Name: Register Number: Class:

For Marker's Use

90

南僑中學

NAN CHIAU HIGH SCHOOL

Parents' signature:

PRELIMINARY EXAMINATION 2023
SECONDARY FOUR EXPRESS

ADDITIONAL MATHEMATICS 4049/01
Paper 1 22 August 2023, Tuesday

Candidates answer on the Question Paper. **2 hours 15 minutes**

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 (a) Each side of an equilateral triangle measures $\left(\frac{1}{2+\sqrt{6}}\right)$ cm. Find, without using a calculator, the area of the equilateral triangle, in cm^2, in the form $(a\sqrt{3}+b\sqrt{2})$, where a and b are rational numbers. [4]

(b) Solve the equation $\sqrt{x+3} = \frac{1}{\sqrt{x+3}} - \frac{5}{6}$. [3]

2 (a) Differentiate $x^5 \ln x^2$ with respect to x, where $x > 0$. [2]

(b) Hence, find $\int x^4 \ln x \, dx$. [3]

3 (a) Express $-9 - 2x^2 - 4x$ in the form $a(x + b)^2 + c$ and hence state the maximum value of $-9 - 2x^2 - 4x$. [3]

(b) Hence, or otherwise, determine the range of values of k for which $k - 9 - 2x^2 - 4x = -2$ has two real and distinct roots. [2]

4 Express $\dfrac{3x^3+2}{(x+1)(x^2-1)}$ in partial fractions. [6]

5 (a) Solve the equation $\log_7 x + \log_{49} x^2 = 6$. [3]

(b) Given $y = \lg(x^2 + 8x + 15) - \lg(x + 4)$, state the range of values of x for which y exists. [2]

6 Given that $\sin\theta = c$ and $90° < \theta < 180°$,

(a) express $\sin 2\theta$ in terms of c, [2]

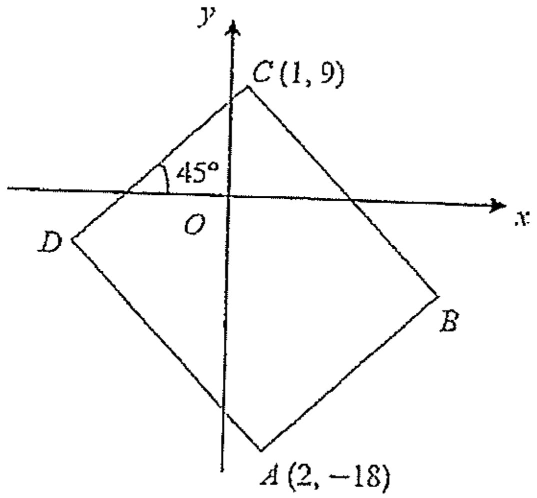
(b) express $\cos(\theta + 30°)$ in terms of c, [3]

(c) find the principal value of $\sin^{-1}(-c)$ in terms of θ. [1]

7 (a) Prove the identity $\dfrac{4\sin\theta+4\sin^2\theta}{\sec\theta+\tan\theta}=2\sin 2\theta$. [4]

(b) Hence, solve the equation $\frac{4\sin\theta+4\sin^2\theta}{\sec\theta+\tan\theta} = 0.7$ for $-\pi \leq \theta \leq \pi$. [3]

8 The diagram shows a rectangle $ABCD$ with A (2, −18) and C (1, 9). DC makes an angle of 45° with the positive direction of the x-axis.



(a) Show that the coordinates of B is (15, −5). [5]

(b) Given that E is a point on DB produced such that $DB = BE$, find the area of triangle ACE. [4]

9 A solid of height x cm has a volume of V cm^3. It has a uniform cross-sectional area given by $(px^2 + qx)$ cm^2, where p and q are constants.

Corresponding values of x and V are shown in the table below.

x	1	2	3	4
V	4	28	90	208

(a) Using suitable variables, draw, on the grid below, a straight line graph and hence estimate the value of p and of q. [5]

Continued working space for (a)

(b) Using your values of p and q obtained in (a), calculate the value of x for which the solid is a cylinder with height half the length of its base radius. [2]

(c) Explain how another straight line drawn on the same grid as (a) can lead to an estimated value of x for which the solid is a cylinder with height half its base radius.

Draw this line and hence verify your value of x found in part (b). [3]

10 Jean trains physically by running along a straight track. At time t seconds after leaving a fixed point O, its velocity v m/s is given by $v = 2.5\sin\left(\frac{1}{2}t\right)$. She makes the first turn back to O when she reaches point A.

(a) Find the acceleration when $t = \frac{\pi}{2}$. [2]

(b) Find the distance OA. [4]

When Jean returns to fixed point O for the fourth time from A, her subsequent velocity towards point A, v m/s is given by $v = 0.25t - 4\pi$ for $t > k$, where k is a constant.

(c) Show that $k = 16\pi$ and explain why Jean did not return to O subsequently. [2]

(d) Find the total distance covered during the first 60 seconds. [4]

11 The equation of a curve is $y = 3x^2e^{-\frac{1}{2}x}$.

(a) Find the range of values of x for which y is increasing. [4]

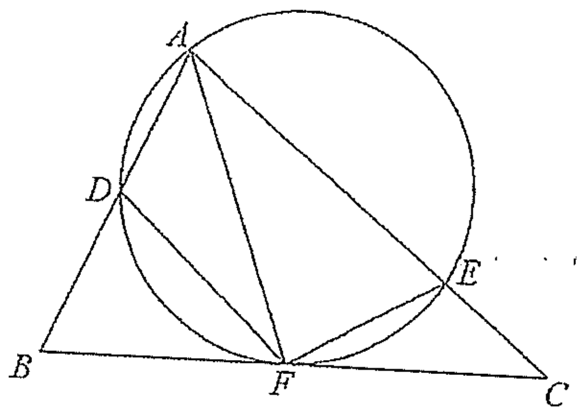
(b) Show that the origin (0, 0) is a minimum point of the curve. [2]

A point, P, lies on the curve such that the normal to the curve at P has a negative gradient.

(c) State the range of values of x-coordinates of P. [1]

(d) Find the equation of the normal at P, given that the x-coordinate of P is 1. [4]

12



In the diagram, the line BC is a tangent to the circle at F. The points A, D, E and F lie on the circumference of the circle. D and F are the midpoints of line AB and BC respectively.

(a) Show that $ADFE$ is a trapezium.

[2]

(b) Show that triangle DFA is similar to triangle EFC.

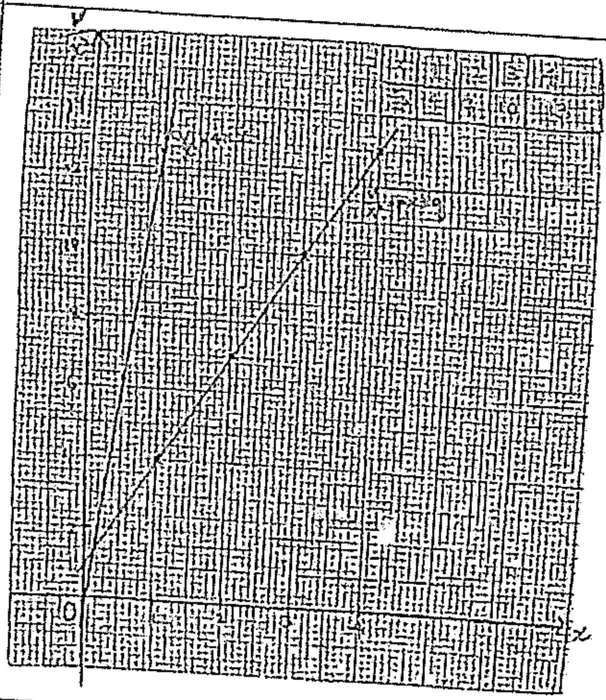
[3]

(c) Prove that $\frac{1}{2} \times AC \times EC = DA \times EF$.

[2]

End of paper

Answer Key:

1a	$\frac{5\sqrt{3}}{8}-\frac{3\sqrt{2}}{4}$
1b	$x=-\frac{23}{9}$
2a	$2x^4+10x^4 lnx$
2b	$\frac{(x^5 lnx)}{5}-\frac{x^5}{25}+c3$
3a	$-2(x+1)^2-7$, $max\ value = -7$
3b	$k>5$
4	$3+\frac{5}{4(x-1)}-\frac{17}{4(x+1)}+\frac{1}{2(x+1)^2}$
5a	343
5b	$x>-3$
6a	$-2c\sqrt{1-c^2}$
6b	$\frac{-1}{2}c-\frac{\sqrt{3-3c^2}}{2}$
6c	θ−180°
7a	As proven
7b	$\theta=0.179, 1.39, -1.75, -2.96$
8a	As shown
8b	546 $units^2$
9a	

	$\frac{y}{x^2} = px + q$ $gradient = p$ $= \frac{10-1}{3-0} = 3$ $q = 1$
9b	0.105
9c	$draw\ \frac{V}{x^2} = 4\pi x$ $From\ graph, x = 0.1$ *This is close to the x value obtained in part (b).*
10a	$\frac{0.884m}{s^2}\ or\ \frac{5\sqrt{2}}{8}m/s^2$
10b	$10m$
10c	As shown and any valid argument eg. Proving v>0
10d	$91.8m$
11a	$0 < x < 4$
11b	1st or 2nd derivative test or justifying components of expressions clearly, with proving of y-coordinate
11c	$0 < x < 4$
11d	$y = -\frac{2e^{\frac{1}{2}}}{9}x + \frac{2e^{\frac{1}{2}}}{9} + 3e^{-\frac{1}{2}}$ $or\ y = -0.366x + 2.19$
12a	Midpoint theorem
12b	$\angle EFC = \angle FAE (alt\ segment\ theorem)$ $\angle FAE = \angle AFD (alt\ \angle s, DF \parallel AE)$ $\therefore \angle EFC = \angle AFD$ $180° - \angle ADF = \angle FEA\ (\angle s\ in\ opp\ segment)$ $\angle FEC = 180° - \angle FEA (adj\ \angle s\ on\ a\ str.line)$ $= 180° - (180° - \angle ADF)$ $= \angle ADF$ $\therefore \Delta DFA$ *is similar to* ΔEFC *(AA Similarity test)*

12c	$\frac{DA}{EC} = \frac{DF}{EF}$ (corresponding sides of similar triangles are proportional) $DF \times EC = DA \times EF$ $By\ midpt\ theorem, DF = \frac{1}{2}AC$ $\frac{1}{2}AC \times EC = DA \times EF$

NCHS 2023 AM Prelim P1 solutions for students

Q1 (a)

Area of triangle

$=\frac{1}{2}\left(\frac{1}{2+\sqrt{6}}\right)^2 \sin 60°$

$=\frac{1}{2}\left(\frac{1}{4+6+4\sqrt{6}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$=\left(\frac{1}{10+4\sqrt{6}}\right)\left(\frac{\sqrt{3}}{4}\right)$

$=\left(\frac{1}{2}\right)\left(\frac{1}{5+2\sqrt{6}}\right)\left(\frac{\sqrt{3}}{4}\right)$

$=\left(\frac{1}{5+2\sqrt{6}}\right)\left(\frac{\sqrt{3}}{8}\right)$

$=\left(\frac{\sqrt{3}}{8}\right)\left(\frac{1}{5+2\sqrt{6}}\right)\left(\frac{(5-2\sqrt{6})}{5-2\sqrt{6}}\right)$

$=\left(\frac{\sqrt{3}}{8}\right)\left(\frac{5-2\sqrt{6}}{25-24}\right)$

$=\left(\frac{\sqrt{3}}{8}\right)\left(\frac{5-2\sqrt{6}}{1}\right)$

$=\frac{5\sqrt{3}}{8}-\frac{\sqrt{18}}{4}$

$=\frac{5\sqrt{3}}{8}-\frac{3\sqrt{2}}{4}$

Q1(b)

<u>Mtd 1</u>

$$let\ y = \sqrt{x+3}$$
$$y = \frac{1}{y}-\frac{5}{6}$$
$$y^2 = 1-\frac{5}{6}y$$
$$y^2-1+\frac{5}{6}y = 0$$
$$6y^2+5y-6 = 0$$
$$(3y-2)(2y+3) = 0$$
$$y = \frac{2}{3}\ or\ y = -\frac{3}{2}$$
$$\sqrt{x+3} = \frac{2}{3}\ or\ \sqrt{x+3} = -\frac{3}{2}\ (rej)$$
$$x+3 = \frac{4}{9}$$
$$x = -\frac{23}{9}$$

Or

$$6(x+3) = 6-5\sqrt{x+3}$$
$$6x+18-6 = -5\sqrt{x+3}$$
$$(6x+12)^2 = \left(-5\sqrt{x+3}\right)^2$$
$$36x^2+144x+144 = 25(x+3)$$
$$36x^2+144x+144 = 25x+75$$
$$36x^2+119x+69 = 0$$
$$(4x+3)(9x+23) = 0$$
$$x = -\frac{3}{4}\ (rej) or -\frac{23}{9}$$

Q2(a)

$$\frac{d}{dx}(x^5 lnx^2) = x^5\left(\frac{1}{x^2}\right)(2x)+5x\text{^}4lnx^2$$
$$= 2x^4+5x^4lnx^2 = 2x^4+10x^4lnx$$

2(b) Method 1

$$\int x^4 lnx\,dx = \frac{1}{10}\int 10x^4lnx+2x^4-2x^4dx$$
$$= \frac{1}{10}\left[x^5lnx^2-\frac{2x^5}{5}\right]+c$$
$$= \frac{1}{5}x^5lnx-\frac{1}{25}x^5+c$$

2(b) Method 2

$$\int 10x^4lnx+2x^4dx = x^5lnx^2+c$$
$$10\int x^4lnxdx+\int 2x^4dx = x^5lnx^2+c$$
$$10\int x^4lnxdx = x^5lnx^2+c-\frac{2x^5}{5}+c1$$
$$= x^5lnx^2-\frac{2x^5}{5}+c2$$
$$\int x^4lnxdx = \frac{(x^5lnx^2)}{10}-\frac{x^5}{25}+c3$$
$$or\ \frac{(x^5lnx)}{5}-\frac{x^5}{25}+c3$$

Q3(a)

$$-9-2x^2-4x$$
$$=-2x^2-4x-9$$
$$=-2(x^2+2x)-9$$
$$=-2[(x+1)^2-1]-9$$
$$=-2(x+1)^2-7$$

$$max\ value=-7$$

Q3(b)

Hence,

$$-9-2x^2-4x=-2-k$$
$$since \text{ max } =-7$$
$$-2-k<-7$$
$$k>5$$

Otherwise,

$$-2x^2-4x+k-7=0$$
$$discriminant>0$$
$$(-4)^2-4(-2)(k-7)>0$$
$$16+8k-56>0$$
$$8k-40>0$$
$$k>5$$

Q4

$$(x+1)(x^2-1)=x^3$$

$$\begin{array}{r} 3 \\ x^3+x^2-x-1\overline{)\,3x^3+0x^2+0x+2} \\ -(3x^3+3x^2-3x-3) \\ \hline -3x^2+3x+5 \end{array}$$

$$\frac{3x^3+2}{(x+1)(x^2-1)}=3+\frac{3x+5-3x^2}{(x+1)^2(x-1)}$$

$$\frac{3x+5-3x^2}{(x+1)^2(x-1)}=\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$$

$$3x+5-3x^2=A(x+1)^2+B(x-1)(x+1)$$
$$+c(x-1)$$

$$sub\ x=1,$$
$$3+5-3=A(2)^2$$
$$4A=5$$
$$A=\frac{5}{4}$$
$$sub\ x=-1,$$
$$-3+5-3=C(-2)$$
$$-1=-2C$$
$$C=\frac{1}{2}$$
$$comparing\ coefficient\ of\ x^2:-3=A+B$$
$$-3=\frac{5}{4}+B$$
$$B=-\frac{17}{4}$$
$$\therefore 3+\frac{5}{4(x-1)}-\frac{17}{4(x+1)}+\frac{1}{2(x+1)^2}$$

Q5(a) $\log_7 x + \frac{\log_7 x^2}{\log_7 7^2} = 6$ $\log_7 x + \frac{2\log_7 x}{2} = 6$ $\log_7 x + \log_7 x = 6$ $2\log_7 x = 6$ $\log_7 x = 3$ $x = 7^3$ $= 343$	Q5(b) $x^2 + 8x + 15 > 0 \; and \; x + 4 > 0$ $(x+5)(x+3) > 0 \; and \; x > -4$ $x > -3, x < -5 \; and \; x > -4$ $\therefore x > -3$
Q6(a) $\sin 2\theta = 2\sin\theta\cos\theta$ $= 2c(-\sqrt{1-c^2})$ $= -2c\sqrt{1-c^2}$ →	Q6(b) $\cos(\theta + 30°)$ $= \cos\theta\cos 30° - \sin\theta\sin 30°$ $= \left(-\sqrt{1-c^2}\right)\left(\frac{\sqrt{3}}{2}\right) - (c)\left(\frac{1}{2}\right)$ $= \frac{-1}{2}c - \frac{\sqrt{3-3c^2}}{2}$
Q6(c) $-(180° - \theta)$ or $\theta - 180°$	
Q7(a) $\frac{4\sin\theta + 4\sin^2\theta}{\sec\theta + \tan\theta}$ $= 4\sin\theta(1+\sin\theta) \div \left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)$ $= 4\sin\theta(1+\sin\theta) \div \left(\frac{1+\sin\theta}{\cos\theta}\right)$ $= 4\sin\theta(1+\sin\theta) \times \frac{\cos\theta}{1+\sin\theta}$ $= 4\sin\theta\cos\theta$ $= 2(2\sin\theta\cos\theta)$ $= 2\sin 2\theta$ →	Q7(b) $2\sin 2\theta = 0.7$ $\sin 2\theta = \frac{7}{20}$ $-\pi \le \theta \le \pi$ $-2\pi \le 2\theta \le 2\pi$ $\alpha = \sin^{-1}\frac{7}{20}$ $= 0.35757$ $2\theta \; is \; in \; the \; 1st \; and \; 2nd \; quadrant.$ $2\theta = 0.35757, \pi - 0.35757, -(\pi + 0.35757), -(2\pi - 0.35757)$ $\theta = 0.179, 1.39, -1.75, -2.96$

Q8(a)

$$gradient\ of\ DC = \tan 45° = 1$$

$$gradient\ of\ BC = -1$$

$$\frac{y-9}{x-1} = -1$$

$$y - 9 = -x + 1$$

Equation of BC: $y = -x + 10$

$$\frac{y+18}{x-2} = 1$$

$$y + 18 = x - 2$$

Equation of AB: $y = x - 20$

$$-x + 10 = x - 20$$

$$2x = 30$$

$$x = 15$$

$$y = -15 + 10 = -5 \quad \therefore B(15, -5)(shown)$$

Q8(b)

$$midpt\ AC = midpt\ DB$$

$$\left(\frac{2+1}{2}, \frac{9-18}{2}\right) = \left(\frac{x+15}{2}, \frac{-5+y}{2}\right)$$

$$3 = x + 15$$
$$x = -12$$
$$9 - 18 = -5 + y$$
$$y = -4$$
$$D(-12, -4)$$

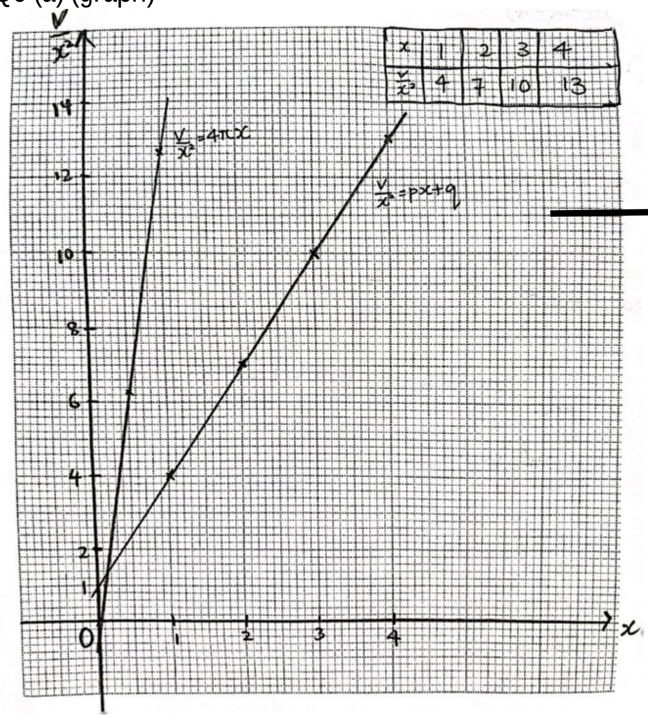
$$B\ is\ the\ midpt\ of\ DE$$

$$(15, -5) = \left(\frac{-12+x}{2}, \frac{-4+y}{2}\right)$$

$$30 = -12 + x$$
$$x = 42$$
$$-10 = -4 + y$$
$$y = -6$$
$$E(42, -6)$$

$$Area = \frac{1}{2}\begin{vmatrix} 2 & 42 & 1 & 2 \\ -18 & -6 & 9 & -18 \end{vmatrix}$$
$$= \frac{1}{2}(-12 + 378 - 18 - 18 + 6 + 758)$$
$$= 546\ units^2$$

Q9 (a) (graph)



$$v = x(px^2 + qx)$$

$$v = px^3 + qx^2$$

$$\frac{v}{x^2} = px + q$$

$$plotted\ points$$

$$join\ with\ a\ straight\ line$$

$$gradient = p$$

$$= \frac{10-1}{3-0} = 3$$

$$q = 1$$

Q9(b)

$$\frac{V}{x^2} = 3x + 1$$

$$V = x(3x^2 + x)$$

$$radius = 2x$$

$$base\ area = \pi(2x)^2 = 4\pi x^2$$

$$4\pi x^2 = 3x^2 + x$$

$$(4\pi - 3)x^2 - x = 0$$

$$x[(4\pi - 3)x - 1] = 0$$

$$x = \frac{1}{4\pi - 3} = 0.1045$$

$$= 0.105$$

Q9(c)

$$V = x(4\pi x^2)$$

$$= 4\pi x^3$$

$$draw\ \frac{V}{x^2} = 4\pi x$$

The volume of the cylinder is expressed in terms of x and then rearranged into the same linear form as the equation of the straight line drawn.

The x − coordinate of the intersection therefore gives the value of x for which the solid is a cylinder with height half its base radius.

x	0	0.5	1
$\frac{V}{x^2}$	0	6.3	12.6

$$From\ graph, x = 0.1$$

This is close to the x value obtained in part (b).

*verify means to confirm if something is accurate

Q10a

$$a = 2.5\cos\left(\frac{1}{2}t\right) \times \frac{1}{2}$$

$$= 1.25\cos\left(\frac{1}{2}t\right)\ or \frac{5}{4}\cos\left(\frac{1}{2}t\right)$$

$$when\ t = \frac{\pi}{2},$$

$$a = 1.25\cos\left(\frac{\pi}{2} \times \frac{1}{2}\right)$$

$$= 0.88388$$

$$= \frac{0.884m}{s^2 or}\ or\ \frac{5\sqrt{2}}{8} m/s^2$$

Q10b

$$when\ v = o,$$

$$2.5\sin\left(\frac{1}{2}t\right) = 0$$

$$\sin\left(\frac{1}{2}t\right) = 0$$

$$\frac{1}{2}t = \pi$$

$$t = 2\pi$$

$$s = -2.5\cos\left(\frac{1}{2}t\right) \times 2 + c$$

$$= -5\cos\left(\frac{1}{2}t\right) + c$$

$$when\ t = 0, s = 0,$$

$$0 = -5\cos(0) + c$$

$$c = 5$$

$$s = -5\cos\left(\frac{1}{2}t\right) + 5$$

$$when\ t = 2\pi,$$

$$s = -5\cos(\pi) + 5$$

$$= -5(-1) + 5 = 10m$$

Q10c

$$When\ Jean\ returns\ for\ the\ 4th\ time, s = 0$$

$$-5\cos\left(\frac{1}{2}t\right) + 5 = 0$$

$$\cos\left(\frac{1}{2}t\right) = 1$$

$$\frac{1}{2}t = 0, 2\pi, 4\pi, 6\pi, 8\pi$$

$When\ Jean\ returns\ for\ the\ 4th\ time, t = 2 \times 8\pi = 16\pi.$

$$\therefore k = 16\pi$$

$$When\ t > 16\pi,$$
$$0.25t > 4\pi$$
$$0.25 - 4\pi > 0$$
$$v > 0$$

$Jean\ will\ always\ be\ moving\ away\ from\ O\ when\ t > 16\pi$

$$or \frac{dv}{dt} = 0.25 > 0$$

$This\ implies\ velocity\ is\ an\ increasing\ function\ starting\ from\ v = 0\ at\ t = 16\pi\ so\ it\ will\ never\ become\ zero. Hence\ Jean\ will\ not\ be\ turning\ back\ towards\ O.$

Q10d

$$total\ distance$$

$$= 20 \times 4 + \int_{16\pi}^{60} 0.25t - 4\pi\, dt$$

$$= 80 + \left[\frac{0.25t^2}{2} - 4\pi t\right]_{16\pi}^{60}$$

$$= 80 + \frac{1}{8}(60)^2 - 4\pi(60) - \frac{1}{8}(16\pi)^2 + 4\pi(16\pi)$$

$$= 91.845m$$
$$= 91.8m$$

Q11a

$$for\ y\ increasing, \frac{dy}{dx} > 0$$

$$\frac{dy}{dx} = 3x^2e^{-\frac{1}{2}x}\left(-\frac{1}{2}\right) + 6xe^{-\frac{1}{2}x}$$

$$= e^{-\frac{1}{2}x}\left(-\frac{3x^2}{2} + 6x\right)$$

$$Since\ e^{-\frac{1}{2}x} > 0\ for\ x \in \mathbb{R},$$

$$-\frac{3x^2}{2} + 6x > 0$$

$$x\left(-\frac{3}{2}x + 6\right) > 0$$

$$0 < x < 4$$

Q11b

1st derivative test

$$when\ x = 0, y = 3(0)^2e^{-\frac{1}{2}(0)} = 0$$

x	0^{-1}	0	0^{+}
$\frac{dy}{dx}$	<0	0	>0
Sketch	\	—	/

Second derivative test

$$When\ x = 0, \frac{dy}{dx} = 0. \therefore x = 0\ is\ a\ stationary\ pt.$$

$$\frac{d^2y}{dx^2} = e^{-\frac{1}{2}x}(3x + 6) - \frac{1}{2}e^{-\frac{1}{2}x}\left(-\frac{3}{2}x^2 + 6x\right)$$

$$when\ x = 0,$$

$$\frac{d^2y}{dx^2} = 6 > 0\ \therefore \min pt$$

$$when\ x = 0, y = 0$$

$$\therefore (0,0)\ \text{is} \min pt$$

Q11c

$$0 < x < 4$$

Q11d

$$when\ x = 1, \frac{dy}{dx} = e^{-\frac{1}{2}}\left(-\frac{3}{2} + 6\right)$$

$$= \frac{9}{2}e^{-\frac{1}{2}}$$

$$gradient = \frac{-2e^{\frac{1}{2}}}{9}$$

$$when\ x = 1, y = 3e^{-\frac{1}{2}},$$

$$y - 3e^{-\frac{1}{2}} = \frac{-2e^{\frac{1}{2}}}{9}(x - 1)$$

$$y = -\frac{2e^{\frac{1}{2}}}{9}x + \frac{2e^{\frac{1}{2}}}{9} + 3e^{-\frac{1}{2}}$$

$$or\ y = -0.366x + 2.19$$

Q12a $Since\ D\ \&\ T\ are\ the\ midpts\ of\ BA\ and\ BC,$ $by\ midpt\ theorem, DF // AC.$ $Since\ there\ is\ one\ pair\ of\ parallel\ sides, ADFE\ is\ a\ trapezium$
Q12b $\angle EFC = \angle FAE (alt\ segment\ theorem)$ $\angle FAE = \angle AFD (alt\ \angle s, DF \parallel AE)$ $\therefore \angle EFC = \angle AFD$ $180° - \angle ADF = \angle FEA\ (\angle s\ in\ opp\ segment)$ $\angle FEC = 180° - \angle FEA (adj\ \angle s\ on\ a\ str. line)$ $= 180° - (180° - \angle ADF)$ $= \angle ADF$ or $\angle DFB = \angle DAF (alt\ segment\ theorem)$ $\angle DFB = \angle FCE (corrs\ \angle s, AC \parallel DF)$ $\therefore \angle DAF = \angle FCE$ $\therefore \Delta DFA\ is\ similar\ to\ \Delta EFC\ (AA\ Similarity\ test)$
Q12c $\frac{DA}{EC} = \frac{DF}{EF}$ (corresponding sides of similar triangles are proportional) $DF \times EC = DA \times EF$ $By\ midpt\ theorem, DF = \frac{1}{2} AC$ $\frac{1}{2} AC \times EC = DA \times EF$

Name: ______________________ Class: ________

For Marker's Use
90

南僑中學

NAN CHIAU HIGH SCHOOL

PRELIMINARY EXAMINATION 2023
SECONDARY FOUR EXPRESS

Parents' signature: ____________

ADDITIONAL MATHEMATICS 4049/02
Paper 2 23 August 2023, Wednesday

Candidates answer on the Question Paper. **2 hours 15 minutes**

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \cdots + \binom{n}{r} a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

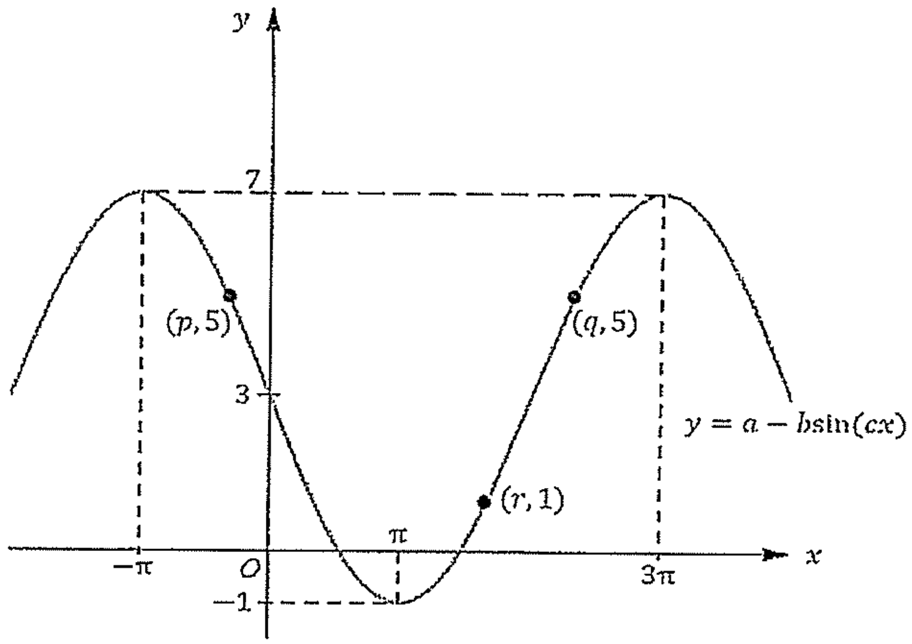
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1



The diagram above shows part of the curve $y = a - b\sin(cx)$, where a, b and c are constants.

(a) Write down the values of a, b and c. [3]

The curve passes through the points $(p, 5)$, $(q, 5)$ and $(r, 1)$, where p, q and r are constants.

(b) Find an equation, in terms of π, connecting

(i) p and q, [1]

(ii) q and r. [1]

2 (a) Explain with the aid of a sketch why $\int_0^a \sqrt{a^2 - x^2}\ \mathrm{d}x = \frac{\pi a^2}{4}$, for $a > 0$. [2]

(b) Hence, find, in terms of a and π,

(i) $\int_a^0 \sqrt{4(a^2 - x^2)}\ \mathrm{d}x$, [2]

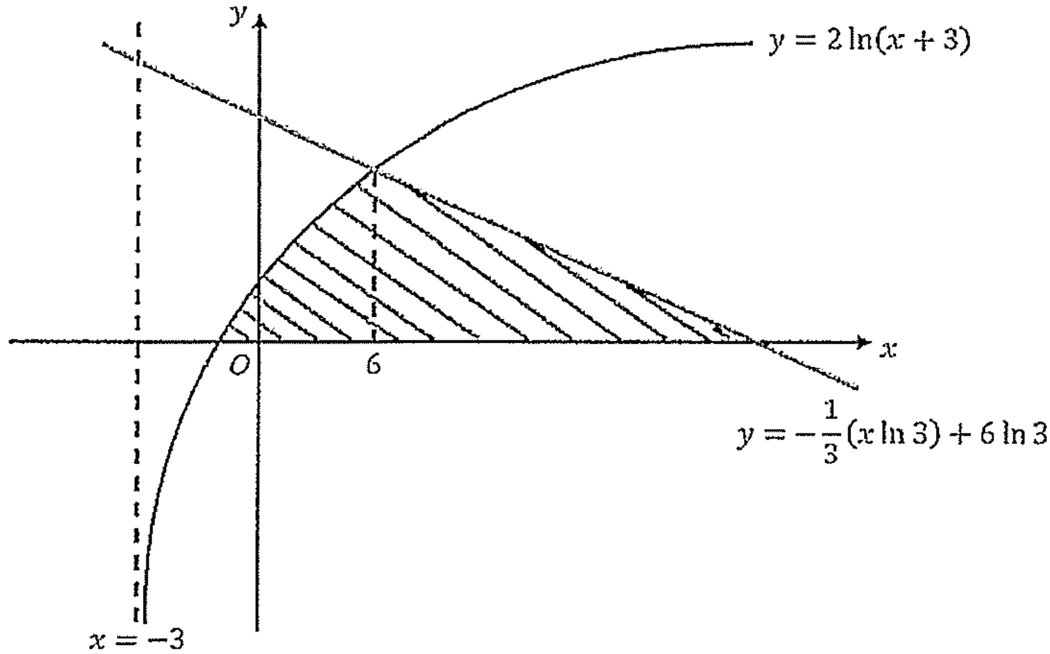
(ii) $\int_{-a}^a \sqrt{a^2 - x^2}\ \mathrm{d}x$. [2]

3 Air is being pumped into a spherical elastic ball at a constant rate of 10 cm^3 per second. It is assumed that the ball maintains a spherical shape throughout the process.

(a) Find the rate of increase of the radius of the ball at the instant when the radius is 5 cm. [The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.] [3]

(b) Find the rate of increase of the surface area of the ball at the instant when the radius of the ball is increasing at the rate of $\frac{5}{72\pi}$ cm per second.
[The surface area of a sphere of radius r is $4\pi r^2$.] [5]

4



The diagram shows part of the curve $y = 2\ln(x+3)$ and a line $y = -\frac{1}{3}(x\ln 3) + 6\ln 3$. The curve and the line intersect at $x = 6$. Find the exact area of the shaded region bounded by the curve, the line and the x-axis. Express your answer in the form $a\ln 3 + b$, where a and b are integers to be determined. [9]

Continuation of working space for question 4.

5 On 01 January 2010, there were 1500 predators of a particular species in a habitat. Ecologists believe that the population of the predator, P in thousands, can be modelled by the formula $P = N - 3.5(e^{kt})$, where N and k are positive constants and t is the time in years after 01 January 2010.

(a) Show that $N = 5$. [1]

(b) Comment on the feasibility of the ecologist's model for the population of the predator in the long run. [2]

(c) The population of the prey, Q in thousands, can be modelled by the formula $Q = 8e^{-t}$, where t is the time in years after 01 January 2010. The population of predator and prey first became equal on 01 January 2020. Find the value of k. [3]

(d) A mathematician believes that the population of the predator, P in thousands, should be modelled by the formula $P = 5 - 3.5(e^{-2t})$, where t is the time in years after 01 January 2010, instead. Determine the year and month in which the population of the predator first doubles the population of the prey under this model. [4]

6 The polynomial $f(x)$ is given by $f(x) = 2x^3 - 6x^2 + Ax + B$, where A and B are constants. Find the values of A and B such that

(a) $x^2 - 9$ is a factor, [4]

(b) $x^2 + 9$ is a factor, [4]

(c) the curve $y = f(x)$ cuts the x-axis at $x = -1$ and just touches the x-axis at $x = 2$. [3]

7 (a) (i) By considering the general term in the binomial expansion of $\left(\frac{1}{2}x^2 + \frac{\sqrt{2}}{x^2}\right)^8$, explain why there are no terms with odd powers of x. [3]

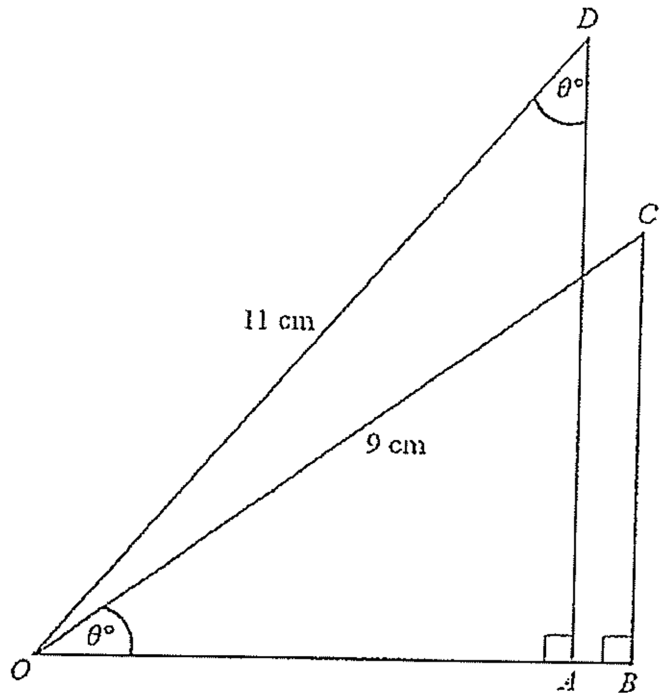
(ii) Find the term independent of x in the expansion of $\left(\frac{1}{2}x^2 + \frac{\sqrt{2}}{x^2}\right)^8 (3x - 7)^2$. [3]

(b) In the binomial expansion of $\left(1+3\sqrt{x}\right)^n$, the coefficient of $x\sqrt{x}$ is 7 times the coefficient of x.

(i) Show that $3\binom{n}{3} = 7\binom{n}{2}$. [3]

(ii) Hence, find the value of n. [2]

8



In the diagram, triangles OBC and OAD are right-angled triangles such that $OC = 9$ cm and $OD = 11$ cm. Angles BOC and ODA are each equal to $\theta°$, where $0° < \theta < 90°$.

(a) Express AB in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [4]

(b) Find the minimum value of AB and the corresponding value of θ. [3]

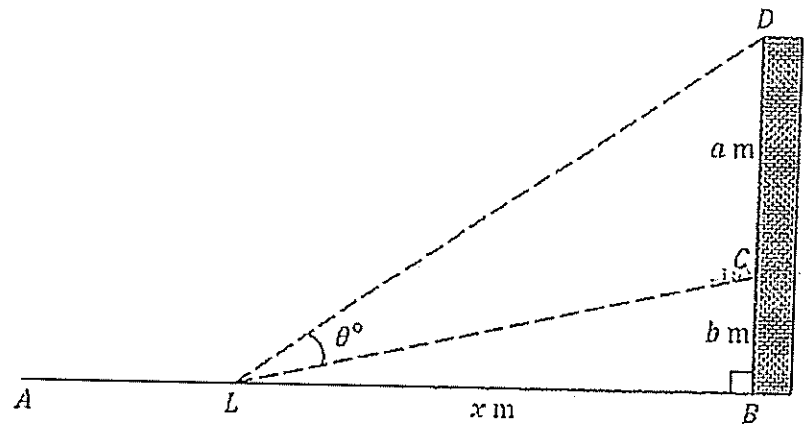
(c) Find the range of values of θ for which A is between O and B. [3]

9 The equation of a circle is $x^2 + y^2 - 10x - 4y + 19 = 0$.

(a) Point P lies on the circle, and the tangent to the circle at point P has a gradient of -3. Find the possible coordinates of point P. [5]

(b) Another two tangents to the circle intersect at the origin. Find the gradients of these two tangents. [5]

10



In the diagram, A and B are two fixed points on a horizontal ground and a projector is positioned on the ground at L which is x m away from B. The projector casts a beam of light on a screen CD, of fixed height a m. C is the bottom of the screen, where $BC = b$ m. Angle CLD is $\theta°$. Assume that the thickness of the screen is negligible.

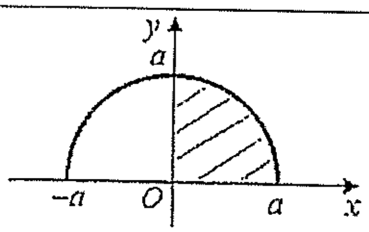
(a) Express $\tan(\angle DLB)$ and $\tan(\angle CLB)$ in terms of a, b or/and x. Hence, show that $\tan\theta = \frac{ax}{x^2+ab+b^2}$. [4]

(b) Given that x can vary, find, in terms of a and b, the value of x for which $\tan\theta$ is stationary. [4]

(c) Given that $a = 10$ and $b = 3$, find the value of θ which gives the stationary value of $\tan\theta$ found in part (b). [2]

End of Paper

Answer Key

1(a)	$a = 3,\ b = 4,\ c = \frac{1}{2}$
1(b)(i)	$p + q = 2\pi$
1(b)(ii)	$q + r = 4\pi$
2(a)	 $\int_0^a \sqrt{a^2 - x^2}\ \mathrm{d}x$ $= Area\ of\ quarter\ circle$ $= \frac{\pi a^2}{4}$
2(b)(i)	$-\frac{\pi a^2}{2}$
2(b)(ii)	$\frac{\pi a^2}{2}$
3(a)	$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{10\pi}$ cm/s Accept: 0.0318 cm/s (3 s.f.)
3(b)	$\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{10}{3}$ cm^2/s *Accept*: 3.33 cm^2/s (3 s.f.)
4	$(60 \ln 3 - 16)$ units^2
5(a)	$N = 5$ (*shown*)
5(b)	$P = 5 - 3.5(e^{kt})$ *As* $t \to \infty$, $3.5(e^{kt}) \to \infty$ for $k > 0$ $\therefore P \to -\infty$ Since the number of predator become <u>negative</u> in the long run, the ecologist's model <u>may not be feasible</u> in the long run.
5(c)	0.0357 (3 s.f.)
5(d)	Year 2011, March

6(a)	$B = 54,\ A = -18$
6(b)	$B = -54,\ A = 18$
6(c)	$B = 8,\ A = 0$
7(a)(i)	Since $16 - 4r = 4(4 - r)$ is a multiple of 4 / even for all real values of $r, 0 \leq r \leq 8$, there are no terms with odd powers of x.
7(a)(ii)	857.5
7(b)(i)	Shown Question
7(b)(ii)	$n = 9$
8(a)	$\therefore AB = 14.2\cos(\theta + 50.7°)$
8(b)	minimum value of $AB = 0$ when $\theta + 50.7° = 90°$ $\theta = 39.3°$ (1 d.p.)
8(c)	$\therefore 0° < \theta < 39.3°$
9(a)	$\therefore P(2,1)$ *or* $P(8,3)$
9(b)	1.59 *or* -0.252 (3 s.f.)
10(a)	$\tan(\angle DLB) = \frac{a+b}{x}$ $\tan(\angle CLB) = \frac{b}{x}$
10(b)	$x = \sqrt{ab + b^2}$ $(x > 0)$
10(c)	38.7° (1 d.p.)