



**TANJONG KATONG GIRLS' SCHOOL  
PRELIMINARY EXAMINATION  
SECONDARY FOUR EXPRESS**

CANDIDATE  
NAME

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CLASS

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INDEX  
NUMBER

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**ADDITIONAL MATHEMATICS**

PAPER 1

**4049/01**

**25 August 2022**

**2 hour 15 minutes**

Candidates answer on the Question Paper

**READ THESE INSTRUCTIONS FIRST**

Write your index number and name on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown **with** the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For  $\pi$ , use either your calculator value or 3.142, unless the question requires the answer in terms of  $\pi$ .

The number of marks is given in brackets [ ] at the end of each question or part question.

The total marks for this paper is 90.

**For Examiner's use**

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Setters: Ms Yeo

Markers: Mrs Pang, Mrs Loy, Ms Koo, Ms Yeo

This document consists of 23 printed pages and 1 blank page.

## 1. ALGEBRA

### Quadratic Equation

For the equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

### Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \dots + \binom{n}{r} a^{n-r}b^r + \dots + b^n,$$

where  $n$  is a positive integer and  $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

## 2. TRIGONOMETRY

### Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

### Formula for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

- 1 (i) Rewrite the expression  $-2x^2 - 4x - m - 6$  in the form  $a(x+b)^2 + c$ , where  $a$  and  $b$  are constants, and  $c$  is an expression in  $m$ . [3]

- (ii) Deduce the range of values of  $m$  such that the curve lies entirely below the  $x$ -axis. [1]

- 2 (a) Given that  $14^{2x} = 2^{3+x}7^{x-2}$ , find the value of  $14^x$ .

[3]

TKGS

- (b) The value,  $\$V$ , of a car decreases exponentially so that its value  $t$  years after 1 January 2001 is given by  $V = Ae^{-0.1104t}$ , where  $A$  is a constant. The initial value of the car was  $\$70000$ . Mary intends to sell the car when the value falls below  $\$42000$ . Determine, with working, if Mary will sell her car in the year 2005. [5]

TJKGS

3

(i) Using long division, divide  $4x^3 + 8x - 3$  by  $2x^2 - 3x$ .

[2]

TJKGS

- (ii) Express  $\frac{4x^3 + 8x - 3}{2x^2 - 3x}$  as the sum of a polynomial and partial fractions. [5]

TJKGS

4 The equation of a curve is  $y = \frac{4}{x^2 + 1}$ .

(i) Show that  $y$  is decreasing for  $x > 0$ .

[3]



(ii) Find the range of values of  $x$  for which the gradient of the curve is increasing. [5]



5 (i) Sketch the graph of  $y = 4(x-1)^2 + 3$ . [2]

(ii) In the case where the line  $y = mx - 9$  is a tangent to the curve  $y = 4(x-1)^2 + 3$  at  $x = 2$ , find the value of  $m$ . [2]

(iii) In the case where the line  $y = mx - 9$ , where  $m > 0$ , does not intersect the curve, find the range of values of  $m$ . Use your result from 5(ii). [1]

- 6 Solve the equation  $3\cot^2 A + 7\operatorname{cosec} A = 3$  for  $-180^\circ \leq A \leq 180^\circ$ . [5]

- 7 Find the coordinates of the stationary points of the curve  $y = x^4 - 8x^3 + 1$  and determine the nature of each stationary point.

[8]

TJKGS

Continuation of working space for question 7.



- 8 The expression  $2x^3 - ax^2 - x + b$ , where  $a$  and  $b$  are constants, has a factor  $2x - 1$ . The remainder when  $2x^3 - ax^2 - x + b$  is divided by  $x - 3$  is 25 times the remainder when  $2x^3 - ax^2 - x + b$  is divided by  $x$ .

(i) Show that  $a = 3$  and  $b = 1$ .

[3]

TJKGS

(ii) Solve the equation  $2x^3 - 3x^2 - x + 1 = 0$ .

If a root is not exact, express it in the form  $\frac{c \pm \sqrt{d}}{2}$ , where  $c$  and  $d$  are integers. [3]

TJKGS

9 The equation of a circle is  $x^2 + y^2 + 2x - 6y = 15$ .

- (i) Determine, with working, whether the line  $3y = 2x + 9$  passes through the centre of the circle.

[3]

- (ii) The line  $y = x + 4$  is the perpendicular bisector of chord  $AB$ . The point  $\left(-\frac{1}{2}, \frac{7}{2}\right)$  lies on the chord  $AB$ . Find the equation of  $AB$ .

[2]

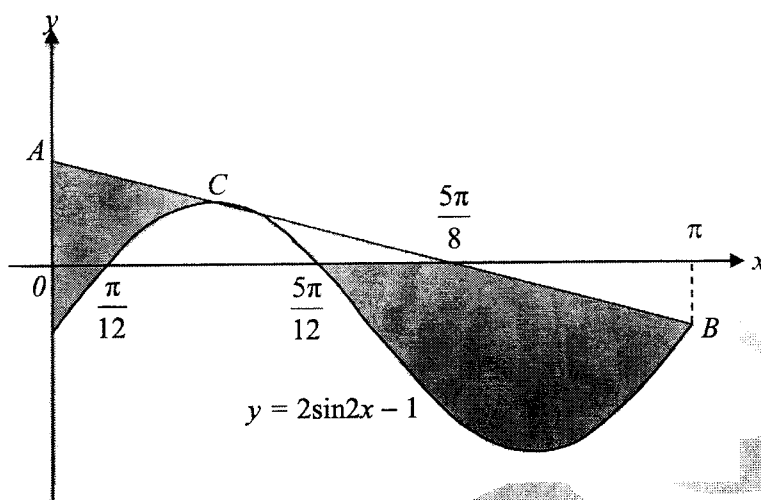


(iii) Find the possible values for the  $x$ -coordinate of  $A$ .

[3]



10



The diagram shows for  $0 \leq x \leq \pi$ , the curve  $y = 2\sin 2x - 1$  and a line segment  $AB$ .

The equation of  $AB$  is  $y = -\frac{8}{3\pi}x + \frac{5}{3}$ . The line segment  $AB$  passes through  $C$ , a maximum point on the curve  $y = 2\sin 2x - 1$ . The line segment also meets the curve at point  $B$  on the curve where  $x = \pi$ . The line  $AB$  intersects the  $x$ -axis at  $x = \frac{5}{8}\pi$ .

The curve intersects the  $x$ -axis at  $x = \frac{\pi}{12}$  and  $x = \frac{5\pi}{12}$ .

(i) Find the coordinates of  $C$ .

[2]

- (ii) Find the area of the region bounded by the curve  $y = 2\sin 2x - 1$ , the line  $AB$  and the  $y$ -axis. [5]

- (iii) Find the exact value of the area of the region bounded by the curve  $y = 2\sin 2x - 1$ , the line  $AB$  and the  $x$ -axis. [4]

11 A particle travelling in a straight line passes through a fixed point  $O$  with a velocity of  $18 \text{ m/s}$ . The acceleration,  $a \text{ m/s}^2$ , of the particle,  $t \text{ s}$  after passing through  $O$ , is given by  $a = -4e^{-0.2t}$ . The particle comes to instantaneous rest at the point  $R$ .

(i) Find an expression for the velocity,  $v \text{ m/s}$ , of the particle in terms of  $t$ . [3]

(ii) Show that the particle reaches  $R$  when  $t = 5 \ln 10$ . [2]

- (iii) Calculate the displacement of the particle from  $O$  when the particle is at  $R$ . [5]

12 (i) Show that  $\sin(x - 30^\circ) = -\cos(x + 60^\circ)$ . [3]

(ii) Hence find the values of  $x$  between  $0^\circ$  and  $360^\circ$  for which  $4\sin(x - 30^\circ) + \cos(x + 60^\circ) = 1$ . [4]

- (iii) Given that  $\sin(x-30^\circ) = \frac{1}{2}(\sqrt{3}\sin x - \cos x)$ , use it to find the exact value of  $\sin 15^\circ$ .

[3]

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PRELIMINARY EXAMINATION  
SECONDARY FOUR EXPRESS**

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**ADDITIONAL MATHEMATICS**

**4049/02**

Paper 2

**30 August 2022**

**2 hour 15 minutes**

Candidates answer on the Question Paper.

No Additional Materials are required.

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Setters: Mrs Loy

Markers: Mrs Pang, Ms Yeo, Mrs Loy, Ms Koo

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3

- 1 (a) The line  $x = y + 5$  intersects the curve  $x^2 + y^2 = 39 + xy$  at points  $A$  and  $B$ . Find the  $x$ -coordinates of  $A$  and  $B$ . [3]

- (b) Find the greatest integer value of  $p$  for which  $px(x-2) + px + 1$  is always positive for all real value of  $x$ . [4]

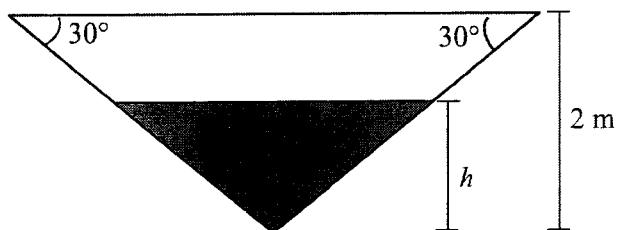
- (c) Find the range of values of  $x$  such that the square of  $(2x-1)$  is at least equal to  $6\frac{1}{4}$ . Represent your solution on a number line. [4]

- 2 (i) In the expansion of  $\left(x^2 - \frac{2}{x}\right)^{14}$ , explain with clear working why every term is dependent on  $x$ . [3]

- (ii) In the expansion of  $\left(x^2 - \frac{2}{x}\right)^{14} \left(1 + x + \frac{1}{2}x^2\right)$ , find the constant term. [3]

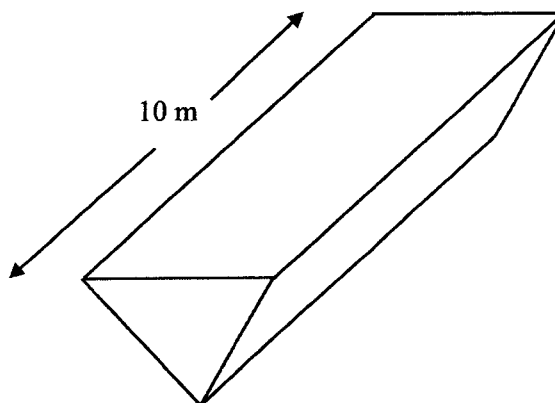
- 3 (a) The diagram shows a vertical cross-section of a water trough in the form of an isosceles triangle with base angles  $30^\circ$  and height 2 m.

The trough is initially empty and is then filled with water at a constant rate of  $5 \text{ m}^3 / \text{min}$  to a height of  $h \text{ m}$ .



Vertical cross-section

The length of the trough is 10 m and the volume of the water is  $V \text{ m}^3$ .



Water trough

- (i) Show that  $V = 10\sqrt{3} h^2$ . [2]

7

- (ii) Find the rate of change of  $h$  when  $h = 0.8$  m. [3]

- (b) Solve the equation  $2e^{2x} = \sqrt{1-3e^{2x}}$ . [5]

4 It is given that  $P = 8\sin\theta + 12\cos\theta$ .

(i) Express  $P$  in the form  $a\sqrt{13}\sin(\theta + \alpha)$ , where  $a$  is an integer and  $0 < \alpha < \frac{\pi}{2}$ . [3]

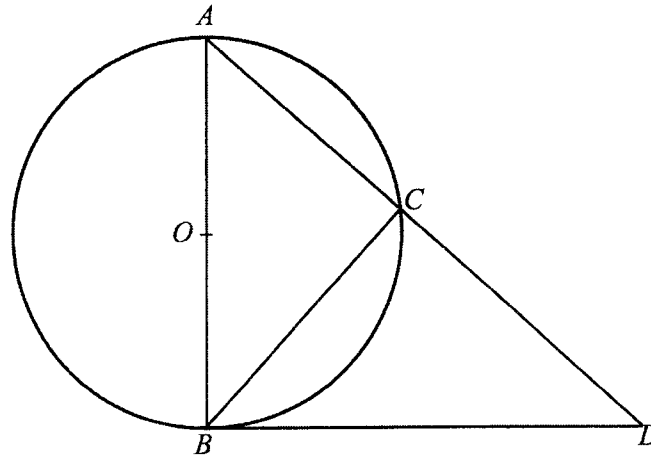


(ii) Find the maximum value of  $(2-P)$  and the corresponding value of  $\theta$  for

$$\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{2}.$$

[4]

5



$A$ ,  $B$  and  $C$  are points on the circle with centre  $O$  and  $AB$  is the diameter.  
 $AC$  is produced to  $D$  and  $BD$  is a tangent to the circle at  $B$ .

- (i) Stating your reasons clearly, show that triangle  $ABC$  is similar to triangle  $BDC$ . [4]

11

Given that  $AC = 4.5$  cm and  $BC = 6$  cm,

(ii) find  $CD$ .

[2]

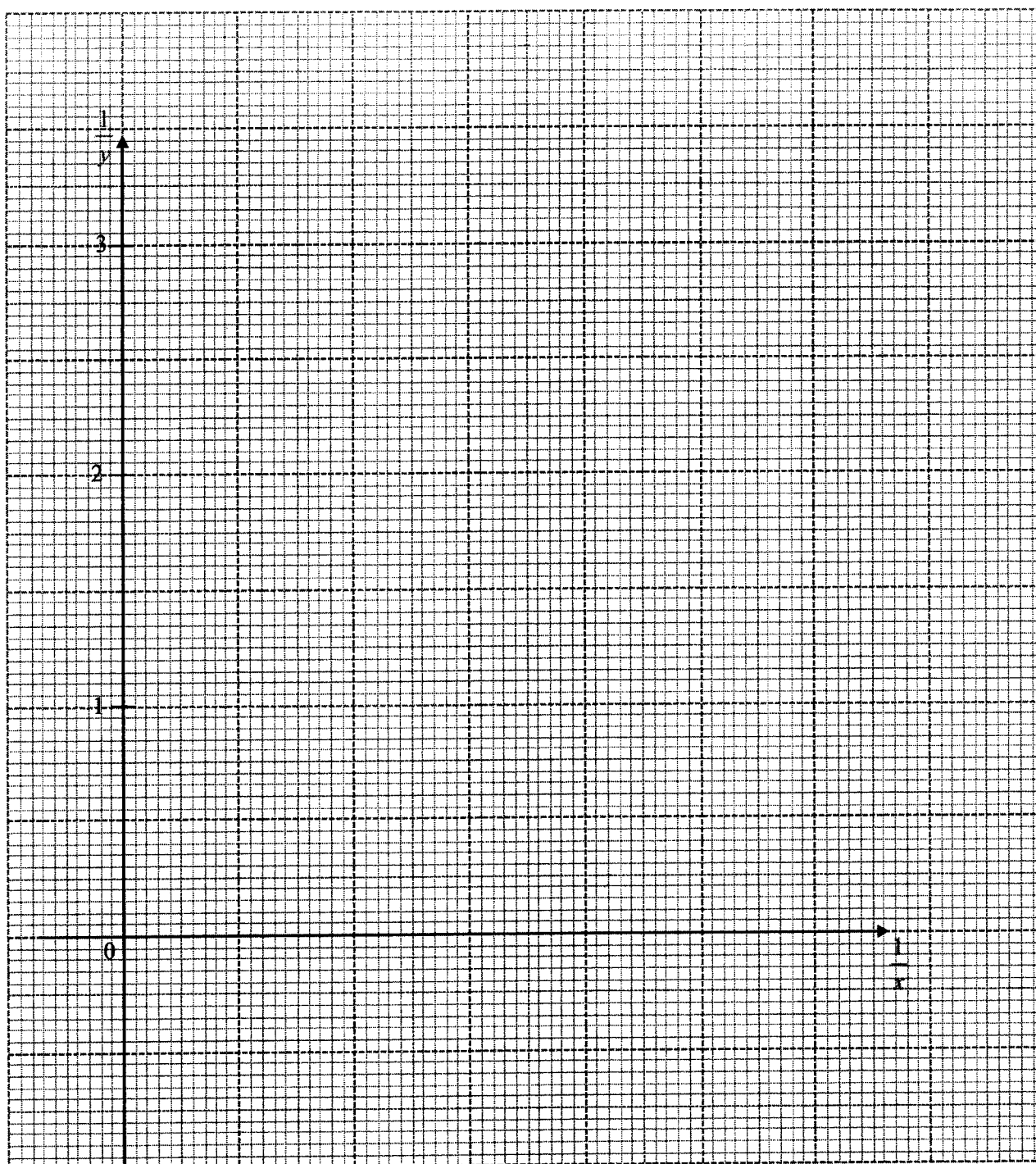
(iii) Determine whether  $OC = \frac{1}{2}BD$ .

[3]

- 6 The table shows experimental values of two variables  $x$  and  $y$ .

$x$	1	2	3	4	5
$y$	0.34	0.85	1.72	3.57	9.65

- (i) On the grid lines given, plot  $\frac{1}{y}$  against  $\frac{1}{x}$  and draw a straight-line graph. [3]



- (ii) Find the gradient of the line and the equation of the line in the form

$$\frac{1}{y} = A\left(\frac{1}{x}\right) + B, \text{ where } A \text{ and } B \text{ are constants.}$$

[3]

7 (a) Solve  $\log_3(6-3x) - 4\log_9 x = 1$ .

[5]

- (b) To obtain a graphical solution of the equation,  $x = \ln\left(\frac{x-4}{2}\right)^3$ , a suitable straight line can be drawn on the same axes as the graph of  $y = e^{\frac{x}{3}} + 3$ . Find the equation of this straight line. [3]

8 (i) Show that  $\frac{d}{dx} \left[ 2x\sqrt{(3x-1)^5} \right] = (21x-2)(3x-1)^{\frac{3}{2}}$ . [4]

(ii) Find  $\int 2(3x-1)^{\frac{3}{2}} dx$ . [2]



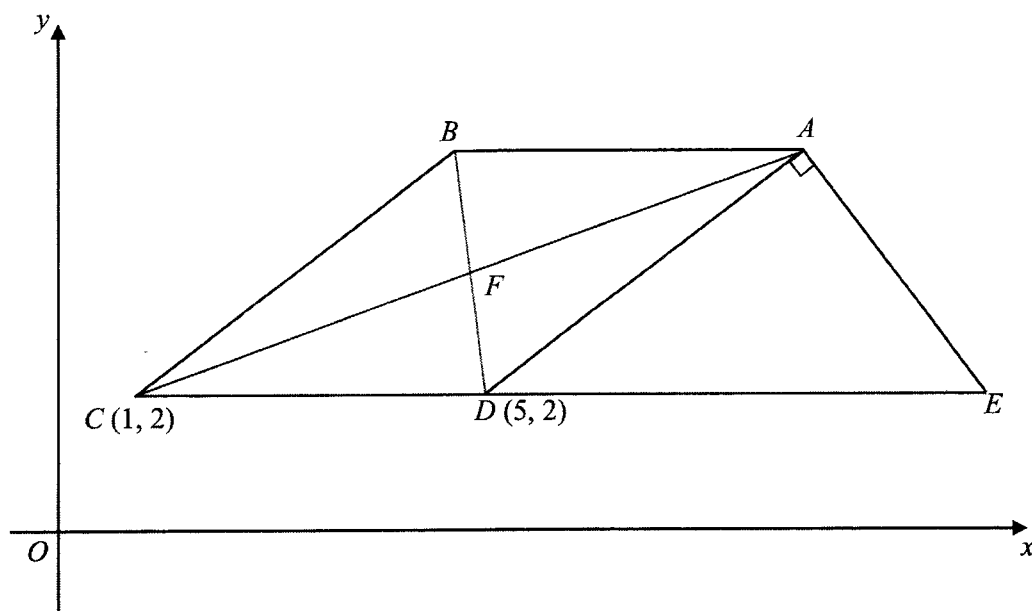
(iii) Using part (i) and (ii), find the value of  $\int_2^1 3x(3x-1)^{\frac{3}{2}} dx$ . [6]

- 9 The function  $f$  is defined for  $x \in \mathbb{R}$  and is such that  $f'(x) = 2x - e^{1-2x}$ .  
The line  $y = \frac{5}{12}$  is the tangent to the curve  $y = f(x)$  at the point where  $x = \frac{1}{2}$ .

(i) Find the equation of the curve,  $y = f(x)$ . [8]

- (ii) Show that the equation of the tangent to the curve  $y = f(x)$  at the point where the curve intersects the  $y$ -axis can be written as  $4y = (2e - 3)x + 4 - e$ . [4]

10



In the diagram,  $ABCDE$  is a trapezium consisting of a parallelogram  $ABCD$  and a triangle  $ADE$  with angle  $DAE$  equal to  $90^\circ$ . The coordinates of points  $C$  and  $D$  are  $(1, 2)$  and  $(5, 2)$  respectively.  $BD$  intersects  $AC$  at the point  $F\left(\frac{9}{2}, \frac{7}{2}\right)$ .

(i) Show that the coordinates of point  $B$  are  $(4, 5)$ .

[2]

- (ii) Find the perimeter of the trapezium  $ABCDE$  in the form  $(a\sqrt{2}+b)$  units, where  $a$  and  $b$  are integers. [7]

**END OF PAPER**

TKGS

Answer Key :

1(i)  $-2(x+1)^2 - m - 4$

(ii)  $m > -4$

2(a)  $14^x = \frac{8}{49}$

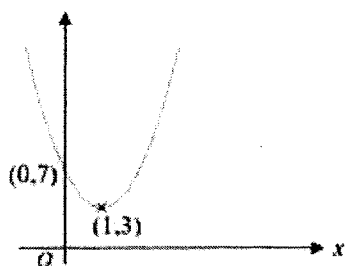
(b) Mary will sell her car in the year 2005

3(i)

(ii)  $\frac{4x^3 + 8x - 3}{2x^2 - 3x} = 2x + 3 + \frac{1}{x} + \frac{15}{2x - 3}$

4(ii)  $x < -\frac{\sqrt{3}}{3}$  or  $x > \frac{\sqrt{3}}{3}$

5(i)



(ii) 8

(iii)  $0 < m < 8$

6.  $-19.5^\circ, -160.5^\circ$

7. (0, 1) and (6, -431)

There is a minimum point at (6, -431).

There is a stationary point of inflexion at (0, 1).

$$8(\text{ii}) \quad \frac{1}{2}, \frac{1+\sqrt{5}}{2} \text{ or } \frac{1-\sqrt{5}}{2}$$

9(i) The line does not pass through the centre.

(ii)  $y = -x + 3$

(iii)  $-4$  or  $3$

$$10(\text{i}) \quad \left(\frac{\pi}{4}, 1\right)$$

(ii)  $0.833$  square units

$$(\text{iii}) \quad \left(1 + \frac{\sqrt{3}}{2} + \frac{19}{48}\pi\right) \text{square units}$$

$$11(\text{i}) \quad v = 20e^{-0.2t} - 2$$

(iii)  $67.0$  m

$$12(\text{ii}) \quad 49.5^\circ, 190.5^\circ$$

$$(\text{iii}) \quad \frac{1}{4}(\sqrt{6} - \sqrt{2})$$

**Answer Key:**

1a	$x = 7, x = -2$	1b	3												
1c	$x \leq -\frac{3}{4}, x \geq \frac{7}{4}$														
2i	Since $r$ is not a positive integer, there is no constant term in the expansion. Hence, every term is dependent on $x$ . $r = \frac{28}{3}$	2ii	Constant term = 512 512												
3aii	$\frac{dh}{dt} = 0.18042$ the height of the water increases at a height of 0.180 m per minute	3b	$x = -0.693$												
4i	$P = 4\sqrt{13} \sin(\theta + 0.983)$	4ii	Max value = 16.4 $\theta = 3.73$												
5ii	CD = 8	5iii	No												
6i	<table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td><math>\frac{1}{x}</math></td> <td>1</td> <td>0.5</td> <td>0.33</td> <td>0.25</td> <td>0.2</td> </tr> <tr> <td><math>\frac{1}{y}</math></td> <td>2.94</td> <td>1.18</td> <td>0.58</td> <td>0.28</td> <td>0.10</td> </tr> </table>	$\frac{1}{x}$	1	0.5	0.33	0.25	0.2	$\frac{1}{y}$	2.94	1.18	0.58	0.28	0.10	6ii	Gradient = 3.55 $\frac{1}{y} = 3.55\left(\frac{1}{x}\right) - 0.6$
$\frac{1}{x}$	1	0.5	0.33	0.25	0.2										
$\frac{1}{y}$	2.94	1.18	0.58	0.28	0.10										
7a	$x = 1$	7b	Equation of line : $y = \frac{x}{2} + 1$												
8ii	$\frac{4}{15}(3x-1)^{\frac{5}{2}} + c$ , where $c$ is a constant	8iii	- 32.2												
9i	$y = \frac{x^3}{3} - \frac{1}{4}e^{1-2x} - \frac{3}{4}x + 1$														
10ii	$6\sqrt{2} + 14$ units														