Class Index Number

Name : ________________________________

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2023
Secondary 4

Friday	**ADDITIONAL MATHEMATICS**	**4049/01**
18 August 2023	**Paper 1**	2 h 15 min

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST
Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figure, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

/ 90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

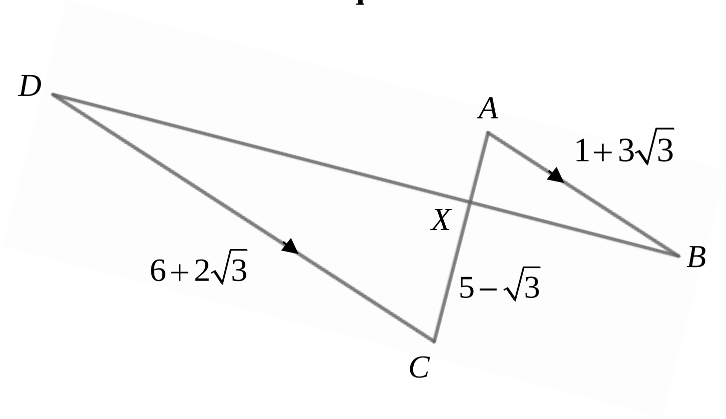
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **A calculator must not be used for this question.**



In the diagram, *AB* is parallel to *DC*, and *AC* meets *BD* at *X*.
Given that $AB = \left(1+3\sqrt{3}\right)$ cm, $CX = \left(5-\sqrt{3}\right)$cm and $CD = \left(6+2\sqrt{3}\right)$ cm, find the exact length of *AX*. [5]

2 A curve has the equation $y = \dfrac{3-x}{e^{1-2x}}$.

(i) Show that the curve is an increasing function of *x* for $x < \dfrac{5}{2}$. [5]

(ii) Given that *y* is decreasing at a constant rate of 2 units per second, find the rate of change of *x* when $x = 0$. [2]

3 The function $f(x) = 2x^3 + ax^2 + bx + 6$, where a and b are constants, is exactly divisible by $x + 2$. When f (x) is divided by $x - 2$, the remainder is -12.

(i) Find the value of a and of b. [4]

(ii) Factorise f (x) completely and hence solve f (x) = 0. [3]

4 It is given that $y = 1 - 3\sin 4x$, for $0^\circ \le x \le 180^\circ$.

(i) State the amplitude and period of y. [2]

(ii) Sketch the graph of $y = 1 - 3\sin 4x$. [3]

(iii) Hence, state the number of solutions for $3\sin 4x = 2$. [1]

5 **(a)** The equation of a curve is $y=\frac{1}{3}\ln(px+3)$, where p is a constant to be determined. The gradient of the tangent to the curve at $x=-\frac{1}{2}$ is parallel to $3y = x$. Find the equation of the normal to the curve at $x=-\frac{1}{2}$.

[4]

(b) **(i)** Express $-2x^2+4x-5$ in the form $a(x+p)^2+q$. [2]

(ii) Hence, determine, with explanation, if the graph of $y=-2x^2+4x-5$ will intersect the x–axis. [2]

6 **(a)** In the expansion of $(2+x)^n$, where $n > 0$, the coefficient of x^2 is twice the coefficient of x. Find the value of n. [4]

(b) Find the value of the term that is independent of x in the expansion of $\left(2x - \frac{1}{4x^4}\right)^{15}$. [3]

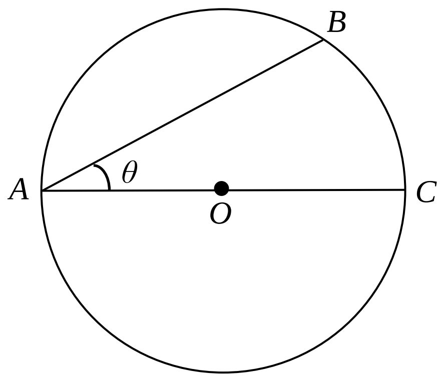
7 It is given that $y = \log_p(px) + 2\log_p(4x-3) - 1$, where p is a positive integer.

(i) Write down the set of values of x for which y is defined. [1]

(ii) Show that y can be written as $\log_p(16x^3 - 24x^2 + 9x)$. [3]

(iii) Find the value of x for which $y = \dfrac{1}{\log_{9x} p}$. [3]

8 The figure shows a circular lake, centre O, of radius 2 km. A duck swims across the lake from A to B at 3 km/h and then walks clockwise around the edge of the lake from B to C at 4 km/h. If angle $BAC = \theta$ radians and the total time taken is T hours,



(i) show that $T = \frac{1}{3}\left(4\cos\theta + 3\theta\right)$, [4]

8 **(ii)** find the maximum value of *T*. [5]

9 **(a)** It is given that f $(x) = (2x-3)^5+4$.

Find $f'(x)$ and hence, determine the nature of the stationary point for $y = f(x)$.

[5]

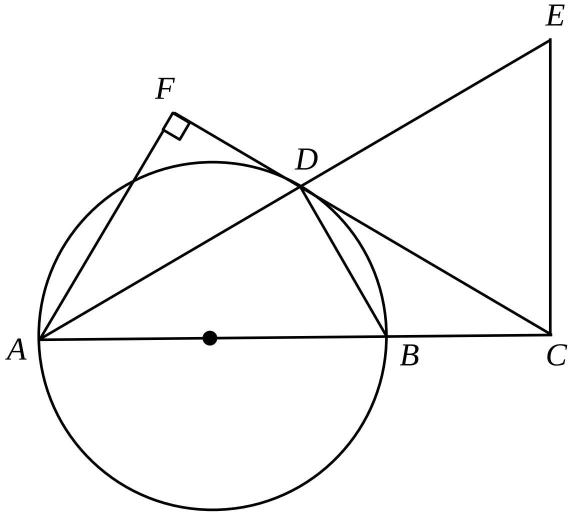
9 **(b)** The gradient function of a curve varies directly with the square of $(x-2)$. Given that the gradient of the curve at $(1,9)$ is 6, find the coordinates of the point at which the curve meets the *y*–axis. [5]

10 **(a)** If $\cos 56^o = p$, express $\sin 28^o$ in terms of *p*. [2]

(b) **(i)** Prove the identity $\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = 2\tan 2x$. [4]

(ii) Determine the values of *x* in the interval $180^\circ < x < 360^\circ$ for which the identity is not valid. [2]

11



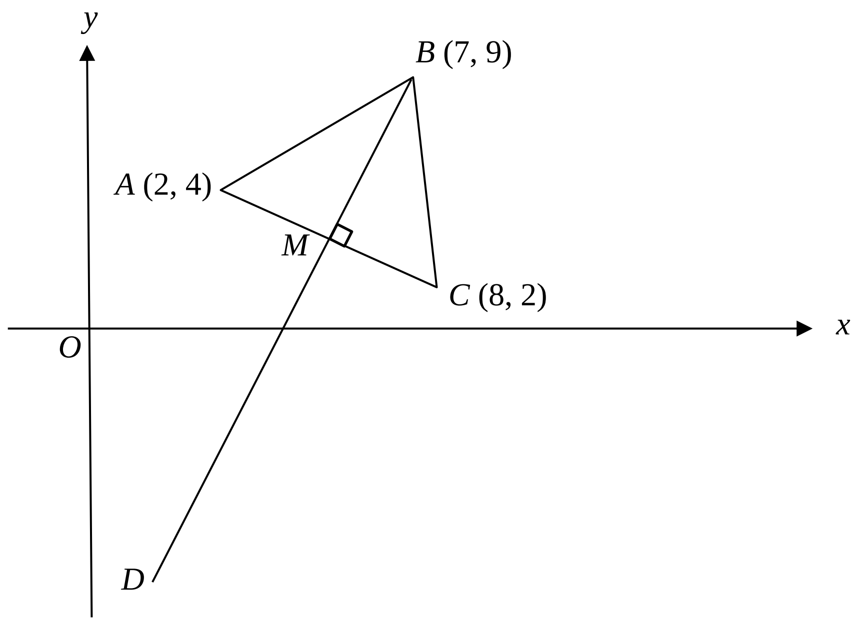
In the diagram, the diameter *AB* is produced to *C* and the line *CD* is the tangent to the circle at *D*. The line *AD* is produced to *E* such that a circle can be drawn passing through *B*, *C*, *E* and *D*. The foot of the perpendicular from *A* to *CD* produced is *F*.

Prove that

(i) triangle *CDE* is isosceles, [4]

(ii) *AE* bisects angle *FAB*. [3]

12 In the diagram, the points *A*, *B* and *C* have coordinates $(2,4)$, $(7,9)$ and $(8,2)$ respectively. The point *M* lies on *AC* such that the line *BMD* is perpendicular to *AC*.



(i) Show that *AB* = *BC*. [2]

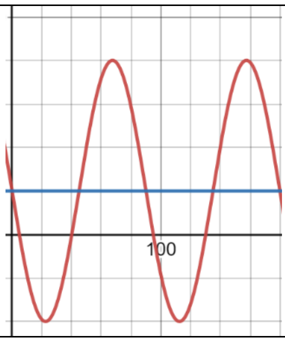
(ii) Find the equation of *BD*. [3]

(iii) Given that the ratio of $DM : MB = 2 : 1$, find the coordinates of *D*. [2]

(iv) Find the area of quadrilateral *ABCD*. [2]

End of Paper.

Answer Key

1	$\dfrac{23\sqrt{3}-27}{6}$		
2	**(ii)**	–1.09	
3	**(i)**	$a=-3$, $b=-11$	
	(ii)	$x=-2,\ \dfrac{1}{2}$ or 3	
4	**(i)**	Amplitude = 3 Period = 90°	
	(ii)	 	
	(iii)	4	
5	**(a)**	$y=-3x-\dfrac{3}{2}+\dfrac{1}{3}\ln 2$	
	(b)	**(i)**	$-2(x-1)^2-3$
		(ii)	Since coefficient of $x^2<0$, graph has maximum turning point at $(1,\ -3)$. Maximum value of y is -3 which is less than 0.
6	**(a)**	9	
	(b)	–29120	
7	**(i)**	$x>\dfrac{3}{4}$	
	(iii)	$x=\dfrac{3}{2}$	
8	**(ii)**	1.73 h	
9	**(a)**	$10(2x-3)^4$, point of inflexion	
	(b)	(0, –5)	
10	**(a)**	**(i)**	$\sqrt{\dfrac{1-p}{2}}$
		(ii)	$225^\circ, 315^\circ$
12	**(ii)**	$y=3x-12$	
	(iii)	$(1,-9)$	
	(iii)	60 units^2	

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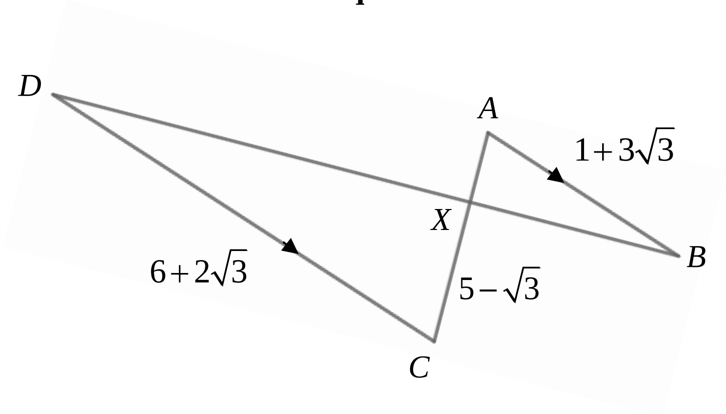

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$$\frac{AX}{5-\sqrt{3}} = \frac{1+3\sqrt{3}}{6+2\sqrt{3}}$$

$$\begin{aligned} &AX \\ &= \frac{1+3\sqrt{3}}{6+2\sqrt{3}} \times \left(5-\sqrt{3}\right) \\ &= \frac{5-\sqrt{3}+15\sqrt{3}-9}{6+2\sqrt{3}} \\ &= \frac{14\sqrt{3}-4}{6+2\sqrt{3}} \\ &= \frac{7\sqrt{3}-2}{3+\sqrt{3}} \times \frac{3-\sqrt{3}}{3-\sqrt{3}} \\ &= \frac{21\sqrt{3}-21-6+2\sqrt{3}}{9-3} \\ &= \frac{23\sqrt{3}-27}{6} \ or \ \frac{23}{6}\sqrt{3}-\frac{9}{2} \end{aligned}$$

2 A curve has the equation $y = \dfrac{3-x}{e^{1-2x}}$.

(i) Show that the curve is an increasing function of x for $x < \dfrac{5}{2}$. [5]

$$\frac{dy}{dx} = \frac{e^{1-2x}(-1) - (3-x)e^{1-2x}(-2)}{\left(e^{1-2x}\right)^2}$$

$$= \frac{-1+6-2x}{e^{1-2x}}$$

$$= \frac{5-2x}{e^{1-2x}}$$

For $x < \dfrac{5}{2}$, $e^{1-2x} > 0$

$x < \dfrac{5}{2}$

$2x - 5 < 0$

$5 - 2x > 0$

$\therefore \dfrac{dy}{dx} > 0$ for $x < \dfrac{5}{2}$

Hence, curve is an increasing function for $x < \dfrac{5}{2}$

(ii) Given that y is decreasing at a constant rate of 2 units per second, find the rate of change of x when $x = 0$. [2]

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$-2 = \frac{5-2x}{e^{1-2x}} \times \frac{dx}{dt}$$

$$-2 = \frac{5}{e} \times \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{-2e}{5} = -1.09 \text{(3sf)}$$

3 The function $\mathrm{f}(x) = 2x^3 + ax^2 + bx + 6$, where a and b are constants, is exactly divisible by $x + 2$. When f (x) is divided by $x - 2$, the remainder is –12.

(i) Find the value of a and of b. [4]

$f(-2) = 0$

$-2(-2)^3 + a(-2)^2 + b(-2) + 6 = 0$

$4a - 2b = 10$

$b = 2a - 5$ --------- (1)

$f(2) = -12$

$2(2)^3 + a(2)^2 + b(2) + 6 = -12$

$4a + 2b = -34$

$2a + b = -17$ ------ (2)

(2) into (1),

$2a + 2a - 5 = -17$

$4a = -12$

$a = -3$

$b = -11$

(ii) Factorise f (x) completely and hence solve f (x) = 0. [3]

$2x^3 - 3x^2 - 11x + 6 = (x+2)(2x^2 + px + 3)$

compare coeff of x^2,

$p + 4 = -3$

$p = -7$

$(x+2)(2x^2 - 7x + 3) = 0$

$(x+2)(2x-1)(x-3) = 0$

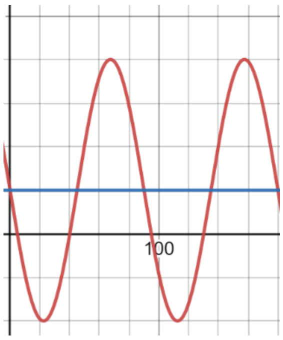
$x = -2,\ \frac{1}{2}$ or 3

4 It is given that $y = 1 - 3\sin 4x$, for $0^\circ \le x \le 180^\circ$.

(i) State the amplitude and period of y. [2]

Amplitude = 3
Period = 90°

(ii) Sketch the graph of $y = 1 - 3\sin 4x$. [3]



(iii) Hence, state the number of solutions for $3\sin 4x = 2$ where $0^\circ \le x \le 180^\circ$. [1]

4

5 **(a)** The equation of a curve is $y=\frac{1}{3}\ln(px+3)$, where p is a constant to be determined. The gradient of the tangent to the curve at $x=-\frac{1}{2}$ is parallel to $3y = x$. Find the equation of the normal to the curve at $x=-\frac{1}{2}$. [4]

$$\frac{dy}{dx}=\frac{1}{3}\left(\frac{1}{px+3}\right)p$$

$$\frac{p}{3\left(-\frac{1}{2}p+3\right)}=\frac{1}{3}$$

$$p=-\frac{1}{2}p+3$$

$$\frac{3}{2}p=3$$

$$p=2$$

$x=-\frac{1}{2},\ y=\frac{1}{3}\ln 2$

grad of normal = -3

$$y-\frac{1}{3}\ln 2=-3\left(x+\frac{1}{2}\right)$$

$$y=-3x-\frac{3}{2}+\frac{1}{3}\ln 2$$

(b) **(i)** Express $-2x^2+4x-5$ in the form $a(x+p)^2+q$. [2]

$$-2x^2+4x-5$$
$$=-2\left(x^2-2x+\frac{5}{2}\right)$$
$$=-2\left[(x-1)^2-1+\frac{5}{2}\right]$$
$$=-2(x-1)^2-3$$

(ii) Hence, determine, with explanation, if the graph of $y=-2x^2+4x-5$ will intersect the x–axis. [2]

Since coefficient of x^2<0, graph has maximum turning point at (1, -3).
Maximum value of y is -3 which is less than 0.
OR The maximum point is below the x-axis.
Hence, graph does not intersect x-axis.

6 **(a)** In the expansion of $(2+x)^n$, where $n > 0$, the coefficient of x^2 is twice the coefficient of x. Find the value of n. [4]

$$(2+x)^n = 2^n + \binom{n}{1}2^{n-1}x + \binom{n}{2}2^{n-2}x^2 + \ldots$$

$$\binom{n}{2}2^{n-2} = 2\binom{n}{1}2^{n-1}$$

$$\frac{n(n-1)}{2}\left(2^{n-2}\right) = 2n\left(2^{n-1}\right)$$

$$n^2 - n = \frac{4n\left(2^{n-1}\right)}{2^{n-2}}$$

$$n^2 - n = 8n$$

$$n^2 - 9n = 0$$

$$n(n-9) = 0$$

$$n = 0 \quad \text{or} \quad n = 9$$

(NA)

(b) Find the value of the term that is independent of x in the expansion of $\left(2x - \frac{1}{4x^4}\right)^{15}$. [3]

general term

$$= \binom{15}{r}(2x)^{15-r}\left(-\frac{1}{4x^4}\right)^r$$

$$= \binom{15}{r}2^{15-r}x^{15-r}\left(-\frac{1}{4}\right)^r x^{-4r}$$

$$= \binom{15}{r}2^{15-r}\left(-\frac{1}{4}\right)^r x^{15-5r}$$

$$15 - 5r = 0$$

$$r = 3$$

$$\text{value} = \binom{15}{3}2^{12}\left(-\frac{1}{4}\right)^3 = -29120$$

7 It is given that $y = \log_p(px) + 2\log_p(4x-3) - 1$, where p is a positive integer.

(i) Write down the values of x for which y is defined. [1]

$$x > \frac{3}{4}$$

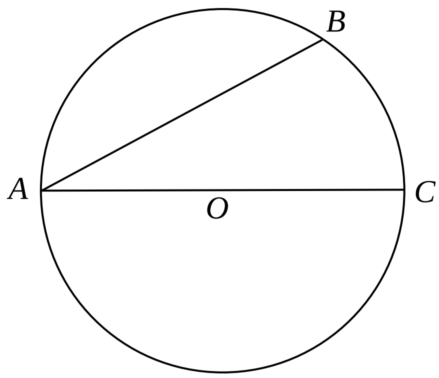
(ii) Show that y can be written as $\log_p(16x^3 - 24x^2 + 9x)$. [3]

$$y = \log_p(px) + 2\log_p(4x-3) - 1$$
$$= \log_p(px) + \log_p(4x-3)^2 - \log_p p$$
$$= \log_p \frac{px(4x-3)^2}{p}$$
$$= \log_p\left[x\left(16x^2 - 24x + 9\right)\right]$$
$$= \log_p(16x^3 - 24x^2 + 9x)$$

(iii) Find the value of x for which $y = \dfrac{1}{\log_{9x} p}$. [3]

$$\log_p(16x^3 - 24x^2 + 9x) = \frac{1}{\log_{9x} p}$$
$$\log_p(16x^3 - 24x^2 + 9x) = 1 \div \frac{\log_p p}{\log_p 9x}$$
$$\log_p(16x^3 - 24x^2 + 9x) = \log_p 9x$$
$$16x^3 - 24x^2 + 9x = 9x$$
$$16x^3 - 24x^2 = 0$$
$$8x^2(2x-3) = 0$$
$$x = 0\left(\text{NA}\right) \quad \text{or} \quad x = \frac{3}{2}$$

8 The figure shows a circular lake, centre O, of radius 2 km. A duck swims across the lake from A to B at 3 km/h and then walks clockwise around the edge of the lake from B to C at 4 km/h. If angle $BAC = \theta$ radians and the total time taken is T hours,



(i) show that $T = \frac{1}{3}(4\cos\theta + 3\theta)$, [4]

$\angle ABC = \frac{\pi}{2}$ (angle in semicircle)

$\cos\theta = \frac{AB}{2(2)}$

$AB = 4\cos\theta$

$\angle BOC = 2\theta$ (angle at centre = $2\times$ angle at circumference)

arc $BC = 2(2\theta) = 4\theta$

time

$= \frac{4\cos\theta}{3} + \frac{4\theta}{4}$

$= \frac{4\cos\theta}{3} + \theta$

$= \frac{1}{3}(4\cos\theta + 3\theta)$

8 **(ii)** find the maximum value of T. [5]

$$T = \frac{4}{3}\cos\theta + \theta$$

$$\frac{dT}{d\theta} = \frac{4}{3}(-\sin\theta) + 1$$

$$-\frac{4}{3}\sin\theta + 1 = 0$$

$$\sin\theta = \frac{3}{4}$$

$$\theta = \sin^{-1}\frac{3}{4} = 0.84806$$

$$\frac{d^2\theta}{d\theta^2} = -\frac{4}{3}\cos\theta$$

$\theta = 0.84806,$

$$\frac{d^2\theta}{d\theta^2} = -\frac{4}{3}\cos(0.84806) = -0.8819 < 0$$

T is a maximum.

$$\text{Max } T = \frac{4}{3}\cos 0.84806 + 0.84806$$

$= 1.73$ h (3sf)

9 **(a)** It is given that $f(x) = (2x-3)^5 + 4$.

Find $f'(x)$ and hence, determine the nature of the stationary point for $y = f(x)$. [5]

$$f'(x) = 5(2x-3)^4(2)$$
$$= 10(2x-3)^4$$
$$10(2x-3)^4 = 0$$
$$x = \frac{3}{2}$$

x	$\left(\frac{3}{2}\right)^-$	$\frac{3}{2}$	$\left(\frac{3}{2}\right)^+$
$f'(x)$	> 0	0	> 0
sketch of tangent	/	–	/

At $\left(\frac{3}{2}, 4\right)$, it is a point of inflexion.

9 **(b)** The gradient function of a curve varies directly with the square of $(x-2)$. Given that the gradient of the curve at $(1,9)$ is 6, find the coordinates of the point at which the curve meets the *y*–axis. [5]

$\frac{dy}{dx}=k(x-2)^2$

At $x = 1$, $\frac{dy}{dx}=6$

$k=6$

$\frac{dy}{dx}=6(x-2)^2$

$y=\int 6(x-2)^2\,dx$

$=\frac{6(x-2)^3}{3}+c$

$=2(x-2)^3+c$

$9=2(1-2)^3+c$

$c=11$

$y=2(x-2)^3+11$

At $x=0,\ y=-5$

coodinates are $(0,-5)$

10 **(a)** If $\cos 56^o = p$, express $\sin 28^o$ in terms of *p*. [2]

$$\cos 56^\circ = p$$
$$\cos 2(28^\circ) = p$$
$$1 - 2\sin^2 28^\circ = p$$
$$\sin^2 28^\circ = \frac{1-p}{2}$$
$$\sin 28^\circ = \sqrt{\frac{1-p}{2}} \quad \text{or} \quad -\sqrt{\frac{1-p}{2}} \ \text{(NA)}$$

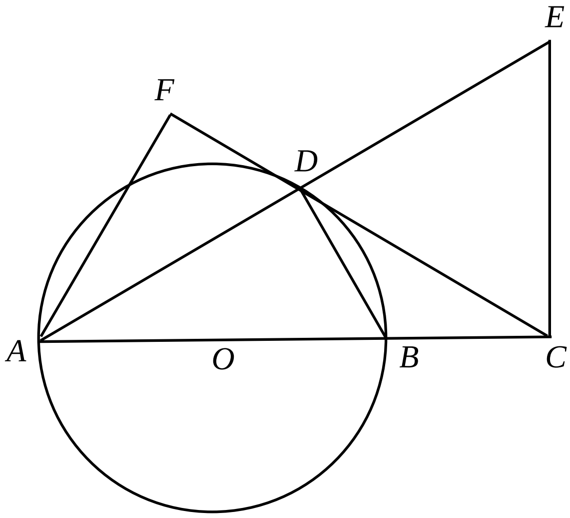
(b) **(i)** Prove the identity $\dfrac{\cos x + \sin x}{\cos x - \sin x} - \dfrac{\cos x - \sin x}{\cos x + \sin x} = 2\tan 2x$. [4]

$$\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x}$$
$$= \frac{\cos^2 x + 2\cos x \sin x + \sin^2 x - (\cos^2 x - 2\cos x \sin x + \sin^2 x)}{\cos^2 x - \sin^2 x}$$
$$= \frac{2(2\sin x \cos x)}{\cos 2x}$$
$$= \frac{2\sin 2x}{\cos 2x}$$
$$= 2\tan 2x$$

(ii) Determine the values of *x* in the interval $180^\circ < x < 360^\circ$ for which the identity is not valid. [2]

$$2x = 90^\circ, 270^\circ, 450^\circ, 630^\circ$$
$$x = 45^\circ, 135^\circ, 225^\circ, 310^\circ$$
For $180^\circ < x < 360^\circ$,
$$x = 225^\circ, 315^\circ$$

11



In the diagram, O is the centre of the circle. The diameter AB is produced to C and the line CD is the tangent to the circle at D. The line AD is produced to E such that a circle can be drawn passing through B, C, E and D. The foot of the perpendicular from A to CD produced is F.

Prove that

(i) triangle CDE is isosceles, [4]

Let $\angle ADF = x$

$\angle ADF = \angle ABD = x$ (alt seg thm)

$\angle ADF = \angle CDE = x$ (vert opp $\angle$)

$\angle CED = \angle ABD = x$ (ext $\angle$, cyclicquad)

$\therefore \angle CED = \angle CDE$

$\therefore \Delta CDE$ is isosceles.

(ii) AE bisects angle FAB. [3]

$\angle ADB = 90^\circ$ (rt $\angle$ in semicircle)

$\angle DAB = 180^\circ - 90^\circ - x = 90^\circ - x$ ($\angle$ sum of Δ)

$\angle AFD = 90^\circ$ (given)

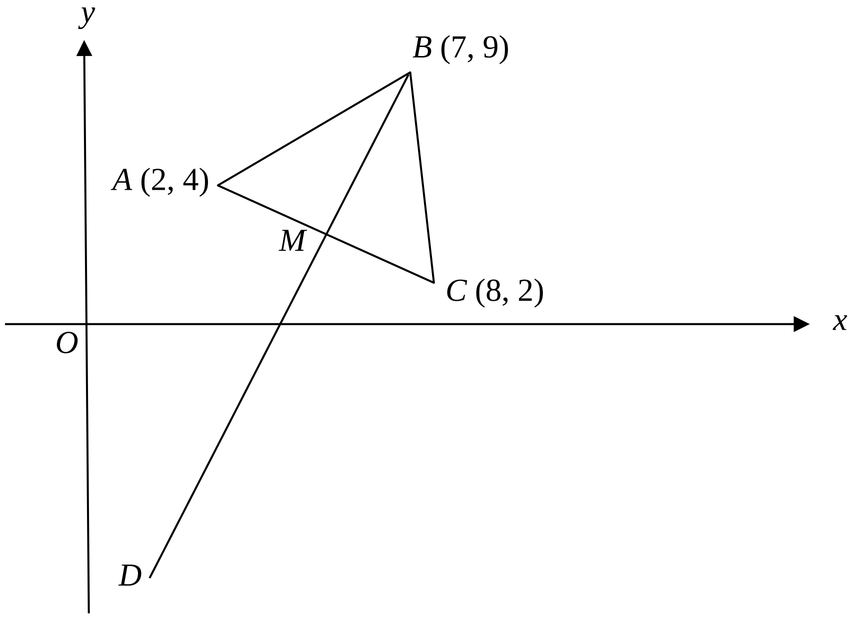
$\angle FAD = 180^\circ - 90^\circ - x = 90^\circ - x$ ($\angle$ sum of Δ)

$\therefore \angle FAD = \angle DAB$

$\therefore AE$ bisects $\angle FAB$.

A1

12 In the diagram, the points A, B and C have coordinates $(2,4)$, $(7,9)$ and $(8,2)$ respectively. The point M lies on AC such that the line BMD is perpendicular to AC.



(i) Show that $AB = BC$. [2]

$$AC = \sqrt{(7-2)^2+(9-4)^2} = \sqrt{50}$$
$$BC = \sqrt{(7-8)^2+(9-2)^2} = \sqrt{50}$$
$$\therefore AC = BC \text{ (shown)}$$

(ii) Find the equation of BD. [3]

grad of AC $= \dfrac{4-2}{2-8} = -\dfrac{1}{3}$

grad of BD $= 3$

Eqn of BD is

$y-9=3(x-7)$

$y=3x-21+9$

$y=3x-12$

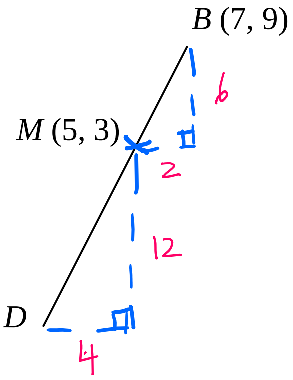
(iii) Given that the ratio of $DM : MB = 2:1,$ find the coordinates of D. [2]

coord of $M = \left(\frac{2+8}{2}, \frac{4+2}{2}\right) = (5,3)$

$x-$coord of $D = 5-2(2) = 1$

$y-$coord of $D = 3-2(6) = -9$

coord D is $(1,-9)$



OR

translation $\overrightarrow{BM} = \begin{pmatrix} -2 \\ -6 \end{pmatrix}$

translation $\overrightarrow{MD} = \begin{pmatrix} -4 \\ -12 \end{pmatrix}$

$\overrightarrow{OD} = \overrightarrow{OD} + \overrightarrow{MD}$

$= \begin{pmatrix} 5 \\ 3 \end{pmatrix} + \begin{pmatrix} -4 \\ -12 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ -9 \end{pmatrix}$

coord D is $(1,-9)$

(iv) Find the area of quadrilateral $ABCD$. [2]

$\frac{1}{2}\begin{vmatrix} 2 & 1 & 8 & 7 & 2 \\ 4 & -9 & 2 & 9 & 4 \end{vmatrix}$

$= \frac{1}{2}\left[-18+2+72+28-(4-72+14+18)\right]$

$= 60 \text{ units}^2$

End of Paper.

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Class	Index Number

Name : ____________________________________

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2023
Secondary 4

Monday

ADDITIONAL MATHEMATICS PAPER 2

4049/02

21 August 2023

2 hours 15 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST
Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figure, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

/ 90

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1. **(i)** A prism has a volume of $\left(3x^2+8x+1\right)$ cm^3 and a cross-sectional area of $\left(x^2+2x+1\right)$ cm^2. Write down an expression for the height of the prism, in the form $A+\dfrac{B}{(x+1)}+\dfrac{C}{(x+1)^2}$. [4]

(ii) Find $\int A+\dfrac{B}{(x+1)}+\dfrac{C}{(x+1)^2}\,\mathrm{d}x$. [3]

2. A circle C passes through the point $P(4, -3)$ and has the same centre as the circle $x^2 + y^2 + 4x - 2y - 1 = 0$.

(i) Find the equation of the circle C. [3]

(ii) Find the equation of the tangent to the circle C at the point $P(4, -3)$. [2]

(iii) Another point $Q(4, q)$ which lies on the circle C, is the same distance from the y-axis as the point P. Find the coordinates of the point Q. [2]

3. **(i)** Given that $y=(x-1)\sqrt{4x+1}$, show that $\frac{\mathrm{dy}}{\mathrm{dx}}=\frac{6x-1}{\sqrt{4x+1}}$. [3]

(ii) Hence, evaluate $\int_1^2 \frac{6x-5}{\sqrt{4x+1}}\,dx$. [4]

4. **(a)** Express $2\sin\theta - 1.5\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R > 0 and $0 < \alpha < \frac{\pi}{2}$. [3]

(b) John models the height of sea water level, H metres, on a particular day by the equation $H = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right)$, $0 \le t < 12$, where t hours is the number of hours after midday. Using this model, calculate

(i) the maximum possible height of sea water level and the value of t, to 2 decimal places, when it occurs. [3]

4 **(b)** **(ii)** the first time when the height of the sea water is 7 metres, leaving your answer correct to the nearest minute. [4]

5. The temperature, θ °C, of an oven t minutes after it was switched on is given by $\theta = 300 - 280e^{-0.05t}$, $t \geq 0$.

(i) State the initial temperature of the oven. [1]

(ii) Find the value of t when the temperature of the oven reaches 160 °C. [2]

(iii) Determine, with justification, the maximum temperature that this oven can approach. [2]

5. The temperature, θ °C, of another oven t minutes after it was switched on is given by $\theta = 250 - 230e^{-0.1t}$, $t \geq 0$.

(iv) Assuming that both ovens are switched on at the same time, find the time when both ovens will have the same temperature since they were switched on. [5]

6. **(i)** Prove that $\sin(A+B)\sin(A-B) = (\sin A + \sin B)(\sin A - \sin B)$. [3]

(ii) Given that $\sin A = \frac{1}{2}$, find all the values of B for $-\pi \le B \le \pi$ that satisfy the equation $\sin(A+B)\sin(A-B) = 0$. [4]

7. **(a)** Find the range of values of k for which $y=(k-6)x^2-8x+k$ cuts the x – axis at two distinct points and has a minimum point. [4]

(b) Given that the line $y=5x+c$ is a tangent to the curve $y=2x^2+bx$, show that c is never positive. [4]

8. The table below shows experimental values of two variables, x and y, which are connected by an equation of the form $y=\frac{a}{x+b}$, where a and b are constants.

x	0.1	0.4	1.0	2.0	3.0
y	8.0	6.0	4.0	2.6	1.9

(i) Draw the graph of x against $\frac{1}{y}$, using a scale of 5 cm to 1 unit on the x-axis and a scale of 2 cm to 0.1 unit on the $\frac{1}{y}$-axis on the graph paper in the next page. [3]

(ii) Use your graph to estimate the value of a and of b. [4]

An alternative method for obtaining a straight line graph for the equation $y=\frac{a}{x+b}$ is to plot y against xy.

(iii) Using answers of a and b in part **(ii)** estimate the gradient and vertical intercept of the graph of y plotted against xy.

[3]

9. A particle moves in a straight line so that t seconds after passing through a fixed point O, its velocity, v cm/s, is given by $v = 3t^2 - 7t + 2$.

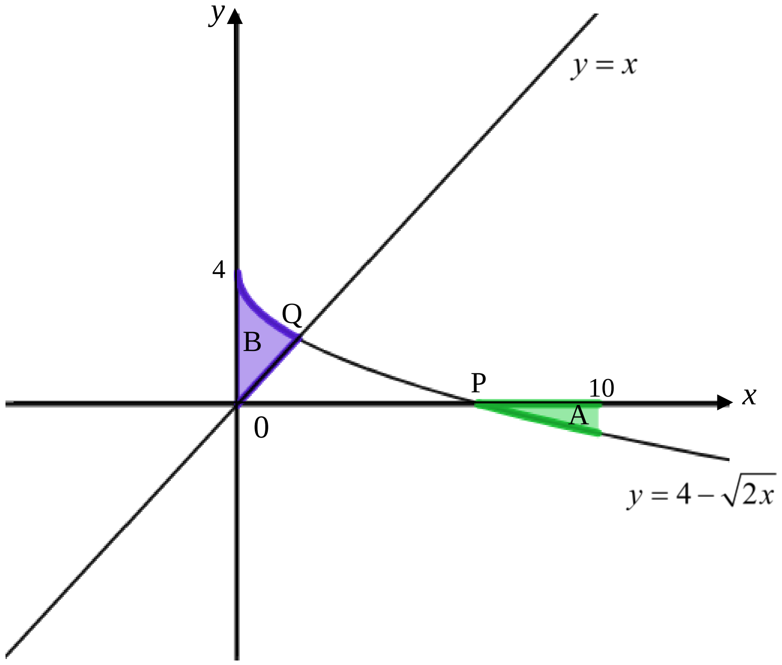
(i) Find the value of t at the instant when the acceleration is -1 cm/s^2. [2]

(ii) Find the values of t at which the particle is instantaneously at rest. [2]

9. **(iii)** Find the total distance travelled by the particle during the first 2 seconds. [3]

(iv) Find the time(s) when the particle returns to O. [3]

10. The diagram shows part of the curve $y = 4 - \sqrt{2x}$ and the line $y = x$. The curve cuts the x – axis at the point P and the line intersects the curve at the point Q.



(i) Show that the x- coordinate of the point P is 8. [2]

(ii) Find the coordinates of the point Q. [3]

10. **(iii)** Find the area of the region A, bounded by the curve $y = 4 - \sqrt{2x}$, the x axis and the line $x = 8$ and $x = 10$. [3]

(iv) Find the area of the region B, bounded by the straight line $x - y = 0$, the curve $y = 4 - \sqrt{2x}$ and the y – axis. [4]

(v) If $\int_k^{10} \left(4 - \sqrt{2x}\right) \mathrm{dx} = 0$, where $2 < k < 8$, explain what this result implies about the curve $y = 4 - \sqrt{2x}$. [2]

End of Paper

Answer Key

Qn 1 (i)	Height, $h=3+\frac{2}{x+1}-\frac{4}{(x+1)^2}$	6(ii)	$\angle B=-\frac{5\pi}{6},-\frac{\pi}{6},\frac{\pi}{6},\frac{5\pi}{6}$
(ii)	$3x+2\ln(x+1)+\frac{4}{(x+1)^2}+C$	7(a)	$\therefore 6<k<8$
Qn 2(i)	$(x+2)^2+(y-1)^2=52$ or $x^2+y^2+4x-2y-47=0$	(b)	since $(b-5)^2\geq 0$ $-8c\geq 0$ $c\leq 0$
(ii)	$y=\frac{3}{2}x-9$	8(ii)	b = – vertical intercept =0.8 Range: {0.74 to 0.84 } a = gradient = 7.27 Range: {7.0 to 7.4}
(iii)	Q(4, 5)	(iii)	$y=-\frac{1}{b}xy+\frac{a}{b}$ Gradient = – 1.25 Vertical intercept = 9.025 Range: {9.025 to 9.1}
Qn 3(ii)	1.47	9 (i)	t = 1
Qn 4(a)	$2\sin\theta-1.5\cos\theta=\frac{5}{2}\sin(\theta-0.644)$	(ii)	$t=\frac{1}{3}$ or t = 2
4(b)(i)	t = 4.41	(iii)	Total distance = $2\frac{17}{27}$ or 2.63
4(b)(ii)	First time = 14 06 or 2.06 pm	(iv)	t = 0 (NA), 2.78 or 0.719
Qn 5 (i)	$20°C$	10(i)	x = 8
(ii)	t = 13.9 mins	(ii)	$\therefore Q(2,2)$
(iii)	As t increase, $e^{-0.05t}$ approaches zero. Hence, maximum temperature that the oven can reach = $300°C$	(iii)	A = 0.481
(iv)	t = 30.5 s	(iv)	$3\frac{1}{3}$
		(v)	The area bounded by the curve x = k, x = 8 and x-axis lies above the x axis and it is equal to the area of region A, which is area bounded by the curve x = 8, x = 10 and x-axis. This is area below the x axis.

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Class Index Number

Name : ____________________________________

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2023
Secondary 4

Monday | **ADDITIONAL MATHEMATICS** | **4049/02**

21 August 2023 | **PAPER 2 Solution** | 2 hours 15 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

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The total number of marks for this paper is 90.

90

This question paper consists of 17 printed pages and 3 blank pages.

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$.

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$$\sin 2A = 2\sin A \cos A$$
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1. **(i)** A prism has a volume of $\left(3x^2+8x+1\right)$ cm^3 and a cross-sectional area of $\left(x^2+2x+1\right)$ cm^2. Write down an expression for the height of the prism, in the form $A+\frac{B}{(x+1)}+\frac{C}{(x+1)^2}$. [4]

Height, $h=\frac{3x^2+8x+1}{x^2+2x+1}=3+\frac{2x-2}{x^2+2x+1}$ B1

$$\frac{2x-2}{x^2+2x+1}=\frac{B}{(x+1)}+\frac{C}{(x+1)^2}$$

$x=-1$ | $x=0$ M1

$2(-2)=C$ | $-2=B-4$

$C=-4$ B1 | $B=2$

Height, $h=3+\frac{2}{x+1}-\frac{4}{(x+1)^2}$ B1

(ii) Find $\int A+\frac{B}{(x+1)}+\frac{C}{(x+1)^2}\,\mathrm{d}x$. [3]

$$\int 3+\frac{2}{(x+1)}-\frac{4}{(x+1)^2}\,\mathrm{d}x$$

$$=3x+2\ln(x+1)+\frac{4}{(x+1)^2}+C$$

B1 B1 B1

where C is an arbitrary constant

2. A circle C passes through the point $P(4,-3)$ and has the same centre as the circle $x^2+y^2+4x-2y-1=0$.

(i) Find the equation of the circle C. [3]

$$x^2+y^2+4x-2y-1=0$$
$$(x+2)^2-4+(y-1)^2-1=1$$
$$(x+2)^2+(y-1)^2=6$$

Centre of circle = $(-2,1)$ B1

Radius of circle = $\sqrt{(4+2)^2+(-3-1)^2}=\sqrt{52}$ B1

Equation of circle is $(x+2)^2+(y-1)^2=52$ or $x^2+y^2+4x-2y-47=0$ A1

(ii) Find the equation of the tangent to the circle C at the point $P(4,-3)$. [2]

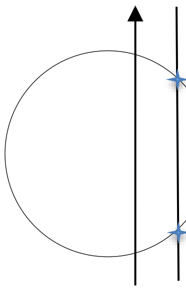
Gradient of P to the centre $=\frac{-3-1}{4+2}=\frac{-4}{6}=-\frac{2}{3}$

Gradient of tangent $=\frac{3}{2}$ M1

Equation of tangent at P is

$$y+3=\frac{3}{2}(x-4)$$
$$y=\frac{3}{2}x-6-3$$
$$y=\frac{3}{2}x-9$$ A1

(iii) Another point $Q(4, q)$ which lies on the circle C, is the same distance from the y-axis as the point P. Find the coordinates of the point Q. [2]



Subs $x = 4$ into equation of circle

$(4+2)^2 + (y-1)^2 = 52$

M1

$(y-1)^2 = 52 - 36$

$y - 1 = \pm 4$

$y = 5$ or $y = -3$

Q(4, 5) A1

<u>Alt Mtd :</u> Since perp bisector of chord passes through centre of circle,

$$\frac{q-3}{2} = 1$$

$$\therefore q = 5$$

Q(4, 5)

3. **(i)** Given that $y = (x-1)\sqrt{4x+1}$, show that $\frac{\text{dy}}{\text{dx}} = \frac{6x-1}{\sqrt{4x+1}}$. [3]

$$\frac{dy}{dx} = (x-1)\frac{1}{2\sqrt{4x+1}} \times 4 + \sqrt{4x+1}$$ M1

$$= \frac{2(x-1)}{\sqrt{4x+1}} + \frac{4x+1}{\sqrt{4x+1}}$$ M1

$$= \frac{6x-1}{\sqrt{4x+1}}$$ A1

(ii) Hence, evaluate $\int_1^2 \frac{6x-5}{\sqrt{4x+1}}\,dx.$ [4]

$$\int_1^2 \frac{6x-5}{\sqrt{4x+1}}\,dx = \int_1^2 \frac{6x-1-4}{\sqrt{4x+1}}\,dx$$

$$= \int_1^2 \frac{dy}{dx}\,dx - \int_1^2 \frac{4}{\sqrt{4x+1}}\,dx$$ M1

$$= \left[(x-1)\sqrt{4x+1}\right]_1^2 - \left[\frac{4\times 2}{4}\sqrt{4x+1}\right]_1^2$$ M1

$$= \left[\sqrt{9}-0\right] - 2\left[\sqrt{9}-\sqrt{5}\right]$$ M1

$$= 2\sqrt{5}-3$$

$$= 1.47$$

A1

4. **(a)** Express $2\sin\theta - 1.5\cos\theta$ in the form $R\sin(\theta-\alpha)$, where R > 0 and $0 < \alpha < \frac{\pi}{2}$. [3]

$$2\sin\theta - 1.5\cos\theta = R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$$
$$2 = R\cos\alpha$$
$$1.5 = R\sin\alpha$$

$$R^2 = 2^2 + (1.5)^2$$ B1
$$R = \frac{5}{2}$$

$$\tan\alpha = \frac{1.5}{2}$$
$$\alpha = 0.644$$ B1

$$2\sin\theta - 1.5\cos\theta = \frac{5}{2}\sin(\theta - 0.644)$$
A1

(b) John models the height of sea water level, H metres, on a particular day by the equation $H = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right)$, $0 \le t < 12$, where t hours is the number of hours after midday. Using this model, calculate

(i) the maximum possible height of sea water level and the value of t, to 2 decimal places, when it occurs. [3]

$$H = 6 + 2\sin\left(\frac{4\pi t}{25}\right) - 1.5\cos\left(\frac{4\pi t}{25}\right)$$

$$\max H = 6 + \frac{5}{2}\sin\left(\frac{4\pi t}{25} - 0.6435\right)$$

$$= 6 + \frac{5}{2}$$

$$= 8.5 \text{ m}$$ B1

$$\sin\left(\frac{4\pi t}{25} - 0.643501\right) = 1$$ M1

It occurs when $\frac{4\pi t}{25} - 0.643501 = \frac{\pi}{2}$

$$t = 4.405204$$

$$t = 4.41$$ A1

4 **(ii)** the first time when the height of the sea water is 7 metres, leaving your answer correct to the nearest minute. [4]

$$7 = 6 + \frac{5}{2}\sin\left(\frac{4\pi t}{25} - 0.643501\right)$$ M1

$$\frac{2}{5} = \sin\left(\frac{4\pi t}{25} - 0.643501\right)$$ M1

$$t = 2.09889$$

$$= 125.933 \text{ mins}$$ A1

$$= 126 \text{ mins}$$

First time = 14 06 or 2.06 pm B1

5. The temperature, θ °C, of an oven t minutes after it was switched on is given by $\theta = 300 - 280e^{-0.05t}$, $t \geq 0$.

(i) State the initial temperature of the oven. [1]

Initial temperature = $300 - 280 = 20°C$ B1

(ii) Find the value of t when the temperature of the oven reaches 160 °C. [2]

$$300 - 280e^{-0.05t} = 160$$
$$280e^{-0.05t} = 140$$
$$e^{-0.05t} = \frac{1}{2}$$

M1

$$-0.05t = \ln\left(\frac{1}{2}\right)$$
$$t = 13.8629$$
$$= 13.9 \text{ mins}$$

A1

(iii) Determine, with justification, the maximum temperature that this oven can approach. [2]

As t increase, $e^{-0.05t}$ approaches zero. B1

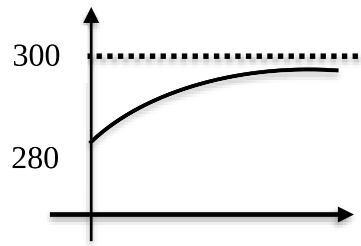
Hence, maximum temperature that the oven can reach = $300°C$

A1

Alt Mtd :

$$e^{-0.05t} > 0$$
$$-280e^{-0.05t} < 0$$
$$300 - 280e^{-0.05t} < 300$$
$$\therefore \theta < 300$$

or



Since $\theta = 300$ is an asymptote, θ approaches 300 as t increases.

5. The temperature, θ °C, of another oven t minutes after it was switched on is given by $\theta = 250 - 230e^{-0.1t}$, $t \geq 0$.

(iv) Assuming that both ovens are switched on at the same time, find the time when both ovens will have the same temperature since they were switched on. [5]

$$250 - 230e^{-0.1t} = 300 - 280e^{-0.05t}$$
$$280e^{-0.05t} - 230e^{-0.1t} = 50$$
$$-28e^{-0.05t} + 23e^{-0.1t} + 5 = 0$$ M1

Let $e^{-0.05t} = p$

$$23e^{(-0.05t)^2} - 28e^{-0.05t} + 5 = 0$$ M1
$$23p^2 - 28p + 5 = 0$$
$$(23p - 5)(p - 1) = 0$$ M1

$$p = \frac{5}{23}$$
$$e^{-0.05t} = \frac{5}{23}$$
$$t = 30.5211$$ A1
$$= 30.5$$

$$p = 1$$
$$e^{-0.05t} = 1$$ A1
$$-0.05t = 0$$
(NA)

6. **(i)** Prove that $\sin(A+B)\sin(A-B)=(\sin A+\sin B)(\sin A-\sin B)$. [3]

$$\sin(A+B)\sin(A-B)=(\sin A\cos B+\cos A\sin B)(\sin A\cos B-\cos A\sin B)$$
$$=\sin^2 A\cos^2 B-\cos^2 A\sin^2 B$$ M1
$$=\sin^2 A(1-\sin^2 B)-\sin^2 B(1-\sin^2 A)$$ M1
$$=\sin^2 A-\sin^2 A\sin^2 B-\sin^2 B+\sin^2 A\sin^2 B$$
$$=\sin^2 A-\sin^2 B$$ A1
$$=(\sin A+\sin B)(\sin A-\sin B)$$

(ii) Given that $\sin A=\frac{1}{2}$, find all the values of B for $-\pi\le B\le\pi$ that satisfy the equation $\sin(A+B)\sin(A-B)=0$. [4]

$$\sin(A+B)\sin(A-B)=\sin^2 A-\sin^2 B$$ M1
$$=0$$

From (i),
$$\sin^2 A=\sin^2 B$$
$$\sin^2 B=\frac{1}{4}$$
$$\sin B=\pm\frac{1}{2}$$ M1

Angle B lies in all 4 quadrants

basic angle, $\alpha=\frac{\pi}{6}$

$$\angle B=-\frac{5\pi}{6},-\frac{\pi}{6},\frac{\pi}{6},\frac{5\pi}{6}$$

or $-2.62, -0.524, 0.524, 2.62$

A1 A1

7. **(a)** Find the range of values of k for which $y=(k-6)x^2-8x+k$ cuts the $x-$axis at two distinct points and has a minimum point. [4]

$$b^2-4ac>0$$

$$(-8)^2-4(k-6)(k)>0$$ M1

$$64-4k^2+24k>0$$

$$k^2-6k-16<0$$ M1

$$(k-8)(k+2)<0$$

M1 $-2<k<8$

For curve to have minimum point, $k-6>0$

$$k>6$$

$\therefore 6<k<8$ A1

(b) Given that the line $y=5x+c$ is a tangent to the curve $y=2x^2+bx$, show that c cannot be positive. [4]

$$5x+c=2x^2+bx$$

$$2x^2+(b-5)x-c=0$$ M1

Since line is a tangent,

$$b^2-4ac=o$$

$$(b-5)^2-4(2)(-c)=0$$ M1

$$(b-5)^2=-8c$$

since $(b-5)^2\geq 0$ M1

$$-8c\geq 0$$

$$c\leq 0$$ A1

Hence c cannot be positive.

8. The table below shows experimental values of two variables, x and y, which are connected by an equation of the form $y = \frac{a}{x+b}$, where a and b are constants.

x	0.1	0.4	1.0	2.0	3.0
y	8.0	6.0	4.0	2.6	1.9

(i) Draw the graph of x against $\frac{1}{y}$, using a scale of 5 cm to 1 unit on the x-axis and a scale of 2 cm to 0.1 unit on the $\frac{1}{y}$-axis on the graph paper in the next page. [3]

x	0.1	0.4	1.0	2.0	3.0
$\frac{1}{y}$	0.125	0.167	0.25	0.385	0.526

M1

(ii) Use your graph to estimate the value of a and of b. [4]

$$y = \frac{a}{x+b}$$
$$y(x+b) = a$$
$$x+b = \frac{a}{y} \quad \text{M1}$$
$$x = a\left(\frac{1}{y}\right) - b$$

vertical intercept $= -0.8$

$b = 0.8$ B1 Range: {0.74 to 0.84 }

gradient $= \frac{2.4+0.8}{0.44-0}$ M1

$a = 7.27$ A1

Range: {7.0 to 7.4}

An alternative method for obtaining a straight line graph for the equation $y = \frac{a}{x+b}$ is to plot y against xy.

(iii) Using answers of a and b in part **(ii)** estimate the gradient and vertical intercept of the graph of y plotted against xy.

[3]

$$y = \frac{a}{x+b}$$
$$xy + by = a$$
$$by = -xy + a$$
$$y = -\frac{1}{b}xy + \frac{a}{b} \quad \text{M1}$$

gradient $= -\frac{1}{b}$

$= -\frac{1}{0.8}$

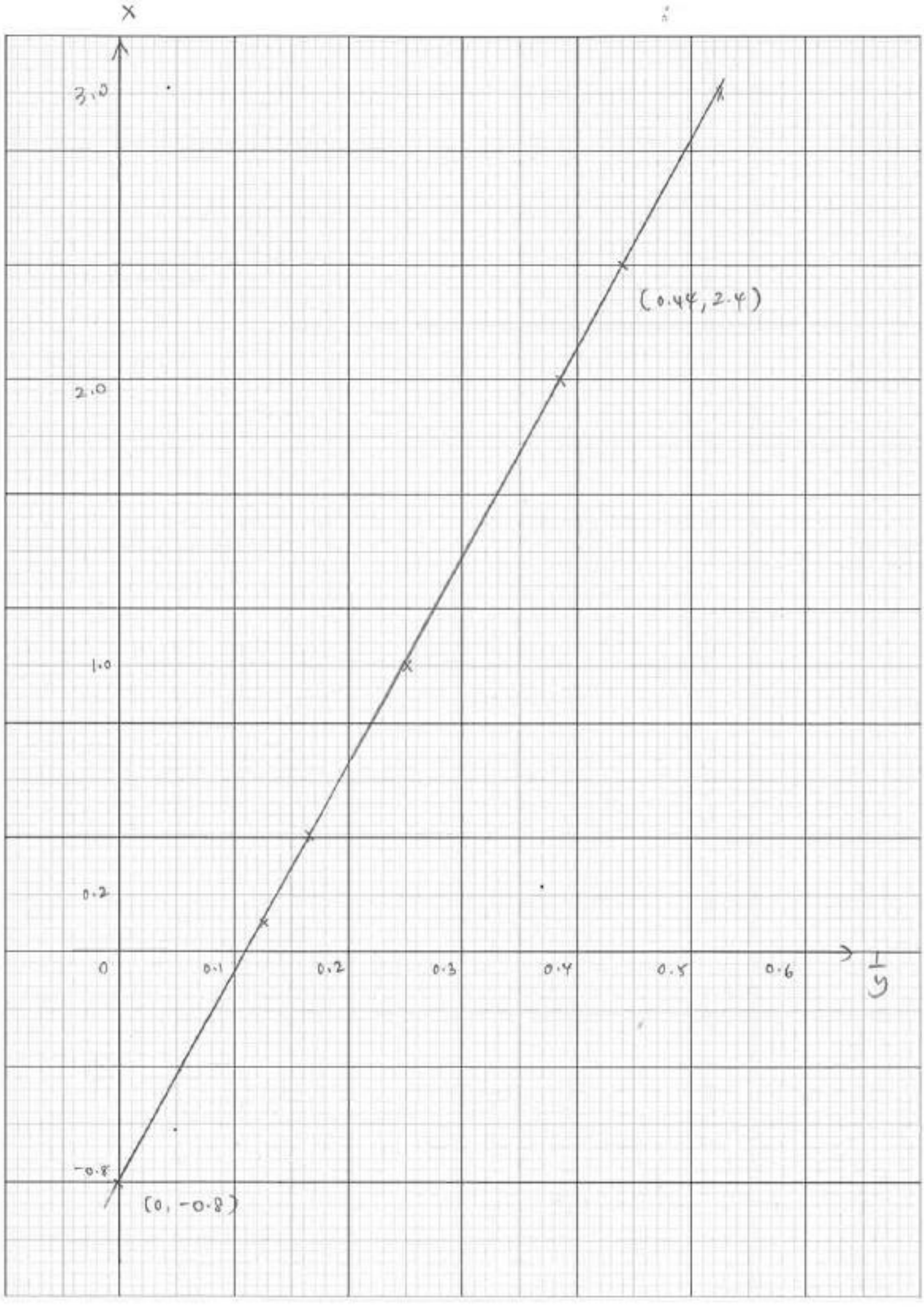
$= -\frac{5}{4}$ B1

vertical intercept $= \frac{a}{b}$

$= \frac{7.22}{0.8}$

$= 9.025$ B1

Range: {9.025 to 9.1}


x
3.0
2.0
1.0
0.2
0
-0.8
0.1
0.2
0.3
0.4
0.5
0.6
1/y
(0.44, 2.4)
(0, -0.8)

9. A particle moves in a straight line so that t seconds after passing through a fixed point O, its velocity, v cm/s, is given by.

(i) Find the value of t at the instant when the acceleration is -1 cm/s^2. [2]

$$v = 3t^2 - 7t + 2$$

$$a = \frac{\mathrm{dv}}{\mathrm{dt}}$$

$$= 6t - 7$$

M1

$$-1 = 6t - 7$$

$$6 = 6t$$

$$t = 1$$

A1

(ii) Find the values of t at which the particle is instantaneously at rest. [2]

$$3t^2 - 7t + 2 = 0$$

$$(3t - 1)(t - 2) = 0$$

M1

$$t = \frac{1}{3} \qquad t = 2$$

A1

9. **(iii)** Find the total distance travelled by the particle during the first 2 seconds. [3]

$$s=\int\left(3t^2-7t+2\right)\,\mathrm{d}t$$

$$s=\frac{3t^3}{3}-\frac{7t^2}{2}+2t+c$$

At t = 0, s = 0 and c =0 M1

$$s=\frac{3t^3}{3}-\frac{7t^2}{2}+2t$$

$$t=\frac{1}{3},\ s=\frac{17}{54}$$ M1

$$t=2,\ s=-2$$

Total distance $=2\left(\frac{17}{54}\right)+2=2\frac{17}{27}$ A1

(iv) Find the time(s) when the particle returns to O. [3]

$$\frac{3t^3}{3}-\frac{7t^2}{2}+2t=0$$

$$t\left(t^2-\frac{7}{2}t+2\right)=0$$ M1

$$\frac{1}{2}t\left(2t^2-7t+4\right)=0$$

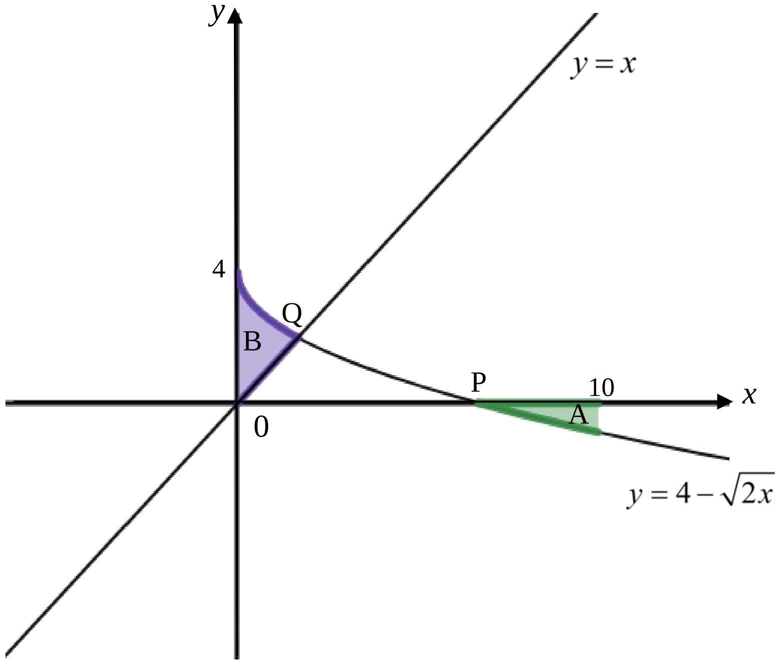
$$2t^2-7t+4=0$$

$t=0$ A1 NA

$$t=\frac{7\pm\sqrt{17}}{4}$$

$=2.78$ or 0.719 A1

10. The diagram shows part of the curve $y=4-\sqrt{2x}$ and the line $y=x$. The curve cuts the x – axis at the point P and the line intersects the curve at the point Q.



(i) Show that the x- coordinate of the point P is 8. [2]

At P, $y = 0$

$4-\sqrt{2x}=0$ M1

$16=2x$

$x=8$ A1

(ii) Find the coordinates of the point Q. [3]

$x=4-\sqrt{2x}$

$\sqrt{2x}=4-x$ M1

$2x=16-8x+x^2$

$x^2-10x+16=0$ M1

$(x-2)(x-8)=0$

$x=2$ or $x=8$ (point P)

$y=2$ $\therefore Q(2,2)$ A1

10. **(iii)** Find the area of the region A, bounded by the curve $y = 4 - \sqrt{2x}$, the x axis and the line $x = 8$ and $x = 10$. [3]

$$A = -\int_8^{10} \left(4 - \sqrt{2x}\right) \text{dx}$$

$$= -\left[4x - \frac{2}{3}\frac{(2x)^{3/2}}{2}\right]_8^{10}$$ M1

$$= -\left[4x - \frac{1}{3}(2x)^{3/2}\right]_8^{10}$$

$$= -\left[10.1857 - \frac{32}{3}\right]$$ M1

$$= 0.480926$$

$$= 0.481$$

A1

(iv) Find the area of the region B, bounded by the straight line $x - y = 0$, the curve $y = 4 - \sqrt{2x}$ and the y – axis. [4]

$$A = \int_0^2 \left(4 - \sqrt{2x}\right) \text{dx} - \frac{1}{2}(2)(2)$$ M1

$$= \left[4x - \frac{1}{3}(2x)^{3/2}\right]_0^2 - 2$$ M1

$$= \frac{16}{3} - 2$$ M1

$$= 3\frac{1}{3}$$ A1

(v) If $\int_k^{10} \left(4 - \sqrt{2x}\right) dx = 0$, where $2 < k < 8$, explain what this result implies about the curve $y = 4 - \sqrt{2x}$. [2]

B1

The area bounded by the curve $x = k$, $x = 8$ and x-axis lies above the x axis and it is **equal** to the area of region A, which is area bounded by the curve $x = 8$, $x = 10$ and x-axis. This is area below the x axis. B1

End of Paper