NAVAL BASE SECONDARY SCHOOL
PRELIMINARY EXAMINATION, 2022

O

Name ______________________________( ) **Class** ______________

ADDITIONAL MATHEMATICS **4049/01**

Paper 1 **29 August 2022**

Candidates answer on the Question Paper **2 hours 15 minutes**
No Additional Materials are required

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

Item	***For Examiner's Use***
Presentation	
Accuracy	
Units	
Total	
Parent's Signature	

This paper consists of **22** printed pages and **2** blank pages. **[Turn over**

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

[Turn over

Answer **all** the questions.

1 The line $x+2y=3$ intersects the curve $x^2+12x+5y^2=18$ at two points.
Find the coordinates of the two points. [4]

[Turn over

2 Express $2x^2 - 3x - 11$ in the form $a(x - h)^2 + k$, where a, h and k are constants.

Hence, state the coordinates of the turning point of the curve $y = 2x^2 - 3x - 11$. [4]

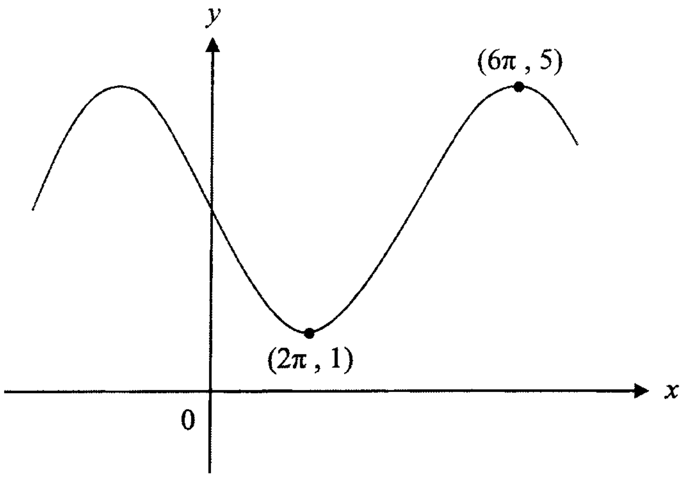
3 (a) Express $\dfrac{10x}{3-5x}$ in the form $a+\dfrac{b}{3-5x}$, where a and b are integers. [2]

(b) Hence, integrate $\dfrac{10x}{3-5x}$ with respect to x. [2]

[Turn over

4 The diagram shows part of the graph of $y = a\sin\frac{x}{b} + c$.

The graph has a maximum point at $(6\pi, 5)$ and a minimum point at $(2\pi, 1)$.



Determine the values of the constants a, b and c. [5]

[Turn over

5 **(a)** Air is being pumped into a spherical balloon at a constant rate of 12 cm^3 per second. Find the rate at which the radius of the balloon is increasing when the radius is 4 cm.

[The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$] [3]

(b) When the radius of the balloon stretches beyond 5 cm, it was found that while air is being pumped in, some air also starts to leak out of the balloon.

Given that the radius of the balloon is increasing at a rate of 0.015 cm per second when the radius is 7 cm, find the rate at which air is leaking out of the balloon. [2]

[Turn over

6 Express $\dfrac{14x^2-15x-29}{(x-1)^2(2x+3)}$ in partial fractions. [6]

[Turn over

7 A polynomial $f(x)$ is given by $4x^3 - 6x^2 + kx + 3$, where k is a constant.

(a) Find the value of k given that $f(x)$ leaves a reminder of 14 when divided by $2x - 1$. [2]

(b) In the case where $k = 2$, the quadratic expression $2x^2 + px + 3$ is a factor of $f(x)$. Find the value of the constant p. [4]

[Turn over

8 **(a)** Solve $3x = \sqrt{1-x} - 1$. [3]

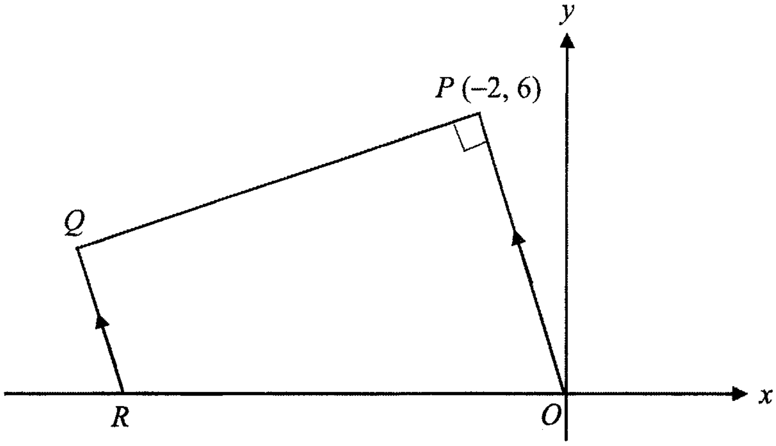
(b) A triangle has a base of length $(1+\sqrt{3})$ cm and an area of $(14+8\sqrt{3})\text{ cm}^2$.

Find, **without using a calculator**, the perpendicular height of the triangle, in cm, in the form $(a+b\sqrt{3})$, where a and b are integers. [4]

[Turn over

9 In the diagram, $OPQR$ is a trapezium with OP parallel to RQ and OP perpendicular to PQ.

The point P has coordinates $(-2, 6)$ and R lies on the x-axis.



(a) Find the equation of the perpendicular bisector of OP. [3]

[Turn over

(b) Given that point R lies on the perpendicular bisector of OP, show that the coordinates of Q is $(-11, 3)$. [3]

(c) Find the area of trapezium $OPQR$. [2]

[Turn over

10 The equation of a curve is $y = 2x^3 + 6x^2 - 18x + 5$.

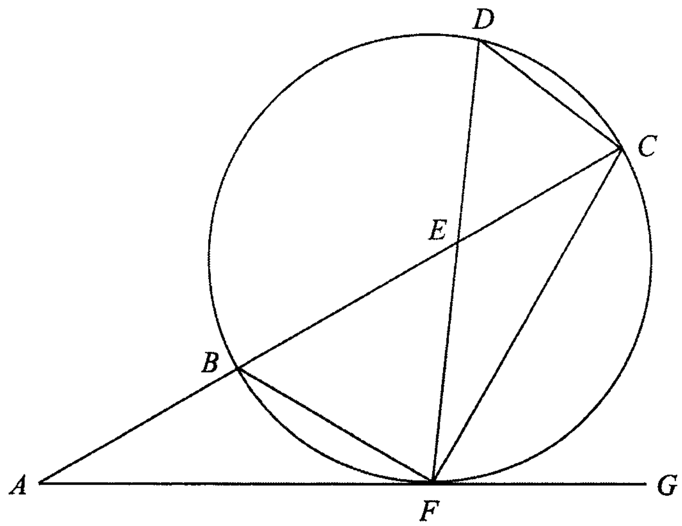
(a) Find the coordinates of the stationary points and determine the nature of these stationary points. [5]

[Turn over

(b) Find the minimum **gradient** of the curve and the value of x when the minimum gradient occurs. [3]

[Turn over

11 In the diagram, BC is a diameter of the circle. ABC is a straight line and AG is a tangent to the circle at point F. The line DF intersects BC at point E.



(a) Prove that triangles ABF and AFC are similar. [3]

(b) Hence, show that $AF^2 = AB \times AC$. [1]

[Turn over

(c) **(i)** Name a triangle similar to triangle *DEC*. [1]

(ii) Given that $3EC = 2EB$, show that $k(BC)^2 = EF \times ED$, where k is a constant to be determined. [3]

12 **(a)** Show that the solution of the equation $3^{2x-1} = 2^{3-x}$ is $x = \frac{\lg 24}{\lg 18}$. [3]

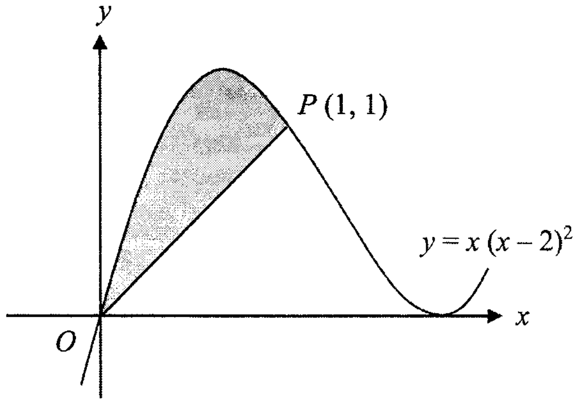
(b) Given that $\log_6 4y = \log_6 3x^2 - 1$, express y in terms of x. [2]

[Turn over

(c) Solve the equation $\log_2 x^2 = 8\log_x 2$. [3]

[Turn over

13 The diagram shows part of the curve of $y = x(x-2)^2$, which passes through the point $P(1,\ 1)$.



(a) Show that the equation of *OP* is $y = x$. [2]

(b) Show that *OP* is the normal to the curve at *P*. [2]

[Turn over

(c) Find the area of the shaded region bounded by the curve and the line *OP*. [4]

[Turn over

14 (a) Prove that $\left(\operatorname{cosec}\theta + \cot\theta\right)^2 = \dfrac{1+\cos\theta}{1-\cos\theta}$. [4]

[Turn over

(b) Hence, solve the equation $(\text{cosec}2\theta+\cot 2\theta)^2(1-\cos 2\theta)=2\cos\theta$ for $0°\le\theta\le 360°$. [5]

----------END OF PAPER----------

NAVAL BASE SECONDARY SCHOOL
PRELIMINARY EXAMINATION, 2022

O

Name ______________________________( ) **Class** ______________

ADDITIONAL MATHEMATICS **4049/02**

Paper 2 **30 August 2022**

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[Turn over

Answer **all** the questions.

1 Solve the equation $3^{2y+1} - 3^{y+2} + 3 = 3^y$. [4]

[Turn over

2 Show that $x+2$ is a solution of the equation $2x^3+5x^2+x-2=0$ and hence solve the equation completely. [5]

3 (a) Differentiate $4xe^{3x}$ with respect to x. [3]

(b) Hence, evaluate $\int_0^3 6xe^{3x}\,dx$, giving your answer in exact form. [4]

4 In the expansion of $\left(x^2 - \frac{m}{2x}\right)^{12}$, where m is a positive constant, the independent term of x is 126 720.

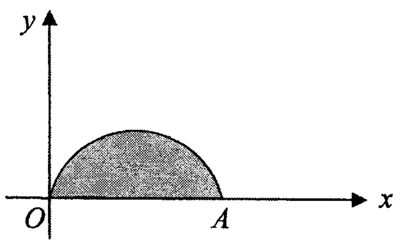
(a) Show that $m = 4$. [4]

(b) Hence, find the coefficient of x^9 in the expansion of $\left(x^2 - \frac{m}{2x}\right)^{12}\left(8x^9 + 5\right)$. [3]

[Turn over

5 **(a)** Show that $\frac{d}{dx}(x\sin x+\cos x)=x\cos x$. [3]

(b) The diagram shows part of the curve $y=x\cos x$.



(i) Find the coordinates of the point A. [3]

(ii) Calculate the area of the shaded region leaving your answer in exact form. [2]

6 (a) (i) Find the range of values of k for which the expression $2x^2 - 7x + k$ is positive for all real values of x. [3]

(ii) Hence, find the range of values of x for which $\frac{(3x-2)(x-4)}{2x^2-7x+8} < 0$. [3]

[Turn over

(b) (i) Find the range of values of p for which the equation $px^2 + x + p(x+1) = 0$ has real and distinct roots. [3]

(ii) State the value(s) of p for which the curve $y = px^2 + x + p(x+1)$ is tangent to the x-axis. [1]

7 The table below shows experimental values of two variables x and y, which are connected by an equation of the form $\frac{a}{y} = \frac{b}{x} + 1$, where a and b are constants.

x	1.25	2	2.5	5
y	1	1.51	1.82	3.08

(a) Plot $\frac{1}{y}$ against $\frac{1}{x}$ and draw a straight line graph. [3]

[Grid provided on page 11]

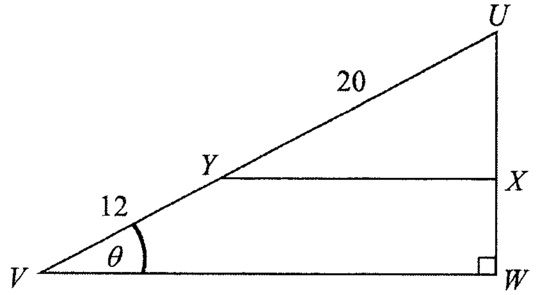
(b) Use your graph to estimate the value of a and of b. [3]

It is found that the values of $\frac{1}{x}$ are rather small. Another straight line graph is proposed by using x as the horizontal axis.

(c) State the variable in the vertical axis for this proposed new line. [2]

(d) Explain how the values of a and b can be found using this proposed new line. [2]

8 The diagram shows a pair of similar triangles YUX and VUW. Point Y lies on the straight line VU such that $VY = 12$ cm and $YU = 20$ cm. Angle $UVW = \theta$ where $0° \leq \theta \leq 90°$ and UW is perpendicular to VW.



(a) Show that L cm, the perimeter of the trapezium $VYXW$, is given by $L = 12\sin\theta + 52\cos\theta + 12$. [3]

(b) Express L in the form $12 + R\sin(\theta + \alpha)$ where $R > 0$ and $0° < \alpha < 90°$. [3]

[Turn over

(c) State the maximum value of L and the corresponding value of θ. [3]

(d) Find the value of θ when $L = 30$. [3]

9 The velocity, v ms^{-1}, of a particle moving in a straight line, t seconds after passing a fixed point O, is given by $v = \frac{3}{t+1} - \frac{t+1}{3}$. It is given that the particle comes instantaneously at rest at point A.

(a) Find the time taken to reach the point A. [3]

(b) Calculate the distance OA. [5]

(c) Show that the particle is again at O at some instant during the fifth second after first passing through O. [3]

(d) Find the acceleration of the particle when $t = 5$. [2]

10 The equation of a circle, S_1, with centre C is given by $x^2 + y^2 - 8x - 16y + 55 = 0$.

(a) Find the coordinates of C and the radius of the circle S_1. [3]

(b) Find the equation of the tangent to S_1 at $A(7, 12)$. [3]

The tangent cuts the x-axis at point B.

(c) State the coordinates of B. [1]

[Turn over

A second circle, S_2, with centre D, passes through C, A and B.

(d) Show that D has coordinates $\left(\frac{27}{2}, 4\right)$ and hence find the equation of S_2. [5]

(e) Determine whether D lies inside or outside S_1. [2]

[Turn over

Secondary 4E Prelims 2022 Add Math Paper 1 (Solution)

Q.	Solution	Remarks
1	$x+2y=3 \rightarrow x=3-2y$ — (1) $x^2+12x+5y^2=18$ — (2) Subst (1) into (2), $(3-2y)^2+12(3-2y)+5y^2=18$ M1 $9-12y+4y^2+36-24y+5y^2-18=0$ $9y^2-36y+27=0$ M1 $y^2-4y+3=0$ $(y-3)(y-1)=0$ $y=3$ or $y=1$ M1 $x=3-2(3)=-3$ $x=3-2(1)=1$ Coordinates are $(-3, 3)$ and $(1, 1)$ A1	
2	$2x^2-3x-11$ $=2\left(x^2-\frac{3}{2}x-\frac{11}{2}\right)$ or $2\left(x^2-\frac{3}{2}x\right)-11$ M1 $=2\left[\left(x-\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^2-\frac{11}{2}\right]$ or $2\left[\left(x-\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^2\right]-11$ $=2\left[\left(x-\frac{3}{4}\right)^2-\frac{97}{16}\right]$ or $2\left[\left(x-\frac{3}{4}\right)^2-\frac{9}{16}\right]-11$ M1 $=2\left(x-\frac{3}{4}\right)^2-\frac{97}{8}$ or $2(x-0.75)^2-12.125$ A1 Turning point $=\left(\frac{3}{4}, -\frac{97}{8}\right)$ or $(0.75, -12.125)$ B1	(allow ecf)
3(a)	$-5x+3\overline{)10x+0}$ with quotient -2 M1 $\underline{10x-6}$ 6 $\frac{10x}{3-5x}=-2+\frac{6}{3-5x}$ (using long division) A1 OR $\frac{10x}{3-5x}=\frac{6-(6-10x)}{3-5x}$ M1 $=\frac{6}{3-5x}-\frac{2(3-5x)}{3-5x}=-2+\frac{6}{3-5x}$ A1	

	OR $\frac{10x}{3-5x} = a + \frac{b}{3-5x} = \frac{a(3-5x)+b}{3-5x}$ $\frac{10x}{3-5x} = \frac{-5ax+3a+b}{3-5x}$ Comparing coeff of x, M1 $10 = -5a \rightarrow a = -2$ $3a+b=0 \rightarrow b=6$ $\frac{10x}{3-5x} = -2 + \frac{6}{3-5x}$ A1 $\int \frac{10x}{3-5x}\,dx = \int\left(-2+\frac{6}{3-5x}\right)dx$	
3(b)	$= -2x - \frac{6\ln(3-5x)}{5} + c$ B1 for each term (minus 1m if $+c$ is missing)	(allow ecf)
4	Max: $c + A = 5$ -- (1) , where A is the amplitude Min: $c - A = 1$ -- (2) (1) + (2), $2c = 6 \rightarrow c = 3$ B1 $A = 5-3 = 2$ M1 Since it is a negative sine graph, $a = -2$ A1 Period $= 8\pi$ M1 $\frac{2\pi}{\left(\frac{1}{b}\right)} = 8\pi$ $b = \frac{8\pi}{2\pi} = 4$ A1	
5(a)	$V = \frac{4}{3}\pi r^3$ $\frac{dV}{dr} = 4\pi r^2$ M1 $\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$ $= \frac{1}{4\pi(4)^2} \times 12$ M1 $= 0.0597$ cm per second A1	

5(b)	$\frac{dV}{dt}=\frac{dV}{dr}\times\frac{dr}{dt}$ $=4\pi(7)^2\times0.015$ $=9.236\text{ cm}^3$ per second Hence, air is leaking at a rate of $12-9.236=2.76$ cm^3 per second. OR $12-x=4\pi(7)^2\times0.015$ $x=12-9.236=2.76$ cm^3 per second.	 M1 A1 M1 A1	
6	$\frac{14x^2-15x-29}{(x-1)^2(2x+3)}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(2x+3)}$ $14x^2-15x-29=A(2x+3)(x-1)+B(2x+3)+C(x-1)^2$ Let $x=1$, $14-15-29=B(2+3)\rightarrow B=-6$ Let $x=-1.5$, $14(-1.5)^2-15(-1.5)-29=C(-1.5-1)^2$ $25=6.25C\quad\rightarrow C=4$ Let $x=0$, $-29=A(3)(-1)+B(3)+C(-1)^2$ $-29-(-6)(3)-(4)(-1)^2=-3A\rightarrow A=5$ $\frac{14x^2-15x-29}{(x-1)^2(2x+3)}=\frac{5}{(x-1)}-\frac{6}{(x-1)^2}+\frac{4}{(2x+3)}$ OR $\frac{14x^2-15x-29}{(x-1)^2(2x+3)}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(2x+3)}$ $14x^2-15x-29=A(2x+3)(x-1)+B(2x+3)+C(x-1)^2$ $=2Ax^2+Ax-3A+2Bx+3B+Cx^2-2Cx+C$ Comparing coeff of x^2: $2A+C=14\rightarrow C=14-2A$ -- (1) x: $A+2B-2C=-15$ -- (2) const: $-3A+3B+C=-29$ -- (3)	M1 M1 A1 M1 A1 A1 M1 M1	

	Subst (1) into (2) and (3), $A+2B-2(14-2A)=-15$ → $5A+2B=13$ -- (4) $-3A+3B+(14-2A)=-29$ → $-5A+3B=-43$ -- (5) (4) + (5) $5B=-30$ → $B=-6$ $5A+2(-6)=13$ → $A=5$ $C=14-2(5)$ → $C=4$ $\frac{14x^2-15x-29}{(x-1)^2(2x+3)}=\frac{5}{(x-1)}-\frac{6}{(x-1)^2}+\frac{4}{(2x+3)}$	M1 A1 A1 A1	
7(a)	Let $x=0.5$, Remainder = $4(0.5)^3-6(0.5)^2+k(0.5)+3$ $14=0.5k+2$ $k=24$	 M1 A1	
7(b)	$4x^3-6x^2+2x+3=(2x^2+px+3)(ax+b)$ Comparing coeff of x^3, $4=2a$ → $a=2$ Comparing constant, $3=3b$ → $b=1$ OR $4x^3-6x^2+2x+3=(2x^2+px+3)(2x+1)$ Comparing coeff of x OR coeff of x^2 $2=p+6$ $-6=2+2p$ $p=-4$ $p=-4$ OR (using long division) $$\begin{array}{r} 2x+(-3-p) \\ (2x^2+px+3)\overline{)4x^3-6x^2+2x+3} \\ \underline{4x^3+2px^2+6x} \\ (-6-2p)x^2-4x+3 \\ \underline{(-6-2p)x^2+(-3p-p^2)x+(-9-3p)} \\ (p^2+3p-4)x+(12+3p) \end{array}$$ Since $(2x^2+px+3)$ is a factor, remainder = $0x+0$ Hence, $p^2+3p-4=0$ AND $12+3p=0$ $p=-4$, $p=1$ $p=-4$ → $p=-4$	M1 A1 M2 M1 A1 M1 A1 M1 A1	

8(a)	$3x=\sqrt{1-x}-1$ $3x+1=\sqrt{1-x}$ $(3x+1)^2=1-x$ $9x^2+6x+1-1+x=0$ $9x^2+7x=0$ $x(9x+7)=0$ $x=0$ or $x=-\frac{7}{9}$ (reject)	 M1 A1, A1	
8(b)	Area of triangle = $\frac{1}{2}\times b\times h$ $14+8\sqrt{3}=\frac{(1+\sqrt{3})h}{2}$ $h=\frac{2(14+8\sqrt{3})}{1+\sqrt{3}}$ $=\frac{28+16\sqrt{3}}{1+\sqrt{3}}\ \frac{\times(1-\sqrt{3})}{\times(1-\sqrt{3})}$ $=\frac{28-28\sqrt{3}+16\sqrt{3}-16(3)}{1^2-3}$ $=\frac{-20-12\sqrt{3}}{-2}$ $=(10+6\sqrt{3})$ cm	 M1 M1 M1 A1	
9(a)	Gradient of OP = $=\frac{6-0}{-2-0}=-3$ Gradient of perp bisector = $\frac{1}{3}$ Midpoint $M=\left(\frac{-2+0}{2},\frac{6+0}{2}\right)=(-1,3)$ $y=\frac{1}{3}x+c$ Subst $(-1,3)$ into equation, $3=\frac{1}{3}(-1)+c\ \rightarrow\ c=\frac{10}{3}$ Equation of perp bisector is $y=\frac{1}{3}x+\frac{10}{3}$	 M1 M1 A1	

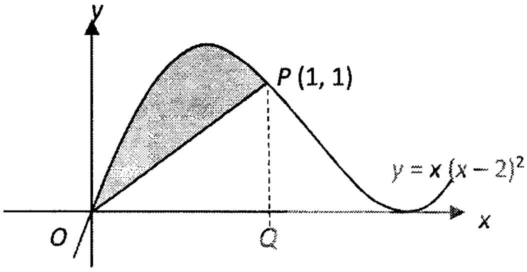
9(b)	Let $y=0$, $0=\frac{1}{3}x+\frac{10}{3}$ $x=-10$ $\quad R(-10,0)$	B1	
	Since R lies on the perp bisector, $MPQR$ is a rectangle. $M(-1,3) \rightarrow P(-2,6)$: $x-1, y+3$ $R(-10,0) \rightarrow Q(-10-1,0+3)$	M1	
	$Q(-11,3)$	A1	
	OR Using point P (–2, 6) and R (–10, 0) Midpoint of diagonals $=\left(\frac{-2-10}{2},\frac{6+0}{2}\right) = (-6,3)$	M1	
	Using point Q (x, y) and M (–1, 3) $(-6,3) = \left(\frac{x-1}{2},\frac{y+3}{2}\right)$ $x=-11, y=3 \rightarrow Q(-11,3)$	A1	
	OR Equation of PQ: $y=\frac{1}{3}x+c$ Subst $P(-2,6)$ into equation, $6=\frac{1}{3}(-2)+c \rightarrow c=\frac{20}{3}$ Equation is $y=\frac{1}{3}x+\frac{20}{3}$ -- (1) Equation of RQ: $y=-3x+c$ Subst $R(-10,0)$ into equation, $0=-3(-10)+c \rightarrow c=-30$ Equation is $y=-3x-30$ -- (2)	M1	
	Solving simultaneous equation, $x=-11, y=3 \rightarrow Q(-11,3)$	A1	

9(c)	$\text{Area} = \frac{1}{2}\begin{vmatrix} 0 & -2 & -11 & -10 & 0 \\ 0 & 6 & 3 & 0 & 0 \end{vmatrix}$ $= \frac{1}{2}[0+(-6)+0+0-0-(-66)-(-30)-0]$ M1 $= 45 \text{ units}^2$ A1 OR Length of $OP = \sqrt{(-2-0)^2+(6-0)^2} = \sqrt{40}$ Length of $QR = \sqrt{(-11+10)^2+(3-0)^2} = \sqrt{10}$ Length of $PQ = \sqrt{(-11+2)^2+(3-6)^2} = \sqrt{90}$ M1 Area of trapezium $= \frac{1}{2}(\sqrt{40}+\sqrt{10})(\sqrt{90})$ $= 45 \text{ units}^2$ A1 OR Area = Area of parallelogram *OMQR* + Area of triangle *MPQ* $= (OR \times \text{perp ht}) + (\frac{1}{2} \times MQ \times \text{perp ht})$ $= (10 \times 3) + (\frac{1}{2} \times 10 \times 3)$ M1 $= 45 \text{ units}^2$ A1	(allow ecf) (allow ecf)
10(a)	$y = 2x^3+6x^2-18x+5$ $\frac{dy}{dx} = 6x^2+12x-18$ M1 Let $\frac{dy}{dx} = 0$, $6x^2+12x-18=0$ M1 $x^2+2x-3=0$ $x=1$ or $x=-3$ $y=2(1)^3+6(1)^2-18(1)+5$ $y=2(-3)^3+6(-3)^2-18(-3)+5$ $y=-5$ $y=59$ A1 $x=1$: x: $(1)^-$, (1), $(1)^+$; $\frac{dy}{dx}$: -ve, 0, +ve (1, –5) Min point (A1) $x=-3$: x: $(-3)^-$, (-3), $(-3)^+$; $\frac{dy}{dx}$: +ve, 0, -ve (–3, 59) Max point (A1)	

	OR $\frac{d^2y}{dx^2} = 12x + 12$ when $x = 1$ $\frac{d^2y}{dx^2} = 24 > 0$ → (1, –5) Min point when $x = -3$ $\frac{d^2y}{dx^2} = -24 < 0$ → (–3, 59) Max point	 A1 A1	
10(b)	gradient, $m = \frac{dy}{dx} = 6x^2 + 12x - 18$ $\frac{dm}{dx} = \frac{d^2y}{dx^2} = 12x + 12$ Let $\frac{dm}{dx} = 0$ $12x + 12 = 0$ $x = -1$ $\frac{d^2m}{dx^2} = 12 > 0$ (minimum gradient) Hence, minimum gradient = $6(-1)^2 + 12(-1) - 18 = -24$	 M1 A1 A1	
11(a)	angle BAF = angle FAC (common angle) angle AFB = angle ACF (alternate segment theorem) hence, triangles ABF and AFC are similar (AA property)	M1 M1 A1	
11(b)	Using similar triangles ABF and AFC, $\frac{AF}{AB} = \frac{AC}{AF}$ → $AF^2 = AB \times AC$	 B1	
11(ci)	triangle BEF (must be in corresponding order)	B1	
(c)(ii)	Using similar triangles DEC and BEF, $\frac{ED}{EB} = \frac{EC}{EF}$ $ED \times EF = EC \times EB$ Given that $3EC = 2EB$, $\frac{EC}{EB} = \frac{2}{3}$ → $\frac{EC}{BC} = \frac{2}{5}$ and $\frac{EB}{BC} = \frac{3}{5}$	 M1 M1	

	Hence, $\left(\frac{2}{5}BC\right)\times\left(\frac{3}{5}BC\right)=EF\times ED$ $\frac{6}{25}(BC)^2=EF\times ED \rightarrow k=\frac{6}{25}$	 A1	
12(a) **12(b)** **12(c)**	$3^{2x-1}=2^{3-x}$ $\frac{3^{2x}}{3^1}=\frac{2^3}{2^x}$ $9^x\times 2^x=8\times 3$ $18^x=24$ $x=\log_{18}24$ $x=\frac{\lg 24}{\lg 18}$ $\log_6 4y=\log_6 3x^2-1$ $\log_6 4y=\log_6 3x^2-\log_6 6$ $\log_6 4y=\log_6\left(\frac{3x^2}{6}\right)$ $4y=\frac{x^2}{2}$ $y=\frac{x^2}{8}$ OR $\log_6 4y=\log_6 3x^2-1$ $\log_6 4y-\log_6 3x^2=-1$ $\log_6\left(\frac{4y}{3x^2}\right)=-1$ $\frac{4y}{3x^2}=6^{-1}$ $y=\frac{3x^2}{4\times 6}$ $y=\frac{x^2}{8}$ $\log_2 x^2=8\log_x 2$ $2\log_2 x=\frac{8\log_2 2}{\log_2 x}$	 M1 M1 A1 M1 A1 M1 A1	

	$2\log_2 x = \frac{8}{\log_2 x}$ Let $u = \log_2 x$, $2u = \frac{8}{u}$ $u^2 = 4$ $u = 2$ or $u = -2$ $\log_2 x = 2$ $\log_2 x = -2$ $x = 2^2 = 4$ $x = 2^{-2} = 0.25$	M1 M1 A1 (for both ans)	
13(a) **13(b)**	Gradient of $OP = \frac{1-0}{1-0} = 1$ $y = x + c$ Subst $P(1, 1)$ into equation, $1 = 1 + c \rightarrow c = 0$ Equation of OP is $y = x$ (shown) $y = x(x-2)^2$ $= x(x^2 - 4x + 4)$ $= x^3 - 4x^2 + 4x$ $\frac{dy}{dx} = 3x^2 - 8x + 4$ OR $u = x$ $v = (x-2)^2$ $\frac{du}{dx} = 1$ $\frac{dv}{dx} = 2(x-2)^1 \times 1 = 2x - 4$ $\frac{dy}{dx} = (x-2)^2(1) + (x)(2x-4)$ $= x^2 - 4x + 4 + 2x^2 - 4x$ $= 3x^2 - 8x + 4$ when $x = 1$ (point P) $\frac{dy}{dx} = 3(1)^2 - 8(1) + 4 = -1$ Gradient of normal $= \frac{-1}{-1} = 1$ = gradient of OP Hence, OP is the normal to the curve at P.	M1 A1 M1 M1 A1	

13(c)		
	Area under curve $= \int_0^1 x(x-2)^2 \, dx$	M1
	$= \int_0^1 (x^3 - 4x^2 + 4x) \, dx$	
	$= \left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{4x^2}{2}\right]_0^1$	
	$= \left[\frac{(1)^4}{4} - \frac{4(1)^3}{3} + 2(1)^2\right] - [0]$	
	$= \frac{11}{12}$ units^2	A1
	Area of triangle $OPQ = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$ units^2	B1
	Shaded area $= \frac{11}{12} - \frac{1}{2} = \frac{5}{12}$ units^2 (or 0.417 units^2)	A1
	OR	
	Area under curve $= \int_0^1 \left[x(x-2)^2 - x\right] dx$	M1, M1
	$= \int_0^1 (x^3 - 4x^2 + 4x - x) \, dx$	
	$= \int_0^1 (x^3 - 4x^2 + 3x) \, dx$	
	$= \left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{3x^2}{2}\right]_0^1$	M1
	$= \left[\frac{(1)^4}{4} - \frac{4(1)^3}{3} + \frac{3(1)^2}{2}\right] - [0]$	
	$= \frac{5}{12}$ units^2	A1
14(a)	$(\operatorname{cosec}\theta + \cot\theta)^2 = \frac{1+\cos\theta}{1-\cos\theta}$	
	LHS $= (\operatorname{cosec}\theta + \cot\theta)^2$	
	$= \left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)^2$	M1

	$=\left(\frac{1+\cos\theta}{\sin\theta}\right)^2$ $=\frac{(1+\cos\theta)^2}{\sin^2\theta}$ $=\frac{(1+\cos\theta)^2}{1-\cos^2\theta}$ $=\frac{(1+\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)}$ $=\frac{1+\cos\theta}{1-\cos\theta}$ = RHS (proved) OR LHS $=(\text{cosec}\,\theta+\cot\theta)^2$ $=\text{cosec}^2\theta+2\,\text{cosec}\,\theta\cot\theta+\cot^2\theta$ $=\frac{1}{\sin^2\theta}+\left(\frac{2}{\sin\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right)+\frac{\cos^2\theta}{\sin^2\theta}$ $=\frac{1+2\cos\theta+\cos^2\theta}{\sin^2\theta}$ $=\frac{(1+\cos\theta)^2}{1-\cos^2\theta}$ $=\frac{(1+\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)}$ $=\frac{1+\cos\theta}{1-\cos\theta}$ = RHS (proved)	 M1 M1 A1 M1 M1 M1 A1	
14(b)	$(\text{cosec}\,2\theta+\cot 2\theta)^2(1-\cos 2\theta)=2\cos\theta$ $1+\cos 2\theta=2\cos\theta$ $1+2\cos^2\theta-1=2\cos\theta$ $2\cos^2\theta-2\cos\theta=0$ $\cos\theta(\cos\theta-1)=0$ $\cos\theta=0$ or $\cos\theta=1$ $\alpha=\cos^{-1}0=90°$ $\quad$ $\alpha=\cos^{-1}1=0°$ $\theta=90°$ or $\theta=360°-90°$ $\quad$ $\theta=0°$ or $\theta=360°-0°$ $\theta=90°,270°$ (A1) $\quad$ $\theta=0°,360°$ (A1)	 M1 M1 M1	

	2022 Prelim 4E AMath Paper 2 Solutions	
1	$3^{2y+1}-3^{y+2}+3=3^y$ Let $u=3^y$ $3u^2-9u+3=u$ $u=3, u=\frac{1}{3}$ $y=1, y=-1$	 M1 M1 A2[4]
2	$\text{f}(x)=2x^3+5x^2+x-2$ $\text{f}(-2)=0$ $x+2$ is a factor <u>Method 1</u> $2x^3+5x^2+x-2=(x+2)(ax^2+bx+c)$ Equating coeff of x^3, $a=2$ Equating constants, $c=-1$ Equating coeff of x, $c+2b=1$ $b=1$ OR Equating coeff of x^2, $4+b=5$ $b=1$ $2x^3+5x^2+x-2=(x+2)(2x^2+x-1)$ $2x^3+5x^2+x-2=(x+2)(2x-1)(x+1)$ $(x+2)(2x-1)(x+1)=0$ $x=-2,-1,\frac{1}{2}$ <u>Method 2</u> Show long division $2x^3+5x^2+x-2=(x+2)(2x^2+x-1)$ $2x^3+5x^2+x-2=(x+2)(2x-1)(x+1)$ $(x+2)(2x-1)(x+1)=0$ $x=-2,-1,\frac{1}{2}$	 B1 M1 M1 M1 A1[5] M1 M1 M1 A1[5]

	<u>Method 3</u> Show synthetic division $2x^3+5x^2+x-2=(x+2)(2x^2+x-1)$ $2x^3+5x^2+x-2=(x+2)(2x-1)(x+1)$ $(x+2)(2x-1)(x+1)=0$ $x=-2,-1,\frac{1}{2}$	M1 M1 M1 A1[5]

3	(a)	$Let\ y = 4xe^{3x}$ $\frac{dy}{dx} = 4xe^{3x}(3) + e^{3x}(4)$ $= 12xe^{3x} + 4e^{3x}$	 M2 (for each term) A1
	(b)	$\int 12xe^{3x} + 4e^{3x}dx = 4xe^{3x}$ $\int 12xe^{3x}dx = \int -4e^{3x}dx + 4xe^{3x}$ $\int 6xe^{3x}dx = \frac{\int -4e^{3x}dx + 4xe^{3x}}{2}$ $\int_0^3 6xe^{3x}dx = \frac{\int_0^3 -4e^{3x}dx + \left[4xe^{3x}\right]_0^3}{2}$ $= \frac{\left[\frac{-4}{3}e^{3x}\right]_0^3 + \left[4xe^{3x}\right]_0^3}{2}$ $= \frac{\frac{-4}{3}e^9 + \frac{4}{3} + 12e^9}{2}$ $= \frac{2}{3} + \frac{16}{3}e^9$	M1 M1 M1 A1[7]

4	(a)	$T_{r+1}=\binom{12}{r}(x^2)^{12-r}\left(-\frac{m}{2x}\right)^r$ $=\binom{12}{r}\left(-\frac{m}{2}\right)^r x^{24-2r-r}$ $0=24-3r$ $r=8$ $\binom{12}{8}\left(-\frac{m}{2}\right)^8=126720$ $m^8=65536$ $m=4$	 M1 A1 M1 A1
	(b)	For $\left(x^2-\frac{m}{2x}\right)^{12}\left(8x^9+5\right)$, term in $x^9=\left(8x^9\right)(126720)+5(?\,x^9)$ For $\left(x^2-\frac{m}{2x}\right)^{12}$,term in x^9 $24-3r=9$ $r=5$ $\binom{12}{5}\left(-\frac{4}{2}\right)^5 x^9=-25344$ Term in $x^9=\left(8x^9\right)(126720)+5(-25344x^9)$ coefficient of $x^9=887040$	 B1 M1 A1[7]

5	(a)	$\frac{d}{dx}(x\sin x+\cos x)$ $=x\cos x+\sin x-\sin x$ $=x\cos x$	M2 A1
	(b)	$0=x\cos x$ $x=0, \cos x=0$ $A(\frac{\pi}{2},0)$	M1 M1 A1
	(c)	$\int_0^{\frac{\pi}{2}} x\cos x \, dx$ $=[x\sin x+\cos x]_0^{\frac{\pi}{2}}$ $=\left(\frac{\pi}{2}-1\right)units^2$	M1 A1

6	(a) (i)	$2x^2 - 7x + k$ $b^2 - 4ac < 0$ $(-7)^2 - 4(2)(k) < 0$ $k > \frac{49}{8}$	M2 A1
	(ii)	$\frac{(3x-2)(x-4)}{2x^2 - 7x + 8} < 0$ $2x^2 - 7x + 8 > 0$ $(3x-2)(x-4) < 0$ $\frac{2}{3} < x < 4$	M1 M1 A1
	(b) (i)	$px^2 + x + p(x+1) = 0$ $px^2 + (1+p)x + p = 0$ $b^2 - 4ac > 0$ $(1+p)^2 - 4(p)(p) > 0$ $1 + 2p - 3p^2 > 0$ $-\frac{1}{3} < p < 1$	M1 M1 A1
	(ii)	$b^2 - 4ac = 0$ $1 + 2p - 3p^2 = 0$ $p = 1, -\frac{1}{3}$	B1[10]

7	(a)	Appropriate scale and correct axes used All points plotted correctly Best fit line	P1 P1 P1
	(b)	$\frac{1}{a} = 0.1(\pm 0.01)$ $a = 10$ $\frac{b}{a} = \frac{1.0-0.1}{0.8-0}(\pm 1)$ $b = \frac{90}{8}$ or 11.25	B1 M1 A1
	(c)	$\frac{a}{y} = \frac{b}{x} + 1$ $\frac{xa}{y} = b + x$ $\frac{x}{y} = \frac{1}{a}x + \frac{b}{a}$ vertical axis= $\frac{x}{y}$	M1 A1
	(d)	$\frac{a}{y} = \frac{b}{x} + 1$ $\frac{xa}{y} = b + x$ $\frac{x}{y} = \frac{1}{a}x + \frac{b}{a}$ $a = \frac{1}{gradient}$ $b = (\frac{x}{y}$ intercept$)(\frac{1}{\text{gradient}})$	B1 B1[10]

8	(a)	$L = VY + YX + XW + VW$ $= 12 + 20\cos\theta + (32\sin\theta - 20\sin\theta) + 32\cos\theta$ $= 12 + 52\cos\theta + 12\sin\theta$ $= 12\sin\theta + 52\cos\theta + 12 (shown)$	M1 – for YX or VW M1 – for XW A1
	(b)	$12\sin\theta + 52\cos\theta = R\sin(\theta + \alpha)$ $R = \sqrt{52^2 + 12^2}$ $= 53.367$ $\theta = \tan^{-1}\frac{52}{12}$ $= 77.005$ $L = 12 + 53.4\sin(\theta + 77.0^\circ)$	B1 B1 B1
	(c)	$L = 12 + 53.367$ $= 65.367$ max value L = 65.4 $65.367 = 12 + 53.367\sin(\theta + 77.0^\circ)$ $1 = \sin(\theta + 77.0^\circ)$ $\theta = \sin^{-1}1 - 77.005^\circ$ $= 12.995$ $= 13.0^\circ$	B1 M1 A1
	(d)	$30 = 12 + 53.367\sin(\theta + 77.005^\circ)$ $0.33729 = \sin(\theta + 77.005^\circ)$ $\alpha = 19.712^\circ$ $\theta + 77.005^\circ = 19.712^\circ, 160.29^\circ$ $\theta = -57.3^\circ (rejected), 83.3^\circ$	M1 A1 A1[12]

9	(a)	$\frac{3}{t+1}-\frac{t+1}{3}=0$ $\frac{3}{t+1}=\frac{t+1}{3}$ $(t+1)^2=9$ $t+1=3,-3$ $t=2,-4(rejected)$	M1 A2
	(b)	$s=\int\frac{3}{t+1}-\frac{t+1}{3}\,dt$ $=3\ln(t+1)-\frac{(t+1)^2}{3(2)}+c$ $0=3\ln(1)-\frac{(0+1)^2}{3(2)}+c$ $c=\frac{1}{6}$ $s=3\ln(t+1)-\frac{(t+1)^2}{3(2)}+\frac{1}{6}$ $s=3\ln(2+1)-\frac{(2+1)^2}{3(2)}+\frac{1}{6}$ $=1.9625$ $=1.96m$ OR $s=\int\frac{3}{t+1}-\frac{t}{3}-\frac{1}{3}\,dt$ $=3\ln(t+1)-\frac{t^2}{6}-\frac{t}{3}+c$ $t=0,s=0$ $0=3\ln(1)-0-0+c$ $c=0$ $s=3\ln(t+1)-\frac{t^2}{6}-\frac{t}{3}$ $s=3\ln(2+1)-\frac{(2)^2}{6}-\frac{2}{3}$ $=1.96m$	M1 M1 M1 A1 A1 M1 M1 M1 A1 A1

	(c)	$s = 3\ln(t+1) - \frac{(t+1)^2}{3(2)} + \frac{1}{6}$ When $t = 4$, $s = 3\ln(4+1) - \frac{(4+1)^2}{3(2)} + \frac{1}{6}$ $= 0.82831$ When $t = 5$ $s = 3\ln(5+1) - \frac{(5+1)^2}{3(2)} + \frac{1}{6}$ $= -0.45805$ $\therefore$ particle is again at O at some instant during the fifth second.	M1 M1 A1
	(d)	$v = \frac{3}{t+1} - \frac{t+1}{3}$ $\frac{dv}{dt} = -3(t+1)^{-2} - \frac{1}{3}$ $= -\frac{5}{12} m/s^2$	M1 A1[13]
10	(a)	Method 1 $x^2 + y^2 - 8x - 16y + 55 = 0.$ $(x-4)^2 + (y-8)^2 = -55 + 16 + 64$ $(x-4)^2 + (y-8)^2 = 5^2$ $C(4,8)$ radius $= 5$ units Method 2 $2g = -8$ $g = -4$ $2f = -16$ $f = -8$ $C(4,8)$ $r = \sqrt{f^2 + g^2 - c}$ $= \sqrt{(-8)^2 + (-4)^2 - 55}$ $= 5units$	M1 A1 A1 B1 M1 A1

	(b)	gradient $CA = \frac{12-8}{7-4}$ $= \frac{4}{3}$ gradient of tangent $= -\frac{3}{4}$ $12 = -\frac{3}{4}(7) + c$ $c = 17\frac{1}{4}$ $y = -\frac{3}{4}x + 17\frac{1}{4}$ *OR* $y = -\frac{3}{4}x + \frac{69}{4}$ *OR* $y = -0.75x + 17.25$	M1 M1 A1
	(c)	$0 = -\frac{3}{4}x + 17\frac{1}{4}$ $x = 23$ $B(23, 0)$	B1

	(d)	Method 1 Since $\angle CAB$ is 90° (rad ⊥ tan) BC is the diameter of S_2 (rt ∠ in semicircle) D is the midpoint of $CB = \left(\frac{4+23}{2}, \frac{8+0}{2}\right)$	M1
		centre of $S_2 = D\left(\frac{27}{2}, 4\right)$	B1
		Radius of $S_2 = \sqrt{\left(23-\frac{27}{2}\right)^2+(0-4)^2}$	M1
		$= \frac{5\sqrt{17}}{2}$ OR $\sqrt{106.25}$	A1
		Therefore, equation of circle is $\left(x-\frac{27}{2}\right)^2+(y-4)^2=\frac{425}{4}$ *OR* $(x-13.5)^2+(y-4)^2=106.25$	A1
		Method 2 Gradient of perpendicular bisector of $AB = \frac{4}{3}$ Midpoint AB = (15,6) Eqn of perpendicular bisector $6=\frac{4}{3}(15)+c$ $y=\frac{4}{3}x-14$	M1
		Gradient of perpendicular bisector of $CB = \frac{19}{8}$ Midpoint CB = (13.5,4) Eqn of perpendicular bisector $4=\frac{19}{8}(13.5)+c$ $y=\frac{19}{8}x-28\frac{1}{16}$	M1
		OR Gradient of perpendicular bisector of $AC = -\frac{3}{4}$ Midpoint AC = (5.5,10) Eqn of perpendicular bisector	

		$10 = -\frac{3}{4}(5.5) + c$ $y = -\frac{3}{4}x + \frac{113}{8}$	M1
		$y = \frac{4}{3}x - 14$ $y = \frac{19}{8}x - 28\frac{1}{16}$ $centre\left(\frac{27}{2}, 4\right)$	A1
		$radius = \sqrt{\left(\frac{27}{2} - 23\right)^2 + (4 - 0)^2}$ $= \sqrt{106.25}$ $\left(x - \frac{27}{2}\right)^2 + (y - 4)^2 = 106.25$	M1 A1
	(e)	$C(4, 8), radius = 5$ $\text{distance } CD = \sqrt{\left(\frac{27}{2} - 4\right)^2 + (4 - 8)^2}$	M1
		$= 10.308$ > 5 D lies outside	A1[14]