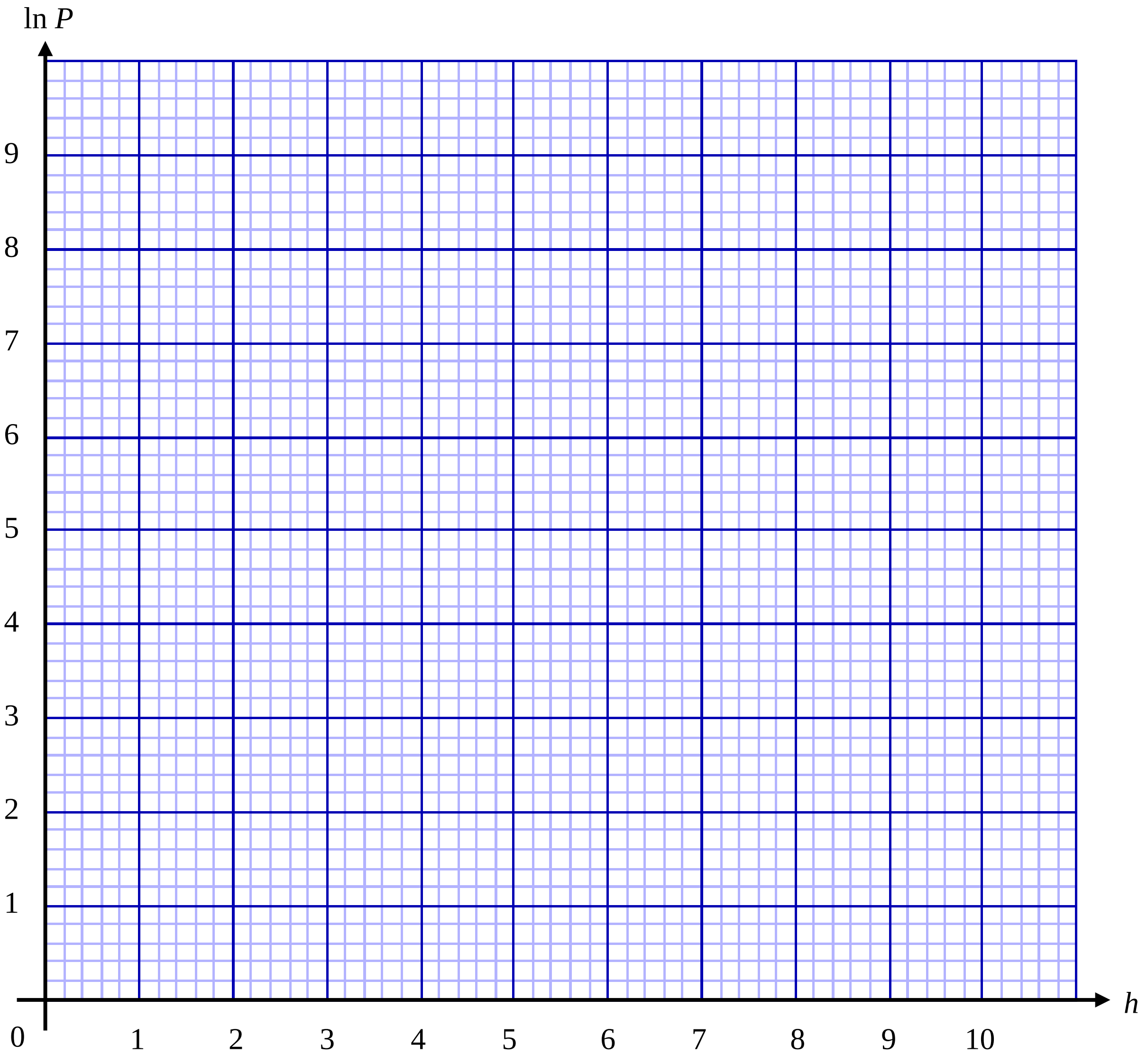
9 Atmospheric pressure is a measure of the force exerted by the mass of air on an object. Altitude is the vertical height above sea level.

The atmospheric pressure, P millibars, exerted at the altitude h kilometres is related by the equation $P = Ae^{bh}$, where A and b are constants.

The following table shows the mean atmospheric pressure at various altitudes.

h (kilometres)	2	4	6	8	10
P (millibars)	810	595	446	340	262

(a) Plot $\ln P$ against h and draw a straight line graph to illustrate the information. [2]



(b) Express the equation $P = Ae^{bh}$ in a form that will yield the straight line graph in **part (a)**.

Hence explain how the graph may be used to determine the value of A and of b. [3]

(c) Use your graph to estimate the atmospheric pressure, to the nearest millibar, when an object is at sea level. [1]

(d) The atmospheric pressure at the summit of Mount Everest is 300 millibars. Use your graph to estimate the altitude of Mount Everest. [1]

10 A patient's blood pressure, $\mathrm{P}(t)$ in mmHg, can be modelled by the function

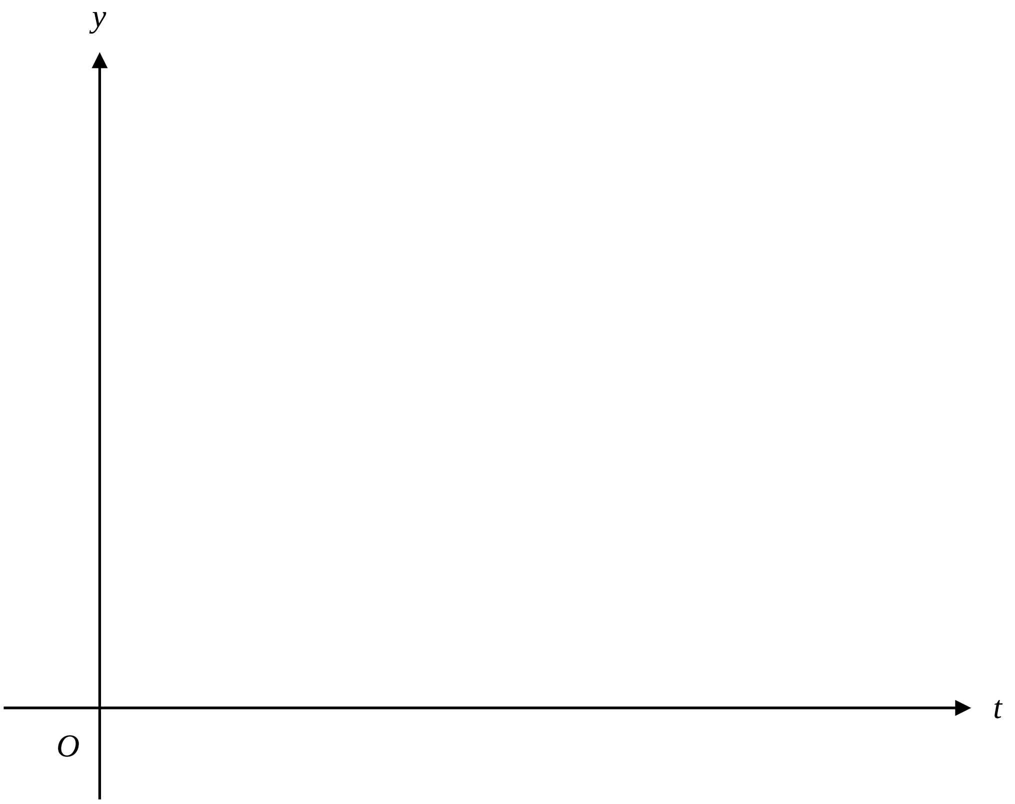
$$\mathrm{P}(t) = 22\cos(2.5\pi t) + 116,$$

where t is the time in seconds.

The systolic pressure (highest pressure) occurs when the heart beats, and the diastolic pressure (lowest pressure) occurs when the heart is at rest between beats.

(a) State the amplitude and period of $\mathrm{P}(t) = 22\cos(2.5\pi t) + 116$. [2]

(b) Sketch the graph of $y = \mathrm{P}(t)$ for $0 \le t \le 2$. [2]



(c) The pulse rate is the number of times a heart beats per minute.
A normal resting pulse rate should be between 60 to 100 beats per minute.
Show that the patient's pulse rate is normal. [2]

(d) According to health guidelines, someone with systolic pressure above 140 mmHg or diastolic pressure above 90 mmHg has high blood pressure and should see a doctor.
Determine whether the patient needs to see a doctor. Justify your answer. [1]

11 A particle moves in a straight line so that t seconds after passing through a fixed point O, its velocity v m/s is given by $v = 5\cos\left(\frac{t}{2}\right)$. Find

(a) the initial velocity of the particle, [1]

(b) the value of t, in terms of π, when the particle first comes to instantaneous rest, [3]

(c) the distance travelled by the particle in the first 5 seconds, after passing through O. [4]

12 A curve has the equation $y = 3 + \left(\frac{x}{2} - 1\right)^4$. The point (p, q) is the stationary point on the curve.

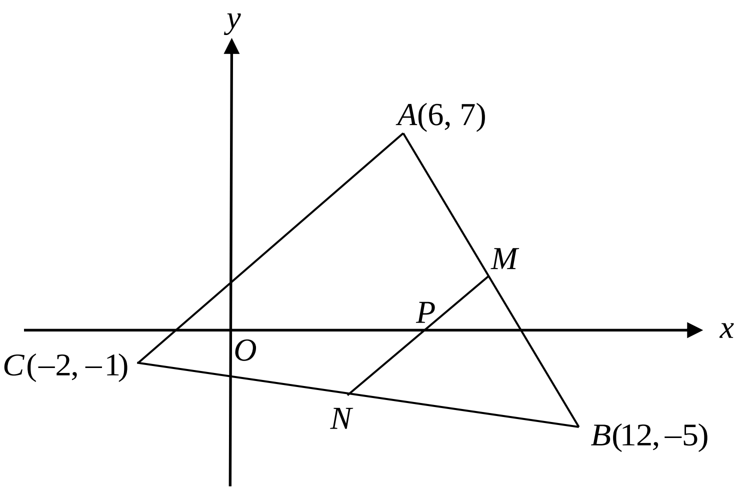
(a) Determine the coordinates of the stationary point (p, q). [4]

(b) **(i)** Justify whether y is increasing or decreasing for values of x less than p. [2]

(ii) Hence infer whether y is increasing or decreasing for values of x greater than p. [1]

(c) What do the results of **part (b)** imply about the stationary point? [1]

13 **Solutions to this question by accurate drawing will not be accepted.**



The diagram above shows a triangle *ABC* with vertices at $A(6, 7)$, $B(12, -5)$ and $C(-2, -1)$.

M and *N* are the mid-points of *AB* and *BC* respectively. The line *MN* cuts the *x*-axis at *P*.

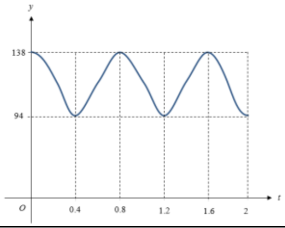
(a) Find the coordinates of *P*. [4]

(b) Find the ratio *AC* : *MN*. [1]

(c) Find the area of the quadrilateral *ACNM*. [2]

(d) Explain why quadrilateral *ACNM* is a trapezium. [2]

END OF PAPER

Qn	Answers
1	$p = 8$
2	$y = 3e^{2x} + e^{-x} - x + 1$
3a	$4k^2 - 8k - 12$
3b	$4(k-1)^2 - 16$
3c	$-1 < k < 3$
4b	$f(x) = (2x+3)(x-2)^2$
4c	$y = 1$
5b	$\dfrac{2x^2+1}{x^3-2x^2+5x-10} = \dfrac{1}{x-2} + \dfrac{x+2}{x^2+5}$
6a	$\left(2+\dfrac{ax}{4}\right)^8 = 256 + 256ax + 112a^2x^2 + ...$
6b	$a = \dfrac{2}{3}$ *or* $a = \dfrac{6}{7}$
8a	$P = (1, -3)$
8b	10.5 units^2
9b	$\ln P = bh + \ln A$ The value of A can be determined by finding the **<u>vertical intercept</u>** of the graph. The value of b can be determined by finding the **<u>gradient</u>** of the graph.
9c	$P \approx 1097$ millibars (nearest whole)
9d	$h = 8.8$ km
10a	Amplitude = 22 Period = 0.8
10b	
10c	Patient's pulse rate = 75 beats per minute Hence the patient's pulse rate is normal.
10d	Since the **<u>diastolic pressure</u>** (94 mmHg) is **<u>above 90 mmHg</u>**, the patient has high blood pressure and **<u>should see the doctor</u>**.
11a	5 m/s
11b	$t = \pi$ s
11c	Distance ≈ 14.0 m
12a	Stationary point = $(2, 3)$
12bi	y is **<u>decreasing</u>** when $x < 2$.
12bii	y is **<u>increasing</u>** when $x > 2$.
12c	The stationary point is a **<u>minimum point</u>**.
13a	$P = (8, 0)$
13b	$AC : MN = 2 : 1$
13c	Area of trapezium $ACNM$ $= 54$ units^2

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PRELIMINARY EXAMINATIONS

MARK SCHEME

Cynthia – Q1 to 10
Sabrina – Q11 to 13

1 The line $y = 2x + 15$ intersects the curve $y = x^2 + 6x + 3$ at points A and B.
Find the value of p for which the distance AB can be expressed as $p\sqrt{5}$. [5]

$x^2 + 6x + 3 = 2x + 15$	M1 (equate curve to line)
$x^2 + 4x - 12 = 0$	
$(x-2)(x+6) = 0$	M1 (factorise)
$x = 2$ *or* $x = -6$	
$y = 19$ *or* $y = 3$	M1 (find y)
$AB = \sqrt{(2-(-6))^2 + (19-3)^2}$	M1 (apply distance formula)
$AB = \sqrt{320}$	
$AB = 8\sqrt{5}$	
$p = 8$	A1

2 A curve is such that $\frac{d^2y}{dx^2} = 12e^{2x} + e^{-x}$. The curve intersects the y-axis at $P(0, 5)$ and the tangent to the curve at P is parallel to $y = 4x + 3$. Find the equation of the curve. [6]

$\frac{dy}{dx} = \int \left(12e^{2x} + e^{-x}\right)dx = 6e^{2x} - e^{-x} + c_1$	M1 (any 2 correct terms)
At $(0,5)$, $\frac{dy}{dx} = 4$	M1 (seen gradient at $P = 4$)
$6e^{2(0)} - e^{-(0)} + c_1 = 4$	M1 (sub. gradient at $x = 0$, attempt to find c_1)
$\Rightarrow c_1 = -1$	
$y = \int \left(6e^{2x} - e^{-x} - 1\right)dx = 3e^{2x} + e^{-x} - x + c$	M1 (any 2 correct terms)
At $(0,5)$, $3e^{2(0)} + e^{-(0)} - 0 + c = 5$	M1 (sub. $x = 0$ & $y = 5$, attempt to find c)
$\Rightarrow c = 1$	
$\therefore y = 3e^{2x} + e^{-x} - x + 1$	A1

3 A function is defined by $f(x) = x^2 + 2kx + 2k + 3$ for all real values of x, where k is a constant.

(a) Find the discriminant of $f(x)$ in terms of k. [2]

For $f(x) = x^2 + 2kx + 2k + 3$,

$b^2 - 4ac = (2k)^2 - 4(1)(2k+3)$ M1 (apply discriminant)

$= 4k^2 - 8k - 12$ A1

(b) Show that the discriminant of $f(x)$ in **part (a)** can be expressed in the form $4(k-a)^2 - b$, where a and b are integers. [2]

$4k^2 - 8k - 12 = 4\left[k^2 - 2k + 1^2 - 1^2\right] - 12$

$= 4\left[(k-1)^2 - 1\right] - 12$ M1 (completing the square)

$= 4(k-1)^2 - 16$ A1

(c) Find the range of values of k for which $f(x) = 0$ has no real roots. [3]

$b^2 - 4ac < 0$

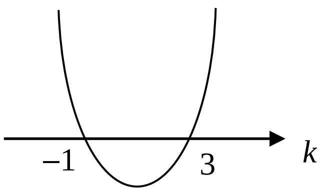
$4(k-1)^2 - 16 < 0$ M1 (apply discriminant < 0)

$(k-1)^2 - 2^2 < 0$

$(k-1+2)(k-1-2) < 0$

$(k+1)(k-3) < 0$ M1 (factorise)

$-1 < k < 3$ A1



4 It is given that $\mathrm{f}(x) = 2x^3 - 5x^2 - 4x + 12$.

(a) Show that $2x+3$ is a factor of $\mathrm{f}(x)$. [2]

$$\mathrm{f}\left(-\frac{3}{2}\right) = 2\left(-\frac{3}{2}\right)^3 - 5\left(-\frac{3}{2}\right)^2 - 4\left(-\frac{3}{2}\right) + 12$$ M1 (apply factor theorem)

$$= -\frac{27}{4} - \frac{45}{4} + 6 + 12$$

$$= 0$$

By the Factor Theorem, $(2x+3)$ is a factor of $\mathrm{f}(x)$. (shown) AG1

(b) Factorise $\mathrm{f}(x)$ completely. [2]

$$\mathrm{f}(x) = 2x^3 - 5x^2 - 4x + 12$$

$$= (2x+3)\left(x^2 - 4x + 4\right)$$ M1 (long division or comparing coefficients)

$$= (2x+3)(x-2)^2$$ A1

(c) Hence find the roots of the equation $2\left(2^{3y}\right) - 5\left(2^{2y}\right) - 4\left(2^{y}\right) + 12 = 0$. [3]

$$2\left(2^y\right)^3 - 5\left(2^y\right)^2 - 4\left(2^y\right) + 12 = 0$$

Let $x = 2^y$,

$$2\left(2^y\right)^3 - 5\left(2^y\right)^2 - 4\left(2^y\right) + 12 = 0$$

$$\left[2\left(2^y\right) + 3\right]\left[\left(2^y\right) - 2\right]^2 = 0$$

$2\left(2^y\right) + 3 = 0$ *or* $\left(2^y\right) - 2 = 0$ M1 (attempt to let $x = 2^y$ and solve)

$2^y = -\frac{3}{2}$ *or* $2^y = 2$ M1 (seen either one)

(rejected)

$\therefore y = 1$ A1

5 **(a)** Using long division, show that $\frac{x^3-2x^2+5x-10}{x^2+5}=x-2$. [2]

$$\begin{array}{r l} & x-2 \\ x^2+5 \overline{)} & x^3-2x^2+5x-10 \\ & \underline{-(x^3 \qquad\quad +5x)} \\ & -2x^2-10 \\ & \underline{-(-2x^2-10)} \\ & 0 \end{array}$$

M1 (attempt to use long division)

$\therefore \frac{x^3-2x^2+5x-10}{x^2+5}=x-2$ (shown) A1

(b) Hence, by first expressing the denominator as a product of two factors, express $\frac{2x^2+1}{x^3-2x^2+5x-10}$ in partial fractions. [5]

$$\frac{2x^2+1}{x^3-2x^2+5x-10}=\frac{2x^2+1}{(x-2)(x^2+5)}$$

$$\frac{2x^2+1}{(x-2)(x^2+5)}=\frac{A}{x-2}+\frac{Bx+C}{x^2+5}$$

M1 (seen both partial fractions)

$$2x^2+1=A(x^2+5)+(Bx+C)(x-2)$$

Sub. $x=2$,

$9=9A$

$A=1$

M1 (seen substitution or comparing coefficients)

Comparing constant term,

$1=5A-2C$

$1=5-2C$

$C=2$

Comparing x^2 term,

$2=A+B$

$2=1+B$ A2 (any 2 correct)

$B=1$

$\therefore \frac{2x^2+1}{x^3-2x^2+5x-10}=\frac{1}{x-2}+\frac{x+2}{x^2+5}$ A1

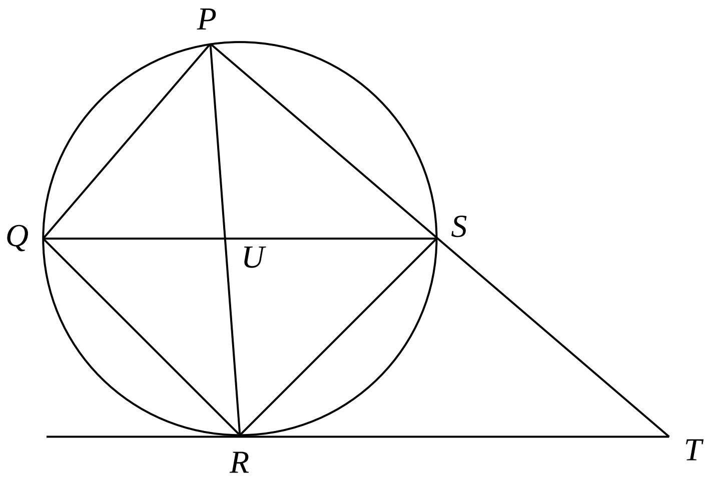
6 **(a)** Find the first 3 terms, in ascending powers of *x*, of the binomial expansion of $\left(2+\frac{ax}{4}\right)^8$, where *a* is a non-zero constant. Give each term in its simplest form. [2]

$\left(2+\frac{ax}{4}\right)^8 = 2^8 + \binom{8}{1}(2)^7\left(\frac{ax}{4}\right) + \binom{8}{2}(2)^6\left(\frac{ax}{4}\right)^2 + \ldots$	M1 (apply Binomial theorem)
$\left(2+\frac{ax}{4}\right)^8 = 256 + 256ax + 112a^2x^2 + \ldots$	A1

(b) Given that the coefficient of x^2 is –320 in the expansion of $(3-x)^2\left(2+\frac{ax}{4}\right)^8$, find the possible value(s) of *a*. [4]

$(3-x)^2\left(2+\frac{ax}{4}\right)^8 = \left(9-6x+x^2\right)\left[256+256ax+112a^2x^2+\ldots\right]$	M1 (expansion)
$(9)\left(112a^2\right)+(-6)\left(256a\right)+(1)(256) = -320$	M1 (comparing)
$1008a^2 - 1536a + 576 = 0$	
$21a^2 - 32a + 12 = 0$	
$(3a-2)(7a-6) = 0$	
$a = \frac{2}{3}$ *or* $a = \frac{6}{7}$	A1, A1

7



The diagram shows a quadrilateral *PQRS* whose vertices lie on the circumference of a circle.

The diagonals *PR* and *QS* intersect at *U*. The tangent at *R* meets *PS* produced at *T*.

If *QR* = *RS*, prove that

(a) *QS* // *RT*, [3]

$\angle RQS = \angle RSQ$ (base $\angle$s of isos. Δ)	B1
$\angle RQS = \angle TRS$ (alt. segment theorem)	B1
Since $\angle RSQ = \angle TRS$, $\therefore QR // RT$ (alt. $\angle$s are equal)	AG1

(b) triangle *PQR* is similar to triangle *QUR*. [3]

$\angle RPQ = \angle RSQ$ ($\angle$s in the same segment)	B1
$\angle RPQ = \angle RSQ = \angle RQS$ (from part (a))	
$\angle PRQ = \angle QRU$ (common $\angle$)	B1
Triangle *PQR* is similar to triangle *QUR*. (AA similarity)	AG1

8 **(a)** The equation of a curve is $y = \ln\left(xe^{-3x}\right)$. The normal to the curve at the point P has a gradient of $\frac{1}{2}$. Find the coordinates of P. [4]

$y = \ln\left(xe^{-3x}\right) = \ln x - 3x$

$\frac{dy}{dx} = \frac{1}{x} - 3$ M1

Gradient at point $P = -1 \div \frac{1}{2} = -2$ M1

$-2 = \frac{1}{x} - 3$

$1 = \frac{1}{x}$

$x = 1$

M1 (equate $\frac{dy}{dx} = -2$ & attempt to solve for x)

$y = \ln\left(e^{-3}\right) = -3$

Coordinates of P = (1, –3) A1

(b) The normal to the curve at P meets the x-axis at Q.
Find the area of triangle OQP, where O is the origin. [3]

$y - (-3) = \frac{1}{2}(x - 1)$

$y = \frac{1}{2}x - \frac{7}{2}$ M1 (find equation of normal)

At Q, $\frac{1}{2}x - \frac{7}{2} = 0$

$x = 7$ M1 (find x-intercept)

Area of triangle OQP

$= \frac{1}{2} \times 7 \times 3 = 10.5 \text{ units}^2$ A1

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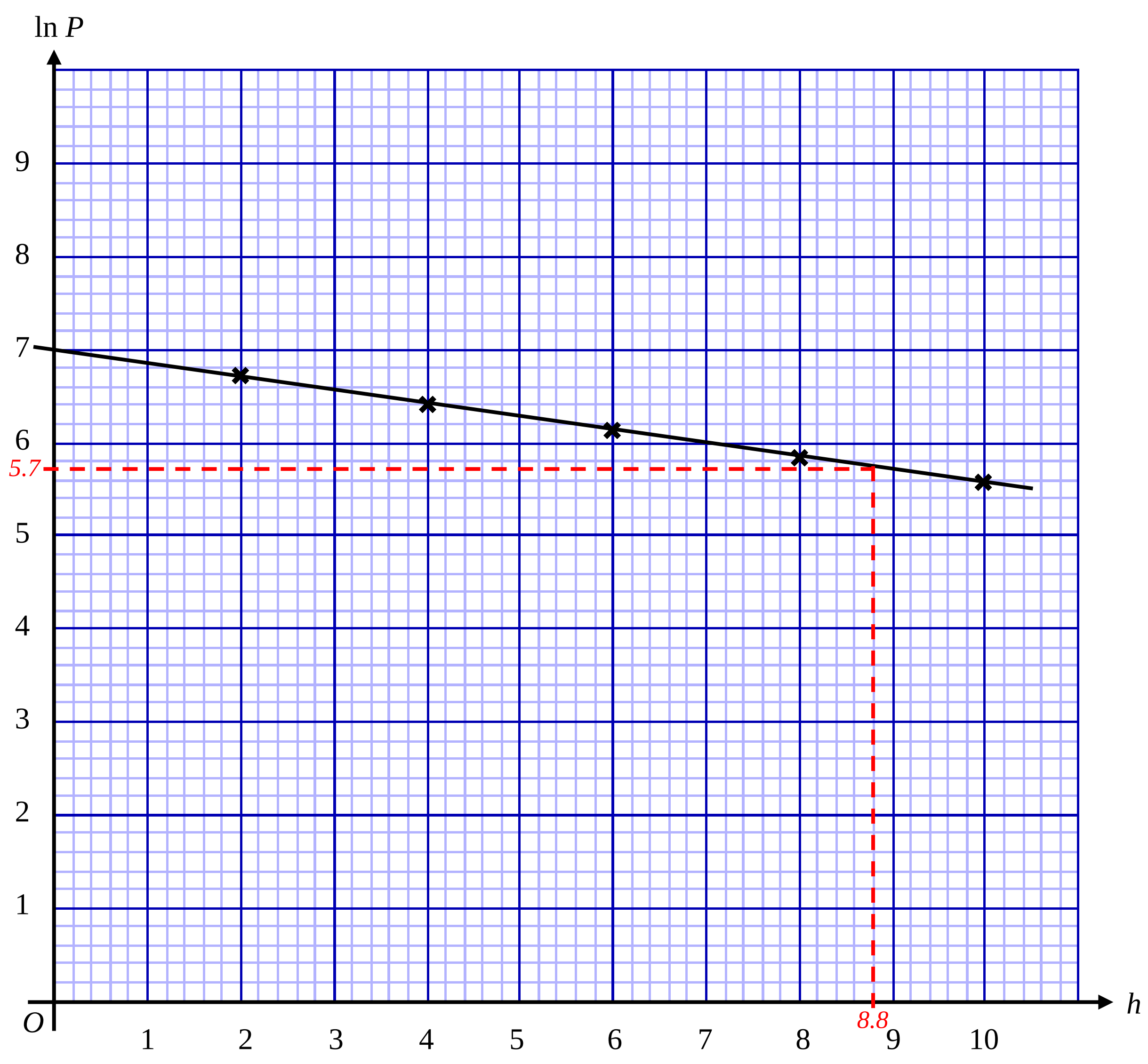
The atmospheric pressure, P millibars, exerted at the altitude h kilometres is related by the equation $P = Ae^{bh}$, where A and b are constants.

The following table shows the mean atmospheric pressure at various altitudes.

h (kilometres)	2	4	6	8	10
P (millibars)	810	595	446	340	262

(a) Plot $\ln P$ against h and draw a straight line graph to illustrate the information. [2]

h	2	4	6	8	10
$\ln P$	6.70	6.39	6.10	5.83	5.57



B1 (plot at least 3 correct points) B1 (best fit line)

(b) Express the equation $P = Ae^{bh}$ in a form that will yield the straight line graph in **part (a)**.
Hence explain how the graph may be used to determine the value of A and of b. [3]

$P = Ae^{bh}$
$\ln P = \ln Ae^{bh}$
$\ln P = \ln A + \ln e^{bh}$
$\ln P = bh + \ln A$ B1

The value of A can be determined by finding the **vertical intercept** of the graph. B1

The value of b can be determined by finding the **gradient** of the graph. B1

(c) Use your graph to estimate the atmospheric pressure, to the nearest millibar, when an object is at sea level. [1]

At sea level, $h = 0$,
$\ln P = 7$
$P = e^7 = 1096.633 \approx 1097$ millibars (nearest whole) B1

(d) The atmospheric pressure at the summit of Mount Everest is 300 millibars.
Use your graph to estimate the altitude of Mount Everest. [1]

When $P = 300$,
$\ln P = \ln 300 = 5.70$

From the graph,
$h = 8.8$ km B1

10 A patient's blood pressure, $\mathrm{P}(t)$ in mmHg, can be modelled by the function

$$\mathrm{P}(t) = 22\cos(2.5\pi t) + 116,$$

where t is the time in seconds.

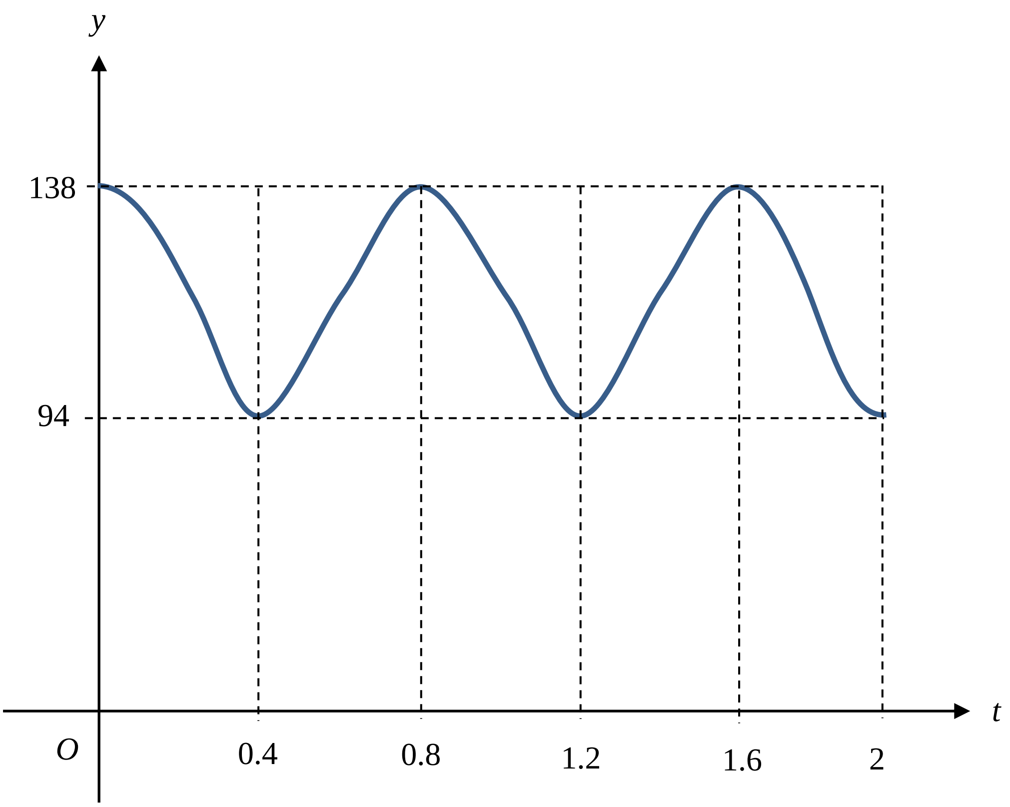
The systolic pressure (highest pressure) occurs when the heart beats, and the diastolic pressure (lowest pressure) occurs when the heart is at rest between beats.

(a) State the amplitude and period of $\mathrm{P}(t) = 22\cos(2.5\pi t) + 116$. [2]

Amplitude = 22 B1

Period = $\dfrac{2\pi}{2.5\pi} = 0.8$ B1

(b) Sketch the graph of $y = \mathrm{P}(t)$ for $0 \le t \le 2$. [2]



B1 (correct shape) B1 (correct amplitude & period)

(c) The pulse rate is the number of times a heart beats per minute.
A normal resting pulse rate should be between 60 to 100 beats per minute.
Show that the patient's pulse rate is normal. [2]

Since the duration of 1 heart beat is 0.8 sec, M1 Patient's pulse rate = $\frac{60}{0.8} = 75$ beats per minute Hence the patient's pulse rate is normal. } AG1

(d) According to health guidelines, someone with systolic pressure above 140 mmHg or diastolic pressure above 90 mmHg has high blood pressure and should see a doctor.
Determine whether the patient needs to see a doctor. Justify your answer. [1]

Since the **diastolic pressure** (94 mmHg) is **above 90 mmHg**, the patient has high blood pressure and **should see the doctor**. B1

11 A particle moves in a straight line so that t seconds after passing through a fixed point O, its velocity v m/s is given by $v = 5\cos\left(\frac{t}{2}\right)$. Find

(a) the initial velocity of the particle, [1]

Initial velocity = $5\cos\left(\frac{0}{2}\right) = 5$ m/s	B1

(b) the value of t, in terms of π, when the particle first comes to instantaneous rest, [3]

At instantaneous rest, $5\cos\left(\frac{t}{2}\right) = 0$	M1
$\frac{t}{2} = \cos^{-1}(0) = \frac{\pi}{2}$	M1
$t = \pi$ s	A1

(c) the distance travelled by the particle in the first 5 seconds, after passing through O. [4]

(a)

$s = \int 5\cos\left(\frac{t}{2}\right)dt = \frac{5\sin\left(\frac{t}{2}\right)}{\frac{1}{2}} + c$	M1
$s = 10\sin\left(\frac{t}{2}\right) + c$	
When $t = 0$, $s = 0$, $\Rightarrow c = 0$	M1
When $t = \pi$, $s = 10\sin\left(\frac{\pi}{2}\right) = 10$	M1(seen either one)
When $t = 5$, $s = 10\sin\left(\frac{5}{2}\right) = 5.984$	
Distance $= 10 + (10 - 5.984)$	
$= 14.016$	
≈ 14.0 m	A1

12 A curve has the equation $y=3+\left(\frac{x}{2}-1\right)^4$. The point (p,q) is the stationary point on the curve.

(a) Determine the coordinates of the stationary point (p, q). [4]

$y=3+\left(\frac{x}{2}-1\right)^4$

$\frac{dy}{dx}=4\left(\frac{x}{2}-1\right)^3\bullet\frac{1}{2}=2\left(\frac{x}{2}-1\right)^3$ M1 (find 1st derivative)

Let $\frac{dy}{dx}=0$,

$2\left(\frac{x}{2}-1\right)^3=0$

$\frac{x}{2}-1=0$ M1 (equate to zero and attempt to find x)

$x=2$

$\Rightarrow y=3$

Stationary point = $(2, 3)$ A1, A1 (correct pair of coordinates)

(b) **(i)** Justify whether y is increasing or decreasing for values of x less than p. [2]

For $x<2$,

$\left(\frac{x}{2}-1\right)^3<0$

$\frac{dy}{dx}=2\left(\frac{x}{2}-1\right)^3<0$

} M1 (use $\left(\frac{x}{2}-1\right)^3<0$ to show $dy/dx > 0$)

Therefore, y is **<u>decreasing</u>** when $x < 2$. A1

(ii) Hence infer whether y is increasing or decreasing for values of x greater than p. [1]

For $x>2$,

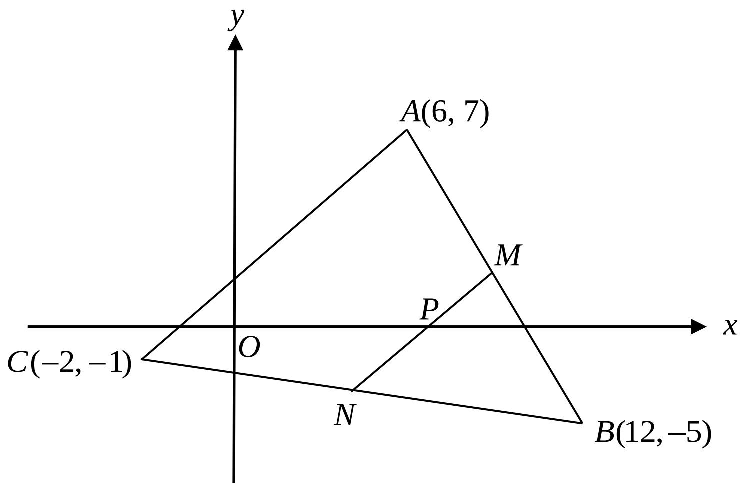
$\frac{dy}{dx}=\left(\frac{x}{2}-1\right)^3>0$

Therefore, y is **<u>increasing</u>** when $x > 2$. B1

(c) What do the results of **part (b)** imply about the stationary point? [1]

The stationary point is a **<u>minimum point</u>**. B1

13 Solutions to this question by accurate drawing will not be accepted.



The diagram above shows a triangle *ABC* with vertices at $A(6, 7)$, $B(12, -5)$ and $C(-2, -1)$. *M* and *N* are the mid-points of *AB* and *BC* respectively. The line *MN* cuts the *x*-axis at *P*.

(a) Find the coordinates of *P*. [4]

$M = \left(\frac{12+6}{2}, \frac{-5+7}{2}\right) = (9,1)$ and $N = \left(\frac{12+(-2)}{2}, \frac{-5+(-1)}{2}\right) = (5,-3)$ M1

gradient of $MN = \frac{1-(-3)}{9-5} = 1$ M1 (apply gradient formula)

Let $P = (x, 0)$, gradient of $NP = \frac{0-(-3)}{x-5} = 1$ M1 (find *x*)

$\Rightarrow x = 8$

$\therefore P = (8,0)$ A1

(b) Find the ratio *AC* : *MN*. [1]

$AC : MN = 2:1$ B1

(c) Find the area of the quadrilateral *ACNM*. [2]

Area of trapezium *ACNM* $= \frac{1}{2}\begin{vmatrix} 6 & -2 & 5 & 9 & 6 \\ 7 & -1 & -3 & 1 & 7 \end{vmatrix}$

$= \frac{1}{2}[-6+6+5+63-(-14)-(-5)-(-27)-6]$ M1

$= 54 \text{ units}^2$ A1

(d) Explain why quadrilateral *ACNM* is a trapezium. [2]

By midpoint theorem, $AC // MN$ **<u>OR</u>** $\text{gradient}_{AC} = \text{gradient}_{MN} = 1 \Rightarrow AC // MN$ M1

Since quadrilateral *ACNM* has **one pair of parallel sides**, it is a trapezium. AG1

Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS **4049/02**
Paper 2

21 August 2023 Monday 2 hours 15 mins

2023 SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC) PRELIMINARY EXAMINATIONS

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

INSTRUCTIONS TO CANDIDATES
Write your name, index number and class in the spaces provided above.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided below the questions.
Give non-exact numerical answers correct to 3 significant figures or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use														
Qn	1	2	3	4	5	6	7	8	9	10				*Marks Deducted*
Marks														

Category	Accuracy	Units	Symbols	Others
Question No.				

TOTAL MARKS
/ 90

Setter: Tan Chee Wee
Vetter: Tan Lip Sing

This question paper consists of **21** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

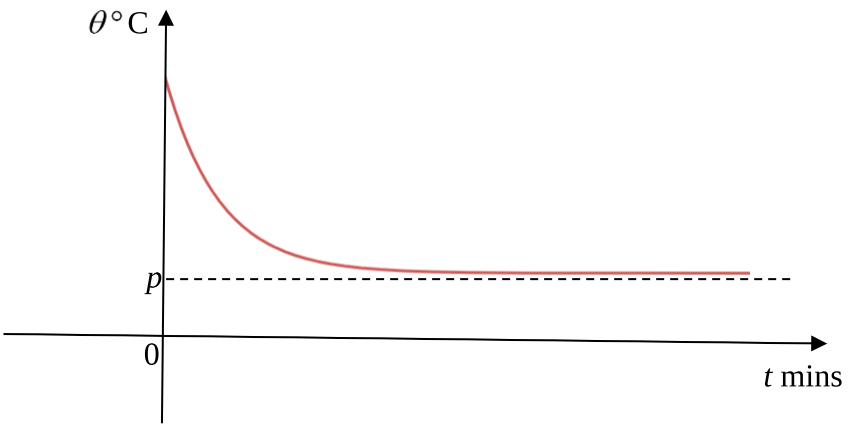
1 An object is heated in an oven until it reaches a temperature of X °C. It is then allowed to cool. Its temperature, θ °C, when it has cooled for time t minutes, is given by $\theta = 30 + 100(0.8)^{\frac{t}{6}}$.

(a) Find the value of X. [1]

(b) Find the value of θ when $t = 8$. [1]

(c) Find the value of t when $\theta = 95$. [3]

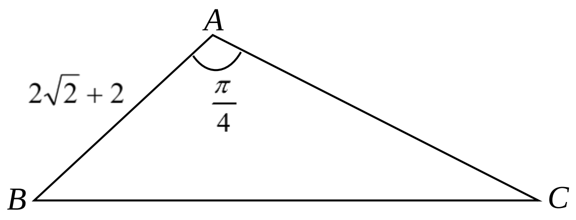
(d) A sketch of the graph of θ against t is given below.



State the value of p. [1]

A calculator must not be used in this question.

2 **(a)** In the diagram, triangle *ABC* has an area of $\left(8\sqrt{2}+4\right)$ cm^2, angle $BAC = \frac{\pi}{4}$ radian and $AB = \left(2\sqrt{2}+2\right)$ cm. Find the length of *AC*, leaving your answer in the form $\left(p\sqrt{2}+q\right)$ cm, where *p* and *q* are integers. [5]



(b) Find $\cos 75°$, giving your answer in the form $\dfrac{\sqrt{a}-\sqrt{b}}{4}$, where a and b are integers. [3]

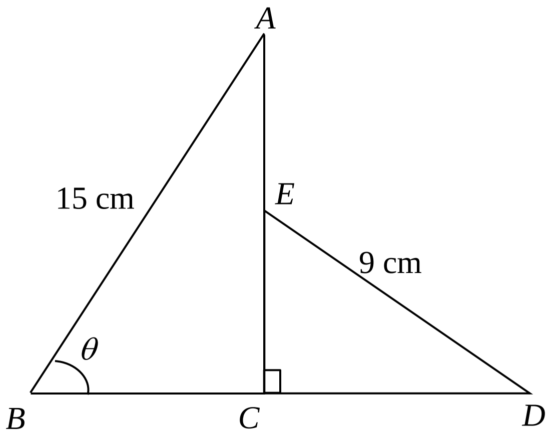
3 **(a)** Prove that $\operatorname{cosec} 2x - \cot 2x = \tan x$. [3]

(b) Hence solve $\operatorname{cosec}2x - \cot 2x = 2\sec^2 x - 3$ for $0° \le x \le 360°$. [5]

4 **(a)** Solve $9^x + 5 = 2\left(3^{x+1}\right)$. [5]

(b) Solve $2\log_4\left[\log_{100}(x^2+9)-\log_{100}x\right]=-1$. [5]

5 The diagram shows a quadrilateral *ABCDE* where triangle *ABC* is similar to triangle *DEC*. *AB* = 15 cm, *DE* = 9 cm, angle *ACD* = 90 ° and angle *ABC* is a variable angle θ, where 0 ° < θ < 90 °.

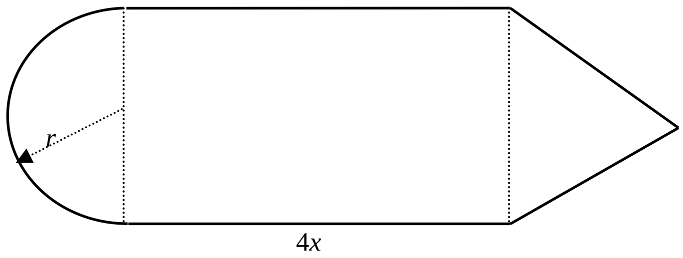


(a) Show that the perimeter, *P* cm, of the quadrilateral is given by $P = 24 + 24\sin\theta + 6\cos\theta$. [4]

(b) Express P in the form $R\sin(\theta+\alpha)+k$. [4]

(c) Find the value of θ when the perimeter is 38 cm. [2]

6 A piece of wire 60 cm long is bent to form the shape shown in the figure. This shape consists of a semi-circular arc, radius, r cm, and an equilateral triangle on the opposite ends of a rectangle of length $4x$ cm.



(a) Express x in term of r. [2]

(b) Hence show that the area enclosed, A cm^2, is given by

$$A = 60r + r^2(\sqrt{3} - 4 - \frac{\pi}{2}).$$ [3]

(c) Calculate the value of r for which A has a stationary value. Find this value of A and determine whether it is a maximum or a minimum. [5]

7 The equation of the curve is $y=(2x+1)\left(\sqrt{x-3}\right)$.

(a) Show that $\frac{dy}{dx}$ can be written in the form $\frac{6x-11}{2\sqrt{x-3}}$. [4]

(b) A particle moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 3 units per second. Find the rate of change of y when $x = 7$. [2]

(c) Use the result from **(a)** to evaluate $\int_4^7 \frac{3(6x-11)}{\sqrt{x-3}}\,dx$. [4]

8 **(a)** Factorise $x^3 - 27k^3$ as a product of a linear and a quadratic factor. [2]

(b) Factorise $x^2 - (3k - 1)x - 3k$. [1]

(c) The equation $x^3 - 27k^3 = x^2 - (3k - 1)x - 3k$ has only 1 real root. Find the set of values of the constant k. [6]

9 The equation of the circle, *C*, is $x^2 + y^2 - 6x + 10y - 66 = 0$.

(a) Find the coordinates of the centre of *C* and the radius of *C*. [4]

(b) Write down an equation of a vertical tangent to the circle. [1]

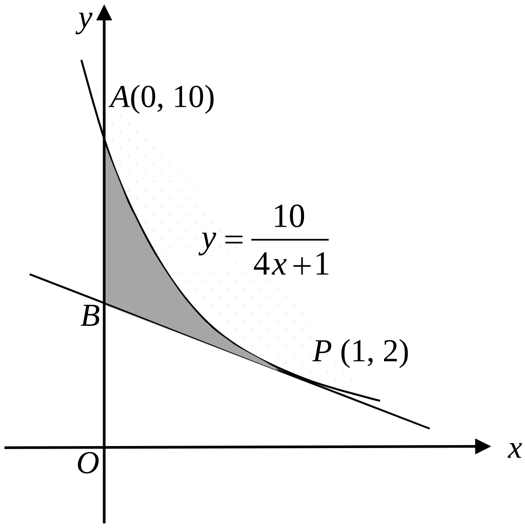
The point $A(-5, 1)$ lies on the circle.

(c) Find the equation of the tangent to the circle at point A. [3]

(d) AB is the diameter of the circle and P is the point (0, 6). Explain why the angle APB is an acute angle. [2]

10 The diagram shows part of the curve $y = \dfrac{10}{4x+1}$ intersecting the y-axis at $A(0, 10)$.

The tangent to the curve at the point $P(1, 2)$ intersects the y-axis at B.



(a) Show that the coordinates of B is $(0, 3.6)$. [4]

(b) Find the **exact** area of the shaded region. [5]

END OF PAPER

Answer Key

1a	130
b	104
c	11.6
d	30
2a	$12\sqrt{2}-8$
b	$\frac{\sqrt{6}-\sqrt{2}}{4}$
3b	$45°, 153.4°, 225°, 333.4°$
4a	$0, 1.46$
b	$1, 9$
5b	$\sqrt{612}\sin(\theta+14.0°)+24$
c	$20.4°$
6a	$x=\frac{60-4r-\pi r}{8}$
c	$r=7.82$, $A=234$, A is a maximum value.
7b	23.25 unit/s
7c	126
8a	$(x-3k)\left(x^2+3kx+9k^2\right)$
b	$(x+1)(x-3k)$
c	$k<-\frac{5}{9}$ or $k>\frac{1}{3}$
9a	$(3,-5)$, 10 unit
b	$x=-7$ or $x=13$
c	$3y=4x+23$
10b	$\frac{5}{2}\ln 5-2.8$ unit ^{2}

Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS **4049/02**
Paper 2

21 August 2023 Monday 2 hrs 15 min

MARKING SCHEME

Setter: Tan Chee Wee
Vetter: Tan Lip Sing

This question paper consists of **18** printed pages and **0** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

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Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 An object is heated in an oven until it reaches a temperature of X °C. It is then allowed to cool. Its temperature, θ °C, when it has cooled for time t minutes, is given by $\theta = 30 + 100(0.8)^{\frac{t}{6}}$.

(a) Find the value of X. [1]

$X = 30 + 100(0.8)^{\frac{0}{6}}$

$X = 130$ B1

(b) Find the value of θ when $t = 8$. [1]

$\theta = 30 + 100(0.8)^{\frac{8}{6}}$

$\theta = 104$ B1

(c) Find the value of t when $\theta = 95$. [3]

$95 = 30 + 100(0.8)^{\frac{t}{6}}$

$65 = 100(0.8)^{\frac{t}{6}}$

$(0.8)^{\frac{t}{6}} = 0.65$ M1

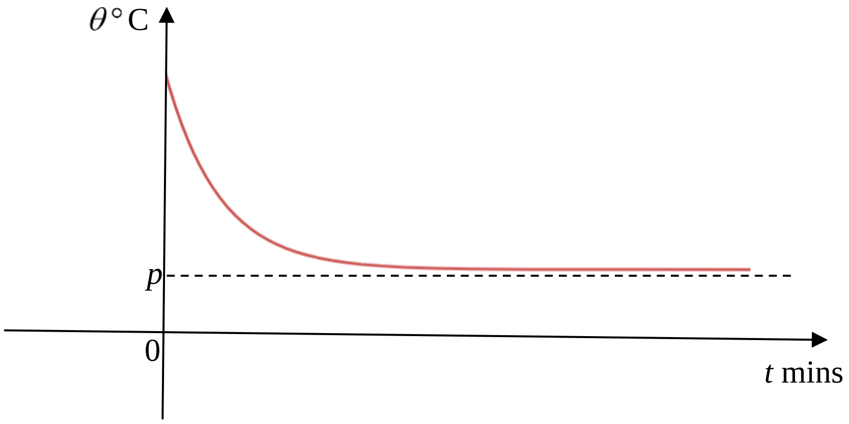
$\lg(0.8)^{\frac{t}{6}} = \lg 0.65$

$\frac{t}{6}\lg(0.8) = \lg 0.65$ M1

$t = \frac{6\lg 0.65}{\lg 0.8}$

$t = 11.6$ A1

(d) A sketch of the graph of θ against t is given below.

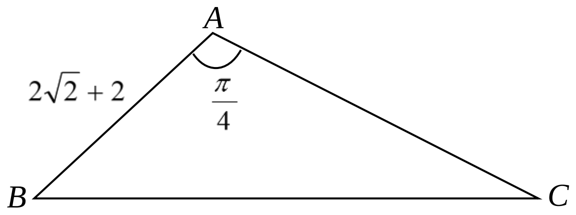


State the value of p. [1]

$p = 30$ B1

A calculator must not be used in this question.

2 **(a)** In the diagram, triangle *ABC* has an area of $\left(8\sqrt{2}+4\right)$ cm², angle $BAC = \dfrac{\pi}{4}$ radian and $AB = \left(2\sqrt{2}+2\right)$ cm. Find the length of *AC*, leaving your answer in the form $\left(p\sqrt{2}+q\right)$ cm, where *p* and *q* are integers. [5]



Area = $\dfrac{1}{2}\times AB\times AC\times\sin\angle BAC$

$8\sqrt{2}+4=\dfrac{1}{2}\left(2\sqrt{2}+2\right)\left(AC\right)\left(\dfrac{\sqrt{2}}{2}\right)$ M1

$8\sqrt{2}+4=\dfrac{1}{2}\left(2+\sqrt{2}\right)\left(AC\right)$

$16\sqrt{2}+8=\left(2+\sqrt{2}\right)\left(AC\right)$

$AC=\dfrac{16\sqrt{2}+8}{2+\sqrt{2}}$ M1

$=\dfrac{16\sqrt{2}+8}{2+\sqrt{2}}\times\dfrac{2-\sqrt{2}}{2-\sqrt{2}}$ M1

$=\dfrac{32\sqrt{2}-32+16-8\sqrt{2}}{4-2}$ M1

$=\dfrac{24\sqrt{2}-16}{2}$

$=12\sqrt{2}-8$ A1

(b) Find $\cos 75°$, giving your answer in the form $\dfrac{\sqrt{a}-\sqrt{b}}{4}$, where *a* and *b* are integers. [3]

$\cos 75° = \cos(30°+45°)$

$\cos 75° = \cos 30°\cos 45° - \sin 30°\sin 45°$ M1

$\cos 75° = \dfrac{\sqrt{3}}{2}\times\dfrac{\sqrt{2}}{2}-\dfrac{1}{2}\times\dfrac{\sqrt{2}}{2}$ M1

$\cos 75° = \dfrac{\sqrt{6}-\sqrt{2}}{4}$ A1

3 **(a)** Prove that $\operatorname{cosec}2x-\cot 2x=\tan x$. [3]

$$\cos ec2x-\cot 2x=\frac{1}{\sin 2x}-\frac{\cos 2x}{\sin 2x}$$

$$\cos ec2x-\cot 2x=\frac{1-\cos 2x}{\sin 2x}$$ M1

$$\cos ec2x-\cot 2x=\frac{2\sin^2 x}{2\sin x\cos x}$$ M1 for either formula

$$\cos ec2x-\cot 2x=\frac{\sin x}{\cos x}$$

$$\cos ec2x-\cot 2x=\tan x$$ AG1

(a) Hence solve $\operatorname{cosec}2x-\cot 2x=2\sec^2 x-3$ for $0°\le x\le 360°$. [5]

$\cos ec2x-\cot 2x=2\sec^2 x-3$

$\tan x=2\sec^2 x-3$

$\tan x=2\left(1+\tan^2 x\right)-3$ M1

$\tan x=2+2\tan^2 x-3$

$2\tan^2 x-\tan x-1=0$ M1

$(2\tan x+1)(\tan x-1)=0$

$\tan x=-0.5$ or $\tan x=1$ M1

Basic angle $=26.6°$ or $=45°$

$x=180-26.6°, 360°-26.6°$ $x=45°, 180°+45°$

$x=45°, 153.4°, 225°, 333.4°$ A1, A1

4 **(a)** Solve $9^x + 5 = 2\left(3^{x+1}\right)$. [5]

$3^{2x} + 5 = 2\left(3^x \times 3\right)$

Let $u = 3^x$

$u^2 + 5 = 6u$ M1

$u^2 - 6u + 5 = 0$

$(u-1)(u-5) = 0$

$u = 1$ or $u = 5$ M1

$3^x = 1$ or $3^x = 5$

$x = 0$ $x = \dfrac{\lg 5}{\lg 3}$ M1

$x = 0$ $x = 1.46$ A1, A1

(b) Solve $2\log_4\left[\log_{100}(x^2+9)-\log_{100}x\right]=-1$. [5]

$$2\log_4\left[\log_{100}(x^2+9)-\log_{100}x\right]=-1$$

$$\log_4\left[\log_{100}(x^2+9)-\log_{100}x\right]=-\frac{1}{2}$$

$$\left[\log_{100}(x^2+9)-\log_{100}x\right]=4^{-\frac{1}{2}}$$ M1

$$\log_{100}\frac{x^2+9}{x}=\frac{1}{2}$$ M1 quotient law

$$\frac{x^2+9}{x}=100^{\frac{1}{2}}$$

$$\frac{x^2+9}{x}=10$$ M1

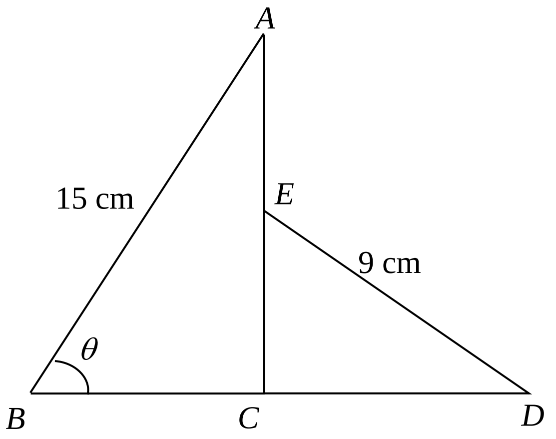
$$x^2+9=10x$$

$$x^2-10x+9=0$$ M1

$$(x-1)(x-9)=0$$

$x=1$ or $x=9$ A1

5 The diagram shows a quadrilateral *ABCDE* where triangle *ABC* is similar to triangle *DEC*. *AB* = 15 cm, *DE* = 9 cm, angle *ACD* = 90 ^{o} and angle *ABC* is a variable angle θ, where $0^{o} < \theta < 90^{o}$.



(a) Show that the perimeter, *P* cm, of the quadrilateral is given by $P = 24 + 24\sin\theta + 6\cos\theta$. [4]

In ΔABC, $\cos\theta = \dfrac{BC}{15}$ M1 either

$BC = 15\cos\theta$

$\sin\theta = \dfrac{AC}{15}$

$AC = 15\sin\theta$

In ΔDCE, $\cos\theta = \dfrac{EC}{9}$ M1either

$EC = 9\cos\theta$

$\sin\theta = \dfrac{DC}{9}$

$DC = 9\sin\theta$

Therefore $P = 15 + AE + 9 + DB$

$P = 24 + 15\sin\theta - 9\cos\theta + 9\sin\theta + 15\cos\theta$ M1

$P = 24 + 24\sin\theta + 6\cos\theta$ (shown) a.g. A1

(b) Express P in the form $R\sin(\theta+\alpha)+k$. [4]

$24\sin\theta+6\cos\theta=R\sin(\theta+\alpha)$

$R=\sqrt{6^2+24^2}=\sqrt{612}\ or\ 6\sqrt{17}$ M1

$\tan\alpha=\dfrac{6}{24}$ M1

$\alpha=14.0°$ M1

$P=\sqrt{612}\sin(\theta+14.0°)+24$ A1

(c) Find the value of θ when the perimeter is 38 cm. [2]

$24+\sqrt{612}\sin(\theta+14.03°)=38$

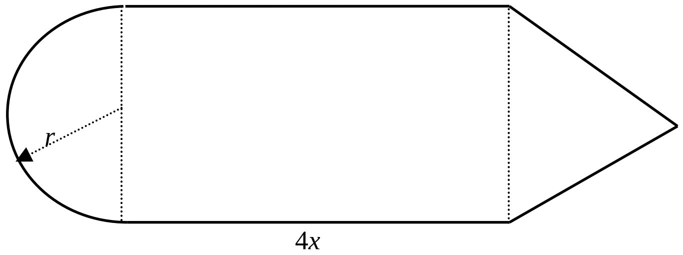
$\sqrt{612}\sin(\theta+14.03°)=14$ M1

$\sin(\theta+14.03°)=\dfrac{14}{\sqrt{612}}$

$\theta+14.03°=34.46°$

$\theta=20.4°$ (1 d.p.) A1

6 A piece of wire 60 cm long is bent to form the shape shown in the figure. This shape consists of a semi-circular arc, radius, r cm, and an equilateral triangle on the opposite ends of a rectangle of length $4x$ cm.



(a) Express x in term of r. [2]

$$2(2r)+2(4x)+\pi r=60$$ M1

$$4r+\pi r+8x=60$$

$$8x=60-4r-\pi r$$

$$x=\frac{60-4r-\pi r}{8}$$ A1

(b) Hence show that the area enclosed, A cm^2, is given by

$$A=60r+r^2(\sqrt{3}-4-\frac{\pi}{2})$$ [3]

$$A=\frac{1}{2}\times 2r\times 2r\times \sin 60°+4x\times 2r+\frac{1}{2}\pi r^2$$ M1 for 2 areas, M2 for all 3 areas

$$A=2r^2\times\frac{\sqrt{3}}{2}+8r\times\frac{60-4r-\pi r}{8}+\frac{1}{2}\pi r^2$$

$$A=\sqrt{3}r^2+60r-4r^2-\pi r^2+\frac{1}{2}\pi r^2$$

$$A=60r+\sqrt{3}r^2-4r^2-\frac{1}{2}\pi r^2$$

$$A=60r+r^2(\sqrt{3}-4-\frac{\pi}{2})\ \text{(shown)}$$ AG1

(c) Calculate the value of r for which A has a stationary value. Find this value of A and determine whether it is a maximum or a minimum. [5]

$$\frac{dA}{dr} = 60 + 2r(\sqrt{3} - 4 - \frac{\pi}{2})$$ M1

$$\frac{dA}{dr} = 0 \Rightarrow 60 + 2r(\sqrt{3} - 4 - \frac{\pi}{2}) = 0$$ M1

$$2r(\sqrt{3} - 4 - \frac{\pi}{2}) = -60$$

$$r = \frac{-60}{2(\sqrt{3} - 4 - \frac{\pi}{2})}$$

$r = 7.82$ cm A1

$$A = 60(7.815) + (7.815)^2(\sqrt{3} - 4 - \frac{\pi}{2})$$

$A = 234\ cm^2$ A1

$$\frac{d^2A}{dr^2} = 2(\sqrt{3} - 4 - \frac{\pi}{2}) < 0$$

Therefore, the area is maximum

} A1

7 The equation of the curve is $y=(2x+1)\left(\sqrt{x-3}\right)$.

(a) Show that $\frac{dy}{dx}$ can be written in the form $\frac{6x-11}{2\sqrt{x-3}}$. [4]

$\frac{dy}{dx}=(2x+1)\times\frac{1}{2}(x-3)^{-\frac{1}{2}}+(x-3)^{\frac{1}{2}}(2)$ M1, M1

$\frac{dy}{dx}=\frac{1}{2}(x-3)^{-\frac{1}{2}}\left((2x+1)+4(x-3)\right)$

$\frac{dy}{dx}=\frac{1}{2}(x-3)^{-\frac{1}{2}}\left(2x+1+4x-12\right)$ M1

$\frac{dy}{dx}=\frac{1}{2}(x-3)^{-\frac{1}{2}}\left(6x-11\right)$

$\frac{dy}{dx}=\frac{6x-11}{2\sqrt{x-3}}$ AG1

(b) A particle moves along the curve in such a way that the *x*-coordinate is increasing at a constant rate of 3 units per second. Find the rate of change of *y* when $x = 7$. [2]

$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$

$\frac{dy}{dt}=\frac{6(7)-11}{2\sqrt{7-3}}\times3$ M1

$\frac{dy}{dt}=23.25$ unit/s A1

(c) Use the result from **(a)** to evaluate $\int_4^7 \frac{3(6x-11)}{\sqrt{x-3}}\,dx$. [4]

$\int_4^7 \frac{(6x-11)}{2\sqrt{x-3}}\,dx = \left[(2x+1)\sqrt{x-3}\right]_4^7$ M1

$6\int_4^7 \frac{(6x-11)}{2\sqrt{x-3}}\,dx = 6\left[(2x+1)\sqrt{x-3}\right]_4^7$ M1

$\int_4^7 \frac{3(6x-11)}{\sqrt{x-3}}\,dx = 6\left[(2(7)+1)\sqrt{7-3}-(2(4)+1)\sqrt{4-3}\right]$ M1

$\int_4^7 \frac{3(6x-11)}{\sqrt{x-3}}\,dx = 126$ A1

8 **(a)** Factorise $x^3 - 27k^3$ as a product of a linear and a quadratic factor. [2]

$(x-3k)\left(x^2+3kx+9k^2\right)$ B1, B1

(b) Factorise $x^2-(3k-1)x-3k$. [1]

$(x+1)(x-3k)$ B1

(c) The equation $x^3-27k^3=x^2-(3k-1)x-3k$ has only 1 real root. Find the set of values of the constant *k*. [6]

$x^3-27k^3=x^2-(3k-1)x-3k$

$(x-3k)\left(x^2+3kx+9k^2\right)=(x+1)(x-3k)$ M1

$(x-3k)\left(x^2+3kx+9k^2\right)-(x+1)(x-3k)=0$

$(x-3k)\left(x^2+(3k-1)x+9k^2-1\right)=0$ M1

Since only 1 real root

$(3k-1)^2-4(9k^2-1)<0$ M1

$9k^2-6k+1-36k^2+4<0$

$-27k^2-6k+5<0$ M1

$27k^2+6k-5>0$

$(9k+5)(3k-1)>0$ M1 for $-\frac{5}{9}$ *and* $\frac{1}{3}$ seen

$k<-\frac{5}{9}$ *or* $k>\frac{1}{3}$ A1

9 The equation of the circle, C, is $x^2 + y^2 - 6x + 10y - 66 = 0$.

(a) Find the coordinates of the centre of C and the radius of C. [4]

Centre $= \left(\frac{-6}{-2}, \frac{10}{-2}\right)$ M1

Centre is $(3, -5)$ A1

Radius $= \sqrt{(3)^2 + (-5)^2 - (-66)}$ M1

$= 10$ units A1

(b) Write down an equation of a vertical tangent to the circle. [1]

$x = -7$ or $x = 13$ B1

The point *A*(-5, 1) lies on the circle.

(c) Find the equation of the tangent to the circle at point *A*. [3]

$m = \dfrac{1-(-5)}{-5-(3)}$ M1

$m_{AB} = -\dfrac{3}{4}$

Gradient of tangent $m_{AB} = -\dfrac{1}{-\frac{3}{4}} = \dfrac{4}{3}$

$y - 1 = \dfrac{4}{3}(x - (-5))$ M1

$3y - 3 = 4x + 20$

$3y = 4x + 23$ A1

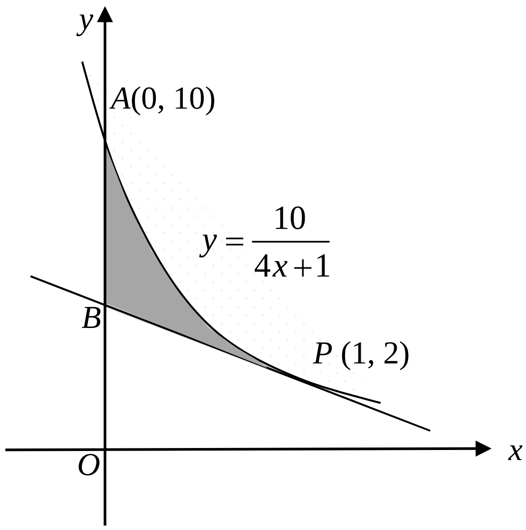
(d) *AB* is the diameter of the circle and *P* is the point (0, 6). Explain why the angle *APB* is an acute angle. [2]

Distance of *P* from centre $\sqrt{(3-0)^2 + (-5-6)^2} = \sqrt{130} > 10$ M1

P is outside the circle, angle *APB* is an acute angle A1

10 The diagram shows part of the curve $y = \dfrac{10}{4x+1}$ intersecting the y-axis at $A(0, 10)$.

The tangent to the curve at the point $P(1, 2)$ intersects the y-axis at B.



(a) Show the coordinates of B is (0 , 3.6). [4]

$y = \dfrac{10}{4x+1} = 10(4x+1)^{-1}$

$\dfrac{dy}{dx} = -10(4x+1)^{-2}(4)$ M1

$\dfrac{dy}{dx} = -40(4x+1)^{-2}$

When $x = 1$ $\dfrac{dy}{dx} = -40(4(1)+1)^{-2}$

$\dfrac{dy}{dx} = -1.6$ M1

$\dfrac{y-2}{0-1} = -1.6$ M1

$y - 2 = 1.6$

$y = 3.6$

Coordinate of B is (0, 3.6) AG1

(b) Find the **exact** area of the shaded region. [5]

$$Area = \int_0^1 \frac{10}{4x+1}\,dx - \frac{1}{2}(3.6+2)(1)$$ M1, M1

$$Area = \left[\frac{10\ln(4x+1)}{4}\right]_0^1 - 2.8$$ M1

$$Area = \left[\frac{10\ln(4+1)}{4} - \frac{10\ln(1)}{4}\right] - 2.8$$ M1

$$Area = \frac{5}{2}\ln 5 - 2.8 \text{ unit}^2$$ A1