AHMAD IBRAHIM SECONDARY SCHOOL
GCE O-LEVEL PRELIMINARY EXAMINATION 2023

SECONDARY 4 EXPRESS

Name:	Class:	Register No.:

ADDITIONAL MATHEMATICS **4049/01**
Paper 1 **7 August 2023**

Candidates answer on the Question Paper**.** **2 hours 15 minutes**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
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Answer **all** questions.
Give non-exact numerical answers to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
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The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use
/90

This document consists of **19** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

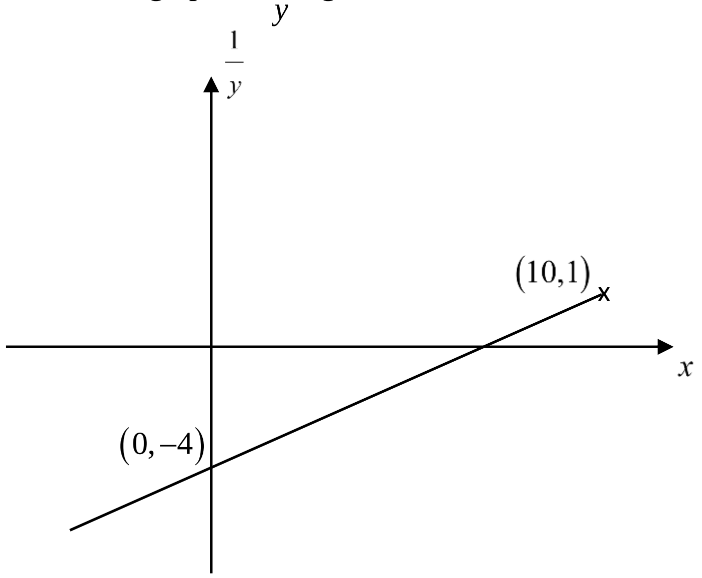
$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\text{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 The variables x and y are related by the equation $y = \dfrac{h}{2x - k}$. The diagram below shows the graph of $\dfrac{1}{y}$ against x.



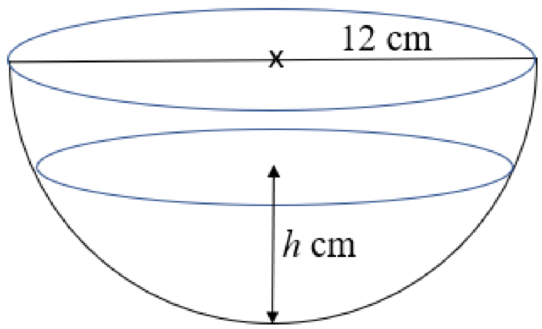
Calculate the value of h and of k. [4]

2 The Richter scale measures the intensity of an earthquake using the formula $M = \lg\left(\frac{I}{I_0}\right)$, where M is the magnitude of the earthquake, I is the intensity of the earthquake, and I_0 is the intensity of the smallest earthquake that can be measured.

(a) Calculate the magnitude of an earthquake if its intensity is 1000 times the intensity of the smallest earthquake that can be measured. [1]

(b) In February 2011, an earthquake with magnitude 6.2 was recorded in Christchurch, New Zealand. Few weeks later, an earthquake with magnitude 9.0 was detected in Fukushima, Japan. How many times stronger in intensity was the Japan's earthquake as compared to the New Zealand's earthquake? Give your answer to 2 decimal places. [3]

3 The diagram shows a hemispherical bowl of radius 12 cm. Water is poured into the bowl and at any time *t* seconds, the height of the water level from the lowest point of the hemisphere is *h* cm. The rate of change of the height of the water level is 0.4 cm/s.



(a) Show that the area of the water surface, *A*, is given by $A = \pi h(24 - h)$. [2]

(b) Find the rate of change of *A* when $h = 5$ cm.
Leave your answer in terms of π. [3]

4 **(a)** Explain why there is only one solution to the equation $\log_5\left(13-4x\right)=\log_{\sqrt{5}}\left(2-x\right)$. [5]

(b) Solve the simultaneous equations

$$4^{x+3} = 32\left(2^{x+y}\right),$$

$$9^{x} + 3^{y} = 10.$$

[7]

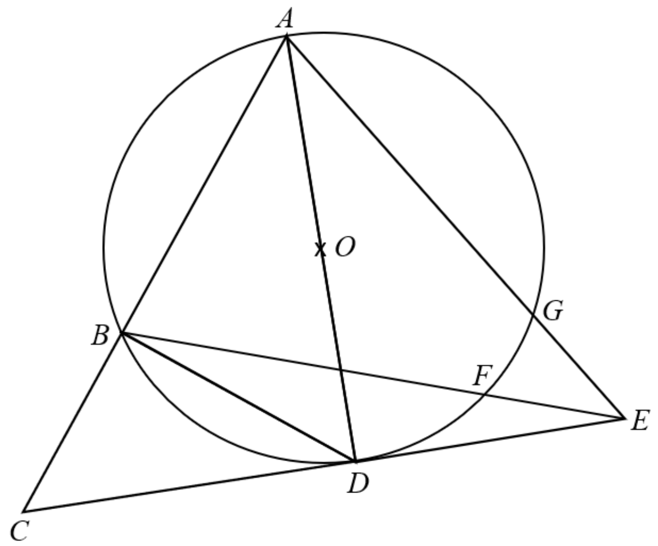
5 **(a)** Prove the identity $\cot 2x = \dfrac{1}{2\tan x} - \dfrac{1}{2}\tan x$. [2]

(b) Hence solve the equation $\tan x\left(3 - 4\cot 2x\right) = 3$ for $0° \le x \le 360°$. [5]

(c) Without further solving, explain why there are 6 roots to the equation $\tan\dfrac{x}{2}\left(3 - 4\cot x\right) = 3$ for $-360° \le x \le 720°$. [2]

6 The curve $y = e^{2x}\sqrt{1-3x}$ intersects the y-axis at the point P. The tangent and the normal to the curve at P meet the x-axis at A and B respectively. Find the exact area of triangle PAB. [7]

7 In the diagram, *CE* is a tangent that touches the circle of centre *O* at *D*. *AD* is the diameter of the circle, *EA* cuts the circle at points *G* and *A*, and *EB* cuts the circle at points *F* and *B*.



(a) Given that *ABC* is a straight line, show that triangle *ABD* and triangle *DBC* are similar. [3]

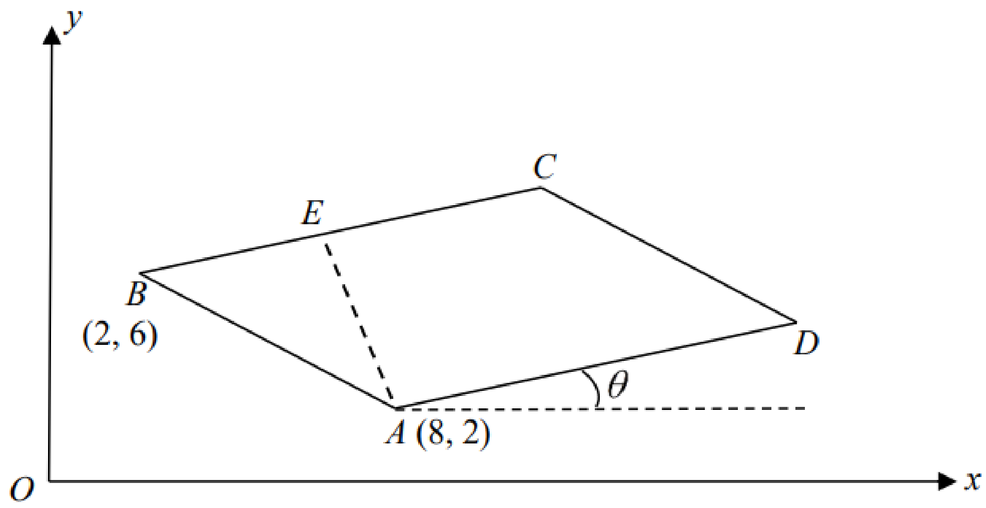
(b) If $BE = AE$, show that $EF = EG$. [4]

8 **(a)** Write down the first three terms in the expansion, in ascending powers of x, of $\left(2-\frac{x}{4}\right)^n$, where n is a positive integer greater than 2. [3]

(b) The first two terms in the expansion, in ascending powers of x, of $(1+x)^2\left(2-\frac{x}{4}\right)^n$ are $a+bx^2$, where a and b are constants.
Find the value of n. [3]

(c) Hence find the value of a and of b. [3]

9 The diagram shows a parallelogram *ABCD* in which the coordinates of the points *A* and *B* are (8, 2) and (2, 6) respectively. The line *AD* makes an angle θ with the horizontal and $\tan\theta = 0.5$. The point *E* lies on *BC* such that *AE* is the shortest distance from *A* to *BC*.



(a) Show that the equation of line *BC* is $2y = x + 10$. [2]

(b) Find the equation of line *AE* and the coordinates of *E*. [3]

(c) Given that $\frac{BE}{BC} = \frac{1}{5}$, find the coordinates of *C* and *D*. [4]

(d) Find the area of the figure *OBEA*, where *O* is the origin. [2]

10 **(a)** Solve the equation $2\cos 3x + 1 = 0$ for $0 \le x \le \pi$. [3]

(b) Sketch the graph of $y = 2\cos 3x + 1$ for $0 \le x \le \pi$. [3]

(c) The equation of a curve is $y = \dfrac{\sin 3x}{2 + \cos 3x}$, where $0 \le x \le \pi$.

Using **(a)** and **(b)**, find the range of values of x for which y is a decreasing function. [5]

11 **(a)** Express $\dfrac{3x^2+4x-20}{(2x+1)\left(x^2+4\right)}$ in partial fractions. [5]

(b) Differentiate $\ln\left(x^2+4\right)$ with respect to x. [2]

(c) The gradient function of a curve is $\dfrac{3x^2+4x-20}{(2x+1)\left(x^2+4\right)}$.

Given that the y-intercept of the curve is $\left(0, \ln 4\right)$, using part **(a)** and **(b)**, find the equation of the curve. [4]

END OF PAPER

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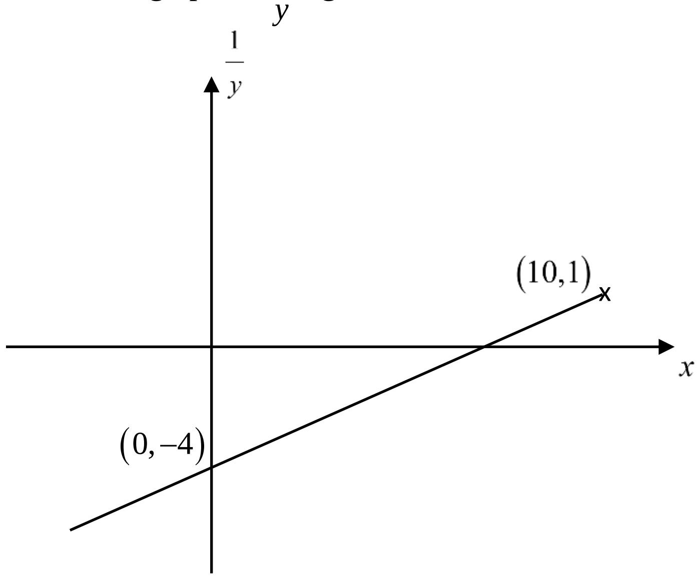

Calculate the value of h and of k. [4]

$$y = \frac{h}{2x-k}$$

$$\frac{1}{y} = \frac{2x-k}{h} = \frac{2}{h}x - \frac{k}{h}$$

B1: setting up linear form

Gradient of line = $\dfrac{1-(-4)}{10-0} = \dfrac{1}{2} = \dfrac{2}{h}$

M1: finding gradient

$$h = 4$$

A1

y-intercept at $-4 = -\dfrac{k}{h}$

$$k = 16$$

A1

2 The Richter scale measures the intensity of an earthquake using the formula $M = \lg\left(\frac{I}{I_0}\right)$, where M is the magnitude of the earthquake, I is the intensity of the earthquake, and I_0 is the intensity of the smallest earthquake that can be measured.

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$$M = \lg\left(\frac{I}{I_0}\right)$$

$$M = \lg\left(\frac{1000I_0}{I_0}\right) = 3$$ B1

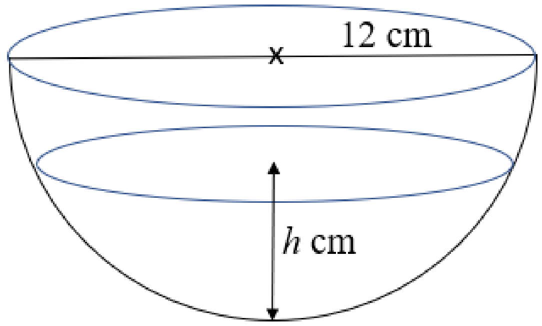
$$6.2 = \lg\left(\frac{I_{NZ}}{I_0}\right) \Rightarrow I_{NZ} = 10^{6.2} I_0$$ M1: substitution and making I the subject

$$9.0 = \lg\left(\frac{I_J}{I_0}\right) \Rightarrow I_J = 10^9 I_0$$

$$\frac{I_J}{I_{NZ}} = 10^{9-6.2} = 630.96 \text{ (2 d.p.)}$$ M1, A1

The Japan's earthquake is 630.96 times stronger than the New Zealand's earthquake.

3 The diagram shows a hemispherical bowl of radius 12 cm. Water is poured into the bowl and at any time t seconds, the height of the water level from the lowest point of the hemisphere is h cm. The rate of change of the height of the water level is 0.4 cm/s.



(a) Show that the area of the water surface, A, is given by $A = \pi h(24 - h)$. [2]

Let radius of water surface be r cm

$r^2 + (12-h)^2 = 12^2$

M1: use of Pythagoras thm

$r^2 = 12^2 - (12-h)^2$

$= (12-(12-h))(12+(12-h))$

$= h(24-h)$

M1: simplification of r^2 and getting the result

Area, $A = \pi r^2 = \pi h(24-h)$

(b) Find the rate of change of A when h = 5 cm.
Leave your answer in terms of π. [3]

$\frac{dA}{dh} = \pi(24-2h)$

M1: correct differentiation

$\frac{dA}{dt} = \frac{dA}{dh} \times \frac{dh}{dt}$

$= \pi(24-2h) \times 0.4$

M1: chain rule

$= \pi(24-10) \times 0.4$ when $h = 5$ cm

$= 5.6\pi$

A1: simplification of r^2 and getting the result

Rate of change of surface area = 5.6π cm^2/s

4 **(a)** Explain why there is only one solution to the equation $\log_5(13-4x) = \log_{\sqrt{5}}(2-x)$. [5]

$$\log_5(13-4x) = \log_{\sqrt{5}}(2-x)$$
$$= \frac{\log_5(2-x)}{\log_5\sqrt{5}}$$
$$= \frac{\log_5(2-x)}{\frac{1}{2}}$$
$$= 2\log_5(2-x)$$
$$\log_5(13-4x) = \log_5(2-x)^2$$
$$(13-4x) = (2-x)^2$$
$$(13-4x) = x^2 - 4x + 4$$
$$x^2 = 9$$
$$x = 3 \quad or \quad -3$$

M1: correct change of base

M1: obtaining quadratic equation

M1: correctly solving quadratic eqn

When $x = 3$, $\log_{\sqrt{5}}(2-3) = \log_{\sqrt{5}}(-1)$ is undefined, so reject $x = 3$.
The only solution is $x = -3$ (ans)

A1, A1: explanation of undefined function and concluding only 1 final answer

(b) Solve the simultaneous equations

$$4^{x+3} = 32\left(2^{x+y}\right) ,$$

$$9^x + 3^y = 10 .$$ [7]

$4^{x+3} = 32\left(2^{x+y}\right)$

$2^{2x+6} = 2^5\left(2^{x+y}\right) = 2^{5+x+y}$

$2x + 6 = 5 + x + y$ — M1: linear equation relating x and y

$y = x + 1$ ---------- (1)

$9^x + 3^y = 10$

$3^{2x} + 3^y = 10$

$3^{2x} + 3^{x+1} = 10$ — M1: equation with 1 variable

$\left(3^x\right)^2 + 3\left(3^x\right) - 10 = 0$

Let $u = 3^x$:

$u^2 + 3u - 10 = 0$ — M1: use of substitution

$(u+5)(u-2) = 0$

$u = 2$ or $u = -5$ — M1: solving quadratic equation

$3^x = 2$ or $3^x = -5$ (reject since $3^x > 0$)

$x \lg 3 = \lg 2$ — M1: Taking lg on both sides to find x

$x = \dfrac{\lg 2}{\lg 3} = 0.631$ (3 sf) — A1: correct x

$y = \dfrac{\lg 2}{\lg 3} + 1 = 1.63$ (3 s.f) — A1: correct y

5 **(a)** Prove the identity $\cot 2x = \dfrac{1}{2\tan x} - \dfrac{1}{2}\tan x$. [2]

$$\cot 2x = \frac{1}{2\tan x} - \frac{1}{2}\tan x$$

$$\begin{aligned}
LHS &= \cot 2x \\
&= \frac{1}{\tan 2x} \\
&= \frac{1}{\frac{2\tan x}{1-\tan^2 x}} \\
&= \frac{1-\tan^2 x}{2\tan x} \\
&= \frac{1}{2\tan x} - \frac{\tan^2 x}{2\tan x} \\
&= \frac{1}{2\tan x} - \frac{1}{2}\tan x
\end{aligned}$$

M1: double angle formula

M1: splitting terms and getting result

(b) Hence solve the equation $\tan x(3-4\cot 2x)=3$ for $0° \le x \le 360°$. [5]

$$\tan x(3-4\cot 2x)=3$$

$$\tan x\left(3-4\left(\frac{1}{2\tan x}-\frac{1}{2}\tan x\right)\right)=3$$

$$\tan x\left(3-\frac{2}{\tan x}+2\tan x\right)=3$$

$$3\tan x - 2 + 2\tan^2 x = 3$$

$$2\tan^2 x + 3\tan x - 5 = 0$$

$$(\tan x - 1)(2\tan x + 5) = 0$$

$\tan x = 1$ or $\tan x = -2.5$

basic angle = 45° or 68.199°

$x = 45°, 225°, 111.8°, 291.8°$

M1: simplification to trigo quadratic

M1: 2 answers

M1: correct basic angles

A2: 2 pairs correct answers
A1: any 1 pair correct

(c) Without further solving, explain why there are 6 roots to the equation $\tan\frac{x}{2}(3-4\cot x)=3$ for $-360° \le x \le 720°$. [2]

There are 4 roots to the equation $\tan x(3-4\cot 2x)=3$ for $0° \le x \le 360°$ from (b).

Since the **period** of $\tan\frac{x}{2}(3-4\cot x)$ is **doubled** of $\tan x(3-4\cot 2x)$, there will be 4/2 = 2 roots to the equation for $0° \le x \le 360°$. [B1]

For $-360° \le x \le 720°$, the graph of $\tan\frac{x}{2}(3-4\cot x)$ would have **repeated 3 cycles**, thereby giving 3 x 2 = 6 roots to the equation. [B1]

OR

$\tan\frac{x}{2}(3-4\cot x)=3$, $-360° \le x \le 720°$

Let $y=\frac{x}{2}$, then $\tan y(3-4\cot 2y)=3$, $-180° \le y \le 360°$

y has 4 solutions in the domain $0° \le y \le 360°$, **1 from each quadrant**, from (b). [B1]

Therefore, for the domain $-180° \le y \le 360°$, the graph would have entered **another half a cycle**, giving rise to 2 additional roots. [B1]

Therefore, there will be 6 solutions for y in the given domain, and thus, 6 roots to the equation.

6 The curve $y = e^{2x}\sqrt{1-3x}$ intersects the y-axis at the point P. The tangent and the normal to the curve at P meet the x-axis at A and B respectively. Find the exact area of triangle PAB. [7]

$y = e^{2x}\sqrt{1-3x}$

At P, $x = 0$: $y = 1$.

M1: coor of P

$$\frac{dy}{dx} = 2e^{2x}\sqrt{1-3x} + e^{2x}\left(\frac{1}{2}\right)(1-3x)^{-1/2}(-3)$$

M1: correct differentiation with product rule

$$= 2e^{2x}\sqrt{1-3x} - \left(\frac{3}{2\sqrt{1-3x}}\right)e^{2x}$$

$$= \frac{e^{2x}}{\sqrt{1-3x}}\left(2(1-3x) - \frac{3}{2}\right)$$

Note: students may not simplify and just subst x values to find gradient

$$= \frac{e^{2x}}{\sqrt{1-3x}}\left(\frac{1}{2} - 6x\right)$$

At P, $\frac{dy}{dx} = \frac{1}{2}$.

M1: either gradient of tangent or normal

Equation of tangent: $y - 1 = \frac{1}{2}(x - 0)$

M1: equation of tangent

$$y = \frac{1}{2}x + 1$$

At A, $y = 0$: $x = -2$

Equation of normal: $y - 1 = -2(x - 0)$

M1: equation of normal

$$y = -2x + 1$$

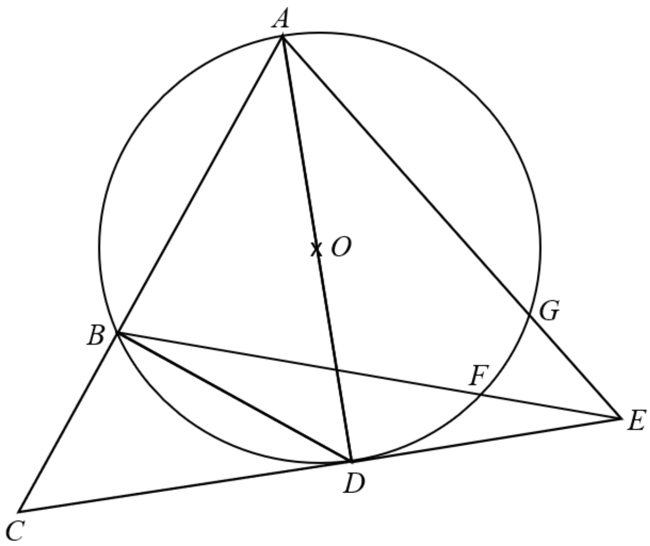
At B, $y = 0$: $x = \frac{1}{2}$

M1: finding coordinates of A and B

Area of triangle PAB = $\frac{1}{2}(1)\left(2 + \frac{1}{2}\right) = \frac{5}{4}$ units^2

A1

7 In the diagram, *CE* is a tangent that touches the circle of centre *O* at *D*. *AD* is the diameter of the circle, *EA* cuts the circle at points *G* and *A*, and *EB* cuts the circle at points *F* and *B*.



(a) Given that *ABC* is a straight line, show that triangle *ABD* and triangle *DBC* are similar. [3]

$\measuredangle ABD = 90°$ (right angle in semicircle)
$\measuredangle DBC = 90°$ (adjacent angles on a straight line)
Therefore $\measuredangle ABD = \measuredangle DBC$.

$\measuredangle CDB = \measuredangle DAB$ (angles in alt. segment)

B3: all 3 reasoning used on correct angles + concluding with correct test

B2: any 2 correct reasoning and angles
B1: any 1 correct reasoning and angle

Since there are 2 pairs of corresponding angles that are equal, triangle *ABD* and triangle *DBC* are similar.

(b) If $BE = AE$, show that $EF = EG$. [4]

$\measuredangle EFG = 180° - \measuredangle BFG$ (adj angles on a str. line)
$= 180° - (180° - \measuredangle BAG)$ (angles in opp. segment)
$= \measuredangle BAG$
$= \measuredangle EAB$

B1: establishing angle EFG = angle EAB

$\measuredangle EGF = 180° - \measuredangle FGA$ (adj angles on a str. line)
$= 180° - (180° - \measuredangle FBA)$ (angles in opp. segment)
$= \measuredangle FBA$
$= \measuredangle EBA$

B1: establishing angle EGF = angle EBA

Given $BE = AE$, triangle EAB is isosceles.
$\measuredangle EBA = \measuredangle EAB$ (base angle of isos. triangle)

B1: establishing angle EBA = angle EAB

Therefore $\measuredangle EGF = \measuredangle EFG$
Hence triangle EGF is isosceles, and $EF = EG$

B1: using base angles of isosceles triangle argument

8 **(a)** Write down the first three terms in the expansion, in ascending powers of x, of $\left(2-\frac{x}{4}\right)^n$, where n is a positive integer greater than 2. [3]

$$\left(2-\frac{x}{4}\right)^n = 2^n + \binom{n}{1}(2)^{n-1}\left(-\frac{x}{4}\right)+\binom{n}{2}(2)^{n-2}\left(-\frac{x}{4}\right)^2+\ldots$$

B1

$$= 2^n - \frac{1}{8}nx\left(2^n\right)+\frac{n(n-1)}{32}\left(2^n\right)x^2+\ldots$$

B1: simplifying binomial coefficient

$$= 2^n - nx\left(2^{n-3}\right)+n(n-1)\left(2^{n-5}\right)x^2+\ldots$$

B1

(b) The first two terms in the expansion, in ascending powers of x, of $(1+x)^2\left(2-\frac{x}{4}\right)^n$ are $a+bx^2$, where a and b are constants.

Find the value of n. [3]

$$(1+x)^2\left(2-\frac{x}{4}\right)^n = \left(1+2x+x^2\right)\left(2^n - nx\left(2^{n-3}\right)+n(n-1)\left(2^{n-5}\right)x^2+\ldots\right)$$

$$= 2^n - nx\left(2^{n-3}\right)+n(n-1)\left(2^{n-5}\right)x^2+2^{n+1}x-nx^2\left(2^{n-2}\right)+\left(2^n\right)x^2$$

$$= 2^n + x\left(2^{n+1}-n\left(2^{n-3}\right)\right)+x^2\left[n(n-1)\left(2^{n-5}\right)-n\left(2^{n-2}\right)+\left(2^n\right)\right]$$

$$= a+bx^2$$

M1: coeff of x

Comparing coefficient of x: $2^{n+1}-n\left(2^{n-3}\right)=0$

M1: equating to 0

$$2^{n-3}\left(2^4-n\right)=0$$

Since $2^{n-3}>0$ for all real values of n, $n=2^4=16$

A1

(c) Hence find the value of a and of b. [3]

Comparing constant: $a=2^{16}=65536$

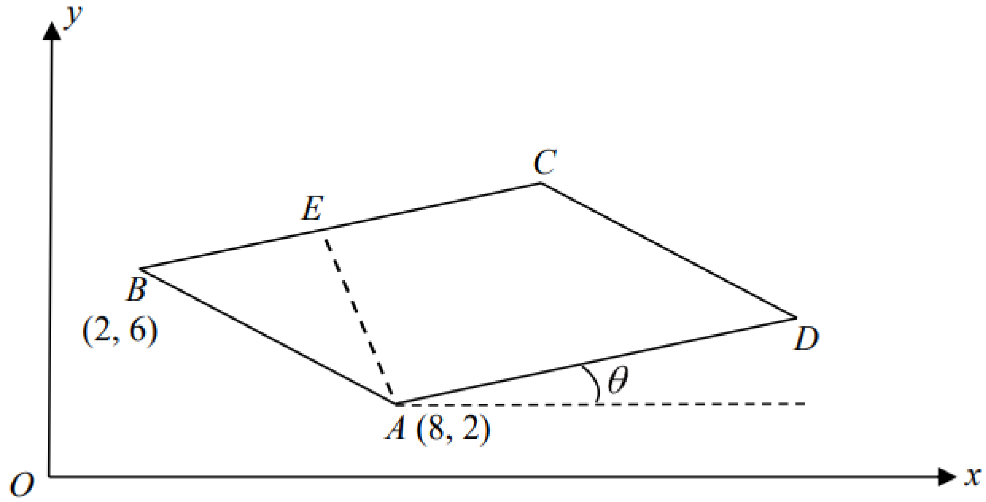
B1

Comparing coefficient of x^2:

$b=16(15)\left(2^{16-5}\right)-16\left(2^{16-2}\right)+\left(2^{16}\right)=294912$

M1: coeff of x^2
A1

9 The diagram shows a parallelogram $ABCD$ in which the coordinates of the points A and B are (8, 2) and (2, 6) respectively. The line AD makes an angle θ with the horizontal and $\tan\theta = 0.5$. The point E lies on BC such that AE is the shortest distance from A to BC.



(a) Show that the equation of line BC is $2y = x + 10$. [2]

Gradient of BC = Gradient of AD ($BC//AD$)

$= \tan\theta = \frac{1}{2}$

M1: identify gradient

Equation of BC: $y - 6 = \frac{1}{2}(x - 2)$

M1: equation formed and simplification to answer given

$y = \frac{1}{2}x - 1 + 6$

$y = \frac{1}{2}x + 5$

$2y = x + 10$

(b) Find the equation of line AE and the coordinates of E. [3]

Gradient of AE = -2

Equation of AE: $y - 2 = -2(x - 8)$

$y = -2x + 18$

M1: equation formed with correct gradient

At intersection: $\frac{1}{2}x + 5 = -2x + 18$

M1: equating and solving x or y

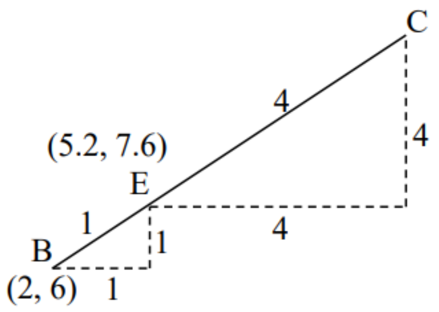
$x = 5.2$, $y = 7.6$

Coordinates of E: $(5.2, 7.6)$

A1

(c) Given that $\frac{BE}{BC} = \frac{1}{5}$, find the coordinates of *C* and *D*. [4]

Let coordinates of *C* be (*x*, *y*).
Using similar triangles,



$\frac{1}{4} = \frac{5.2-2}{x-5.2} = \frac{7.6-6}{y-7.6}$ M1

$x = 4(3.2) + 5.2 = 18$

$y = 4(1.6) + 7.6 = 14$

Coor of C: (18, 14) A1

Midpoint of *AC*: (13, 8) = Midpoint of *BD* (*p*, *q*)

$\left(\frac{2+p}{2}, \frac{6+q}{2}\right) = (13, 8)$ M1

$p = 24,\ q = 10$

Coordinates of D (24, 10) A1

(d) Find the area of the figure *OBEA*, where *O* is the origin. [2]

Area = $\frac{1}{2}\begin{vmatrix} 0 & 8 & 5.2 & 2 & 0 \\ 0 & 2 & 7.6 & 6 & 0 \end{vmatrix}$ M1

$= \frac{1}{2}(60.8 + 31.2 - 10.4 - 15.2) = 33.2 \text{ units}^2$ A1

10 **(a)** Solve the equation $2\cos 3x + 1 = 0$ for $0 \le x \le \pi$. [3]

$2\cos 3x + 1 = 0$

$\cos 3x = -\dfrac{1}{2}$

Basic angle = $\cos^{-1}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{3}$

Since $0 < x < \pi$, $0 < 3x < 3\pi$

$3x = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{8\pi}{3}$

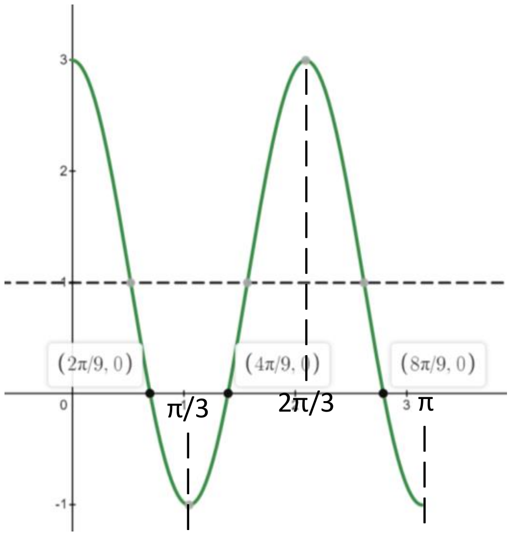
$x = \dfrac{2\pi}{9}, \dfrac{4\pi}{9}, \dfrac{8\pi}{9}$

M1: finding correct basic angle

M1: correct 3x values

A1: correct

(b) Sketch the graph of $y = 2\cos 3x + 1$ for $0 \le x \le \pi$. [3]



B1: correct shape
B1: correct period
B1: correct amplitude

(c) The equation of a curve is $y = \dfrac{\sin 3x}{2 + \cos 3x}$, where $0 \le x \le \pi$.

Using **(a)** and **(b)**, find the range of values of x for which y is a decreasing function. [5]

$$y = \frac{\sin 3x}{2 + \cos 3x}$$

$$\frac{dy}{dx} = \frac{(2 + \cos 3x)3\cos 3x - \sin 3x(-3\sin 3x)}{(2 + \cos 3x)^2}$$

M1: correct differentiation

$$= \frac{6\cos 3x + 3\cos^2 3x + 3\sin^2 3x}{(2 + \cos 3x)^2}$$

$$= \frac{6\cos 3x + 3}{(2 + \cos 3x)^2}$$

For decreasing function, $\dfrac{dy}{dx} < 0$

$$\frac{6\cos 3x + 3}{(2 + \cos 3x)^2} < 0$$

M1: setting to <0

Since $(2 + \cos x)^2 > 0$ for all values of x, $6\cos 3x + 3 < 0$

$$2\cos 3x + 1 < 0$$

M1: simplification to (i) expression

$$\frac{2\pi}{9} < x < \frac{4\pi}{9} \quad \text{or} \quad \frac{8\pi}{9} < x \le \pi$$

A2

11 **(a)** Express $\dfrac{3x^2+4x-20}{(2x+1)\left(x^2+4\right)}$ in partial fractions. [5]

$$\frac{3x^2+4x-20}{(2x+1)\left(x^2+4\right)}=\frac{A}{2x+1}+\frac{Bx+C}{x^2+4}$$

M1: identify correct form

$$3x^2+4x-20=A(x^2+4)+(Bx+C)(2x+1)$$

Let $x = -\dfrac{1}{2}$ $\quad 3\left(-\dfrac{1}{2}\right)^2+4\left(-\dfrac{1}{2}\right)-20=A(\left(-\dfrac{1}{2}\right)^2+4)+0$

M1: substitution or expand and compare coeff

$$A=-5$$

A2: any correct 2 constants

A1: any correct 1 constant

Let $x = 0$ $\quad -20=A(4)+C(1)$

$$C=0$$

Let $x = 1$ $\quad -13=5A+3(B+C)$

$$B=4$$

$$\frac{3x^2+4x-20}{(2x+1)\left(x^2+4\right)}=\frac{4x}{x^2+4}-\frac{5}{2x+1}$$

A1

(b) Differentiate $\ln\left(x^2+4\right)$ with respect to x. [2]

$$\frac{d}{dx}\ln\left(x^2+4\right)=\frac{2x}{x^2+4}$$

B1: chain rule to get denominator,
B1: numerator

(c) The gradient function of a curve is $\dfrac{3x^2+4x-20}{(2x+1)\left(x^2+4\right)}$.

Given that the y-intercept of the curve is $\left(0, \ln 4\right)$, using part **(a)** and **(b)**, find the equation of the curve. [4]

$$\frac{dy}{dx} = \frac{3x^2+4x-20}{(2x+1)\left(x^2+4\right)} = \frac{4x}{x^2+4} - \frac{5}{2x+1}$$

$$y = \int \frac{4x}{x^2+4} - \frac{5}{2x+1}\ dx$$

$$= \int \frac{4x}{x^2+4}\ dx - \int \frac{5}{2x+1}\ dx$$

$$= 2\int \frac{2x}{x^2+4}\ dx - 5\int \frac{1}{2x+1}\ dx$$

$$= 2\ln\left(x^2+4\right) - \frac{5}{2}\ln\left(2x+1\right) + C$$

M1: factorising with intent to use (b)

M1: correctly integrating $\dfrac{1}{2x+1}$

Since $\left(0, \ln 4\right)$ is on the curve: $\ln 4 = 2\ln\left(4\right) - \frac{5}{2}\ln\left(1\right) + C$ M1

$$C = -\ln 4$$

$$y = 2\ln\left(x^2+4\right) - \frac{5}{2}\ln\left(2x+1\right) - \ln 4$$ A1

END OF PAPER

AHMAD IBRAHIM SECONDARY SCHOOL
GCE O-LEVEL PRELIMINARY EXAMINATION 2023

SECONDARY 4 EXPRESS

Name:	Class:	Register No.:

ADDITIONAL MATHEMATICS **4049/02**
Paper 2 **11 August 2023**

Candidates answer on the Question Paper**.** **2 hours 15 minutes**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
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The total number of marks for this paper is 90.

For Examiner's Use
/90

This document consists of **19** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\text{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **(a)** Find the range of values of x for which the expression $3-2x^2$ is negative. [2]

(b) Find the set of values of the constant k for which the curve $y=x^2$ lies entirely above the line $y=k(x+1)$. [3]

2 **(a)** Find the range of values of k such that the line $x+y=3$ intersects the curve $x^2-2x+2y^2=k$. [4]

(b) State a possible value of k if there is no intersection between the line and the curve. [1]

3 A polynomial, P, is $x^{2n} - (k+1)x^2 + k$ where n and k are positive integers.

(a) Explain why $x - 1$ is a factor of P for all values of k. [2]

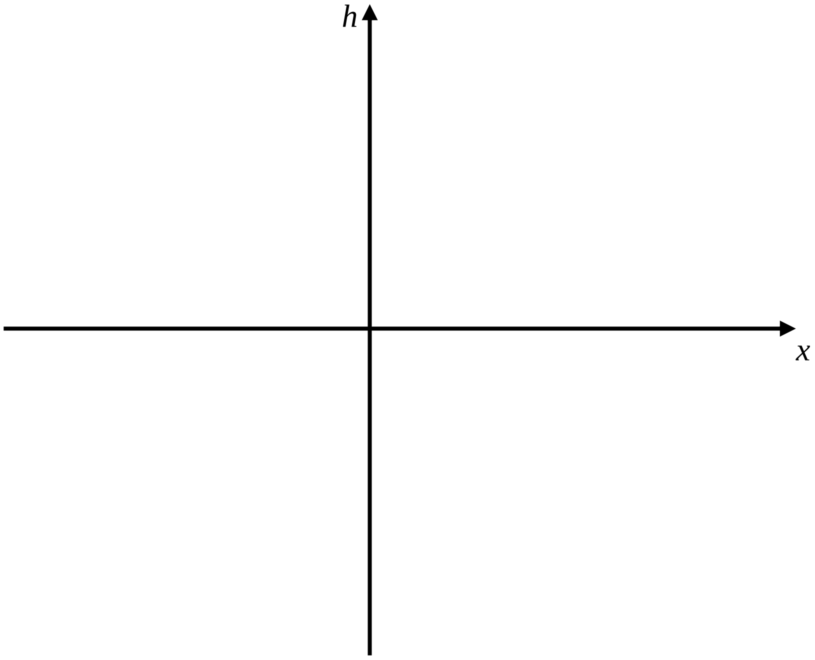
(b) Given that $k = 4$, find the value of n for which $x - 2$ is a factor of P.
Hence factorise P completely. [4]

4 A projectile was launched from a catapult to hit a defence structure on a fort. The height, h metres, of the projectile above ground is given by the equation $h = -2x^2 + 3x + 1.5$, where x metres is the horizontal distance from the catapult.

(i) By expressing the function in the form $h = a(x - m)^2 + n$, where a, m and n are constants, explain whether the projectile can reach a height of 3 metres. [2]

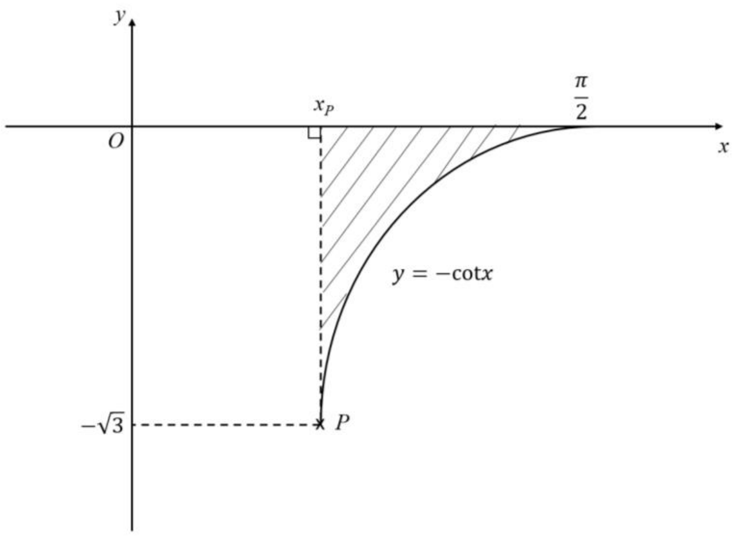
(ii) Given that the defence structure is 1.4 metres horizontally from the catapult and 0.8 metres above the ground, justify if the projectile will hit the structure. [2]

(iii) Sketch the curve of $h = -2t^2 + 3t + 1.5$. [2]



5 **(a)** Differentiate $\ln(\sin x)$ with respect to x. [2]

(b)



The diagram shows part of the curve $y = -\cot x$, cutting the x-axis at $\left(\frac{\pi}{2}, 0\right)$.

The line $y = -\sqrt{3}$ intersects the curve at P.

(i) State the value of x_p, the x-coordinate of P. [1]

(ii) Explain why the expression $\int_{x_p}^{\frac{\pi}{2}} -\cot x \, dx$ does not give the area of the shaded region. [1]

(iii) Find the exact area of the shaded region. [3]

5 **(a)** Without using a calculator, show that $\cos\left(\frac{7\pi}{12}\right)=\frac{1}{4}\left(\sqrt{2}-\sqrt{6}\right)$. [3]

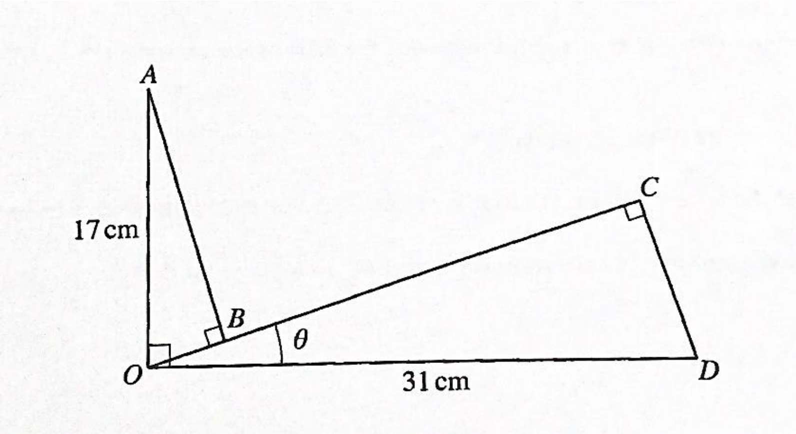
(b) Evaluate $\int_0^{\frac{\pi}{12}} 3\cos^2 x-\sin^2 x \, dx$ exactly. [4]

6 **(a)** **(i)** Factorise $x^6 - 64$ completely. [2]

(ii) Hence solve $x^6 - 64 = \left(x^2 + 4\right)^2 - \left(2x\right)^2$. [3]

(b) Find the values of the integers a and b for which $\dfrac{a+\sqrt{b}}{2}$ is the solution of the equation $2x\sqrt{3} + x\sqrt{125} = x\sqrt{45} + \sqrt{12}.$ [4]

7



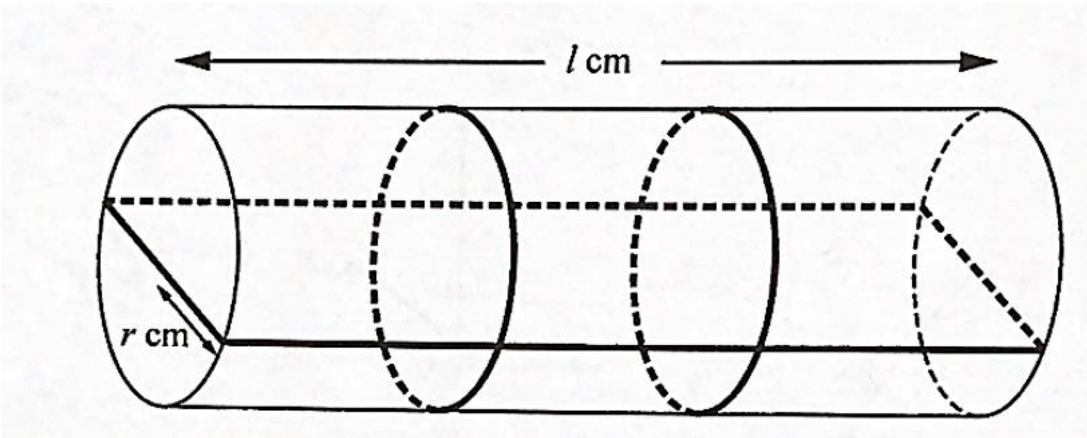
The diagram shows three fixed points *O*, *A* and *D* such that *OA* = 17 cm, *OD* = 31 cm and angle *AOD* = $90°$.
The lines *AB* and *DC* are perpendicular to the line *OC* which makes an angle θ with the line *OD*.
The angle θ can vary in such a way that the point B lies between the points *O* and *C*.

(i) Show that $AB + BC + CD = (48\cos\theta + 14\sin\theta)$ cm. [3]

(ii) Find the values of θ for which $AB + BC + CD = 49$ cm. [6]

(iii) Find the maximum value of $AB + BC + CD$ and the corresponding value of θ. [2]

8 The diagram shows a roll of material in the shape of a cylinder of radius r cm and length l cm.
The roll is held together by three pieces of adhesive tape whose width and thickness may be ignored.
One piece of tape is in the shape of a rectangle, the other two pieces are in the shape of circles.
The total length of tape is 600 cm.



(i) Show that the volume, V cm^3, of the cylinder is given by
$V = \pi r^2 (300 - 2r - 2\pi r)$. [3]

(ii) Given that r can vary, show that V has a stationary value when $r = \frac{k}{1+\pi}$, where k is a constant to be found, and find the corresponding value of l. [5]

(iii) Determine if the volume is a minimum or maximum. [3]

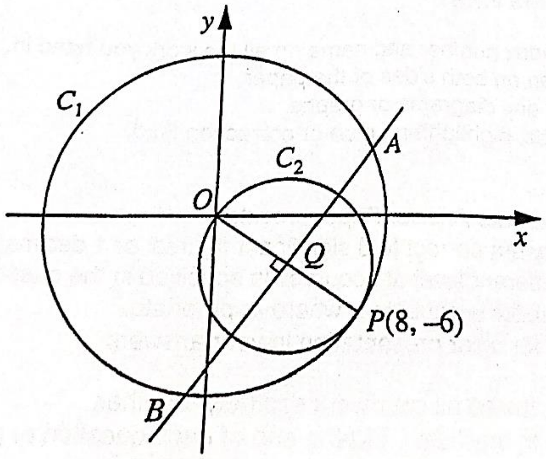
9 A particle travelling in a straight line passes through a fixed point O with a speed of 8 m/s.
The acceleration, a m/s^2 , of the particle t s after passing through O, is given by $a = -e^{-0.1t}$.
The particle comes to instantaneous rest at the point P.

(i) Show that the particle reaches P when $t = 10\ln 5$. [5]

(ii) Calculate the distance OP. [3]

(iii) Explain why the particle is again at *O* at some instant during the fiftieth second after first passing through *O*. [3]

10



The diagram shows two circles C_1 and C_2.
Circle C_1 has its centre at the origin *O*.
Circle C_2 passes through *O* and has its centre at *Q*.
The point $P(8,-6)$ lies on both circles and *OP* is a diameter of C_2.

(a) Find the equation of C_1. [2]

(b) Explain why the equation of C_2 is $x^2+y^2-8x+6y=0$. [3]

(c) The line through Q perpendicular to OP meets the circle C_1 at the point A and B.

Show that the x-coordinates of A and B are $a+b\sqrt{3}$ and $a-b\sqrt{3}$ respectively, where a and b are integers to be found. [7]

END OF PAPER

AHMAD IBRAHIM SECONDARY SCHOOL
GCE O-LEVEL PRELIMINARY EXAMINATION 2023

SECONDARY 4 EXPRESS

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$$\Delta = \frac{1}{2} bc \sin A$$

1 **(a)** Find the range of values of x for which the expression $3-2x^2$ is negative. [2]

$$3-2x^2<0$$
$$2x^2-3>0$$
$$x<-\sqrt{\frac{3}{2}} \quad \text{or} \quad x>\sqrt{\frac{3}{2}} \quad \text{OR}$$
$$x<-\frac{\sqrt{6}}{2} \quad \text{or} \quad x>\frac{\sqrt{6}}{2}$$

M1: finding roots or factorisng

A1

(b) Find the set of values of the constant k for which the curve $y=x^2$ lies entirely above the line $y=k(x+1)$. [3]

$$x^2>k(x+1)$$
$$x^2-kx-k>0$$

For the quadratic expression to be always positive,

$$b^2-4ac<0$$
$$k^2+4k<0 \quad \text{M1}$$
$$k(k+4)<0 \quad \text{M1}$$
$$-4<k<0 \quad \text{A1}$$

2 **(a)** Find the range of values of k such that the line $x+y=3$ intersects the curve $x^2-2x+2y^2=k$. [4]

$x+y=3$

$y=3-x$--------(1)

$x^2-2x+2(3-x)^2=k$

$x^2-2x+2(9-6x+x^2)=k$

$x^2-2x+18-12x+2x^2-k=0$

$3x^2-14x+18-k=0$

Since the line and curve intersects,

$b^2-4ac\geq 0$

$(-14)^2-4(3)(18-k)\geq 0$

$196-216+12k\geq 0$

$-20+12k\geq 0$

$k\geq\frac{5}{3}$

M1 manipulation to get quadratic equation in 1 unknown

M1

M1

A1

(b) State a possible value of k if if there is no intersection between the line and the curve. [1]

Any value that is $<\frac{5}{3}$.

B1

3 A polynomial, P, is $x^{2n}-(k+1)x^2+k$ where n and k are positive integers.

(a) Explain why $x-1$ is a factor of P for all values of k. [2]

let $\mathrm{f}(x)=x^{2n}-(k+1)x^2+k$

$\mathrm{f}(1)=1-k-1+k$

$=0$

$\therefore$ since remainder $=0$, $(x-1)$ is a factor.

M1

A1: must mention remainder = 0, or by factor theorem

(b) Given that $k = 4$, find the value of n for which $x-2$ is a factor of P. Hence factorise P completely. [4]

$\mathrm{f}(x)=x^{2n}-5x^2+4$

$\mathrm{f}(2)=2^{2n}-16$

M1

Since $x-2$ is a factor,

$2^{2n}-16=0$

$n=2$

A1

$\mathrm{f}(x)=x^4-5x^2+4$

$=(x-1)(x-2)(x^2+3x+2)$

M1, must write factors $(x-1)(x-2)$ first since it's a hence question

$=(x-1)(x-2)(x+1)(x+2)$

A1

4 A projectile was launched from a catapult to hit a defence structure on a fort. The height, h metres, of the projectile above ground is given by the equation $h=-2x^2+3x+1.5$, where x metres is the horizontal distance from the catapult.

(i) By expressing the function in the form $h=a(x-m)^2+n$, where a, m and n are constants, explain whether the projectile can reach a height of 3 metres. [2]

$h=-2x^2+3x+1.5$

$=-2(x^2-1.5x-0.75)$

$=-2[(x-0.75)^2-0.75^2-0.75]$

$=-2(x-0.75)^2-2.625$

maximum point is $(0.75,\ 2.625)$

Therefore the projectile cannot reach a height of 3m since the maximum height is 2.625m.

M1 for completing the square

A1 for comparing 2.625 and 3

(ii) Given that the defence structure is 1.4 metres horizontally from the catapult and 0.8 metres above the ground, justify if the projectile will hit the structure. [2]

Subs $(1.4, 0.8)$ into $h=-2x^2+3x+1.5$

$h=-2(1.4)^2+3(1.4)+1.5$

$= 1.78$

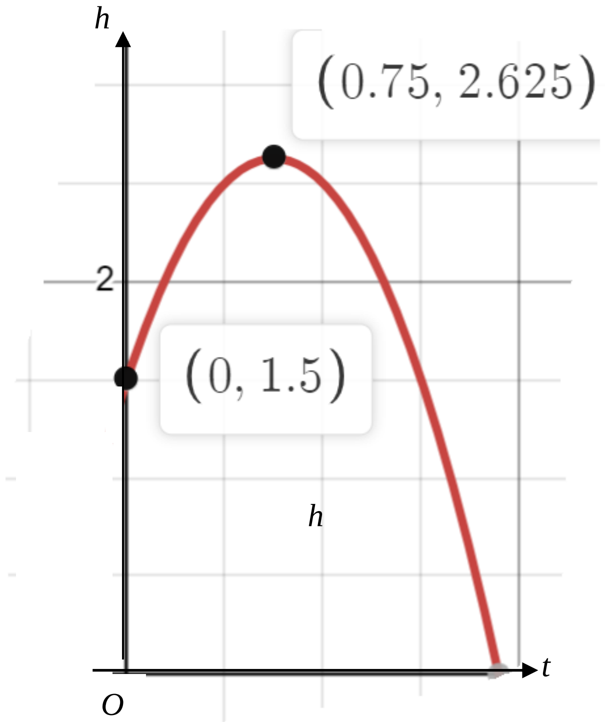
$\neq 0.8$

Since the point $(1.4, 0.8)$ does not lie on the curve $h=-2x^2+3x+1.5$, therefore the projectile will not hit the structure.

M1 for determining if the point lies on the equation

A1

(iii) Sketch the curve of $h = -2x^2 + 3x + 1.5$. [2]



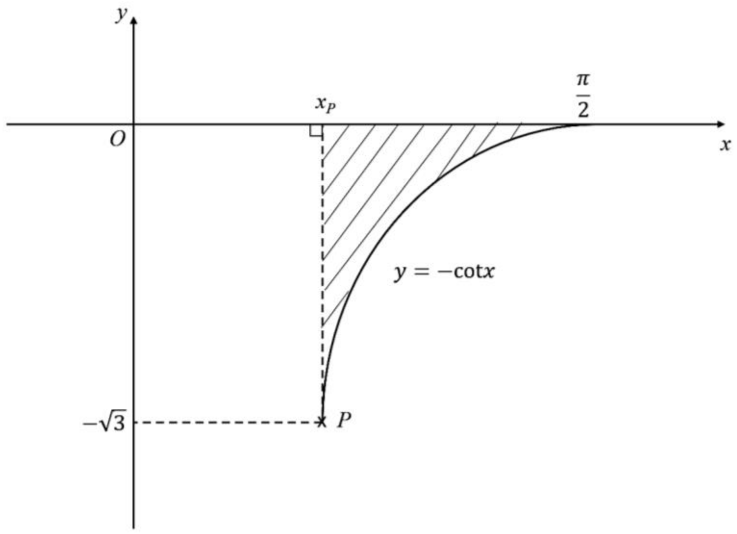
B! correct shape with turning point

B1 correct y-intercept

5 **(a)** Differentiate $\ln(\sin x)$ with respect to x. [2]

$$\frac{d}{dx}\ln(\sin x)$$
$$= \frac{1}{\sin x}\cos x \quad \text{M1}$$
$$= \cot x \quad \text{A1}$$

(b)



The diagram shows part of the curve $y = -\cot x$, cutting the x-axis at $\left(\frac{\pi}{2}, 0\right)$.

The line $y = -\sqrt{3}$ intersects the curve at P.

(i) State the value of x_p, the x-coordinate of P. [1]

$$\sqrt{3} = \frac{1}{\tan x}$$
$$\tan x = \frac{1}{\sqrt{3}}$$
$$x = \frac{\pi}{6} \quad \text{B1}$$

(ii) Explain why the expression $\int_{x_P}^{\frac{\pi}{2}} -\cot x \ \mathrm{dx}$ does not give the area of the shaded region. [1]

The shaded area is below the *x*-axis. If we $\int_{Q_x}^{\frac{\pi}{2}} -\cot x \ \mathrm{dx}$, we will get a **negative value** for the area. Thus $\int_{Q_x}^{\frac{\pi}{2}} -\cot x \ \mathrm{dx}$ does not give area of the shaded region.

B1

(iii) Find the exact area of the shaded region. [3]

$y = -\dfrac{1}{\tan x}$

when $y = -\sqrt{3}$

$-\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} -\cot x \ \mathrm{dx}$ M1

$= [\ln(\sin x)]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ M1

$= \ln 1 - \ln \dfrac{1}{2}$

$= \ln 2 \text{ units}^2 \text{ or } -\ln \dfrac{1}{2} \text{ units}^2$

A1

6 **(a)** Without using a calculator, show that $\cos\left(\frac{7\pi}{12}\right)=\frac{1}{4}\left(\sqrt{2}-\sqrt{6}\right)$. [3]

$$\cos\left(\frac{7\pi}{12}\right)=\cos\left(\frac{\pi}{4}+\frac{\pi}{3}\right)$$

M1: identifying special angles

$$=\cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right)-\sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$$

M1: correct application of formula

$$=\frac{\sqrt{2}}{2}\left(\frac{1}{2}\right)-\frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right)$$

$$=\frac{1}{4}\left(\sqrt{2}-\sqrt{6}\right)$$

M1: recognising exact values and reach result given

(b) Evaluate $\int_0^{\frac{\pi}{12}} 3\cos^2 x-\sin^2 x \text{ d}x$ exactly. [4]

$$\int_0^{\frac{\pi}{12}} 3\cos^2 x-\sin^2 x \text{ d}x=\int_0^{\frac{\pi}{12}} \frac{3}{2}(2\cos^2 x-1+1)+\frac{1}{2}\left(1-2\sin^2 x-1\right) \text{ d}x$$

$$=\int_0^{\frac{\pi}{12}} \frac{3}{2}\cos 2x+\frac{1}{2}\cos 2x+1 \text{ d}x$$

M1: correct application of double angle formula

$$=\int_0^{\frac{\pi}{12}} 2\cos 2x+1 \text{ d}x$$

M1

$$=\left[\sin 2x+x\right]_0^{\frac{\pi}{12}}$$

$$=\sin\frac{\pi}{6}+\frac{\pi}{12}$$

M1

$$=\frac{6+\pi}{12}$$

A1

7 (a) (i) Factorise $x^6 - 64$ completely. [2]

$x^6 - 64$
$= (x^2)^3 - 4^3$
$= (x^2 - 4)(x^4 + 4x^2 + 16)$
$= (x - 2)(x + 2)(x^4 + 4x^2 + 16)$

M1: either cubic factorisation or difference of squares factorisation

A1

OR

$(x^3)^2 - (2^3)^2$
$= (x^3 - 8)(x^3 + 8)$
$= (x^3 - 2^3)(x^3 + 2^3)$
$= (x - 2)(x^2 + 2x + 4)(x + 2)(x^2 - 2x + 4)$
$= (x - 2)(x + 2)(x^2 + 2x + 4)(x^2 - 2x + 4)$

M1

A1

(ii) Hence solve $x^6 - 64 = (x^2 + 4)^2 - (2x)^2$. [3]

$x^6 - 64 = (x^2 + 4)^2 - (2x)^2$
$(x + 2)(x - 2)(x^4 + 4x^2 + 16) = x^4 + 8x^2 + 16 - 4x^2$
$(x + 2)(x - 2)(x^4 + 4x^2 + 16) = x^4 + 4x^2 + 16$
$(x + 2)(x - 2) = 1$
$x^2 - 5 = 0$
$x = \pm\sqrt{5}$

M1 for expanding the RHS of the equation

M1

A1

(b) Find the values of the integers a and b for which $\dfrac{a+\sqrt{b}}{2}$ is the solution of the equation $2x\sqrt{3}+x\sqrt{125}=x\sqrt{45}+\sqrt{12}.$ [4]

$$2x\sqrt{3}+x\sqrt{125}=x\sqrt{45}+\sqrt{12}$$
$$2x\sqrt{3}+5x\sqrt{5}=3x\sqrt{5}+2\sqrt{3}$$
$$2x\sqrt{3}+5x\sqrt{5}-3x\sqrt{5}=2\sqrt{3}$$
$$x\left(2\sqrt{3}+2\sqrt{5}\right)=2\sqrt{3}$$

M1 , isolating x terms and simplifying the surds

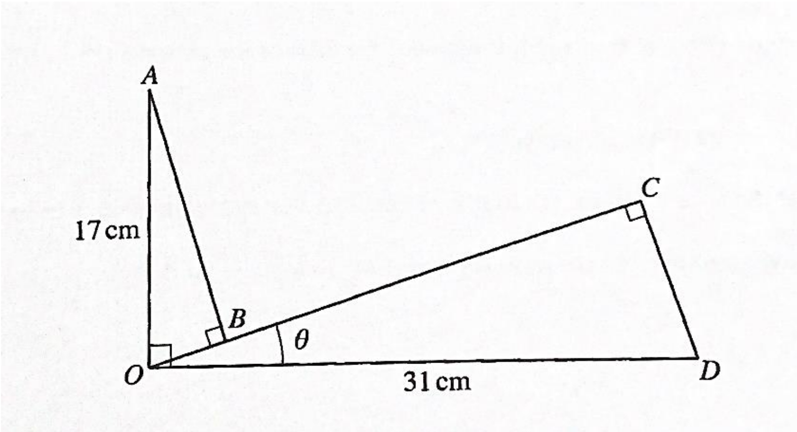
$$x=\frac{2\sqrt{3}}{\left(2\sqrt{3}+2\sqrt{5}\right)}\times\frac{\left(2\sqrt{3}-2\sqrt{5}\right)}{\left(2\sqrt{3}-2\sqrt{5}\right)}$$

M1 for rationalising

$$=\frac{12-4\sqrt{15}}{12-20}$$
$$=\frac{12-4\sqrt{15}}{-8}$$
$$=\frac{-3+\sqrt{15}}{2}$$
$$a=-3,\ b=15$$

A1 , A1

8



The diagram shows three fixed points *O*, *A* and *D* such that *OA* = 17 cm, *OD* = 31 cm and angle *AOD* = 90°.
The lines *AB* and *DC* are perpendicular to the line *OC* which makes an angle θ with the line *OD*.
The angle θ can vary in such a way that the point B lies between the points *O* and *C*.

(i) Show that $AB + BC + CD = (48\cos\theta + 14\sin\theta)$ cm. [3]

$\sin\theta = \frac{CD}{31}$

$CD = 31\sin\theta$

$\sin\theta = \frac{OB}{17}$

$OB = 17\sin\theta$

$\cos\theta = \frac{AB}{17}$

$AB = 17\cos\theta$

M2 for any 2 correct

$\cos\theta = \frac{OC}{31}$

$OC = 31\cos\theta$

$AB + BC + CD$

$= 17\cos\theta + 31\cos\theta - 17\sin\theta + 31\sin\theta$

$= (48\cos\theta + 14\sin\theta)$ cm

A1

(ii) Find the values of θ for which $AB+BC+CD=49$ cm. [6]

$48\cos\theta+14\sin\theta=49$

$R\cos(\theta-\alpha)=49$ M1

$R=\sqrt{48^2+14^2}$

$=50$ M1

$\alpha=\tan^{-1}\left(\frac{14}{48}\right)$

$=16.26°$ M1

$50\cos(\theta-16.26°)=49$

$\cos(\theta-16.26°)=\frac{49}{50}$ M1

Reference angle $=11.48°$

$\theta=27.7°, 4.8°$ A1, A1

(iii) Find the maximum value of $AB+BC+CD$ and the corresponding value of θ. [2]

max $50\cos(\theta-16.26°)$

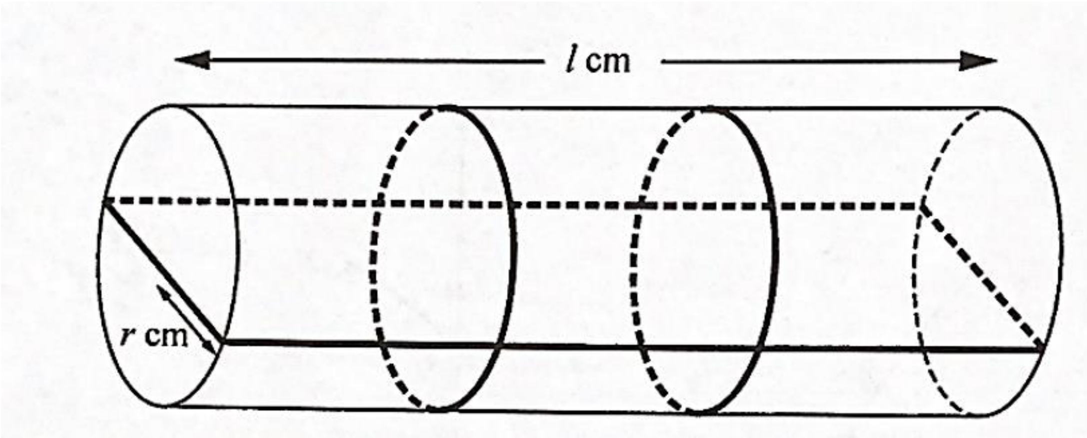
$=50$ B1

occurs when $\cos(\theta-16.26°)=1$

$\theta-16.26°=0$

$\theta=16.3°$ (1 d.p) B1

9 The diagram shows a roll of material in the shape of a cylinder of radius r cm and length l cm.
The roll is held together by three pieces of adhesive tape whose width and thickness may be ignored.
One piece of tape is in the shape of a rectangle, the other two pieces are in the shape of circles.
The total length of tape is 600 cm.



(i) Show that the volume, V cm^3, of the cylinder is given by
$V = \pi r^2 (300 - 2r - 2\pi r)$. [3]

$2(2r) + 2(l) + 2(2\pi r) = 600$ M1

$2r + l + 2\pi r = 300$

$l = 300 - 2r - 2\pi r$ M1

$V = \pi r^2 l$

$= \pi r^2 (300 - 2r - 2\pi r)$ (shown) A1

(ii) Given that r can vary, show that V has a stationary value when $r = \dfrac{k}{1+\pi}$, where k is a constant to be found, and find the corresponding value of l. [5]

$$\frac{dV}{dr} = 600\pi r - 6\pi r^2 - 6\pi^2 r^2$$

$$= 6\pi r(100 - r - \pi r) \quad \text{M1}$$

For stationary value, $\dfrac{dV}{dr} = 0$.

$6\pi r(100 - r - \pi r) = 0$ M1

$r = 0$ rejected because $r > 0$ or

$100 - r - \pi r = 0$

$(1+\pi)r = 100$

$r = \dfrac{100}{1+\pi}$ A1

$\therefore$ V has a stationary value when $r = \dfrac{100}{1+\pi}$, where $k = 100$. (shown)

When $r = \dfrac{100}{1+\pi}$,

$$l = 300 - 2\left(\frac{100}{1+\pi}\right) - 2\pi\left(\frac{100}{1+\pi}\right) \quad \text{M1}$$

$$= 300 - 2\left(\frac{100}{1+\pi}\right)(1+\pi)$$

$$= 300 - 200$$

$$= 100 \quad \text{A1}$$

(iii) Determine if the volume is a minimum or maximum. [3]

$$\frac{dV}{dr} = 6\pi r(100 - r - \pi r)$$

$$\frac{d^2V}{dr^2} = 6\pi r(-1-\pi) + (100 - r - \pi r)(6\pi)$$

$$= 6\pi r(-1-\pi) + (100 - r - \pi r)(6\pi)$$

$$= 6\pi(-r - \pi r + 100 - r - \pi r)$$

$$= 6\pi(-2r - 2\pi r + 100) \quad \text{M1}$$

when $r = \dfrac{100}{1+\pi}$,

$$\frac{d^2V}{dr^2} = 6\pi\left(-\frac{200}{1+\pi} - \frac{200\pi}{1+\pi} + 100\right) \quad \text{M1}$$

$$= -600\pi$$

Since $\dfrac{d^2V}{dr^2} < 0$, V is a maximum. A1

10 A particle travelling in a straight line passes through a fixed point *O* with a speed of 8 m/s.
The acceleration, a m/s^2, of the particle t s after passing through *O*, is given by $a = -e^{-0.1t}$.
The particle comes to instantaneous rest at the point *P*.

(i) Show that the particle reaches *P* when $t = 10\ln 5$. [5]

$$a = -e^{-0.1t}$$
$$v = \int a = -e^{-0.1t} \ \mathrm{d}t$$
$$= \frac{-e^{-0.1t}}{-0.1} + c$$ M1
When $t = 0,\ v = 8$
$$8 = \frac{-1}{-0.1} + c$$
$$8 = 10 + c$$
$$c = -2$$ M1
$$v = 10e^{-0.1t} - 2$$
when $v = 0$ M1
$$10e^{-0.1t} - 2 = 0$$
$$e^{-0.1t} = \frac{1}{5}$$ M1
$$-0.1t = -\ln 5$$ A1
$$t = 10\ln 5 \text{ (shown)}$$

(ii) Calculate the distance *OP*. [3]

$$s = \int_0^{10\ln 5} 10e^{-0.1t} - 2 \ \mathrm{d}t$$
$$= \left[\frac{10e^{-0.1t}}{-0.1} - 2t \right]_0^{10\ln 5}$$ M1
$$= -100e^{-0.1(10\ln 5)} - 20\ln 5 + 100$$ M1
$$= -100e^{-\ln 5} - 20\ln 5 + 100$$
$$= -20 - 20\ln 5 + 100$$ A1
$$= 47.8\text{m}$$

(iii) Explain why the particle is again at *O* at some instant during the fiftieth second after first passing through *O*. [3]

when $t = 49$

$s = -100e^{-0.1(49)} - 2(49) + 100$

$= 1.255m$ M1

when $t = 50$

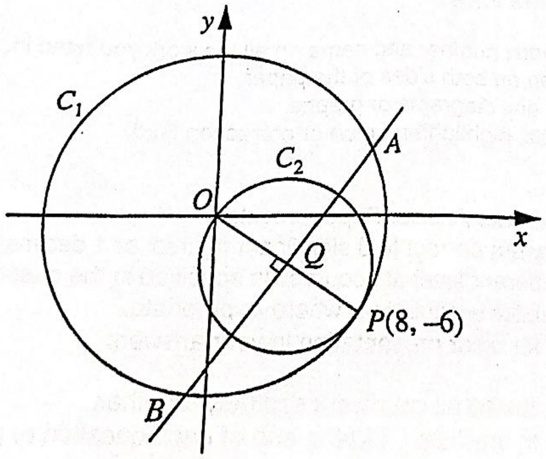
$s = -100e^{-0.1(50)} - 2(50) + 100$

$= -0.674\text{m}$ M1

Since displacement of the particle is positive when $t = 49$ and negative when $t = 50$, this shows that the particle must have passed through O at some point in the fiftieth second.

Thus the particle is again at O at some instant during the fifitieth second after passing through O. A1

11



The diagram shows two circles C_1 and C_2.
Circle C_1 has its centre at the origin O.
Circle C_2 passes through O and has its centre at Q.
The point $P(8,-6)$ lies on both circles and OP is a diameter of C_2.

(i) Find the equation of C_1. [2]

$$|OP| = \sqrt{(8-0)^2 + (6-0)^2}$$
$$= 10$$ M1

Equation of $C_1 : x^2 + y^2 = 100$ A1

(ii) Explain why the equation of C_2 is $x^2 + y^2 - 8x + 6y = 0$. [3]

$$x^2 + y^2 - 8x + 6y = 0$$
$$x^2 - 8x + y^2 + 6y = 0$$
$$(x-4)^2 - 16 + (y+3)^2 - 9 = 0$$
$$(x-4)^2 + (y+3)^2 = 5^2$$ M1

Centre $= (4,-3)$ because it is the mid point of OP A1

Radius is 5 units because it is $\frac{1}{2}|OP|$ A1

(iii) The line through Q perpendicular to OP meets the circle C_1 at the point A and Show that the x-coordinates of A and B are $a+b\sqrt{3}$ and $a-b\sqrt{3}$ respectively, where a and b are integers to be found. [7]

$$\text{gradient } OP = -\frac{6}{8}$$
$$= -\frac{3}{4}$$
$$\text{gradient } AB = \frac{4}{3}$$
$$\frac{4}{3}(x-4) = y+3$$
$$4x-16 = 3y+9$$
$$y = \frac{4}{3}x - \frac{25}{3} \text{..........(1)}$$

M1

$$x^2+y^2 = 100$$
$$y^2 = 100-x^2 \text{.......(2)}$$
$$(1)^2 \quad y^2 = \frac{16}{9}x^2 - \frac{200}{9}x + \frac{625}{9} \text{..........(3)}$$
$$(2)=(3)$$
$$\frac{16}{9}x^2 - \frac{200}{9}x + \frac{625}{9} = 100 - x^2$$

M1 Reasonable attempt at manipulating the equations to obtain the quadratic equation

$$\frac{25}{9}x^2 - \frac{200}{9}x - \frac{275}{9} = 0$$
$$x^2 - 8x - 11 = 0$$

M1

$$x = \frac{8 \pm \sqrt{64+44}}{2}$$
$$= \frac{8 \pm \sqrt{108}}{2}$$

M1

$$= \frac{8 \pm 6\sqrt{3}}{2}$$

M1

$$= 4 \pm 3\sqrt{3}$$

x-coordinate of A is $4+3\sqrt{3}$

A1, A1

x-coordinate of B is $4-3\sqrt{3}$

END OF PAPER