Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS
Paper 1

4049/01

17 August 2022 Wednesday 2 hours 15 min

2022 SECONDARY FOUR EXPRESS
PRELIMINARY EXAMINATIONS

INSTRUCTIONS TO CANDIDATES

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Write your name, index number and class on the spaces provided above.
Write in dark blue or black pen. You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Write your answers on the spaces provided below each question.
Give non exact numerical answers correct to 3 significant figures or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
Omission of essential working will result in loss of marks.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

	For Examiner's Use														
Qn	1	2	3	4	5	6	7	8	9	10	11	12	13	14	*Marks Deducted*
Marks															

Category	Accuracy	Units	Symbols	Others
Question No.				

Total Marks
90

Setter: Mr Tan Lip Sing
Vetter: Mr Gregory Quek

This question paper consists of **23** printed pages and **1** blank pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$
$$\sec^2 A = 1 + \tan^2 A.$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$
$$a^2 = b^2 + c^2 - 2bc\cos A.$$
$$\Delta = \frac{1}{2}ab\sin C.$$

Answer all questions in the space provided.

1 The area of a triangle is given as $1+2\sqrt{5}$ cm^2. The base of the triangle is given as $3-\sqrt{5}$ cm. Without using a calculator, express the height of the triangle, in cm, in the form $a+b\sqrt{5}$, where a and b are rational numbers. [3]

2 Find the y-coordinates of the points for which the line $x-2y=3$ meets the curve $xy+6=2x$. [3]

3 Express $-x^2+8x+5$ in the form $a(x+b)^2+c$ and hence state the coordinates of the turning point of the curve $y=-x^2+8x+5$. [4]

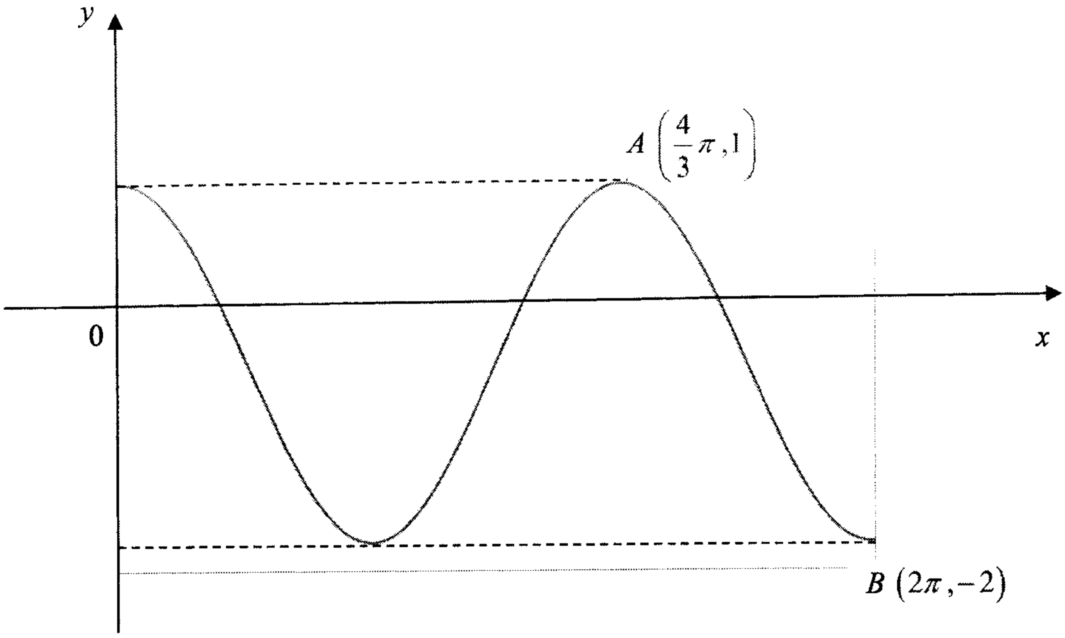
4 Integrate $\dfrac{4}{2-5x}+\dfrac{2}{x^3}+e^{4x}$ with respect to x. [4]

5 Express $\frac{9x^2-34x+27}{(x-1)(x-2)^2}$ as the sum of three partial fractions. [6]

6 **(a)** The expression $ax^3 + 13x^2 + bx - 5$ is exactly divisible by $x-1$ but gives a reminder of 49 when divided by $x-2$. Find the value of a and of b. [4]

(b) The cubic polynomial f(x) is such that the coefficient of x^3 is -2 and the roots of the equation f(x) = 0 are -1, 2 and k. Given that f(x) has a reminder of 80 when divided by $x+3$, find the value of k, given that k is a positive number. [3]

7 The diagram shows the graph of the curve $y = a\cos bx + c$ for $0 \le x \le 2\pi$. The curve has a maximum point at A and a minimum point at B. The coordinates of $A = \left(\frac{4}{3}\pi, 1\right)$ and $B = (2\pi, -2)$.

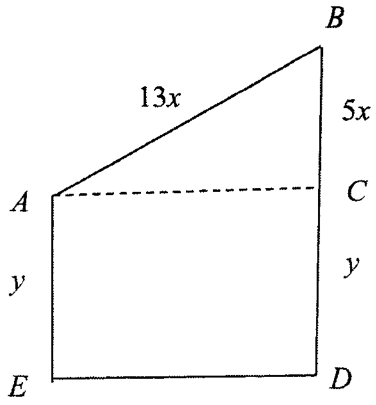


(a) State the period of the curve. [1]

(b) Find the value of a, b and c. [3]

(c) Find the range of values of k for which $a\cos bx + c = k$ has three solutions. [2]

8



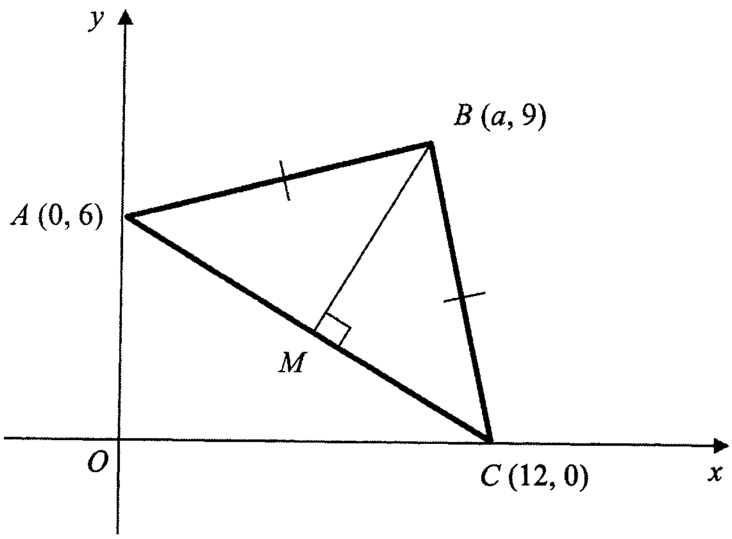
A piece of wire, l cm long, is bent to form the shape $ABCDE$ as shown in the diagram. $ACDE$ is a rectangle with $AE = y$ cm and ΔABC is a right-angled triangle with $AB = 13x$ cm and $BC = 5x$ cm.

(a) Express l in terms of x and y. [1]

(b) Given that the area enclosed is 96 cm^2, show that $l = 25x + \frac{16}{x}$. [3]

(c) Find the value of x for which l has a stationary value and determine the nature of this stationary value. [3]

9



The diagram shows a triangle ABC, where A is $(0,6)$, B is $(a,9)$ and C is $(12,0)$. AB is equal to BC and M is the midpoint of AC.

(a) Find the coordinates of M. [1]

(b) Find the equation of the perpendicular bisector of AC. [4]

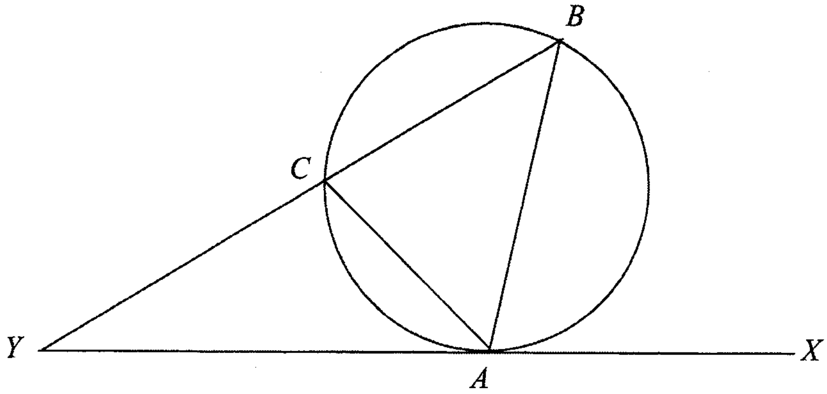
(c) Find the value of a. [2]

(d) Calculate the area of the triangle ABC. [2]

10 **(a)** Prove the identity $\frac{1-\cos 2A}{1+\cos 2A}=\tan^2 A$. [3]

(b) Hence solve the equation $\frac{1-\cos 2A}{1+\cos 2A} = 2\tan A$ for $0° < A < 360°$. [4]

11



The diagram shows a triangle ABC inscribed in a circle. XY is a tangent to the circle at point A and AC bisects angle BAY.

(a) Prove that triangle ABC is isosceles. [2]

(b) Prove that triangle AYC is similar to triangle BYA. [3]

(c) Hence, show that $AY^2 = CY \times BY$ [2]

12 **(a)** Given that $3^{x+1} \times 2^{2x+1} = 2^{x+2}$, evaluate 6^x . [4]

(b) Express y in terms of x if $\log_2 y = \log_8 x - \log_2 4$. [4]

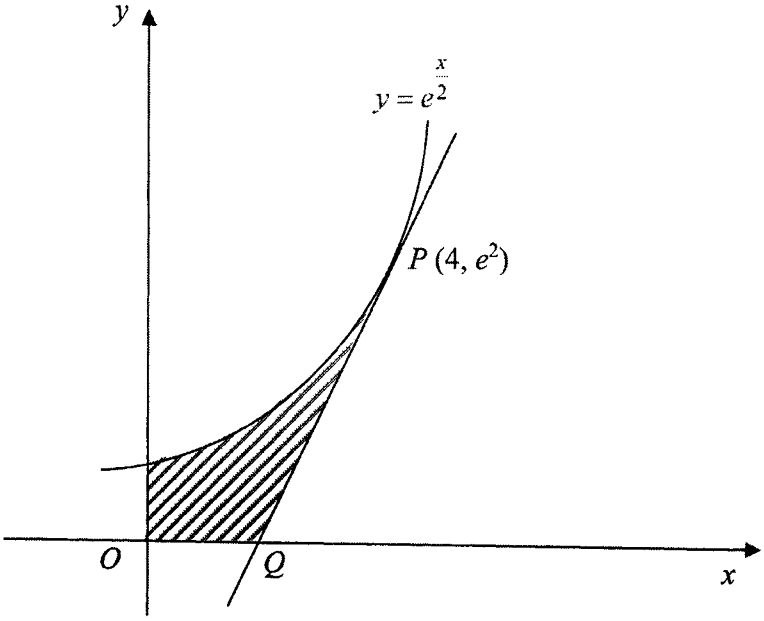
13 A curve has the equation $y=(x-3)\sqrt{2x+3}$, where $x>-\frac{3}{2}$.

(a) Show that $\frac{dy}{dx}$ can be expressed in the form $\frac{kx}{\sqrt{2x+3}}$ and state the value of k. [4]

(b) Find the equation of the tangent when $x = 11$. [3]

(c) Find the rate of change of x at the instant when $x = 11$, given that y is increasing at a rate of 5 units per second at this instant. [2]

14



The diagram shows part of the curve $y = e^{\frac{x}{2}}$. The tangent to the curve at $P\left(4, e^2\right)$ meets the x-axis at Q.

(a) Find the coordinates of Q. [5]

Continuation of working space for question 14a.

(b) Find the area of the shaded region bounded by the curve, the coordinate axes and the tangent to the curve at P, leaving your answer in terms of e. [5]

Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS **4049/02**
Paper 2

18 August 2022 Thursday 2 hours 15 minutes

2022 SECONDARY FOUR EXPRESS / FIVE NORMAL PRELIMINARY EXAMINATIONS

INSTRUCTIONS TO CANDIDATES

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

Qn	1	2	3	4	5	6	7	8	9	10	*Marks Deducted*
Marks											

Category	Accuracy	Units	Symbols	Others
Question No.				

Total Marks
90

Setter: Mr Gregory Quek
Vetter: Mr Tan Lip Sing

This question paper consists of **19** printed pages (including this cover page) and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$
$$\sec^2 A = 1 + \tan^2 A.$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$
$$a^2 = b^2 + c^2 - 2bc\cos A.$$
$$\Delta = \frac{1}{2}ab\sin C.$$

Answer all the questions in the spaces provided.

1 Show that the equation $2e^{x} + 9 = 18e^{-x}$ has only one solution and find its value correct to 2 significant figures. [5]

2 A polynomial $f(x)$ is defined as $x^3 - 13x^2 + 49x - 57$.

(a) Show that $x = 3$ is a root of the equation $f(x) = 0$. [1]

(b) It is given that the two other roots of $f(x) = 0$ are $x_1 = a + b\sqrt{c}$ and $x_2 = a - b\sqrt{c}$, where a, b and c are positive integers. Find the exact values of x_1 and x_2. [4]

(c) Express $x_1^3 - x_2^3$ in the form $d\sqrt{c}$, where d is a positive integer. [3]

3 The equation of a curve is $y^2 + mx^2 = m$, where m is a positive constant.

(a) Find the largest integer value of m for which the line $x - y = 3$ does not meet the curve. [5]

(b) If the line $x - y = 3$ is a tangent to the curve at point P, deduce the value of the constant m. Hence find the coordinates of P. [3]

4 It is given that $\left(x+\dfrac{k}{x^2}\right)^n$ is a binomial expansion, where k and n are positive constants.

(a) Write down the first 4 terms in the expansion of $\left(x+\dfrac{k}{x^2}\right)^5$, in terms of k, in descending powers of x. [2]

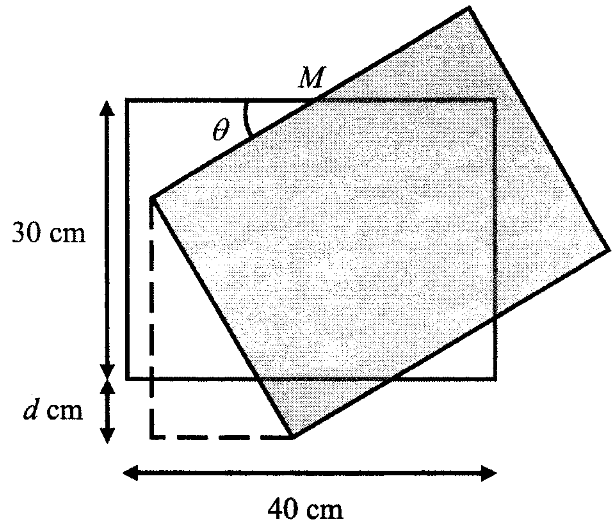
(b) Hence or otherwise, find the value(s) of k if the coefficient of x^2 in the expansion of $\left(5x^3+3\right)\left(x+\dfrac{k}{x^2}\right)^5$ is 5. [3]

(c) By considering the general term in the binomial expansion of $\left(x+\frac{k}{x^2}\right)^n$, show that for the term independent of x, the value of the constant n is a multiple of 3. [3]

5 **(a)** Given that $y = \frac{\ln 2x}{5x}$, show that $\frac{dy}{dx} = \frac{1 - \ln 2x}{5x^2}$. [4]

(b) Hence find the value of $\int_1^2 \frac{\ln 2x}{x^2}\,dx$. [4]

6



The diagram shows a rectangular picture frame 40 cm by 30 cm hung on the wall.
The picture frame is rotated through an angle θ about the midpoint, M of the top edge.

(a) Show that the vertical displacement, d cm, of the picture frame below its original bottom edge is given by

$$d = 20\sin\theta + 30\cos\theta - 30.$$ [2]

(b) Express d in the form $R\sin(\theta+\alpha)-30$, where $R>0$ and $0°\leq\alpha\leq 90°$. [4]

(c) Find the value of d and the corresponding value of θ that will give the greatest vertical displacement of the picture frame below its original bottom edge. [3]

7 A particle moves in a straight line such that its velocity, v m/s, is given by $v=\frac{1}{2}-2e^{-\frac{t}{2}}$, where t is the time in seconds after leaving a fixed point O.

(a) State the value that v approaches as t becomes very large. Justify your answer. [2]

(b) Find the initial acceleration of the particle. [2]

(c) Find the value of t when the particle is instantaneously at rest. [2]

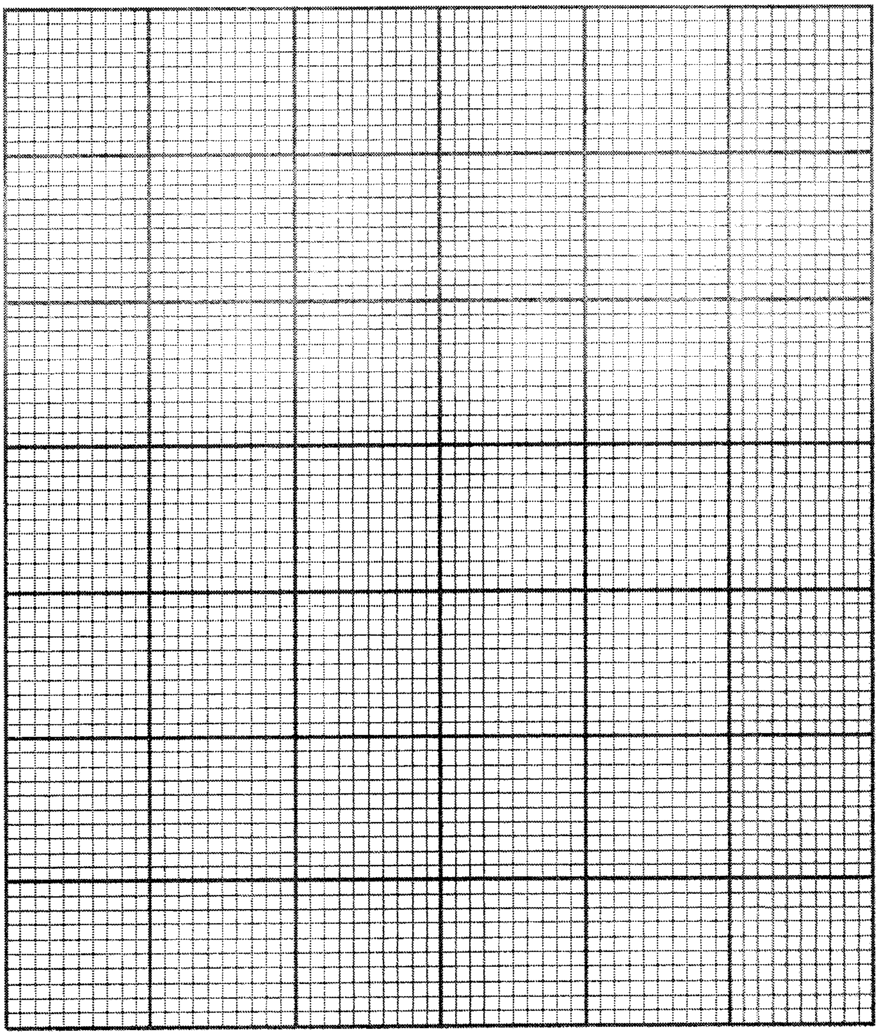
(d) Find the total distance travelled by the particle in the interval $0 \le t \le 10$. [4]

8 The value of a car, $\$V$, after t years following 2014 can be modelled by the formula $V = ab^t$, where a and b are constants. The table shows the value of the car in the years following 2014.

Year	2016	2018	2020	2022
t (years)	2	4	6	8
V (\$)	52100	43800	37100	31600

(a) Given that $\lg V$ is the variable for the vertical axis, express the formula in a form suitable for drawing a straight line graph. [2]

(b) Draw a straight line graph to show that the model is reasonable. [4]

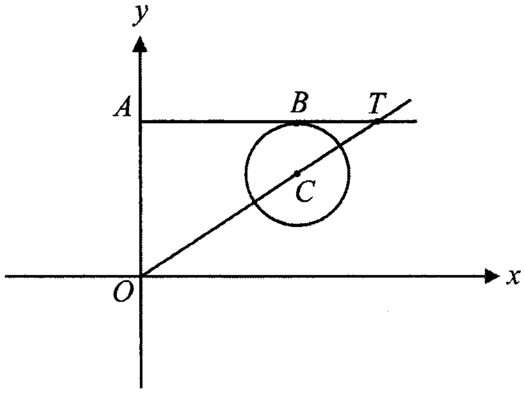


Use the graph in part **(b)** to estimate, correct to 3 significant figures,

(c) the value of the constants a and b, [3]

(d) the value of the car in the year 2024. [2]

9



(a) The equation of circle with centre C is given by $x^2 + y^2 - 6x - 4y + 12 = 0$.
Find the radius of the circle and the coordinates of its centre C. [3]

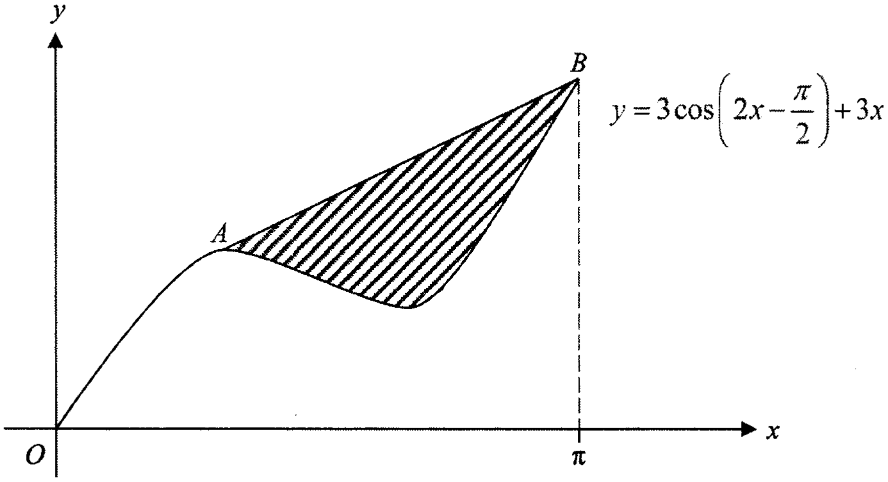
(b) AB is a horizontal tangent to the circle at point B. Given that the line OC produced meets the line AB produced at point T, find the coordinates of T. [3]

(c) Show that triangle *AOT* and triangle *BCT* are similar. [3]

(d) Find the ratio *OC* : *CT*. [1]

(e) Find the angle *ATO* in degrees. [1]

10



The diagram shows the curve $y = 3\cos\left(2x - \frac{\pi}{2}\right) + 3x$ for $0 \le x \le \pi$ radians.

The point A is the maximum point of the curve and AB is a straight line.

(a) Show that the coordinates of A are $\left(\frac{\pi}{3}, \frac{3\sqrt{3}}{2} + \pi\right)$ and coordinates of B are $(\pi, 3\pi)$. [5]

(b) Hence find the area of the shaded region, leaving your answer in terms of π. [7]

END OF PAPER

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS
4049/01

Paper 1

17 August 2022 Wednesday 2 hours 15 min

2022 SECONDARY FOUR EXPRESS
PRELIMINARY EXAMINATIONS

MARKING SCHEME

Question 1

The area of a triangle is given as $1+2\sqrt{5}$ cm^2. The base of the triangle is given as $3-\sqrt{5}$ cm. Without using a calculator, express the height of the triangle, in cm, in the form $a+b\sqrt{5}$, where a and b are rational numbers.

[3]

	Area of triangle $=1+2\sqrt{5}$ $\frac{1}{2}\left(3-\sqrt{5}\right)h=1+2\sqrt{5}$ $h=\frac{2+4\sqrt{5}}{3-\sqrt{5}}$ $h=\frac{2+4\sqrt{5}}{3-\sqrt{5}}\times\frac{3+\sqrt{5}}{3+\sqrt{5}}$ $=\frac{6+2\sqrt{5}+12\sqrt{5}+20}{9-5}$ $=\frac{26+14\sqrt{5}}{4}$ $=\frac{13}{2}+\frac{7}{2}\sqrt{5}\ cm$	M1 (multiply using $\frac{3+\sqrt{5}}{3+\sqrt{5}}$) M1 (expand numerator or denominator correctly) A1

Question 2

Find the y-coordinates of the points for which the line $x-2y=3$ meets the curve $xy+6=2x$.

[3]

	$x-2y=3$ $x=3+2y$ .....(1) $xy+6=2x$ ....(2) Substitute (1) into (2): $(3+2y)y+6=2(3+2y)$	M1 (substitution method)
	$3y+2y^2+6=6+4y$ $2y^2-y=0$ $y(2y-1)=0$	M1 (factorise)
	$y=0 \ or \ y=\frac{1}{2}$	A1

Question 3

Express $-x^2+8x+5$ in the form $a(x+b)^2+c$ and hence state the coordinates of the turning point of the curve $y=-x^2+8x+5$. [4]

	$-x^2+8x+5$ $=-\left[x^2-8x\right]+5$ $=-\left[x^2-8x+\left(\frac{8}{2}\right)^2-\left(\frac{8}{2}\right)^2\right]+5$ $=-(x-4)^2+16+5$ $=-(x-4)^2+21$ Coordinates of turning point $=(4, 21)$	 M1 M1 A1, A1

Question 4

Integrate $\frac{4}{2-5x}+\frac{2}{x^3}+e^{4x}$ with respect to x. [4]

	$\int\left(\frac{4}{2-5x}+\frac{2}{x^3}+e^{4x}\right)dx$ $=-\frac{4}{5}\ln(2-5x)-\frac{1}{x^2}+\frac{1}{4}e^{4x}+c$	B2 [B1 for showing $\ln(2-5x)$] B1, B1

Question 5

Express $\frac{9x^2-34x+27}{(x-1)(x-2)^2}$ as the sum of three partial fractions. [6]

	$\frac{9x^2-34x+27}{(x-1)(x-2)^2}=\frac{A}{x-1}+\frac{B}{x-2}+\frac{C}{(x-2)^2}$	M1
	$9x^2-34x+27=A(x-2)^2+B(x-1)(x-2)+C(x-1)$	M1
	Substitute $x=2$, $9(2)^2-34(2)+27=C(2-1)$ $C=-5$	M1 (using substitution method correctly)
	Substitute $x=1$, $9(1)^2-34(1)+27=A(1-2)^2$ $A=2$	
	Substitute $x=0$, $27=2(4)+2B+5$ $2B=14$ $B=7$	A2 (at least 2 out of 3 correct values for A, B & C)
	$\frac{9x^2-34x+27}{(x-1)(x-2)^2}=\frac{2}{x-1}+\frac{7}{x-2}-\frac{5}{(x-2)^2}$	A1

	Alternative method	
	$\frac{9x^2-34x+27}{(x-1)(x-2)^2}=\frac{A}{x-1}+\frac{B}{x-2}+\frac{C}{(x-2)^2}$	M1
	$9x^2-34x+27=A(x-2)^2+B(x-1)(x-2)+C(x-1)$	M1
	$9x^2-34x+27=A(x^2-4x+4)+B(x^2-3x+2)+C(x-1)$ $=Ax^2-4Ax+4A+Bx^2-3Bx+2B+Cx-C$ $=(A+B)x^2+(c-4A-3B)x+(4A+2B-C)$	
	Comparing the coefficients on both sides, $A+B=9$ ......(1) $C-4A-3B=-34$ ......(2) $4A+2B-C=27$ ......(3)	M1 (using the comparing of coefficient method correctly)
	$(2)+(3): B=7$ $(1): A+7=9$ $A=2$ $(2): C-8-21=-34$ $C=-5$	A2 (at least 2 out of 3 correct values for A, B & C)
	$\frac{9x^2-34x+27}{(x-1)(x-2)^2}=\frac{2}{x-1}+\frac{7}{x-2}-\frac{5}{(x-2)^2}$	A1

Question 6

(a) The expression ax^3+13x^2+bx-5 is exactly divisible by $x-1$ but gives a reminder of 49 when divided by $x-2$. Find the value of a and of b. [4]

(b) The cubic polynomial f(x) is such that the coefficient of x^3 is -2 and the roots of the equation f(x) = 0 are -1, 2 and k. Given that f(x) has a reminder of 80 when divided by $x+3$, find the value of k, given that k is a positive number. [3]

(a)	Let f(x) $= ax^3+13x^2+bx-5$ $f(1)=a(1)^3+13(1)^2+b(1)-5$ $0=a+b+8$ $a+b=-8$ ......(1) $f(2)=a(2)^3+13(2)^2+b(2)-5$ $49=8a+52+2b-5$ $8a+2b=2$ $4a+b=1$ ......(2) Solving (1) and (2), $a=3, b=-11$	M1 (equate to 0) M1 (equate to 49) A1, A1
(b)	$f(x)=-2(x+1)(x-2)(x-k)$ $80=-2(-3+1)(-3-2)(-3-k)$ $80=-20(-3-k)$ $80=60+20k$ $k=1$	M1 M1 A1

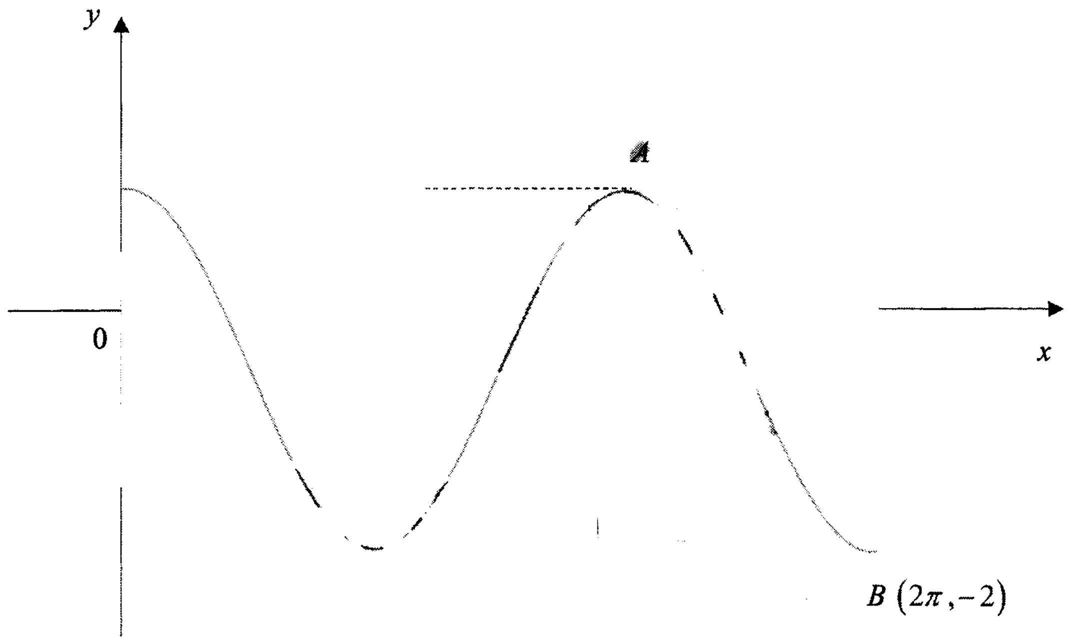
Question 7

The diagram shows the graph of the curve $y = a\cos bx + c$ for $0 \le x \le 2\pi$. The curve has a maximum point at A and a minimum point at B. The coordinates of $A = \left(\frac{4}{3}\pi, 1\right)$ and $B = (2\pi, -2)$.

(a) State the period of the curve. [1]

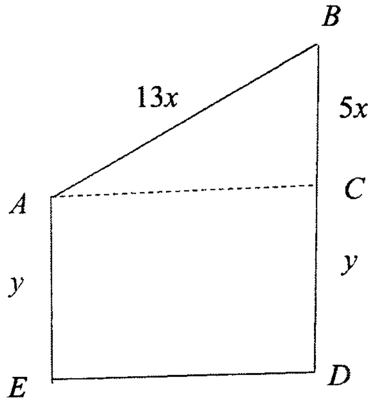
(b) Find the value of a, b and c. [3]

(c) Find the range of values of k for which $a\cos bx + c = k$ has three solutions. [2]



(a)	Period $= \frac{4}{3}\pi$	B1
(b)	$a = \frac{3}{2}, b = \frac{3}{2}, c = -\frac{1}{2}$	B1, B1, B1
(c)	$-2 < k < 1$	B2 B1 (either state $-2 < k$ or $k < 1$)

Question 8



A piece of wire, l cm long, is bent to form the shape as shown in the diagram. $ACDE$ is a rectangle with $AE = y$ cm and ΔABC is a right-angled triangle with $AB = 13x$ cm and $BC = 5x$ cm.

(a) Express l in terms of x and y. [1]

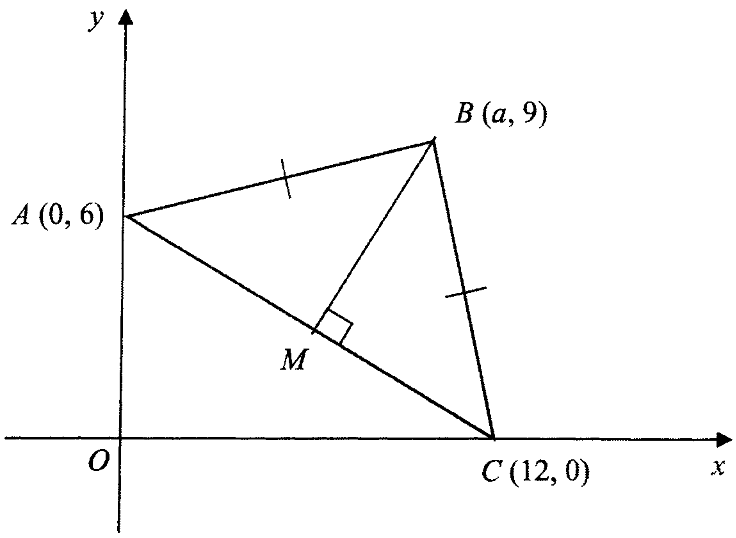
(b) Given that the area enclosed is 96 cm^2, show that $l = 25x + \dfrac{16}{x}$. [3]

(c) Find the value of x for which l has a stationary value and determine the nature of this stationary value. [3]

(a)	$l = 12x + 13x + 5x + y + y$ $l = 30x + 2y$	B1
(b)	$12xy + \frac{1}{2}(12x)(5x) = 96$ $12xy + 30x^2 = 96$ $12xy = 96 - 30x^2$ $y = \frac{8}{x} - \frac{5x}{2}$ $l = 30x + 2\left(\frac{8}{x} - \frac{5x}{2}\right)$ $l = 30x + \frac{16}{x} - 5x$ $l = 25x + \frac{16}{x}$ (*shown*)	M1 M1 A1

(c)	$\frac{dl}{dx} = 25 - \frac{16}{x^2}$ $when\ \frac{dl}{dx} = 0,$ $25 - \frac{16}{x^2} = 0$ $x^2 = \frac{16}{25}$ $x = \frac{4}{5}$ $\frac{d^2l}{dx^2} = \frac{32}{x^3}$ $when\ x = \frac{4}{5},$ $\frac{d^2l}{dx^2} = 62.5 > 0$ l is a minimum value.	 M1 A1 A1

Question 9



The diagram shows a triangle ABC, where A is $(0,6)$, B is $(a,9)$ and C is $(12,0)$. AB is equal to BC and M is the midpoint of AC.

(a) Find the coordinates of M. [1]

(b) Find the equation of the perpendicular bisector of AC. [4]

(c) Find the value of a. [2]

(d) Calculate the area of the triangle ABC. [2]

	Coordinates of $M = (6,3)$	B1
	Gradient of $AC = -\frac{1}{2}$ Gradient of $BM = 2$ Equation of perpendicular bisector of AC $y-3=2(x-6)$ $y=2x-9$	M1 M1 M1 A1
	$9=2a-9$ $2a=18$ $a=9$	M1 A1

	Area of triangle ABC $=\frac{1}{2}\begin{vmatrix} 0 & 12 & 9 & 0 \\ 6 & 0 & 9 & 6 \end{vmatrix}$ $=\frac{1}{2}\left[(0+108+54)-(72+0+0)\right]$ $=\frac{1}{2}(90)$ $=45\,units^2$	M1 A1

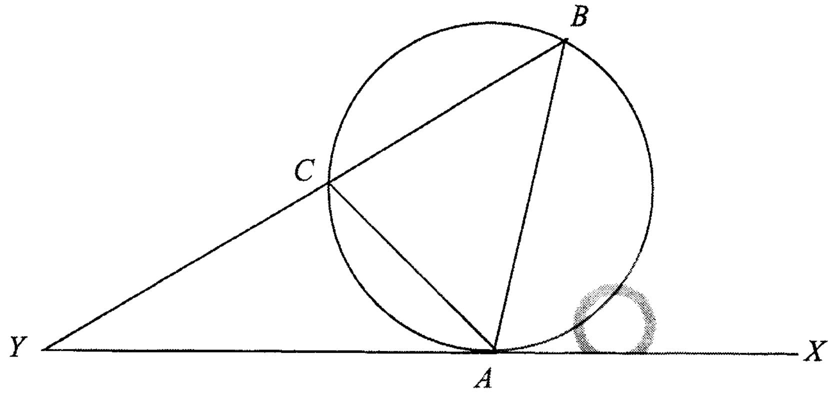
Question 10

(a) Prove the identity $\dfrac{1-\cos 2A}{1+\cos 2A} = \tan^2 A$. [3]

(b) Hence solve the equation $\dfrac{1-\cos 2A}{1+\cos 2A} = 2\tan A$ for $0° < A < 360°$. [4]

(a)	$LHS = \dfrac{1-\cos 2A}{1+\cos 2A}$ $= \dfrac{1-(1-2\sin^2 A)}{1+(2\cos^2 A-1)}$ $= \dfrac{2\sin^2 A}{2\cos^2 A}$ $= \tan^2 A = RHS$	 M1 (either correct numerator or denominator) M1 A1
(b)	$\dfrac{1-\cos 2A}{1+\cos 2A} = 2\tan A$ $\tan^2 A = 2\tan A$ $\tan^2 A - 2\tan A = 0$ $\tan A(\tan A - 2) = 0$ $\tan A = 0$ *or* $\tan A - 2 = 0$ $A = 180°$ $\tan A = 2$ A lies in the first & third quadrants $A = 63.43°, 180° + 63.43°$ $= 63.4°, 243.4°$	 M1 (factorise terms) A1 A1, A1

Question 11



The diagram shows a triangle ABC inscribed in a circle. XY is a tangent to the circle at point A and AC bisects angle BAY.

(a) Prove that triangle ABC is isosceles. [2]

(b) Prove that triangle AYC is similar to triangle BYA. [3]

(c) Hence, show that $AY^2 = CY \times BY$ [2]

(a)	Let $\angle CAY = \angle CAB = \theta$ $\angle CBA = \angle CAY = \theta$ (Alternate Segment Theorem) Since $\angle CAB = \angle CBA = \theta$, ΔABC is isosceles.	 B1 AG1
(b)	In ΔAYC and ΔBYA, $\angle BYA = \angle AYC$ (common angle) $\angle ABY = \angle CAY = \theta$ (Alternate Segment Theorem) So ΔAYC is similar to ΔBYA (AA Similarity)	 B1 B1 AG1
(c)	Since ΔAYC and ΔBYA are similar, $\frac{AY}{CY} = \frac{BY}{AY}$ $AY^2 = CY \times BY$	 B1 AG1

Question 12

(a) Given that $3^{x+1} \times 2^{2x+1} = 2^{x+2}$, evaluate 6^x. [4]

(b) Express y in terms of x if $\log_2 y = \log_8 x - \log_2 4$. [4]

	$3^{x+1} \times 2^{2x+1} = 2^{x+2}$	
	$3^{x+1} \times \frac{2^{2x+1}}{2^{x+2}} = 1$	
	$3^{x+1} \times 2^{2x+1-(x+2)} = 1$	M1 (apply quotient rule)
	$3^{x+1} \times 2^{x-1} = 1$	
	$(3^x)3 \times \frac{2^x}{2} = 1$	M1 (correct expansion)
	$3^x \times 2^x = \frac{2}{3}$	M1 ($3^x \times 2^x = 6^x$)
	$6^x = \frac{2}{3}$	A1
	$\log_2 y = \log_8 x - \log_2 4$	
	$= \frac{\log_2 x}{\log_2 8} - \log_2 4$	M1 (change of base law)
	$= \frac{\log_2 x}{\log_2 2^3} - \log_2 4$	
	$= \frac{\log_2 x}{3} - \log_2 4$	M1 (apply power law)
	$3\log_2 y = \log_2 x - 3\log_2 4$	
	$\log_2 y^3 = \log_2 x - \log_2 4^3$	
	$\log_2 y^3 = \log_2 \frac{x}{64}$	M1 (apply quotient law)
	$y^3 = \frac{x}{64}$	
	$y = \frac{1}{4}x^{\frac{1}{3}}$	A1

Question 13

A curve has the equation $y=(x-3)\sqrt{2x+3}$, where $x>-\frac{3}{2}$.

(a) Show that $\frac{dy}{dx}$ can be expressed in the form $\frac{kx}{\sqrt{2x+3}}$ and state the value of k. [4]

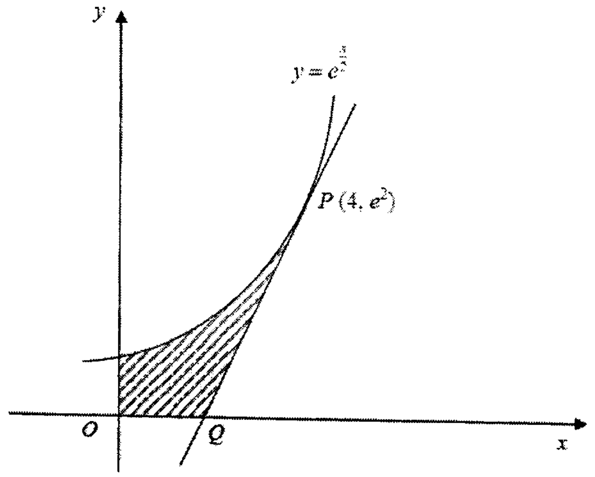
(b) Find the equation of the tangent when $x=11$. [3]

(c) Find the rate of change of x at the instant when $x=11$, given that y is increasing at a rate of 5 units per second at this instant. [2]

(a)	$y=(x-3)\sqrt{2x+3}$ $\frac{dy}{dx}=(2x+3)^{\frac{1}{2}}+\frac{1}{2}(x-3)(2)(2x+3)^{-\frac{1}{2}}$ $=\sqrt{2x+3}+\frac{x-3}{\sqrt{2x+3}}$ $=\frac{2x+3+x-3}{\sqrt{2x+3}}$ $=\frac{3x}{\sqrt{2x+3}}$ $k=3$	M1, M1 M1 A1
(b)	When $x=11$, $y=(11-3)\sqrt{2(11)+3}=40$ $\frac{dy}{dx}=\frac{3(11)}{\sqrt{2(11)+3}}=\frac{33}{5}$ $y=\frac{33}{5}x+c$ $40=\frac{33}{5}(11)+c$ $c=-32.6$ Equation of tangent is $y=6.6x-32.6$	M1 M1 A1

(c)	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $5=\frac{33}{5}\times\frac{dx}{dt}$ $\frac{dx}{dt}=\frac{25}{33}\ units/s$	 M1 A1

Question 14



The diagram shows part of the curve $y = e^{\frac{x}{2}}$. The tangent to the curve at $P\left(4, e^2\right)$ meets the x-axis at Q.

(a) Find the coordinates of Q. [5]

(b) Find the area of the shaded region bounded by the curve, the coordinate axes and the tangent to the curve at P, leaving your answer in terms of e. [5]

(a)	$y = e^{\frac{x}{2}}$ $\frac{dy}{dx} = \frac{1}{2}e^{\frac{x}{2}}$	M1
	At $P\left(4, e^2\right)$, $\frac{dy}{dx} = \frac{1}{2}e^{\frac{4}{2}} = \frac{1}{2}e^2$	M1
	Let $Q = (x, 0)$ Gradient of $PQ = \frac{e^2 - 0}{4 - x}$	M1
	$\frac{1}{2}e^2 = \frac{e^2}{4-x}$ $4 - x = 2$ $x = 2$	M1
	Coordinates of $Q = (2, 0)$	A1

(b)	Area of shaded region	
	$=\int_0^4 e^{\frac{x}{2}}dx-\frac{1}{2}(4-2)e^2$	M1, M1
	$=\left[2e^{\frac{x}{2}}\right]_0^4-e^2$	M1 (integrate correctly)
	$=\left[2e^2-2e^0\right]-e^2$	M1 (correct substitution)
	$=e^2-2$	A1

Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS **4049/02**
Paper 2

18 August 2022 Thursday 2 hours 15 minutes

2022 SECONDARY FOUR EXPRESS / FIVE NORMAL
PRELIMINARY EXAMINATIONS

MARK SCHEME

1 Show that the equation $2e^x + 9 = 18e^{-x}$ has only one solution and find its value correct to 2 significant figures. [5]

$2e^x + 9 = 18e^{-x}$

Let $u = e^x$

$2u + 9 = \frac{18}{u}$ M1 (attempt to form quadratic)

$2u^2 + 9u - 18 = 0$

$(2u-3)(u+6) = 0$ M1 (factorisation, o.e.)

$e^x = \frac{3}{2}$ *or* $e^x = -6$ (rejected) M1 (seen rejected)

$x = \ln\frac{3}{2}$ M1 (ln both sides)

$x = 0.4054 \approx 0.41$ (2sf)

The equation has only one solution $x = 0.41$. (shown) A1

2 A polynomial $f(x)$ is defined as $x^3 - 13x^2 + 49x - 57$.

(a) Show that $x = 3$ is a root of the equation $f(x) = 0$. [1]

(b) It is given that the two other roots of $f(x) = 0$ are $x_1 = a + b\sqrt{c}$ and $x_2 = a - b\sqrt{c}$, where a, b and c are positive integers. Find the exact values of x_1 and x_2. [4]

(c) Express $x_1^3 - x_2^3$ in the form $d\sqrt{c}$, where d is a positive integer. [3]

(a) When $x = 3$,
$(3)^3 - 13(3)^2 + 49(3) - 57 = 0$
Hence $x = 3$ is a solution of the equation. (shown) — AG1

(b) From (a), $x - 3$ is a factor of $f(x)$.

$(x-3)(x^2 - 10x + 19) = 0$ — M1 (long division or comparing coefficients)
M1 (seen $x^2 - 10x + 19$)

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(19)}}{2(1)}$$ M1 (quadratic formula)

$$x = \frac{10 \pm \sqrt{24}}{2}$$

$$x = \frac{10 \pm 2\sqrt{6}}{2}$$

$x_1 = 5 + \sqrt{6}$ *or* $x_2 = 5 - \sqrt{6}$ — A1

(c) $x_1^3 - x_2^3$

$= (x_1 - x_2)(x_1^2 + x_1x_2 + x_2^2)$

$= \left[5 + \sqrt{6} - (5 - \sqrt{6})\right]\left[(5+\sqrt{6})^2 + (5+\sqrt{6})(5-\sqrt{6}) + (5-\sqrt{6})^2\right]$ — M1

$= 2\sqrt{6}\left[(25 + 10\sqrt{6} + 6) + (25 - 6) + (25 - 10\sqrt{6} + 6)\right]$ — M1 (attempt to simplify)

$= 2\sqrt{6}[81]$

$= 162\sqrt{6}$ — A1

3 The equation of a curve is $y^2 + mx^2 = m$, where m is a positive constant.

(a) Find the largest integer value of m for which the line $x - y = 3$ does not meet the curve. [5]

(b) If the line $x - y = 3$ is a tangent to the curve at point P, deduce the value of the constant m. Hence find the coordinates of P. [3]

(a) $x - y = 3 \Rightarrow y = x - 3$

Sub. into the curve,

$(x-3)^2 + mx^2 = m$ — M1 (equate line to curve)

$x^2 - 6x + 9 + mx^2 = m$

$(m+1)x^2 - 6x + 9 - m = 0$ — M1 (reduce to quadratic)

Since the line does not meet the curve,

$(-6)^2 - 4(m+1)(9-m) < 0$ — M1 (apply D < 0)

$36 - 4(-m^2 + 8m + 9) < 0$

$4m^2 - 32m < 0$

$4m(m-8) < 0$

$0 < m < 8$ — M1 (solving quadratic inequality)

Largest integer $m = 7$ — A1

(b) Since the line is tangent to the curve,

$4m(m-8) = 0$

$m = 0$ (rejected) *or* $m = 8$ — B1 (seen $m = 8$)

When $m = 8$, $(8+1)x^2 - 6x + 9 - 8 = 0$

$9x^2 - 6x + 1 = 0$

$(3x-1)^2 = 0$

$x = \frac{1}{3}$ — M1 (attempt to find x)

$y = \frac{1}{3} - 3 = -\frac{8}{3}$

$\therefore P = \left(\frac{1}{3}, -\frac{8}{3}\right)$ — A1

4 It is given that $\left(x+\frac{k}{x^2}\right)^n$ is a binomial expansion, where k and n are positive constants.

(a) Write down the first 4 terms in the expansion of $\left(x+\frac{k}{x^2}\right)^5$, in terms of k, in descending powers of x. [2]

(b) Hence or otherwise, find the value(s) of k if the coefficient of x^2 in the expansion of $(5x^3+3)\left(x+\frac{k}{x^2}\right)^5$ is 5. [3]

(c) By considering the general term in the binomial expansion of $\left(x+\frac{k}{x^2}\right)^n$, show that for the term independent of x, the value of the constant n is a multiple of 3. [3]

(a) $\left(x+\frac{k}{x^2}\right)^5 = x^5 + \binom{5}{1}x^4\left(\frac{k}{x^2}\right) + \binom{5}{2}x^3\left(\frac{k}{x^2}\right)^2 + \binom{5}{3}x^2\left(\frac{k}{x^2}\right)^3 + \ldots$ M1

$\left(x+\frac{k}{x^2}\right)^5 = x^5 + 5kx^2 + \frac{10k^2}{x} + \frac{10k^3}{x^4} + \ldots$ A1

(b) $(5x^3+3)\left(x+\frac{k}{x^2}\right)^5 = (5x^3+3)\left[x^5 + 5kx^2 + \frac{10k^2}{x} + \frac{10k^3}{x^4} + \ldots\right]$

$(5x^3)\left(\frac{10k^2}{x}\right) + 3(5kx^2) = 5x^2$ M1 (equating coefficients of x^2)

$10k^2 + 3k - 1 = 0$

$(2k+1)(5k-1) = 0$ M1 (solving for k)

$k = -\frac{1}{2}$ (rejected) *or* $k = \frac{1}{5}$ A1

(c) General term $= \binom{n}{r}x^{n-r}\left(\frac{k}{x^2}\right)^r$ M1 (substitution into general term)

$= \binom{n}{r}k^r x^{n-3r}$

For the term independent of x,

let $n - 3r = 0$ M1 (equate power to zero)

$\therefore n = 3r$

Since $n = 3r$, where r is an integer, hence the value of n is a multiple of 3. AG1

5 **(a)** Given that $y = \frac{\ln 2x}{5x}$, show that $\frac{dy}{dx} = \frac{1 - \ln 2x}{5x^2}$. [4]

(b) Hence find the value of $\int_1^2 \frac{\ln 2x}{x^2}\,dx$. [4]

(a) $y = \frac{\ln 2x}{5x}$

$\frac{dy}{dx} = \frac{5x\left(\frac{1}{2x}\right)(2) - \ln 2x\,(5)}{(5x)^2}$ M2 (quotient rule & chain rule)

$\frac{dy}{dx} = \frac{5 - 5\ln 2x}{25x^2}$ M1 (simplifying)

$\frac{dy}{dx} = \frac{1 - \ln 2x}{5x^2}$ (shown) AG1

(b) $\int_1^2 \frac{1 - \ln 2x}{5x^2}\,dx = \left[\frac{\ln 2x}{5x}\right]_1^2$ M1 (reverse part (a))

$\int_1^2 \frac{1 - \ln 2x}{x^2}\,dx = \left[\frac{\ln 2x}{x}\right]_1^2$

$\int_1^2 \frac{1}{x^2}\,dx - \int_1^2 \frac{\ln 2x}{x^2}\,dx = \left[\frac{\ln 2x}{x}\right]_1^2$ M1 (separate into two integrals)

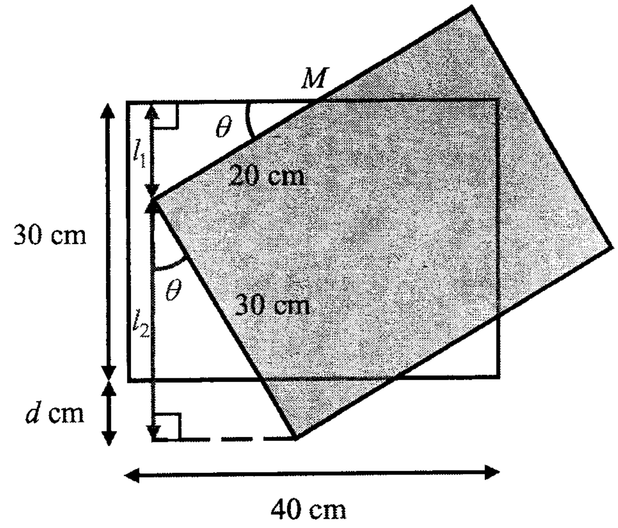
$\int_1^2 \frac{\ln 2x}{x^2}\,dx = \int_1^2 \frac{1}{x^2}\,dx - \left[\frac{\ln 2x}{x}\right]_1^2$

$\int_1^2 \frac{\ln 2x}{x^2}\,dx = \left[-\frac{1}{x}\right]_1^2 - \left[\frac{\ln 2x}{x}\right]_1^2$ M1 (integration of $1/x^2$)

$\int_1^2 \frac{\ln 2x}{x^2}\,dx = \left[-\frac{1}{2} - (-1)\right] - \left[\frac{\ln 4}{2} - \ln 2\right]$

$\int_1^2 \frac{\ln 2x}{x^2}\,dx = \frac{1}{2}$ A1

6



The diagram shows a rectangular picture frame 40 cm by 30 cm hung on the wall.
The picture frame is rotated through an angle θ about the midpoint, M of the top edge.

(a) Show that the vertical displacement, d cm, of the picture frame below its original bottom edge is given by

$$d = 20\sin\theta + 30\cos\theta - 30.$$ [2]

(b) Express d in the form $R\sin(\theta+\alpha)-30$, where $R > 0$ and $0° \le \alpha \le 90°$. [4]

(c) Find the value of d and the corresponding value of θ that will give the greatest vertical displacement of the picture frame below its original bottom edge. [3]

(a) $l_1 = \frac{1}{2}(40)\sin\theta = 20\sin\theta$ and $l_2 = 30\cos\theta$ M1 (identify and find the lengths)

$d + 30 = 20\sin\theta + 30\cos\theta$

$d = 20\sin\theta + 30\cos\theta - 30$ (shown) AG1

(b) Let $20\sin\theta + 30\cos\theta - 30 = R\sin(\theta+\alpha) - 30$

$R = \sqrt{20^2 + 30^2} = \sqrt{1300} = 10\sqrt{13}$ M1 (finding R)

$\alpha = \tan^{-1}\left(\frac{30}{20}\right) = 56.309° \approx 56.3°$ M1 (finding α)

$\therefore d = 10\sqrt{13}\sin(\theta + 56.3°) - 30$ A2 (deduct 1 mark for each incorrect value)

(c) The greatest vertical displacement occurs when $\sin(\theta + 56.3°) = 1$.

$\therefore d = 10\sqrt{13} - 30 = 6.0555 \approx 6.06$ B1

$\theta + 56.3° = 90°$ M1

$\therefore \theta = 33.7°$ A1

7 A particle moves in a straight line such that its velocity, v m/s, is given by $v=\frac{1}{2}-2e^{-\frac{t}{2}}$, where t is the time in seconds after leaving a fixed point O.

(a) State the value that v approaches as t becomes very large. Justify your answer. [2]

(b) Find the initial acceleration of the particle. [2]

(c) Find the value of t when the particle is instantaneously at rest. [2]

(d) Find the total distance travelled by the particle in the interval $0 \le t \le 10$. [4]

(a) The value of v approaches $\frac{1}{2}$. B1

As t becomes very large, $2e^{-\frac{t}{2}}$ approaches zero, so $v \approx \frac{1}{2}$. B1

(b) $a=-2e^{-\frac{t}{2}}\left(-\frac{1}{2}\right)=e^{-\frac{t}{2}}$ M1 (find dv/dt)

When $t=0$, initial acceleration $=e^{-\frac{0}{2}}=1\text{ m/s}^2$ A1

(c) At instantaneous rest,

$v=\frac{1}{2}-2e^{-\frac{t}{2}}=0$ M1 (equate v to zero)

$e^{-\frac{t}{2}}=\frac{1}{4}$

$-\frac{t}{2}=\ln\frac{1}{4}$

$t=-2\ln\frac{1}{4}=\ln 16$

$\therefore t=2.7725 \approx 2.77$ s (3sf) A1 (Accept 4ln2)

(d) $s=\frac{1}{2}t-\frac{2e^{-\frac{t}{2}}}{-\frac{1}{2}}=\frac{1}{2}t+4e^{-\frac{t}{2}}+c$ M1 (correct antiderivative)

$0=\frac{1}{2}(0)+4e^{-\frac{0}{2}}+c\Rightarrow c=-4$ M1 (attempt to find arbitrary constant)

$\Rightarrow s=\frac{1}{2}t+4e^{-\frac{t}{2}}-4$

When $t=\ln 16$, $s=\frac{1}{2}\ln 16+4e^{-\frac{\ln 16}{2}}-4=-1.6137\text{ m}$

When $t=10$, $s=\frac{1}{2}(10)+4e^{-\frac{10}{2}}-4=1.0269\text{ m}$

M1 (attempt to find either one)

Total distance travelled

$=(1.6137)+(1.6137+1.0269)=4.2543\approx 4.25\text{ m (3sf)}$ A1

8 The value of a car, $\$V$, after t years following 2014 can be modelled by the formula $V = ab^t$, where a and b are constants. The table shows the value of the car in the years following 2014.

Year	2016	2018	2020	2022
t (years)	2	4	6	8
V (\$)	52100	43800	37100	31600

(a) Given that $\lg V$ is the variable for the vertical axis, express the formula in a form suitable for drawing a straight line graph. [2]

(b) Draw a straight line graph to show that the model is reasonable. [4]

Use the graph in part **(b)** to estimate, correct to 3 significant figures,

(c) the value of the constants a and b, [3]

(d) the value of the car in the year 2024. [2]

(a) $V = ab^t$

$\lg V = \lg\left(ab^t\right)$

$\lg V = \lg a + \lg b^t$ M1 (seen product law)

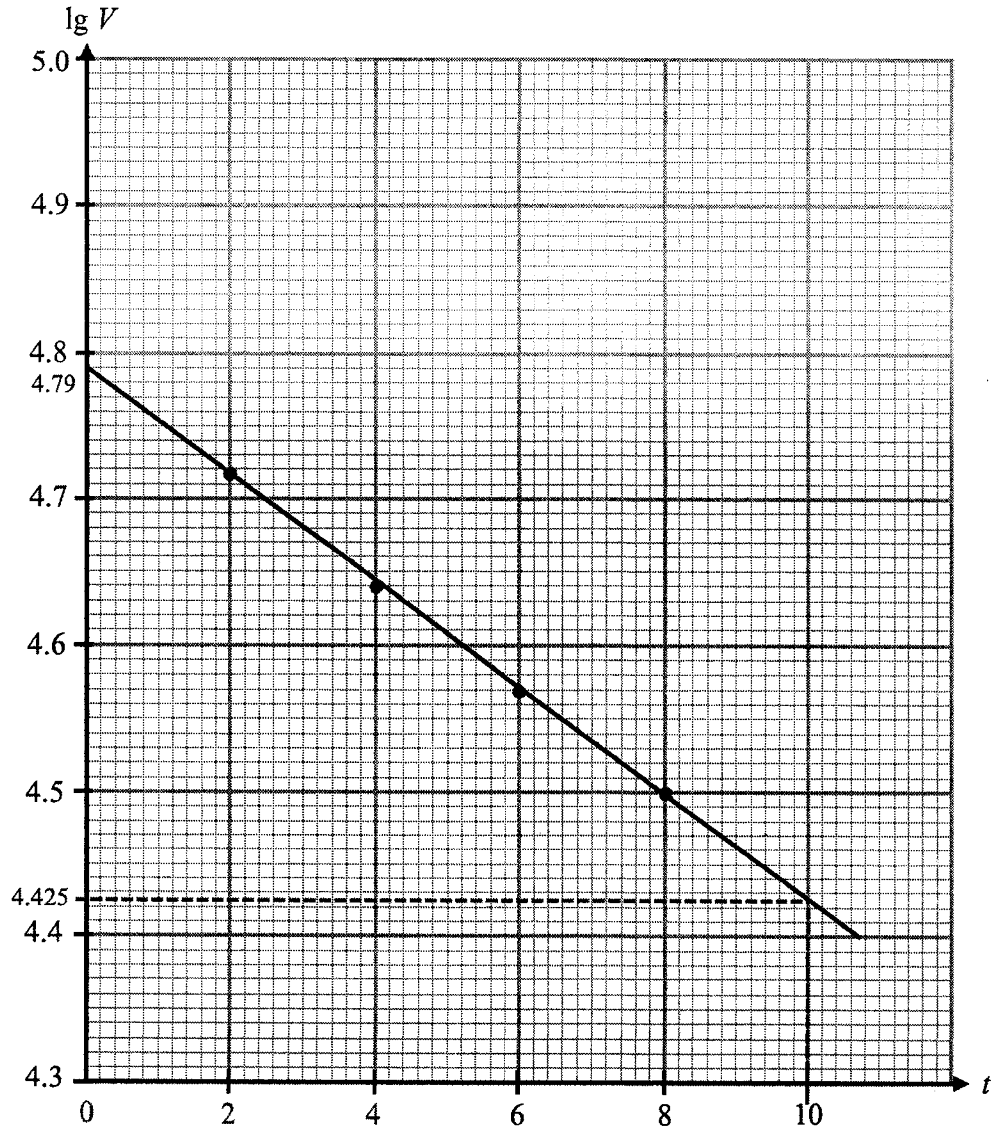
$\lg V = (\lg b)t + \lg a$ A1

(b) Label axes B1 (correct axes with at least 1 point)

All correct points P2 (deduct 1 mark if any point is wrong)

Best fit line C1

(b)



(c) From the $\lg V$ versus t graph,

$\lg a = 4.79$

$\therefore a = 10^{4.79} = 61659 \approx 61700$ (3sf) B1 (Accept $4.78 \le \lg a \le 4.8$)

$\lg b = \dfrac{4.5-4.72}{8-2} = -\dfrac{11}{300} = -0.036666$ M1 (Accept $-0.03 \le \lg b \le -0.04$)

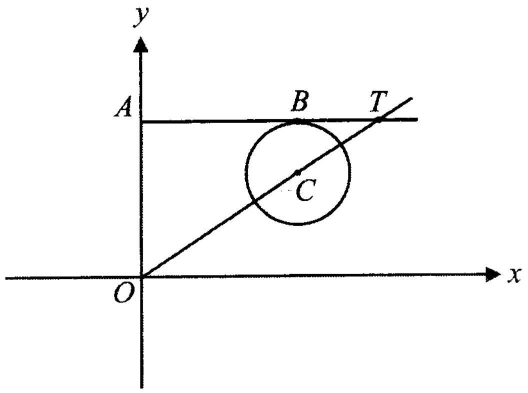
$\therefore b = 10^{-\frac{11}{300}} = 0.91903 \approx 0.919$ (3sf) A1

(d) From the $\lg V$ versus t graph, when $t = 10$,

$\lg V = 4.425$ M1 (Accept $4.415 \le \lg V \le 4.435$)

$\therefore V = 10^{4.425} = 26607 \approx 26600$ (3sf) A1

9



(a) The equation of circle with centre C is given by $x^2 + y^2 - 6x - 4y + 12 = 0$.
Find the radius of the circle and the coordinates of its centre C. [3]

(b) AB is a horizontal tangent to the circle at point B. Given that the line OC produced meets the line AB produced at point T, find the coordinates of T. [3]

(c) Show that triangle AOT and triangle BCT are similar. [3]

(d) Find the ratio $OC : CT$. [1]

(e) Find the angle ATO in degrees. [1]

(a) **Method 1**

$x^2 + y^2 - 6x - 4y + 12 = 0$

$(x-3)^2 - 9 + (y-2)^2 - 4 + 12 = 0$ M1 (complete the square, o.e.)

$(x-3)^2 + (y-2)^2 = 1$

Centre, $P = (3, 2)$ A1

Radius $= 1$ unit A1

Method 2

$x^2 + y^2 - 6x - 4y + 12 = 0$

$g = -3, f = -2, c = 12$

Centre, $P = (3, 2)$ B1, B1

Radius $= 1$ unit A1

(b) *Equation of line AT*: $y = 3$

Equation of line OT: $y = \frac{2}{3}x$ M1

Solving simultaneously: $\frac{2}{3}x = 3$ M1

$\Rightarrow x = 4.5$

$\therefore T = (4.5, 3)$ A1

(c) $\angle BTC = \angle ATO$ (common angle) M1

$\angle TAO = 90^\circ$ (given)

$\angle TBC = 90^\circ$ (tangent $\perp$ radius) M1

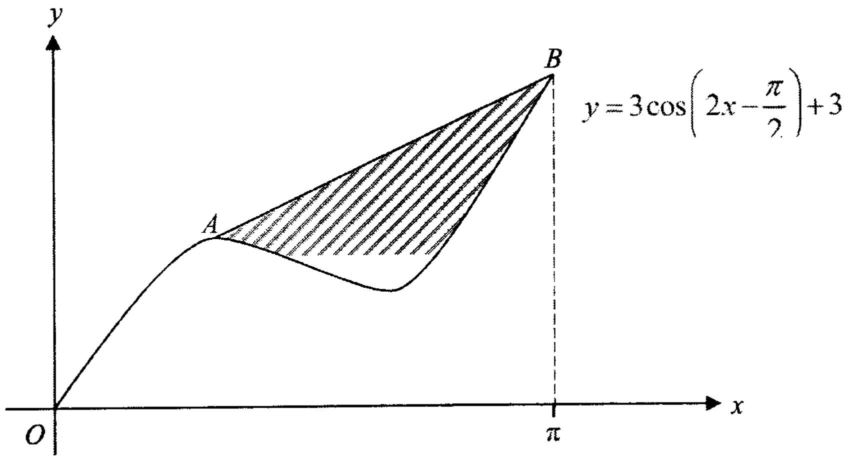
Triangle *AOT* and triangle *BCT* are similar. (AA similarity) A1

(d) $OC : CT = 2 : 1$ B1

(e) $\tan \angle ATO = \frac{3}{4.5}$

$\angle ATO = \tan^{-1}\left(\frac{3}{4.5}\right) = 33.69 \approx 33.7^\circ$ (1dp) B1

10



The diagram shows the curve $y=3\cos\left(2x-\frac{\pi}{2}\right)+3x$ for $0\leq x\leq \pi$ radians.

The point A is the maximum point of the curve and AB is a straight line.

(a) Show that the coordinates of A are $\left(\frac{\pi}{3},\frac{3\sqrt{3}}{2}+\pi\right)$ and coordinates of B are $(\pi, 3\pi)$. [5]

(b) Hence find the area of the shaded region, leaving your answers in terms of π. [7]

(a) $y=3\cos\left(2x-\frac{\pi}{2}\right)+3x$

$\frac{dy}{dx}=-6\sin\left(2x-\frac{\pi}{2}\right)+3$ — M1 (correct dy/dx)

For turning point, $-6\sin\left(2x-\frac{\pi}{2}\right)+3=0$ — M1 (equate dy/dx to zero)

$\sin\left(2x-\frac{\pi}{2}\right)=\frac{1}{2}$

$\alpha=\frac{\pi}{6}$ — M1 (attempt to find reference angle)

$2x-\frac{\pi}{2}=\frac{\pi}{6}$

$x=\frac{\pi}{3}$

When $x=\frac{\pi}{3}$, $y=\frac{3\sqrt{3}}{2}+\pi$

$\Rightarrow A=\left(\frac{\pi}{3},\frac{3\sqrt{3}}{2}+\pi\right)$ — AG1

When $x=\pi$, $y=3\pi$

$\Rightarrow B=(\pi, 3\pi)$ — AG1

(b)

$$\text{Area under the curve} = \int_{\frac{\pi}{3}}^{\pi}\left[3\cos\left(2x-\frac{\pi}{2}\right)+3x\right]dx$$

$$=\left[\frac{3}{2}\sin\left(2x-\frac{\pi}{2}\right)+\frac{3}{2}x^2\right]_{\frac{\pi}{3}}^{\pi}$$ M1 (find antiderivative)

$$=\left[\frac{3}{2}\sin\left(2\pi-\frac{\pi}{2}\right)+\frac{3}{2}\pi^2\right]-\left[\frac{3}{2}\sin\left(\frac{2\pi}{3}-\frac{\pi}{2}\right)+\frac{3}{2}\left(\frac{\pi}{3}\right)^2\right]$$ M1 (substitution of limits)

$$=\left(\frac{4}{3}\pi^2-\frac{9}{4}\right)\text{units}^2$$ A1

$$\text{Area of trapezium} = \frac{1}{2}\left(\frac{3\sqrt{3}}{2}+\pi+3\pi\right)\left(\frac{2\pi}{3}\right)$$ M1 (find area of trapezium)

$$=\left(\frac{\sqrt{3}}{2}\pi+\frac{4}{3}\pi^2\right)\text{unit}^2$$ A1

$$\text{Area of shaded region} = \left(\frac{\sqrt{3}}{2}\pi+\frac{4}{3}\pi^2\right)-\left(\frac{4}{3}\pi^2-\frac{9}{4}\right)$$ M1 (attempt to find shaded area)

$$=\left(\frac{\sqrt{3}}{2}\pi+\frac{9}{4}\right)\text{units}^2$$ A1