NORTH VISTA SECONDARY SCHOOL
Preliminary Examination 2022
Secondary 4 Express / 5 Normal Academic

/ 90

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS

4049/01

Paper 1

24 August 2022

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your centre number, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use	
Category	**Question No.**
Accuracy	
Brackets	
Fractions	
Units	
Others	
Marks Deducted	

This document consists of 19 printed pages and 1 blank page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

Answer **all** the questions.

1 **(a)** Factorise $64x^3 - 125$. [1]

(b) Hence, show that $64x^3 - 125 = 0$ has only one solution and state this value. [3]

2 The minimum value of the curve $y = kx^2 - 9x + 3$ is positive.

(a) A straight line, L_1 of the form $y = mx - 5$, where m is a constant, meets the curve for all real values of x.

State the conditions that must apply to m and k. Justify your answer with reasons. [4]

(b) Give an example of the values of m and k which satisfy the condition found in **(a)**. [1]

(c) Another straight line L_2 meets the curve at one point only.
Given that L_2 is not a tangent to the curve, what can be deduced about L_2? [1]

3 A polynomial, P, is $x^3+kx^2-10x+6$.

(a) Find the value of k given that P leaves a remainder $k-3$ when divided by $(x+k)$. [2]

(b) Given that $x^3+kx^2-10x+6$ and x^2-x-2 have a common factor, find the possible values of k. [3]

[Turn over

4 The curve $\frac{x^2}{4} - \frac{y}{a} = x + b$, where a and b are constants, intersects the y-axis at A and the x-axis at B and C. The coordinates of B are $(-2, 0)$.

Given that the gradient of AB is -3, find the value of a and of b. [4]

5 The mass, M milligrams, of a radioactive substance, present t years after first being observed, is given by $M = 300(0.4 + e^{kt})$.

(a) Find the initial amount of the substance. [1]

(b) Given that it takes 3 years for the mass to decay by 250 mg.
Show that the value of $k = -0.597$, correct to 3 significant figures. [3]

(c) Sketch the graph of M against t. [2]

[Turn over

6 The variables x and y are such that when values of $y\sqrt{x}$ are plotted against x, a straight line is obtained. It is given that the straight line passes through $(1.5,\ 8)$ and $(6.5,\ 18)$.

(a) Find the equation of the line in the form $Y = mX + c$, where X and Y are functions of x and/or y, and m and c are constants. [3]

(b) Find the value of x when $y = 4.5\sqrt{x}$. [2]

7 **(a)** Express $12x^2 - 6x + 5$ in the form $p(x-q)^2 + r$, where p, q and r are constants to be found. [3]

(b) Hence, explain why the graph $y = 12x^2 - 6x + 5$ has no x-intercepts. [2]

(c) Write down the coordinates of the turning point of the graph $y = (12x^2 - 6x + 5)^{-1}$, stating whether it is a maximum or minimum point. [2]

[Turn over

8 Angles A and B are such that $\sin(A-B)=4\cos(A+B)$.

(a) Show that $\tan B=\dfrac{4-\tan A}{4\tan A-1}$. [3]

(b) Given that angle $A=\dfrac{\pi}{6}$, **without using a calculator**, find $\tan B$ in the form $\dfrac{h\sqrt{3}+k}{13}$, where h and k are integers. [4]

9 The function f' is defined, for all values of x, by $f'(x)=(x-2)^2(1-x)$.

(a) Find the set of values of x for which f' is a decreasing function and represent this set on a number line. [5]

(b) A point P moves along the curve $y=f(x)$ in such a way that the y-coordinate of P increases at a constant rate of 0.2 units per second.

Find the rate of change of x when $x=6$. [2]

[Turn over

10 It is given that $y = 3\cos^2 x - \sin^2 x$.

(a) Express $3\cos^2 x - \sin^2 x$ in the form $a + b\cos 2x$, where a and b are constants to be found. [3]

(b) Hence, evaluate $\int_{\frac{\pi}{2}}^{\pi} (3\cos^2 x - \sin^2 x)\, dx$. [3]

10 (c) State the amplitude and period of the graph $y = 3\cos^2 x - \sin^2 x$. [2]

(d) Sketch the graph of $y = 3\cos^2 x - \sin^2 x$ for $0 \le x \le \pi$. [2]

[Turn over

The equation of a curve is $y = \dfrac{2\ln x}{x}$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{2 - 2\ln x}{x^2}$. [2]

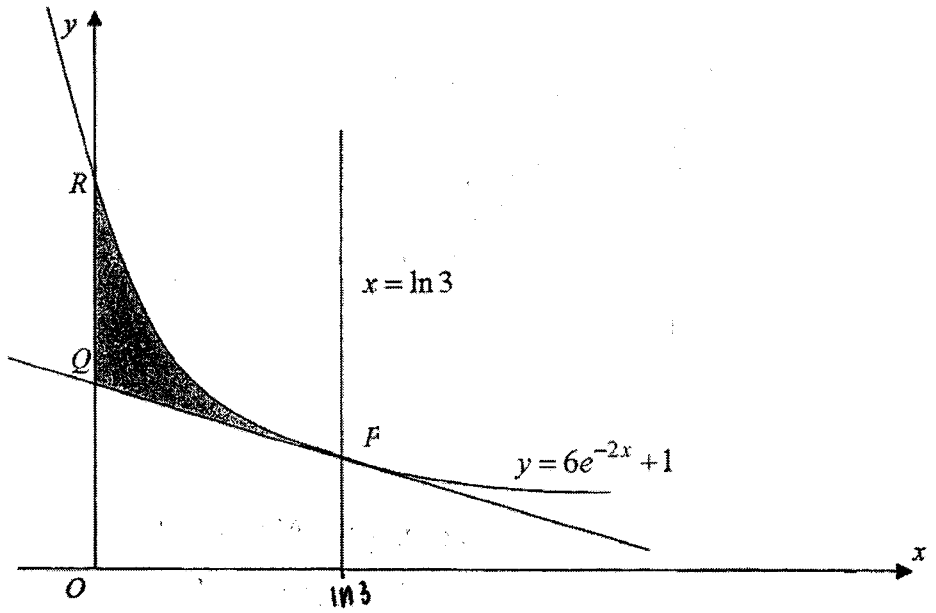
(b) Hence, prove that $\displaystyle\int_1^2 \frac{\ln x}{x^2}\, dx = \frac{1 - \ln 2}{2}$. [4]

11 (c) Find the x-coordinate of the stationary point of the curve and determine its nature.

[4]

[Turn over

12



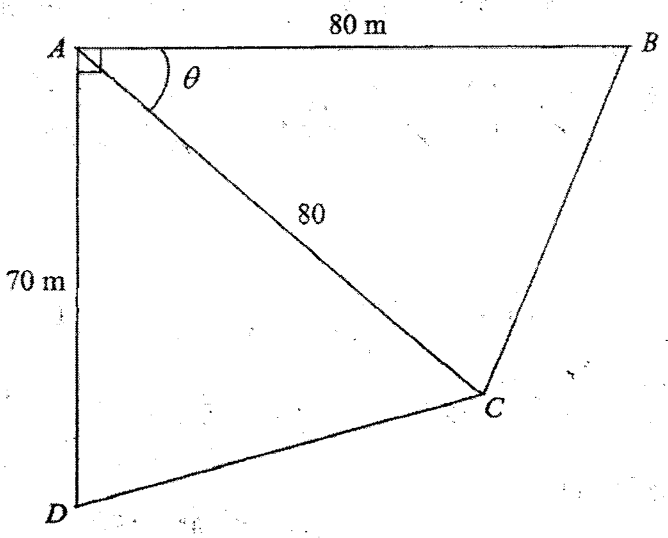
The diagram shows part of the curve $y = 6e^{-2x} + 1$ intersecting the line $x = \ln 3$ at point P. The tangent to the curve at the point P intersects the y-axis at Q.

(a) Show that the coordinates of Q are $\left(0,\ \frac{4}{3}\ln 3 + \frac{5}{3}\right)$. [5]

12 (b) Find the area of the shaded region.

[Turn over

13 The diagram shows two triangular plots of land ABC and ACD.
It is given that $AB = AC = 80$ m and $AD = 70$ m.
Angle BAD is a right angle and angle $BAC = \theta$ radians.



(a) Given that the area of the two plots of land, $A = 3600$ m^2, show that

$$9 = 8\sin\theta + 7\cos\theta.$$

[3]

13 **(b)** Express $8\sin\theta + 7\cos\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [4]

(c) Hence, find the value(s) of θ. [2]

END OF PAPER

NORTH VISTA SECONDARY SCHOOL
Preliminary Examination 2022
Secondary 4 Express / 5 Normal Academic

90

CANDIDATE NAME

CLASS

INDEX NO.

ADDITIONAL MATHEMATICS

4049/02

Paper 2

26 August 2022

Candidates answer on the Question Booklet.

Additional Materials:

2 hour 15 minutes

READ THESE INSTRUCTIONS FIRST

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Write in dark blue or black pen.
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where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

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$$\Delta = \frac{1}{2} bc \sin A$$

1 **(a)** Given that $\int_{p}^{14} \frac{3}{2y-5}\, dy = \ln 27$, find the exact value of p. [4]

(b) Given that $\int_{3}^{4} f(x)\, dx = -2$ and $\int_{4}^{8} f(x)\, dx = 10$, find the value of $\int_{3}^{8} [f(x)+5]\, dx$. [2]

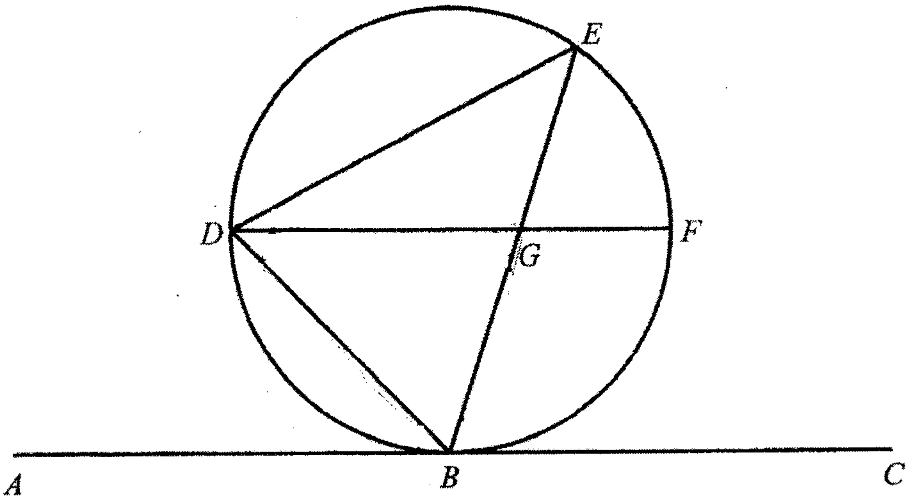
2 A curve has the equation $y = e^x \cos x$, where $0 \le x \le \frac{\pi}{2}$.

Explain why the range of values of x for which y is a decreasing function of x is $\frac{\pi}{4} < x < \frac{\pi}{2}$. [6]

3 (i) Prove that $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$. [4]

(ii) Hence, find the exact values of $\tan\theta$, given that $\cos 2\theta = \frac{1}{3}$. [3]

4



In the diagram, ABC is a tangent to the circle at the point B. The points D, E and F lie on the circle. The chords DF and EB intersect at G and angle EDB = angle DGB.

(i) Prove that triangle EDB is similar to triangle DGB. [2]

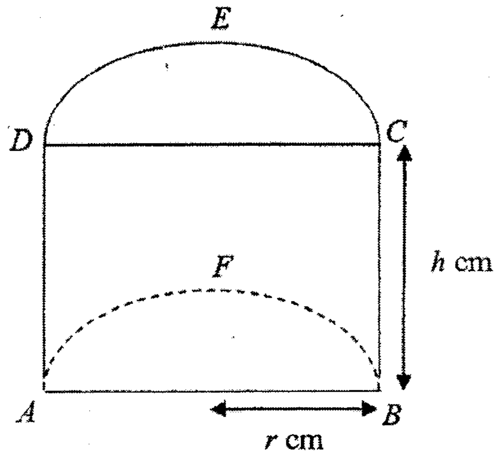
(ii) Prove that $BD^2 = GB \times BE$. [2]

(iii) Explain why DF is parallel to AC. [3]

5 (a) Evaluate $\log_4 5 \times \log_5 6 \times \log_6 7 \times \log_7 8$. [2]

(b) Solve $\log_3(71x+10) - \log_3(x-2) = \dfrac{1}{\log_x 3} + 2$. [5]

6



The figure shows a solid whose two cross-sections *DEC* and *AFB* are each a semicircle of a circle with radius *r* cm. The height of the solid is *h* cm and the volume of the solid is $20\pi\ \text{cm}^3$.

(i) Express *h* in terms of *r*. [2]

(ii) Hence, show that the total surface area, $A\ \text{cm}^2$, of the solid is given by $A = \pi r^2 + \dfrac{80 + 40\pi}{r}$. [2]

6 **(iii)** Find the value of r, correct to 3 significant figures, for which A has a stationary value. [2]

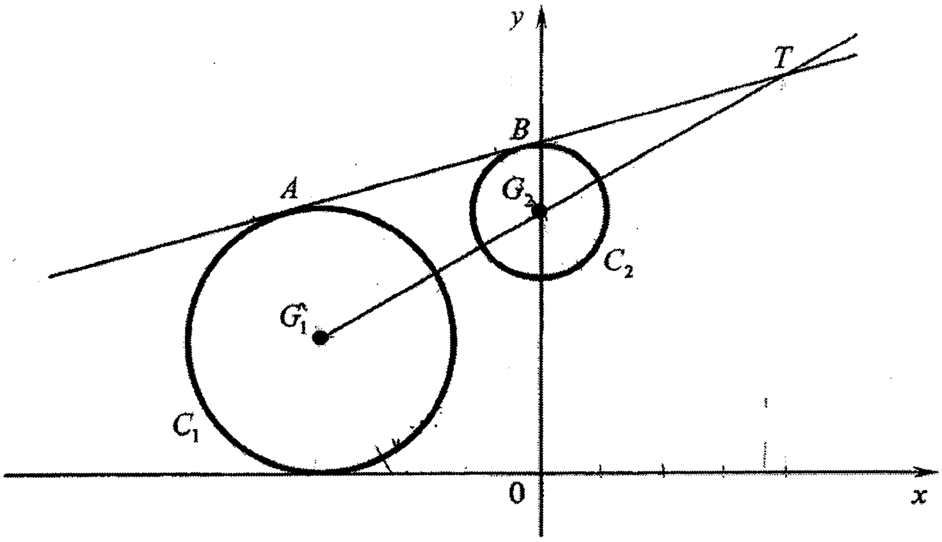
(iv) Determine whether this value of A is a minimum or maximum. [2]

[Turn over

7 In the diagram, ABT is a common tangent of the circles, C_1 and C_2, whose centres are G_1 and G_2 respectively. The line joining G_1 and G_2 meets ABT at T.
The equations of the circles are

$$C_1 : x^2 + y^2 + 6x - 4y + 9 = 0$$
$$C_2 : x^2 + y^2 - 8y + 15 = 0$$

Find the coordinates of T. [9]



8 (a) Expand $\left(3-\frac{x}{30}\right)^6$ in ascending powers of x up to the fourth term.

Hence, find the coefficient of x^3, in the expansion of $\left(3-\frac{x}{30}\right)^6(1-x)$, leaving your answer as a fraction. [4]

(b) In the expansion of $(1+x)^n$, the coefficient of x^2 is the mean of the coefficients of x and x^3. Find the possible value(s) of n. [5]

9 (i) Given that $u = 3^x$, express $9^x = 3^{x+2} - 18$ as an equation in u. [3]

(ii) Hence find the values of x for which $9^x = 3^{x+2} - 18$, giving your answers, where appropriate to 1 decimal place. [4]

(iii) Explain why the equation $9^x = 3^{x+2} - k$ has no solution if $k > \frac{81}{4}$. [2]

[Turn over

10 (i) Express $\dfrac{52x^2 - 20x - 6}{(x-2)(4x+1)^2}$ as the sum of 3 partial fractions. [5]

10 (ii) Find $\int \frac{52x^2-20x-6}{(x-2)(4x+1)^2}\,dx$ and hence evaluate $\int_3^4 \frac{52x^2-20x-6}{(x-2)(4x+1)^2}\,dx$. [5]

[Turn over

11 A particle moves in a straight line so that at time t seconds after leaving a point A, its displacement s metres from a fixed point O, is given by $s = 3e^{\sin\left(\frac{t}{2}\right)} - 4$.

(a) Find the displacement of the particle from O at the initial moment. [1]

(b) What is the initial velocity of the particle? [3]

(c) Find the instances when the particle returns to O during the first 6 seconds. [3]

11 (d) Do you agree that the total distance travelled by the particle during the first 6 seconds is 10.86 m? [5]

End of Paper

North Vista Amath SA2 2022
Paper 1

1a	$(4x-5)(16x^2+20x+25)$
1b	$x=\frac{5}{4}$ and $b^2-4ac<0$
2a	$k>6\frac{3}{4}$ $(-9-m)^2\geq 32k$
2b	$k=7, m=10$ (accept any answers that satisfy the conditions)
2c	It is a vertical line that cuts the curve at only one point.
3a	$k=-1$
3b	$k=-15$ $k=\frac{3}{2}$
4	$a=2$ $b=3$
5a	420 milligrams
5b	$k=-0.597$
6a	$y\sqrt{x}=2x+5$
6b	$x=2$
7a	$12\left(x-\frac{1}{4}\right)^2+\frac{17}{4}$
7b	$\left(x-\frac{1}{4}\right)^2\geq 0$ $12\left(x-\frac{1}{4}\right)^2+\frac{17}{4}\geq\frac{17}{4}$ $y\geq\frac{17}{4}$ Since the minimum value of y is $\frac{17}{4}$, the graph will not have any x-intercepts
7c	Maximum point at $\left(\frac{1}{4},\frac{4}{17}\right)$
8a	Show
8b	$\frac{15\sqrt{3}+8}{13}$
9a	$x<\frac{4}{3}$ or $x>2$
9b	$-\frac{1}{400}$ units/s
10a	$1+2\cos(2x)$
10b	$\frac{\pi}{2}$
10c	Amplitude = 2 and period :
11a	Show
11b	Prove
11c	$x=\pi$, Maximum point
12a	Show
12b	1.13 units^2
13a	Show
13b	$\sqrt{113}\cos(\theta-0.852)$
13c	1.41

North Vista Amath SA2 2022

Paper 2

1a	$\frac{34}{9}$
1b	33
2	$e^x(\cos x - \sin x) < 0$ Since $e^x > 0$, $\cos x - \sin x < 0$ $1 - \tan x < 0$ $\tan x > 1$ Sketch $y = \tan x$ graph to show: $\frac{\pi}{4} < x < \frac{\pi}{2}$ **This is not a very well set question and is slightly outside of syllabus*
3i	Prove
3ii	$\tan\theta = \pm\frac{1}{\sqrt{2}}$
4i	Prove
4ii	Prove
4iii	Prove
5a	$\frac{3}{2}$
5b	10 *or* $-\frac{1}{9}(Rej)$
6i	$h = \frac{40}{r^2}$
6ii	Show
6iii	$r = 3.20$ cm
6iv	Minimum
7	$T(6,6)$ *Using Similar Triangles
8a	$-\frac{137}{100}$
8b	$n = 7$ *or* $2\ (rej)$ *or* $8\ (rej)$
9i	$u^2 - 9u + 18 = 0$
9ii	$x = 1.6$ or $x = 1$
9iii	Explain using discriminant
10i	$\frac{2}{x-2} + \frac{5}{4x+1} - \frac{1}{(4x+1)^2}$
10ii	1.72
11a	$-1m$
11b	$\frac{3}{2}$ m/s
11c	0.594s and 5.70s
11d	Disagree. It is 9.86m