Full Name	Class Index No	Class

Anglo-Chinese School
(Barker Road)

PRELIMINARY EXAMINATION 2023
SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC)

ADDITIONAL MATHEMATICS
4049
PAPER 1

2 HOURS 15 MINUTES

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your index number and name on all the work you hand in.
Write in dark blue or black pen.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

For Examiner's Use

This question paper consists of **18** printed pages and **2** blank pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A$$

$$\operatorname{cosec}^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}bc\sin A$$

1 A cuboid has a square area $\left(6-\sqrt{35}\right)\text{m}^2$. The capacity of the cuboid is $\left(45\sqrt{5}-38\sqrt{7}\right)\text{m}^3$. Find the height of the cuboid, in m, in the form $\left(a\sqrt{5}-b\sqrt{7}\right)$, where *a* and *b* are integers. [3]

2 **(a)** Show that $(x-1)$ is a factor of $\mathrm{f}(x)=x^3-(2h+1)x^2-(k-2h)x+k$,where h and k are constants. [1]

(b) Explain with working why the equation $\mathrm{f}(x)=0$ will have 3 real roots if $h^2+k\geq 0$. [4]

3 The temperature, $T\,°\text{C}$, of glass being heated in a kiln is given by the formula $T = 2030 - 2000e^{-kt}$, where t is the time in hours since the glass was placed in the kiln. The temperature of the glass after 90 minutes is $1000\,°\text{C}$.

(a) Find the temperature of the glass before it was heated. [1]

(b) The glass can be molded into different shapes when its temperature reaches at least $1300\,°\text{C}$. What is the earliest time that the glass can be removed from the kiln and molded? [4]

(c) The instruction manual of the kiln gives the range of the temperature of the kiln as $T < y°\text{C}$. State the value of y. [1]

4 The curve $y = \mathrm{f}(x)$ is such that $\mathrm{f}''(x) = \dfrac{18}{(1-2x)^3}$ and $2\mathrm{f}'(-1) = 1$. Find the equation of the curve given that it passes through the point $(1, 0)$. [6]

5 It is given that $\mathrm{f}(x)=\dfrac{(x+2)^2}{p-x}$, $x \neq p$, where p is an integer.

(a) Obtain and simplify an expression for $\mathrm{f}'(x)$ in terms of p. [3]

(b) The range of values of x for which $\mathrm{f}(x)$ is an increasing function is $-2 < x < 8$. Show that the value of p is 3. [3]

(c) Hence find the equation of the normal to the curve at the point where the curve crosses the x-axis. [3]

6 Given that $5\sin^2 A - 3\cos^2 A = 7\sin 2A$ where $0 \le A \le 90^\circ$.

(a) Show that $\tan A = 3$. [3]

(b) Hence find the value of $\cos\left(60^\circ + A\right)$, leaving your answer in the form $\dfrac{a + b\sqrt{3}}{2\sqrt{10}}$, where *a* and *b* are integers. [3]

(c) Without finding the value of *A*, explain whether $60^\circ + A$ is acute or obtuse. [1]

7 **(a)** Given that $y=(3x-1)\left(\sqrt{6x+1}\right)$, show that $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{27x}{\sqrt{6x+1}}$. [3]

(b) Hence find the value of $\int_0^4 \frac{9x-3}{\sqrt{6x+1}}\,\mathrm{d}x$. [4]

8 **(a)** In the binomial expansion of $\left(2x-\frac{3}{x}\right)^n$, where n is a positive integer, the coefficient of the third term is $\frac{270}{8}\left(2^n\right)$. Show that $n = 6$. [4]

(b) Using the value of n in **part (a)**, find the coefficient of x^4 in the expansion of $\left(1+x^2\right)\left(2x-\frac{3}{x}\right)^n$. [3]

9 It is given that $y = a\cos 2x + b$ for $-180^\circ \le x \le 90^\circ$ where $a < 0$ and $b > 0$. Given that the amplitude of y is 3 and the maximum value of y is 7.

(a) State the value of a and of b. [2]

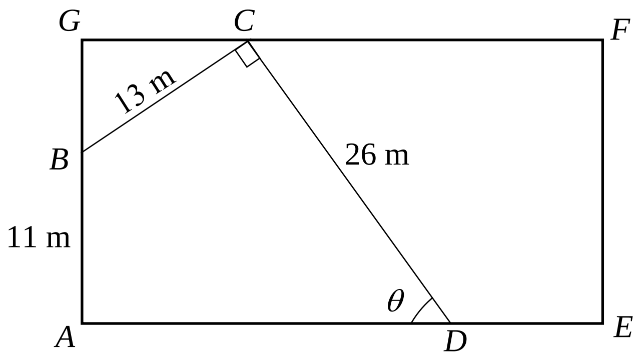
(b) Sketch the graph of $y = a\cos 2x + b$ for $-180^\circ \le x \le 90^\circ$. [3]

(c) Hence state the number of solution(s) of the equation $1 - \frac{1}{2}\cos 2x = 0$ for $-180^\circ \le x \le 90^\circ$. [2]

10 **(a)** Solve $49^x - 7^{x+1} = 18$, leaving your answer in the form $\log_a b$, where a and b are integers to be determined. [5]

(b) Find the largest value of the integer q such that $-3x^2 + qx - 8$ is negative for all real values of x. [3]

11



The diagram shows a rectangular futsal court, *AEFG*.
From a point *A* on the court, players are to run along the straight paths *AB*, *BC*, *CD* and *DA*. The lengths of *AB*, *BC* and *CD* are 11 m, 13 m and 26 m respectively. Angle *ADC* is θ, where $0^0 < \theta < 90°$. The total distance covered by each player is *T* metres.

(a) Show that *T* can be expressed as $50 + 26\cos\theta + 13\sin\theta$. [3]

(b) Express T in the form $50 + R\cos\left(\theta - \alpha\right)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

(c) Bryan claims that he ran a total distance of 85 m. Explain with calculations if this is possible. [2]

12 The line $y = 4$ and the positive y-axis are tangents to a circle C. It is given that the x-coordinate of the centre of C is a, where $a > 0$.

(a) Write down, in terms of a, the largest possible y-coordinate of the centre of C. [1]

The line L is a tangent to C at the point (8, 13) on the circle. The centre of C lies below and to the left of (8, 13).

(b) Find the equation of C. [4]

(c) Find the equation of L. [2]

13 **(a)** Express $\frac{4x^3+2x^2-5}{x^2(2x-1)}$ in partial fractions. [7]

(b) Hence find $\int \frac{4x^3+2x^2-5}{x^2\left(2x-1\right)}\,\mathrm{d}x$. [3]

Anglo-Chinese School
(Barker Road)

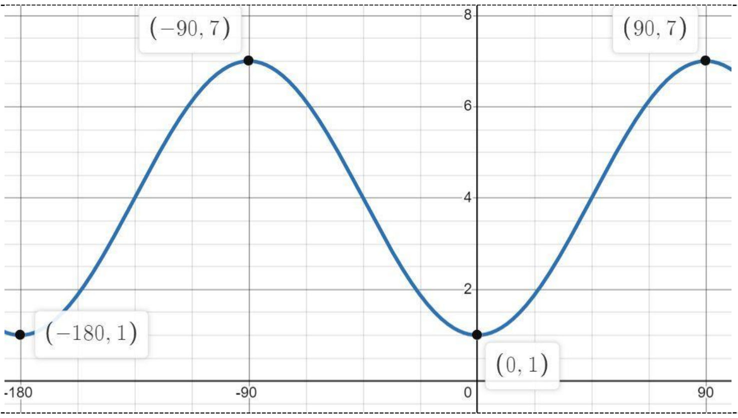
Marking Scheme
Secondary 4 Express / 5 Normal (Academic)
Additional Mathematics Paper 1
Preliminary Examination 2023

1		Height = $\frac{45\sqrt{5}-38\sqrt{7}}{6-\sqrt{35}}\times\frac{6+\sqrt{35}}{6+\sqrt{35}}$			
		$=\frac{270\sqrt{5}+45\sqrt{175}-228\sqrt{7}-38\sqrt{245}}{1}$			
		$=270\sqrt{5}+225\sqrt{7}-228\sqrt{7}-266\sqrt{5}$			
		$=4\sqrt{5}-3\sqrt{7}$			
2	(a)	$\mathrm{f}(x)=x^3-(2h+1)x^2-(k-2h)x+k$			
		$\mathrm{f}(1)=(1)^3-(2h+1)(1)^2-(k-2h)(1)+k$			
		$1-2h-1-k+2h+k=0$ (shown)			
		By factor theorem, $x-1$ is a factor of f(*x*)			
	(b)	$x^3-(2h+1)x^2-(k-2h)x+k=(x-1)(x^2-2hx-k)$			
		$\mathrm{f}(x)=(x-1)(x^2-2hx-k)$			
		Since f(*x*) has three real roots when f(*x*) = 0, $x^2-2hx-k=0$ has two real roots.			
		$b^2-4ac\geq 0$			
		$(-2h)^2-4(1)(-k)\geq 0$			
		$4h^2+4k\geq 0$			
		$h^2+k\geq 0$			
3	(a)	$T=30°\mathrm{C}$			
	(b)	$1000=2030-2000e^{-k(1.5)}$			
		$2000e^{-k(1.5)}=1030$			
		$k=0.442392$			
		$1300=2030-2000e^{-0.442392t}$			
		$2000e^{-0.442392t}=730$			
		$\ln e^{-0.442392t}=\ln\frac{730}{2000}$			
		$t=2.28h$			
	(c)	$y=2030$			

4	(a)	$f''(x)=\frac{18}{(1-2x)^3}$			
		$f'(x)=\frac{18(1-2x)^{-2}}{-2\times-2}+c_1$			
		$=\frac{9(1-2x)^{-2}}{2}+c_1$			
		$2f'(-1)=1$			
		$2\left(\frac{9}{2(1-2(-1))^2}+c_1\right)=1$			
		$\frac{1}{2}+c_1=\frac{1}{2}$			
		$c_1=0$			
		$f'(x)=\frac{9}{2(1-2x)^2}$			
		$f(x)=\frac{9}{4(1-2x)}+c_2$			
		Sub (1,0)			
		$0=\frac{9}{4(1-2(1))}+c_2$			
		$c_2=\frac{9}{4}$			
		$y=\frac{9}{4(1-2x)}+\frac{9}{4}$			
5	(a)	$f'(x)=\frac{(p-x)[2(x+2)(1)]-(x+2)^2(-1)}{(p-x)^2}$			
		$=\frac{2(p-x)(x+2)+(x+2)^2}{(p-x)^2}$			
		$=\frac{(x+2)(2p+2-x)}{(p-x)^2}$			
	(b)	For $f(x)$ is an increasing function, $f'(x)>0$			
		$(x+2)(2p+2-x)>0$			

		$-x^2+2px+4p+4>0$			
		$x^2-2px-4p-4<0$ ---(1)			
		$(x+2)(x-8)<0$			
		$x^2-6x-16<0$ ----(2)			
		Comparing (1) & (2)			
		$-2p=-6$			
		$p=3$			
		Alternative method			
		Comparing this with $(x+2)(2p+2-x)>0$ in (a), $(x+2)(x-8)<0$ $(x+2)(8-x)>0$ $2p+2=8$ $p=3$			
	(c)	$\frac{(x+2)^2}{3-x}=0 \Rightarrow x=-2$			
		$\text{f}'(x)=\frac{(-2+2)(2(4)+2+2)}{(3+2)^2}$ $=0$ (gradient of tangent)			
		Gradient of normal is undefined (vertical line) $\therefore$ Eqn. of normal : $x=-2$			
6	(a)	$5\sin^2 A-3\cos^2 A=7\sin 2A$			
		$5\sin^2 A-3\cos^2 A=7\left(2\sin A\cos A\right)$			
		$5\sin^2 A-14\sin A\cos A-3\cos^2 A=0$			
		$(5\sin x+\cos x)(\sin x-3\cos x)=0$			
		$\frac{\sin A}{\cos A}=-\frac{1}{5}$ or $\frac{\sin A}{\cos A}=3$			
		$\tan A=-\frac{1}{5}$ (reject, A is acute) or $\tan A=3$ (shown)			
	(b)	$\cos(60^{\circ}+A)=\cos 60^{\circ}\cos A-\sin 60^{\circ}\sin A$			
		$=\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{10}}\right)-\left(\frac{\sqrt{3}}{2}\right)\left(\frac{3}{\sqrt{10}}\right)$			

		$= \frac{1-3\sqrt{3}}{2\sqrt{10}}$			
	(c)	From (b), $\cos(60^\circ + A)$ is a negative ratio. So $60^\circ + A$ lies in the 2nd or 4th quadrant. Both 60° and A are both acute, $60^\circ + A$ cannot exceed 180°. Thus $60^\circ + A$ lies in the 2nd quadrant and is obtuse.			
7	(a)	$y = (3x-1)\left(\sqrt{6x+1}\right)$			
		$\frac{\mathrm{d}y}{\mathrm{d}x} = 3\left(\sqrt{6x+1}\right) + \frac{1}{2}(3x-1)(6x+1)^{-\frac{1}{2}}(6)$			
		$= \frac{18x+3+9x-3}{\sqrt{6x+1}}$			
		$= \frac{27x}{\sqrt{6x+1}}$			
	(b)	$\int_0^4 \frac{9x-3}{\sqrt{6x+1}}\,\mathrm{d}x$			
		$= \frac{1}{3}\int_0^4 \frac{27x}{\sqrt{6x+1}}\,\mathrm{d}x - \int_0^4 \frac{3}{\sqrt{6x+1}}\,\mathrm{d}x$			
		$= \frac{1}{3}\left[(3x-1)\sqrt{6x+1}\right]_0^4 - 3\left[\frac{(6x+1)^{\frac{1}{2}}}{6\times\frac{1}{2}}\right]_0^4$			
		$= \frac{1}{3}\left(11\sqrt{25}+\sqrt{1}\right) - \left(\sqrt{25}-\sqrt{1}\right)$			
		$= \frac{44}{3}$ or 16.7 (3s.f)			
8	(a)	$T_3 = \binom{n}{2}(2x)^{n-2}\left(-\frac{3}{x}\right)^2$			
		$= \binom{n}{2}(2)^{n-2}(x)^{n-2}(-3)^2\left(x^{-1}\right)^2$			
		$\frac{n(n-1)}{2}(2)^{n-2}(-3)^2 = \frac{270}{8}\left(2^n\right)$			
		$\frac{n(n-1)}{2}(2)^{-2}(-3)^2 = \frac{270}{8}$			

		$\frac{n(n-1)}{2}=15$			
		$n^2-n-30=0$			
		$(n-6)(n+5)=0$			
		$n=6$ or $n=-5$ (reject)			
8	(b)	$\left(1+x^2\right)\left(2x-\frac{3}{x}\right)^6=\left(1+x^2\right)\left(-576x^4+2160x^2+...\right)$			
		Coefficient of x^4 = $-576+2160=1584$			
9	(a)	$a=-3$			
		$b=4$			
	(b)				
	(c)	$1-\frac{1}{2}\cos 2x=0$			
		$6-\frac{6}{2}\cos 2x=0$			
		$4-3\cos 2x=-2$			
		$y=-2$			
		No solution			
10	(a)	$49^x-7^{x+1}=18$			
		$7^{2x}-7^x\times 7=18$			
		Let y = 7^x			
		$y^2-7y-18=0$			

		$(y-9)(y+2)=0$			
		$y=-2$ or $y=9$			
		(reject)			
		$7^x=9$			
		$x=\log_7 9$			
		$a=7,\ b=9$			
	(b)	$-3x^2+qx-8<0$			
		$b^2-4ac<0$			
		$q^2-4(-3)(-8)<0$			
		$q^2-96<0$			
		$(q-\sqrt{96})(q+\sqrt{96})<0$	M1		
		$-\sqrt{96}<q<\sqrt{96}$			
		$-9.798<q<9.798$			
		Largest value of the integer q = 9	A1		
					[8]
11	(a)	$DA = GC + x$			
		$x=26\cos\theta$			
		$GC=13\sin\theta$			
		$AD = 13\sin\theta+26\cos\theta$			
		T = 11+13 +26 + $26\cos\theta+13\sin\theta$			
		T = 50 + $26\cos\theta+13\sin\theta$			
	(b)	$\sqrt{26^2+13^2}$			
		$=\sqrt{845}$ or $13\sqrt{5}$			
		$\tan^{-1}\dfrac{13}{26}$			
		$T = 50+13\sqrt{5}\cos(\theta-26.6°)$			
	(c)	Max T = 50 + $13\sqrt{5}$			
		=79.1 (3s.f)			
		This is not possible as the maximum distance of T is 79.1 m.			

12	(a)	$a + 4$			
	(b)	centre = $(a, a+4)$			
		$\sqrt{(a-8)^2+(a+4-13)^2}=a$			
		$(a-8)^2+(a-9)^2=a$			
		$a^2-16a+64+a^2-18a+81=a^2$			
		$a^2-34a+145=0$			
		$a=29$ (rejected) or $a=5$			
		centre = $(5,9)$			
		$(x-5)^2+(y-9)^2=25$			
	(c)	Gradient of radius = $\frac{13-9}{8-5}=\frac{4}{3}$			
		Gradient of line *L* = $-\frac{3}{4}$			
		Equation of line *L*: $y=-\frac{3}{4}x+c$			
		$13=-\frac{3}{4}(8)+c$			
		$c=19$			
		$y=-\frac{3}{4}x+19$			
13	(a)	$\frac{4x^3+2x^2-5}{x^2(2x-1)} = 2+\frac{4x^2-5}{x^2(2x-1)}$			
		$\frac{4x^2-5}{x^2(2x-1)} = \frac{A}{x}+\frac{B}{x^2}+\frac{C}{2x-1}$			
		$4x^2-5=A(x)(2x-1)+B(2x-1)+Cx^2$			
		Let $x=0$			
		$-5=B(-1)$			
		$B=5$			
		Let $x=\frac{1}{2}$			

		$-4=\frac{1}{4}C$			
		$C=-16$			
		Let $x=1$			
		$-1=A+5-16$			
		$-1=A+5-16$			
		$A=10$			
		$\frac{4x^3+2x^2-5}{x^2(2x-1)}=2+\frac{10}{x}+\frac{5}{x^2}-\frac{16}{2x-1}$			
	(b)	$\int\frac{4x^3+2x^2-5}{x^2\left(2x-1\right)}\,\mathrm{d}x$			
		$=\int 2+\frac{10}{x}+\frac{5}{x^2}-\frac{16}{2x-1}\,\mathrm{d}x$			
		$=2x+10\ln x-\frac{5}{x}-8\ln\left(2x-1\right)+c$			

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 Mr Tan invested some money in stock in May.
The value of a stock, \$$y$, varies with time, x, can be modelled by the equation
$y=-\frac{1}{3}x^2+2x+1$, where y is in thousands and x is in months.

(a) State the initial value of the stock. [1]

(b) Express y in the form $a(x-b)^2+c$. [2]

(c) Determine the best time to sell off his stocks to make a maximum profit.
Find the value of the stock at this time. [2]

2 It is given that $\mathrm{f}(x) = ax^3 + 2bx^2 - 34x + 12$, where a and b are constants, has a factor of $x - 3$ and leaves a remainder of 32 when divided by $x + 1$.

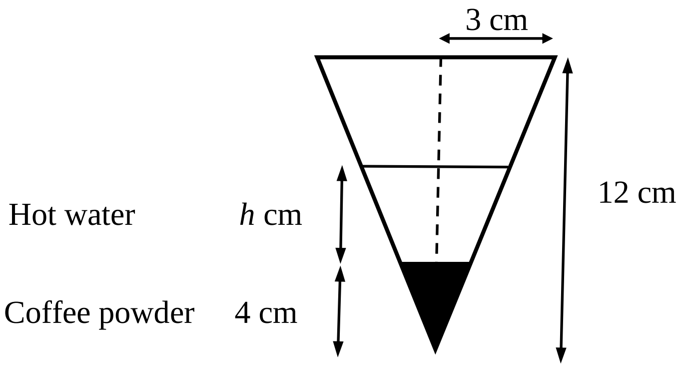
(a) Find the value of a and of b. [4]

(b) Find the range of values of k for which the line $y + 9 = kx$ intersects the curve $x = \sqrt{y - 3x}$ at two distinct points. [4]

3 **(a)** Show that $\dfrac{2\cos^2\theta-\sin\theta\cos\theta+1}{\sin^2\theta}=3\cot^2\theta-\cot\theta+1$. [3]

(b) Hence solve the equation $2\cos^2\theta-\sin\theta\cos\theta+1=\sin^2\theta$ for $-\pi\le\theta\le\pi$. [4]

4 A coffee filter is in the shape of a right circular cone. The cone has a base radius of 3 cm and a height of 12 cm. The coffee powder, as seen in the cross-section of the filter below, has a depth of 4 cm. Let *h* be the height of the hot water in the filter at time *t* seconds. Assume that the amount of water in the coffee powder is negligible.



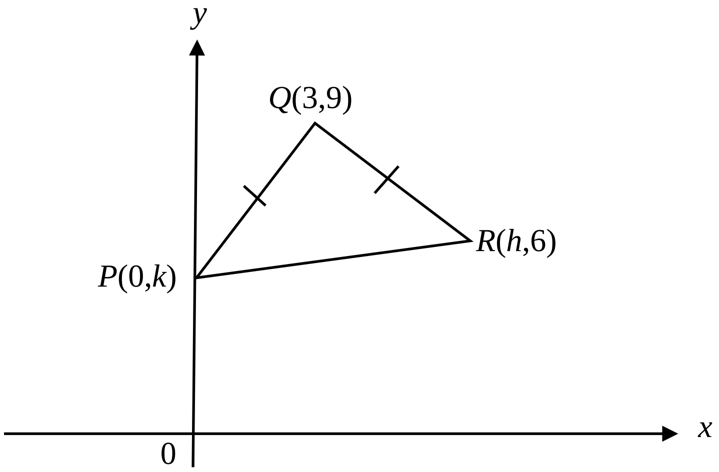
(a) Show that the volume of hot water, *V*, in the filter at the time *t* seconds is given by $V = \frac{\pi}{48}(h+4)^3 - \frac{4\pi}{3}$. [3]

[The volume of a cone of height *h* and base radius *r* is $\frac{1}{3}\pi r^2 h$]

Initially, the filter contains only coffee powder. A machine dispenses hot water into the filter at a constant rate of 5 cm^3 per second and hot water drips out of the filter at a constant rate of 2 cm^3 per second.

(b) Find the exact rate of change of the depth of water when $h = 8$. [4]

5



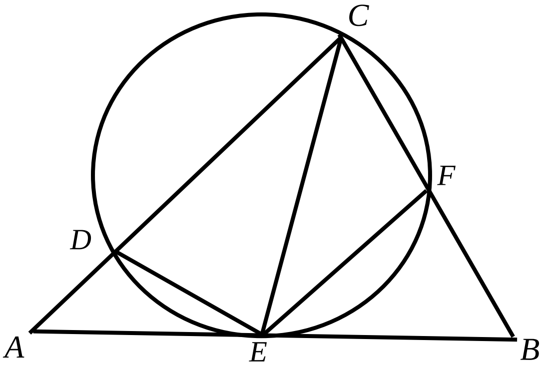
The diagram shows an isosceles triangle *PQR* with vertices at *P*(0, *k*), *Q*(3, 9) and *R*(*h*, 6), where *h* and *k* are positive constants and $h>k$.

(a) Show that $h + k = 12$. [3]

(b) It is now given that $h = 7$.
The point S is such that $PQRS$ is a kite. Given that S lies on the line $21y = 7x - 8$, find the coordinates of S. [4]

(c) Find the area of the kite. [2]

6



In the diagram above, *CDEF* is a circle and the line *AB* is a tangent to the circle at *E*. *ADC* and *CFB* are straight lines and *AE* = *EB* and *CF* = *FB*.

(a) Prove that *AC* is parallel to *EF*. [1]

(b) Prove that angle AED = angle CEF . [2]

(c) Prove that triangle AED is similar to triangle CEF . [2]

7 Theresa bought a house on 2 January 1970. The house was valued by a local estate agent on the same date every 10 years up to 2010.

The valuation price, V, of the house is related to t, the number of years since 1970. The variables V and t can be modelled by the equation $V = pq^t$, where p and q are constants.

The table below gives the values of V and t for some of the years 1970 to 2010.

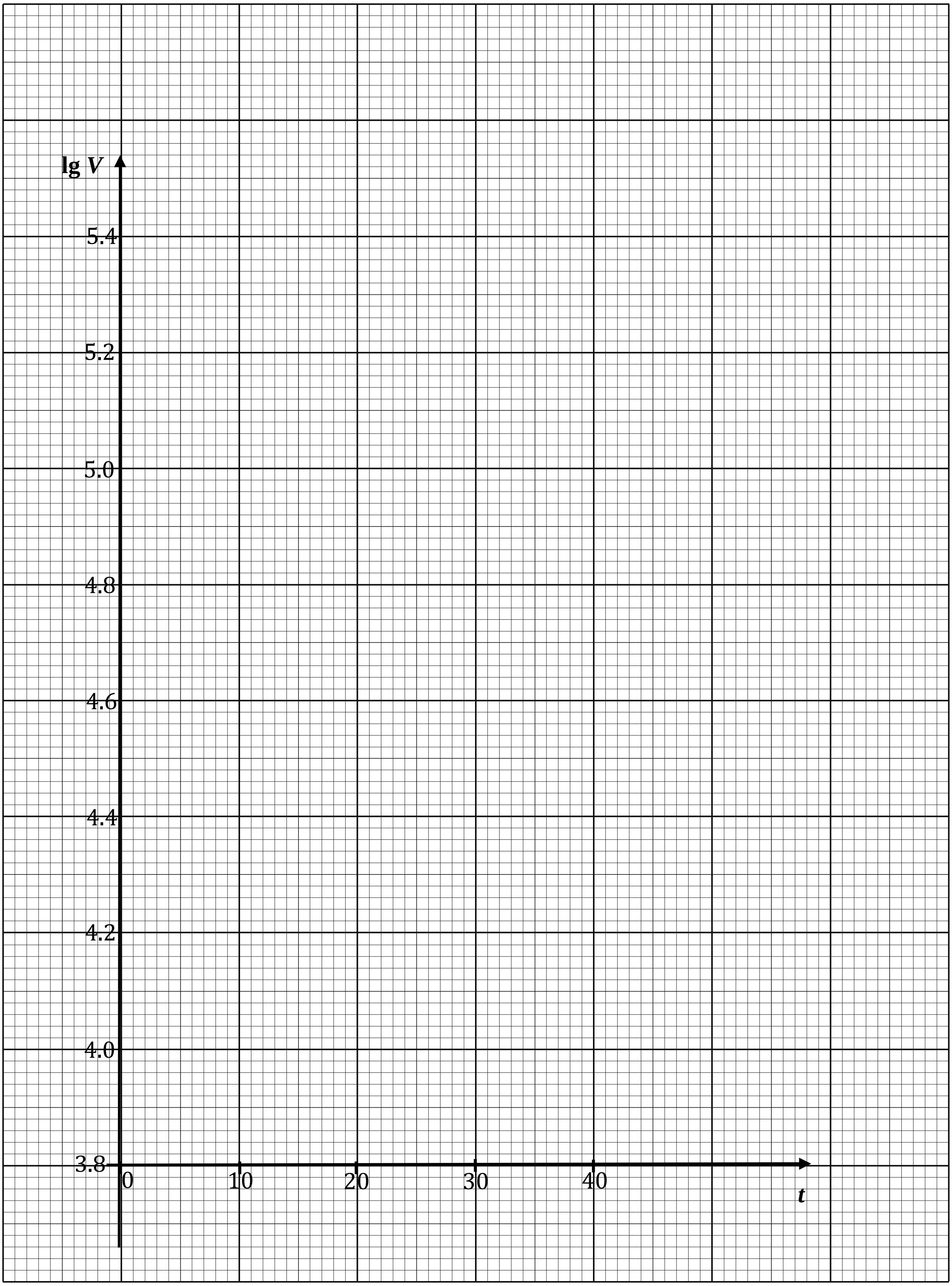
Year	1970	1980	1990	2000	2010
t	0	10	20	30	40
V ($)	8000	17500	38000	83000	190000

(a) Plot lg V against t and draw a straight line graph on the grid on the next page. [2]

(b) Use your graph to estimate the values of p and q. [3]

(c) Determine the year in which Theresa's house will first be worth a million dollars. [3]

(d) Explain whether your answer to **part (c)** is likely to be reliable. [1]

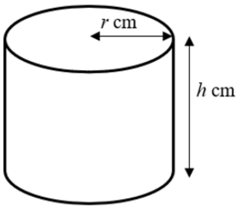

lg V
5.4
5.2
5.0
4.8
4.6
4.4
4.2
4.0
3.8
0
10
20
30
40
t

8 It is given that $\log_3\left(4-x^2\right)-\log_{\sqrt{3}}(x-1)=1$.

(a) Explain clearly why $1<x<2$. [4]

(b) Hence solve $\log_3\left(4-x^2\right)-\log_{\sqrt{3}}(x-1)=1$ and show that it has only one solution. [5]

9 A beverage company makes closed cylindrical cans, each with volume 400cm^3. The materials for the curved surface and base of the cans cost \$0.025 per cm^2 and \$0.03 per cm^2 respectively. Assume that the cost of wasted materials is negligble.

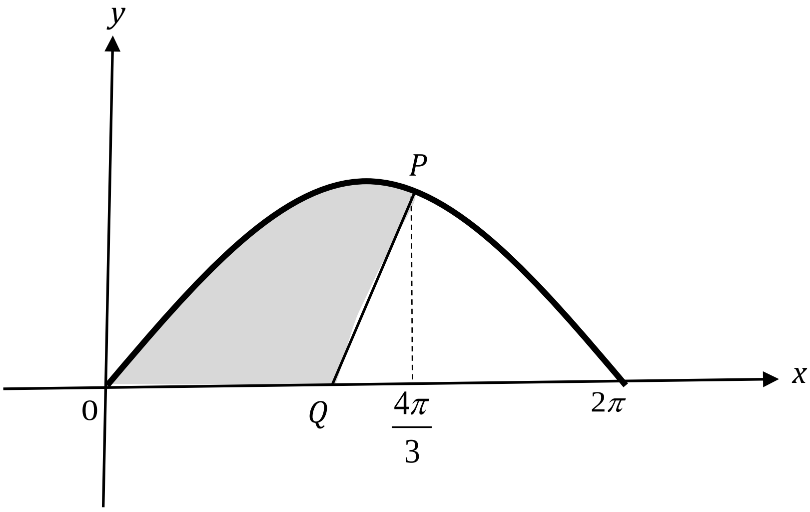


(a) Show that the cost in dollars, C, of the materal required to make the container is $C=\frac{3}{50}\pi r^2+\frac{20}{r}$. [3]

(b) Given that *r* can vary, find the value of *r* for which the cost of material is stationary. [3]

(c) Justify, why the beverage company should choose to use the value of *r* found in **part (b)** in producing the beverage can. [3]

10



The diagram shows part of the curve $y = 3\sin\frac{x}{2}$ that cuts the $x -$ axis at $x = 0$ and $x = 2\pi$. The normal to the curve at $x = \frac{4\pi}{3}$ cuts the x-axis at Q.

(a) Find the coordinates of Q, leaving your answer in exact form. [6]

(b) Find the area of the shaded region bounded by the curve, the normal *PQ* and the coordinate axes. [4]

11 A particle moves in a straight line, so that, t seconds after passing a fixed point A, its velocity, v m/s, is given by $v = 10e^{-0.1t} - 5$. The particle comes to instantaneous rest at the point B.

(a) Find the value of t when the particle reaches B. Express your answer in the form of $p\ln q$, where p and q are integers. [3]

(b) Calculate the distance AB. [4]

(c) Find the acceleration of the particle when $t = 3$. [2]

(d) Show that the particle is again at A at some instant during the sixteenth second after first passing through A. [3]

Anglo-Chinese School
(Barker Road)

Marking Scheme
Secondary 4 Express / 5 Normal (Academic)
Additional Mathematics Paper 2
Preliminary Examination 2023

1	(a)	\$1000			
	(b)	$-\frac{1}{3}x^2+2x+1$			
		$=-\frac{1}{3}\left[x^2-6x-3\right]$			
		$=-\frac{1}{3}\left[\left(x-3\right)^2-\left(3\right)^2\right]+1$			
		$=-\frac{1}{3}\left(x-3\right)^2+3+1$			
		$=-\frac{1}{3}\left(x-3\right)^2+4$			
	(c)	3 months or August			
		\$4000			
2	(a)	$\mathrm{f}(x)=ax^3+2bx^2-34x+12$			
		$\mathrm{f}(3)=0$			
		$a(3)^3+2b(3)^2-34(3)+12=0$			
		$27a+18b=90$			
		$3a+2b=10--(1)$			
		$\mathrm{f}(-1)=32$			
		$a(-1)^3+2b(-1)^2-34(-1)+12=0$			
		$-a+2b=-14--(2)$			
		Solving (1), (2)			
		$a=6$			
		$b=-4$			
	(b)	$y=kx-9 \qquad ---(1)$			
		$x=\sqrt{y-3x} \qquad ---(2)$			
		Sub. (1) into (2):			
		$x=\sqrt{(kx-9)-3x}$			
		$x^2=kx-9-3x$			
		$x^2-(k-3)x+9=0$			
		For the line to intersect the curve at two distinct points,			

		$b^2-4ac>0$			
		$[-(k-3)]^2-4(1)(9)>0$			
		$k^2-6k+9-36>0$			
		$k^2-6k-27>0$			
		$(k+3)(k-9)>0$			
		$\therefore\ k<-3$ or $k>9$			
3	(a)	$\dfrac{2\cos^2\theta-\sin\theta\cos\theta+1}{\sin^2\theta}$			
		$=\dfrac{2\cos^2\theta}{\sin^2\theta}-\dfrac{\sin\theta\cos\theta}{\sin^2\theta}+\dfrac{1}{\sin^2\theta}$			
		$=2\cot^2\theta-\cot\theta+\cos ec^2\theta$			
		$=2\cot^2\theta-\cot\theta+\left(1+\cot^2\theta\right)$			
		$=3\cot^2\theta-\cot\theta+1$			
		Alternative Method			
		$\dfrac{2\cos^2\theta-\sin\theta\cos\theta+\cos^2\theta+\sin^2\theta}{\sin^2\theta}$			
		$\dfrac{3\cos^2\theta-\sin\theta\cos\theta+\sin^2\theta}{\sin^2\theta}$			
		$\dfrac{3\cos^2\theta}{\sin^2\theta}-\dfrac{\sin\theta\cos\theta}{\sin^2\theta}+\dfrac{\sin^2\theta}{\sin^2\theta}$			
		$=3\cot^2\theta-\cot\theta+1$			
	(b)	$3\cot^2\theta-\cot\theta+1=1$			
		$3\cot^2\theta-\cot\theta=0$			
		$\cot\theta\left(3\cot\theta-1\right)=0$			
		$\cot\theta=0$ or $\cot\theta=\dfrac{1}{3}$			
		$\dfrac{\cos\theta}{\sin\theta}=0$			
		$\cos\theta=0$, $\theta=-\dfrac{\pi}{2},\dfrac{\pi}{2}$			
		When $\tan\theta=3,$ basic $\angle=1.249$ $\theta=-1.89\ ,\ 1.25$			
		$\theta=-1.89\ ,-\dfrac{\pi}{2},\ 1.25,\dfrac{\pi}{2}$			

4	(a)	$V=\frac{1}{3}\pi r^2\left(h+4\right)-\frac{1}{3}\pi(1)^2(4)$			
		Using similar triangles			
		$\frac{4}{4+h}=\frac{1}{r}$			
		$r=\frac{1}{4}\left(h+4\right)$			
		$\therefore V=\frac{1}{3}\pi\left(\frac{h+4}{4}\right)^2\left(h+4\right)-\frac{4\pi}{3}$			
		$=\frac{1}{3}\pi\left(\frac{1}{4}\right)^2\left(h+4\right)^2\left(h+4\right)-\frac{4\pi}{3}$			
		$=\frac{\pi}{48}\left(h+4\right)^3-\frac{4\pi}{3}$			
	(b)	$\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{\pi}{16}\left(h+4\right)^2$			
		Using chain rule			
		$\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}$			
		$5-2=\frac{\pi}{16}\left(h+4\right)^2\times\frac{\mathrm{d}h}{\mathrm{d}t}$			
		When h = 8, $3=\frac{\pi}{16}\left(8+4\right)^2\times\frac{\mathrm{d}h}{\mathrm{d}t}$			
		$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{1}{3\pi}\text{cm/s}$			
5	(a)	$\sqrt{\left(3-0\right)^2+\left(9-k\right)^2}=\sqrt{\left(3-h\right)^2+\left(9-6\right)^2}$			
		$3^2+\left(9-k\right)^2=\left(3-h\right)^2+3^2$			
		$\left(9-k\right)^2=\left(3-h\right)^2$			
		$9-k=\pm\left(3-h\right)$			
		$9-k=3-h$ or $9-k=-\left(3-h\right)$			
		$h-k=3-9$ $9+3=h+k$			
		$h-k=-6$ (reject) $h+k=12$ (shown)			
		Since $h-k>0$			

	(b)	Midpoint of *PR* = $\left(\frac{0+7}{2},\frac{5+6}{2}\right)$			
		$=\left(\frac{7}{2},\frac{11}{2}\right)$			
		Gradient of *QS* = $\frac{9-\frac{11}{2}}{3-\frac{7}{2}}=-7$			
		Equation of *QS* : $y-\frac{11}{2}=-7\left(x-\frac{7}{2}\right)$			
		$y=-7x+30---(1)$			
		Sub (1) into $21y=7x-8$			
		$21\left(-7x+30\right)=7x-8$			
		$x=4\frac{1}{7}$			
		Subst. $x=4\frac{1}{7}$ into (1) : $y=1$			
		$S\left(4\frac{1}{7},1\right)$			
	(c)	Area = $\frac{1}{2}\begin{vmatrix}3 & 0 & 4\frac{1}{7} & 7 & 3\\ 9 & 5 & 1 & 6 & 9\end{vmatrix}$			
		=. $\frac{1}{2}\left\{\left[(3)(5)+0+\left(4\frac{1}{7}\right)(6)+(7)(9)\right]-\left[0+(5)\left(4\frac{1}{7}\right)+(1)(7)+(6)(3)\right]\right\}$			
		$=\frac{1}{2}\left(\frac{720}{7}-\frac{320}{7}\right)$			
		$=28\frac{4}{7}$ or 28.6 (3sf)			
6	(a)	Given that *AE* = *EB* and *CF* = *FB*, E is the midpoint of *AB* and *F* is midpoint *FB.*			
		Hence by mid-point theorem, *AC* and *EF* are parallel			
	(b)	angle *AED* = angle *DCE* (tangent chord theorem)			
		angle *DCE* = angle *CEF*			

		(alternate angles, *AC* is parallel to *EF*)			
		Hence angle *AED* = angle *CEF*			
	(c)	In triangle *AED* and triangle *CEF*			
		Let angle CFE = x			
		Angle *CDE* = $180-x$ (angles in opposite segment)			
		Angle *ADE* = $180-(180-x)=x$ (adjacent angles on a a straight line)			
		Angle *CFE* = Angle *ADE*			
		or Angle *CFE* = Angle *ADE* (exterior angle of cyclic quadrilateral)			
		or angle *DAE* = angle *FEB* (corresponding angles, *AC* parallel *EF*)			
		Angle *FEB* = angle *ECF* (tangent chord theorem)			
		Angle *DAE* = angle *ECF*			
		From part (a) angle *AED* = angle *CEF*			
		Triangle *AED* is similar to triangle *CEF* (AA similarity)			
7	(a)	$\lg V = \lg pq^t$			
		$\lg V = \lg p + t\lg q$			
		$\lg V = \lg qt + \lg p$			

t	0	10	20	30	40
V	8 000	17 500	38 000	83 000	190 000
$\lg V$	3.90	4.24	4.58	4.92	5.28

Plot a straight line graph of $\lg V$ against *t*.

	(b)	$\lg V$ intercept: $\lg p = 3.90$ $p = 10^{3.90}$ $p = 7940$ (to 3 s.f.)			
		Gradient: $\lg q = \frac{5.3-4.06}{35-5} = 0.041333$			
		$q = 10^{\frac{5.3-4.06}{35-5}}$ $= 1.10$ (3s.f)			

	(c)	$V = pq^t$			
		$10^6 = 10^{3.90}\left(10^{0.041333t}\right)$			
		$\dfrac{10^6}{10^{3.90}} = 10^{0.041333t}$			
		$\lg\dfrac{10^6}{10^{3.90}} = 0.041333t$			
		$t = 50.81$ years			
		1970 + 50.81 = 2020			
	(d)	2020 is outside range of values in the initial table. House prices may not grow in the same way after 2010. 2020 would be using extrapolation, so is not likely to be reliable.			
8	(a)	$\log_3\left(4-x^2\right)-\log_{\sqrt{3}}(x-1)=1$			
		$4-x^2>0$			
		$(x-2)(x+2)<0$			
		$-2<x<2$			
		$x-1>0$			
		$x>1$			
		$-2<x<2$ and $x>1$			
		$1<x<2$			
	(b)	$\log_3\left(4-x^2\right)-\log_{\sqrt{3}}(x-1)=1$			
		$\log_3\left(4-x^2\right)-\dfrac{\log_3(x-1)}{\log_3\sqrt{3}}=1$			
		$\log_3\left(4-x^2\right)-\log_3(x-1)^2=1$			
		$\log_3\dfrac{\left(4-x^2\right)}{(x-1)^2}=1$			
		$\dfrac{\left(4-x^2\right)}{(x-1)^2}=3$			
		$4x^2-6x-1=0$			
		$x=\dfrac{6\pm\sqrt{36-4(-4)}}{8}$			

		$x = 1.65$ or -0.151(rejected)			

9	(a)	$h = \frac{400}{\pi r^2}$			
		Total surface area $= 2\pi r^2 + 2\pi rh$			
		$= 2\pi r^2 + 2\pi r\left(\frac{400}{\pi r^2}\right)$			
		$= 2\pi r^2 + \frac{800}{r}$			
		$C = 0.03\left(2\pi r^2\right) + 0.025\left(\frac{800}{r}\right)$			
		$= \frac{3}{50}\pi r^2 + \frac{20}{r}$			
	(b)	$\frac{\mathrm{d}C}{\mathrm{d}r} = \frac{3}{25}\pi r - \frac{20}{r^2}$			
		For stationary cost			
		$\frac{\mathrm{d}C}{\mathrm{d}r} = 0$			
		$\frac{3}{25}\pi r - \frac{20}{r^2} = 0$			
		$\frac{3}{25}\pi r = \frac{20}{r^2}$			
		$r^3 = \frac{500}{3\pi}$			
		$r = 3.7575$			
		$r = 3.76$ (3s.f)			
	(c)	$\frac{\mathrm{d}^2C}{\mathrm{d}r^2} = \frac{3}{25}\pi + \frac{40}{r^3}$			
		When $r^3 = \frac{500}{3\pi}$, $\frac{\mathrm{d}^2C}{\mathrm{d}r^2} = 1.13097 > 0$			
		Since the cost is minimum, the company should choose *r* found in (b) .			

10	(a)	$y = 3\sin\frac{x}{2}$			
		$\frac{\mathrm{d}y}{\mathrm{d}x} = 3\left(\frac{1}{2}\right)\cos\frac{x}{2}$ $= \frac{3}{2}\cos\frac{x}{2}$			
		At $x = \frac{4\pi}{3}$,			
		$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{2}\cos\frac{2\pi}{3}$			
		$= -\frac{3}{4}$ (gradient of tangent)			
		Gradient of normal = $= \frac{4}{3}$			
		At $x = \frac{4\pi}{3}, y = \frac{3\sqrt{3}}{2}$			
		Eqn of normal: $\frac{3\sqrt{3}}{2} = \frac{4}{3}\left(\frac{4\pi}{3}\right) + c$			
		$y = \frac{4}{3}x - \frac{16\pi}{9} + \frac{3\sqrt{3}}{2}$			
		$y = 0$ $\frac{16\pi}{9} - \frac{3\sqrt{3}}{2} = \frac{4}{3}x$			
		$x = \frac{3}{4}(\frac{16\pi}{9} - \frac{3\sqrt{3}}{2})$			
		$x = \frac{4\pi}{3} - \frac{9\sqrt{3}}{8}$			
		$Q(\frac{4\pi}{3} - \frac{9\sqrt{3}}{8}, 0)$			
	(b)	Shaded area $= \int_0^{\frac{4\pi}{3}} 3\sin\frac{x}{2}\ \mathrm{d}x - \frac{1}{2} \times \frac{3\sqrt{3}}{2} \times \frac{9\sqrt{3}}{8}$			
		$= \left[-6\cos\frac{x}{2}\right]_0^{\frac{4\pi}{3}} - 2\frac{17}{32}$			
		$= -6\cos\frac{2\pi}{3} - (-6\cos 0) - 2\frac{17}{32}$			

		$=3+6-2\frac{17}{32}$				
		$=6\frac{15}{32}$ or 6.47 units^2				
11	(a)	At B , $v = 0$				
		$10e^{-0.1t}-5=0$				
		$e^{-0.1t}=\frac{1}{2}$				
		$-0.1t=\ln\frac{1}{2}$				
		$0.1t=\ln 2$				
		$t=10\ln 2$				
	(b)	$s=\int v\ dt$				
		$=\int 10e^{-0.1t}-5\ dt$				
		$=-100e^{-0.1t}-5t+c$				
		when t = 0, s = 0, 4 c = 100				
		$s=-100e^{-0.1t}-5t+100$				
		when t = 10 ln 2,				
		$s=-100e^{-\ln 2}-50\ln 2+100$				
		$=-15.3\text{m}$				
		Distance AB = 15.3 m				
		Alternative Method:				
		Distance $AB=\int_0^{10\ln 2}\left(10e^{-0.1t}-5\right)\ dt$				
		$=\left[-100e^{-0.1t}-5t\right]_{0}^{10\ln 2}$				
		$=(-100e^{-\ln 2}-50\ln 2)-(-100-0)$				
		$=-50-50\ln 2+100$				
		= 15.3 m (3s.f)				
	(c)	$a=\frac{\mathrm{d}v}{\mathrm{d}t}=-e^{-0.1t}$				
		When t = 3, $a=-e^{3}=-0.741\text{ m/s}^2$ (3s.f)				
	(d)	$s=-100e^{-0.1t}-5t+100$				

		when t = 15, $s=-100e^{-0.1(15)}-5(15)+100=2.69\text{m}$			
		when t = 16, $s=-100e^{-0.1(16)}-5(16)+100=-0.1897\text{m}$			
		Since the displacement changes from positive 2.69m at t = 15 to negative 0.1897 m at t = 16,			
		4 the particle was at A again during the sixteenth second.			