Name | Reg. No | Class

4EX/5NA

ADDITIONAL MATHEMATICS **4049/01**

Paper 1 [90 marks]

PRELIMINARY EXAMINATION

23 AUGUST 2022

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

INSTRUCTIONS TO CANDIDATES

Do not open this booklet until you are told to do so.

Write your name, register number and class on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **ALL** questions.

Write your answers in the space provided.

If working is needed for any question, it must be shown with the answer.

Omission of essential working will result in loss of marks.

Write the brand and model of your calculator in the space provided below.

INFORMATION FOR CANDIDATES

You are expected to use an electronic calculator to evaluate explicit numerical expressions.

If the degree of accuracy is not specified in the question, and if the answer is not exact, the answer should be given to **three** significant figures. Answers in degrees should be given to **one** decimal place.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

Brand / Model of Calculator

This question paper consists of **17** printed pages and **1** blank page

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Area of $\Delta = \frac{1}{2}ab \sin C$

[Turn over

1 **(a)** Find the remainder when $9x^3 - 26x^2 + 3x + 14$ is divided by $x + 1$. [1]

(b) Given that $f(x) = bx^3 - 5x^2 + 2x + 4$ and $g(x) = bx^3 + 6x - 8$ have a common factor $x - a$, where a is an integer, find the value of b. [4]

2 A curve has the equation $y = \dfrac{2-x}{3x-4}, x \neq \dfrac{4}{3}$.

(i) Find an expression for $\dfrac{dy}{dx}$. [2]

(ii) Find the coordinates of the points on the curve where the normal is parallel to the line $2y - 16x = 3$. [5]

[Turn over

3 **(a)** Find the range of values of a for which the inequality $x^2 + ax - 2 < 2(x-1)^2$ for all real values of x. [4]

(b) The equation of a curve is $y = 2x^3 + (b+1)x^2 - x + b$, where b is a constant.

Show that, for all real values of b, the curve will have two distinct stationary points. [4]

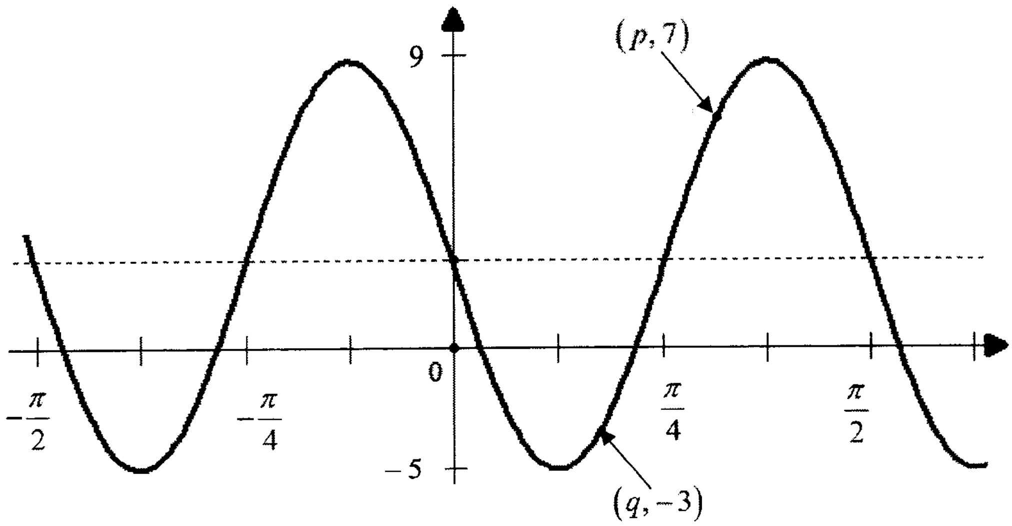
[Turn over

4 (a) State the range of values

(i) for the principal value of $\cos^{-1} x$, [1]

(ii) of y such that $\sin^{-1} y$ is not defined. [1]

(b)



The diagram shows part of the graph of $y = a\sin bx + c$, passing through the points $(p, 7)$ and $(q, -3)$.

(i) Find the value of the constants a, b and c. [3]

(ii) Form an equation connecting p, q and π. [2]

[Turn over

5 **(a)** The population, N, of a certain virus is given by $N = N_o(3^{kt})$, where N_o and k are constants and t is measured in days. Given that the population of the virus increases by 200% at the end of 15 days, calculate the value of k. [3]

(b) The equation $\log_3 x + 2\log_3 5 = \log_{27} x$ has the solution $x = 5^m$. Find the value of m. [4]

[Turn over

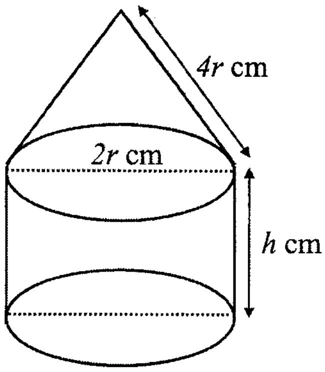
6 Given that $\int_8^n \frac{x-6}{x^2-2x-24}\,dx = \ln\frac{4}{3}$,

(i) state the value(s) of x for the integral to be undefined. [2]

(ii) find the value of n. [4]

[Turn over

7 The diagram shows a **solid** made up by a right circular cone and a cylinder of diameter $2r$ cm. The slant height of the cone is $4r$ cm and height of the cylinder is h cm.



(i) Given that the total surface area of the solid is 300 cm^2, express h in terms of r. [2]

(ii) Show that the volume, V cm^3, of the solid is given by $V = 150r + \left(\frac{\sqrt{15}}{3} - \frac{5}{2}\right)\pi r^3$. [3]

[Turn over

(iii) Given that r can vary, find the stationary value of V and determine whether this value of V is maximum or minimum. [5]

[Turn over

8 **(a)** Prove that $\dfrac{\cos A + \sin A}{\cos A - \sin A} = \sec 2A + \tan 2A$. [3]

(b) The equation of the **gradient** of a curve is $f'(\theta) = \cos 2\theta - 3\sin 2\theta$. Given that the curve $y = f(\theta)$ passes through the point $\left(\dfrac{\pi}{2}, -\dfrac{1}{2}\right)$, find

(i) the equation of the curve, [3]

[Turn over

(ii) the coordinates of the turning points of the curve for $0 \le \theta \le \pi$. [4]

[Turn over

9 **(a)** **(i)** Write down, and simplify, the first three terms in the expansion of $(2-3x)^5$ in ascending powers of x. [2]

(ii) Hence, find the coefficient of x^2 in the expansion of $\left(1-4x^2\right)\left(2-3x\right)^5$. [2]

(b) In the binomial expansion of $\left(\dfrac{k}{x^3}+x\right)^{12}$, where k is a positive constant, the term independent of x is 27500. Show that $k=5$. [4]

[Turn over

10 An ice cube retains its shape during melting. When its length is x mm, the surface area, A, is decreasing at a rate of $10\ \text{mm}^2/\text{s}$. The volume, V, of the ice cube changes at the rate of $-45\ \text{mm}^3/\text{s}$,

(i) find the value of x. [5]

(ii) find the rate of change of x for the **value found in (i)**. [1]

[Turn over

11 **(i)** $\sqrt{48+24\sqrt{3}}$ can be expressed in the form $a+2\sqrt{3}$, where a is an integer. Find the value of a. [2]

(ii) If the area of a given square is $48+24\sqrt{3}$ cm^2, find the length of one side of the square. [1]

(iii) The square in part **(ii)** is the base of a right pyramid with volume $\left(50+20\sqrt{3}\right)$ cm^3. Find the perpendicular height of the pyramid, leaving your answer in the form $\left(b-c\sqrt{3}\right)$ cm, where b and c are constants. [3]

[Turn over

12 A particle moves in a straight line, such that, t seconds after passing a fixed point O, its velocity, v m/s, is given by $v = \frac{2}{5}e^{3t} - 6e^{\frac{1}{2}-t}$. The particle comes to an instantaneous rest at point A.

(i) Show that the particle reaches A when $t = \frac{1}{8}(1 + 2\ln 15)$. [2]

(ii) Find the acceleration of the particle at A. [2]

(iii) Find the distance OA. [4]

[Turn over

(iv) Explain whether the particle will pass through point O **again** at some instant during the 2nd second. [2]

---- End of Paper ----

[Turn over

Name | Reg. No | Class

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4E/5N

ADDITIONAL MATHEMATICS 4049/02

PAPER 2 [90 marks]

PRELIMINARY EXAMINATION

26 August 2022

2 hours 15 minutes

Candidates answer in the Question Paper
No additional material required

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Brand / Model of Calculator

For Examiner's Use

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$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

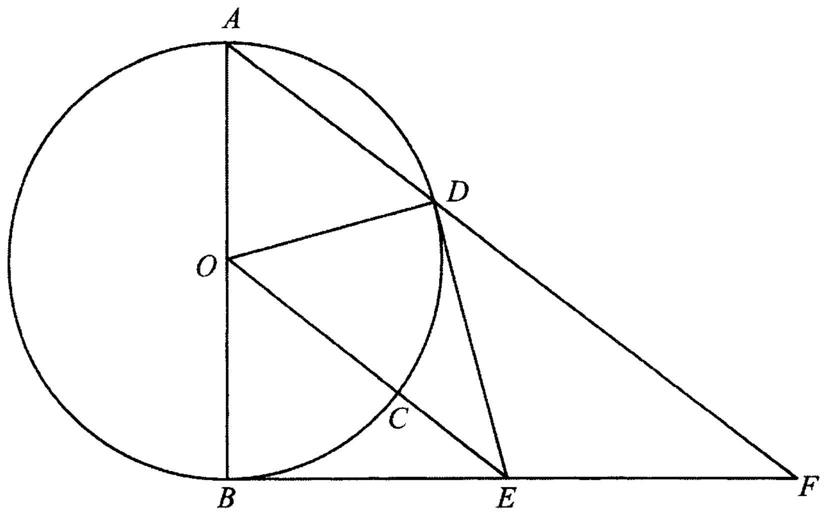
$$\text{Area of } \Delta = \frac{1}{2} ab \sin C$$

1 Express $5+8x-2x^2$ in the form $a(x-b)^2+c$, and hence state the coordinates of the turning point of the curve $y=5+8x-2x^2$. [4]

2 Express $\dfrac{4x^2+5x-11}{(x-2)(x^2+1)}$ in partial fractions. [6]

TURN OVER FOR QUESTION 3

3



A, B, C and D are points on a circle with centre O. AB is the diameter of the circle, ADF and OCE are straight lines and lines BEF and DE are tangents to the circle at B and D respectively.

(i) Prove that triangle OBE is congruent to triangle ODE. [4]

(ii) Show that E is the midpoint of BF. [4]

4 It is known that t and N are related by an equation of the form $N = AB^t$, where A and B are constants. The table shows experimental values of two variables t and N.

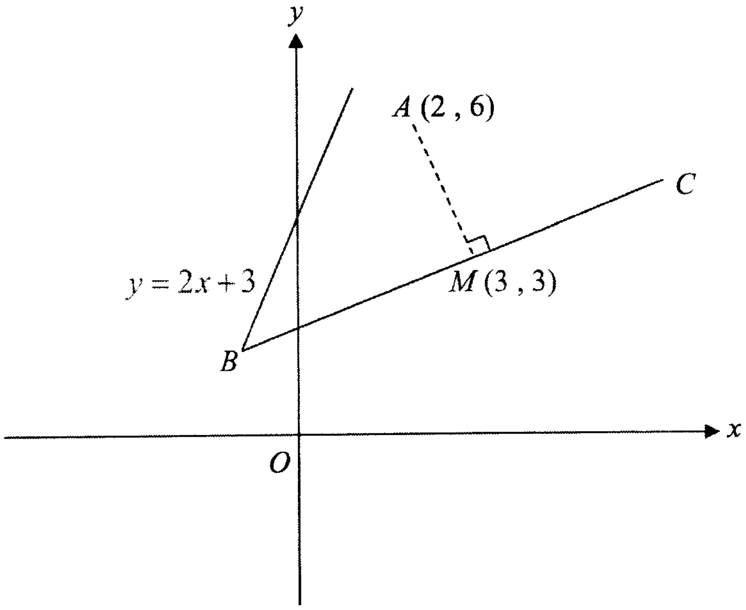
t	1	2	3	4	5
N	199	1258	7943	50118	316227

(i) By plotting lg N against t, draw a straight line to represent the above data. [3]

Continuation of working space for question **4(i)**

(ii) Use your graph to estimate the value of A and of B. [4]

5



Point A is (2 , 6), M is (3 , 3) and line AM is perpendicular to line BC.
Point B lies on the line with equation $y = 2x + 3$.

(i) Find the equation of line BC. [3]

(ii) Find the coordinates of B. [2]

M is the midpoint of BC.

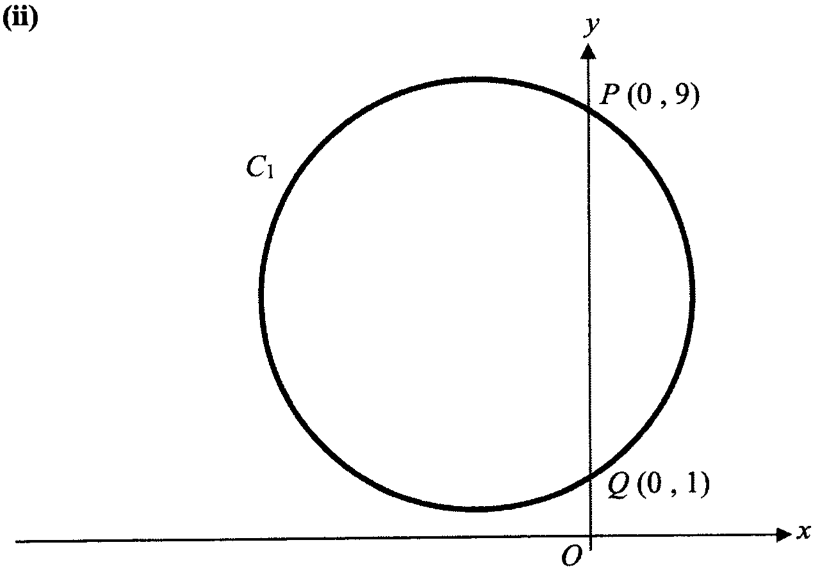
(iii) Find the coordinates of C. [2]

(iv) Calculate the area of triangle OAM. [2]

6 **(a)** A circle, C_1 intersects the y-axis at $P\,(0\,,9)$ and $Q\,(0\,,1)$ and its centre is 3 units to the left of the y-axis.

(i) Find the coordinates of the centre and radius of C_1. [3]

(ii)



Hammond drew a diagram of the circle by using the information given.

Explain why his diagram is wrong. [2]

(b) Another circle, C_2 has equation $x^2+y^2-14x-20y+113=0$. Let L be the centre of C_2.

(i) State the coordinates of L and calculate the radius of C_2. [3]

$M(2, 2)$ is a point outside C_2 and N is a point on C_2.

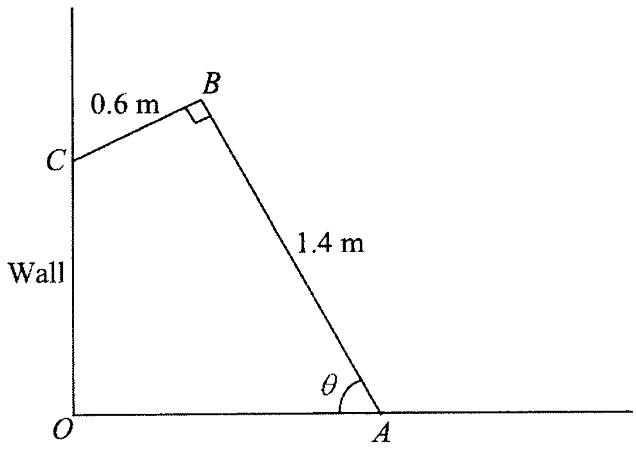
(ii) Find, in degrees, the greatest angle LMN. [3]

7 (a) Differentiate $\ln\left[\dfrac{(x-1)^{\frac{1}{2}}}{x+2}\right]$ with respect to x, for $x > 1$. [3]

(b) **(i)** Given that $y = \dfrac{x}{(3x+2)^2}$, show that $\dfrac{dy}{dx} = \dfrac{2-3x}{(3x+2)^3}$. [4]

(ii) Hence, find the value of $\int_0^1 \dfrac{-3x}{(3x+2)^3}\,dx$. [4]

8



A crowbar ABC is leaning against a vertical wall, where angle $ABC = 90°$, $AB = 1.4$ m and $BC = 0.6$ m. OA is horizontal and angle OAB is θ and is measured in degrees.

(i) Show that $OC = 1.4\sin\theta - 0.6\cos\theta$ [2]

(ii) Express OC in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0° \leq \alpha \leq 90°$ [4]

(iii) Find the value of θ if $OC = 1$ m. [2]

(iv) Explain the significance when OC is minimum and write down the corresponding value of θ. [2]

9 **(a)** **(i)** Prove the identity $\sin 2\theta - \tan\theta\cos 2\theta = \tan\theta$. [3]

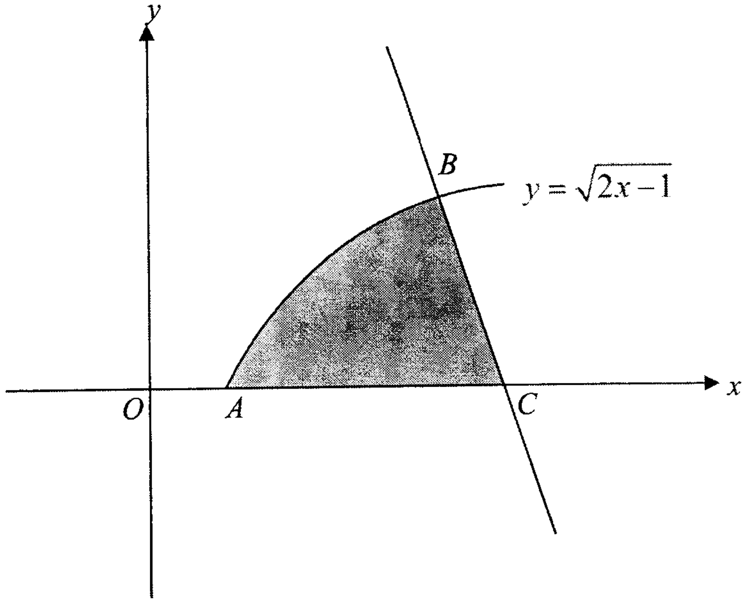
(ii) Hence, solve the equation $\sin 2\theta - \tan\theta\cos 2\theta = \dfrac{3}{\tan\theta}$ for $0 \le \theta \le 2\pi$. [4]

(b) Given that $\sin A = -\frac{3}{5}$, $\cos B = \frac{5}{6}$ and both angle A and angle B are in the same quadrant, find the exact value of

(i) $\cos(A-B)$. [3]

(ii) $\sin\frac{B}{2}$. [2]

10



The diagram shows part of the graph of $y = \sqrt{2x-1}$ intersecting the x-axis at A and has a gradient of $\frac{1}{3}$ at B. The normal of the graph at B intersects the x-axis at C. By first finding the coordinates of A, B and C or otherwise, show that the area of the shaded region is $10\frac{1}{2}$ units^2. [12]

Continuation of working space for question **10**

End of Paper

Marking Scheme
2022 Prelim Exam_4EX_AM P1

Qn.	Solutions	Marks
1(a)	$R = 9(-1)^3 - 26(-1)^2 + 3(-1) + 14$ $= -24$	B1
1(b)	$f(a) = g(a)$ $a^3b - 5a^2 + 2a + 4 = a^3b + 6a - 8$ $-5a^2 - 4a + 12 = 0$ $5a^2 + 4a - 12 = 0$ $(5a-6)(a+2) = 0$	M1 for equating
	$\Rightarrow \quad a = \frac{6}{5} \quad or \quad a = -2$ $\left(\begin{matrix}\textit{rejected since} \\ a \textit{ is an integer}\end{matrix}\right)$	M1 for correct a values
	When $a = -2$, ***factor*** $= x + 2$ $\therefore$ $\mathbf{g}(-2) = \mathbf{0}$ * *or use* $\mathbf{f}(-2) = 0$ $-8b - 12 - 8 = \mathbf{0}$ $\boldsymbol{b} = \frac{20}{-8}$	M1 for correct method to find b
	$= -2\frac{1}{2}$	A1

2(a)	$\frac{dy}{dx}=\frac{(3x-4)(-1)-(2-x)(3)}{(3x-4)^2}$ $=\frac{-3x+4-6+3x}{(3x-4)^2}$ $=-\frac{2}{(3x-4)^2}$	**M1 for correct quotient rule** **A1**
2(b)	$2y-16x=3$ $y=8x+\frac{3}{2}$ *Gradient of normal* $=8$ *Gradient of tangent* $=-\frac{1}{8}$ $\Rightarrow\ -\frac{2}{(3x-4)^2}=-\frac{1}{8}$ $16=(3x-4)^2$ $3x-4=4$ *or* $3x-4=-4$ $x=\frac{8}{3}$ *or* $x=0$ $y=-\frac{1}{6}$ $y=-\frac{1}{2}$ *Coordinates* $=\left(\frac{8}{3},-\frac{1}{6}\right)$ *or* $\left(0,-\frac{1}{2}\right)$	**M1** **M1 (allow e.c.f)** **M1 for both correct *x* values** **A1 + A1**

[Turn over

3(a)	$x^2+ax-2<2(x-1)^2$ $x^2+ax-2<2x^2-4x+2$ $0<x^2-(4+a)x+4$ $x^2-(4+a)x+4>0$ $\Rightarrow\ b^2-4ac<0$ $(4+a)^2-4(1)(4)<0$ $(4+a+4)(4+a-4)<0$ $(8+a)a<0$ $\therefore\ -8<a<0$	**M1** **M1** **M1 for correct factorization** **A1**
3(b)	$\frac{dy}{dx}=6x^2+2(b+1)x-1$ *For stationary pts,* $\frac{dy}{dx}=0$ $\Rightarrow\ 6x^2+2(b+1)x-1=0$ *Now, discriminant* $=4(b+1)^2-4(6)(-1)$ $=4(b+1)^2+24$ *Since* $4(b+1)^2+24>0$ *for all real values of* b, *the curve will have 2 distinct stationary points.*	**M1** **M1** **M1** **A1**
4(a)(i) 4(a)(ii)	$0\le\cos^{-1}x\le\pi\ (or\ 180°)$ $y>1,\quad y<-1$	**B1** **B1**
4(b)(i) 4(b)(ii)	$\lvert a\rvert=\frac{14}{2}=7$ $a=-7$ $b=\frac{2\pi}{\left(\frac{\pi}{2}\right)}=4$ $c=9-7=2$ $p-\frac{\pi}{4}=\frac{\pi}{4}-q$ $p+q=\frac{\pi}{2}$	**B1** **B1** **B1** **M1** **A1**

[Turn over

5(a)	*When* $t = 0, \quad N = N_0$ *When* $t = 15, \quad N = 3N_0$ $\Rightarrow \quad 3N_0 = N_0\left(3^{15k}\right)$ $\Rightarrow \quad 3^1 = 3^{15k}$ $\Rightarrow \quad k = \frac{1}{15}$	 **M1** **M1** **A1**
5(b)	$\log_3 x \quad + \quad \log_3 25 \quad = \quad \frac{\log_3 x}{3}$ $\log_3 25x = \log_3 x^{\frac{1}{3}}$ $\Rightarrow \quad 25x = x^{\frac{1}{3}}$ $\Rightarrow \quad x^{\frac{2}{3}} = \frac{1}{25}$ $\Rightarrow \quad x = \frac{1}{5^3}$ $\Rightarrow \quad x = 5^{-3}$ $\therefore \quad m = -3$ OR $\log_3 x \quad + \quad 2\log_3 5 \quad = \quad \frac{\log_3 x}{3}$ $3\log_3 x \quad + \quad 6\log_3 5 \quad = \quad \log_3 x$ $2\log_3 x = -6\log_3 5$ $\log_3 x = -3\log_3 5$ $\log_3 x = \log_3 5^{-3}$ $\Rightarrow \quad x = 5^{-3}$ $\therefore \quad m = -3$	**M1 for change of base** **M1 for product law** **M1** **A1** **M1 for change of base** **M1** **M1** **A1**

[Turn over

6(i)	$x^2-2x-24\neq 0$ $(x-6)(x+4)\neq 0$ *Hence,* $x\neq 6$ *or* $x\neq -4$	**M1 for factorization** **A1**
6(ii)	$\int_8^n \frac{x-6}{(x-6)(x+4)}\,dx = \ln\frac{4}{3}$ $\int_8^n \frac{1}{x+4}\,dx = \ln\frac{4}{3}$ $\left[\ln(x+4)\right]_8^n = \ln\frac{4}{3}$ $\ln(n+4)-\ln 12 = \ln\frac{4}{3}$ $\ln\left(\frac{n+4}{12}\right) = \ln\frac{4}{3}$ $\Rightarrow \frac{n+4}{12}=\frac{4\times 4}{3\times 4}$ $\Rightarrow n+4=16$ $\Rightarrow n=12$	**M1 for simplifying** **M1 for correct integration** **M1** **A1**

[Turn over

7(i)	$\pi r(4r) + \pi r^2 + 2\pi rh = 300$ $5\pi r^2 + 2\pi rh = 300$ $h = \frac{1}{2\pi r}(300 - 5\pi r^2)$ *or accept* $\frac{150}{\pi r} - \frac{5r}{2}$	M1 A1
7(ii)	$V = \frac{1}{3}\pi r^2 (r\sqrt{15}) + \pi r^2 h$ $= \frac{\sqrt{15}}{3}\pi r^3 + \pi r^2 \left(\frac{1}{2\pi r}\right)(300 - 5\pi r^2)$ $= \frac{\sqrt{15}}{3}\pi r^3 + 150r - \frac{5}{2}\pi r^3$ $= 150r + \left(\frac{\sqrt{15}}{3} - \frac{5}{2}\right)\pi r^3$ (*shown*)	**M1 for finding height of cone,** $r\sqrt{15}$ **M1 for substituting** ***h*** **A1**
7(iii)	$\frac{dV}{dr} = 150 + 3\pi r^2 \left(\frac{\sqrt{15}}{3} - \frac{5}{2}\right)$ *For stationary value of* V, $\frac{dV}{dr} = 0$. $\Rightarrow 150 + 3\pi r^2 \left(\frac{\sqrt{15}}{3} - \frac{5}{2}\right) = 0$ $\Rightarrow r^2 = \frac{-150}{3\pi\left(\frac{\sqrt{15}}{3} - \frac{5}{2}\right)}$ $= 13.16412$ $\Rightarrow r = 3.62824\ cm$ *When* $r = 3.62824$, $V = 362.824\ cm^3$ $\approx 363\ cm^3$ *Now,* $\frac{d^2V}{dr^2} = 6\pi r\left(\frac{\sqrt{15}}{3} - \frac{5}{2}\right) = -82.6848 < 0$ $\Rightarrow$ *V is a maximum.*	**M1 for correct differentiation** **M1 for correct** ***r*** **value** **A1 for correct** ***V*** **M1 for finding 2nd derivative** **A1**

[Turn over

8(a)	$LHS = \dfrac{(\cos A + \sin A)(\cos A + \sin A)}{\cos^2 A - \sin^2 A}$ $= \dfrac{\cos^2 A + \sin^2 A + 2\sin A\cos A}{\cos 2A}$ $= \dfrac{1 + \sin 2A}{\cos 2A}$ $= \sec 2A + \tan 2A = RHS \quad (proven)$	**M1 for multiplying the conjugate** **M1 for either one of the double-angled formula** **A1**
8(b)(i)	$y = \int \cos 2\theta - 3\sin 2\theta \quad d\theta$ $= \frac{1}{2}\sin 2\theta + \frac{3}{2}\cos 2\theta + c$ $When\ \theta = \frac{\pi}{2}, \quad y = -\frac{1}{2}.$ $\therefore \quad -\frac{1}{2} = \frac{1}{2}\sin \pi + \frac{3}{2}\cos \pi + c$ $\Rightarrow \quad -\frac{1}{2} = -\frac{3}{2} + c$ $\Rightarrow \quad c = 1$ $Hence, \quad y = \frac{1}{2}\sin 2\theta + \frac{3}{2}\cos 2\theta + 1$	**M1** **M1 for finding *c*** **A1**
8(b)(ii)	*For turning pts,* $f'(\theta) = 0.$ $\Rightarrow \quad \cos 2\theta - 3\sin 2\theta = 0$ $\Rightarrow \quad \tan 2\theta = \frac{1}{3}$ $(basic \quad = 0.321751)$ $2\theta = 0.321751, \quad 3.463344$ $\theta = 0.160876, \quad 1.731672$ $\therefore\ y = 2.58, \quad -0.581$ *Coordinates of turning pts are* $(0.161,\ 2.58)$ *and* $(1.73,\ -0.581)$	**M1** **M1 for correct values of θ** **A1 + A1**

[Turn over

9(a)(i)	$(2-3x)^5 = 2^5 + 5(2^4)(-3x) + 10(2^3)(-3x)^2 + \ldots$ $\approx 32 - 240x + 720x^2$	M1 A1
9(a)(ii)	$(1-4x^2)(32-240x+720x^2)$ $Coeff\ of\ x^2 = 720 - 4(32)$ $= 592$	M1 A1
9(b)	$(r+1)\,term = {}^{12}C_r \left(\frac{k}{x^3}\right)^{12-r} x^r$ $= {}^{12}C_r \,.\, k^{12-r} \,.\, x^{3r-36+r}$ $= {}^{12}C_r \,.\, k^{12-r} \,.\, x^{4r-36}$ $For\ x^0, \quad 4r - 36 = 0$ $r = 9$ $\Rightarrow \quad {}^{12}C_9\, k^3 = 27500$ $\Rightarrow \quad k^3 = 125$ $\Rightarrow \quad k = 5 \quad (shown)$	M1 M1 for finding r M1 for equating A1

[Turn over

10(i)	$A = 6x^2$ $\quad$ $V = x^3$ $\frac{dA}{dx} = 12x$ $\quad$ $\frac{dV}{dx} = 3x^2$ $\frac{dA}{dt} = \frac{dA}{dx}.\frac{dx}{dt}$ $\quad$ $\frac{dV}{dt} = \frac{dV}{dx}.\frac{dx}{dt}$ $-10 = (12x)\frac{dx}{dt}$ $\quad$ $-45 = (3x^2)\frac{dx}{dt}$ $\frac{dx}{dt} = -\frac{5}{6x}$ $\quad$ $\frac{dx}{dt} = -\frac{15}{x^2}$ $\Rightarrow \quad -\frac{5}{6x} = -\frac{15}{x^2}$ $\Rightarrow \quad 5x^2 = 90x$ $\Rightarrow \quad x(x-18) = 0$ $\Rightarrow \quad x = 0\,(rejected) \quad or \quad x = 18\,mm$	**M1 for either of the correct chain rule** **M1 + M1** **M1 for equating** **A1**
10(ii)	$\frac{dx}{dt} = -\frac{5}{6\times18} = -\frac{5}{108}$	**B1**
11(i)	$48+24\sqrt{3} = \left(a+2\sqrt{3}\right)^2$ $= a^2 + 4a\sqrt{3} + 12$ $\Rightarrow \quad 24 = 4a$ $\Rightarrow \quad a = 6$	**M1 for expansion** **A1**
11(ii)	$6+2\sqrt{3}$	**B1**
11(iii)	$\frac{1}{3}\left(48+24\sqrt{3}\right)h = 50+20\sqrt{3}$ $h = \frac{50+20\sqrt{3}}{16+8\sqrt{3}} \times \frac{16-8\sqrt{3}}{16-8\sqrt{3}}$ $= \frac{800-80\sqrt{3}-480}{64}$ $= \frac{320-80\sqrt{3}}{64}$ $= 5 - \frac{5}{4}\sqrt{3}$	**M1 for correct volume formula** **M1 for multiplying conjugate surd** **A1**

[Turn over

12(i)	*When* $v=0$, $\frac{2}{5}e^{3t}=6e^{\frac{1}{2}-t}$ $e^{4t-\frac{1}{2}}=15$ $4t-\frac{1}{2}=\ln 15$ $t=\frac{1}{4}\left(\frac{1}{2}+\ln 15\right)$ $=\frac{1}{8}(1+2\ln 15) \quad (shown)$	**M1** **A1**
12(ii)	$a=\frac{6}{5}e^{3t}+6e^{\frac{1}{2}-t}$ *When* $t=\frac{1}{8}(1+2\ln 15)\approx 0.802013$ sec, $a=17.74389\approx 17.7\ m/s^2$	**M1** **A1**
12(iii)	$s=\int \frac{2}{5}e^{3t}-6e^{\frac{1}{2}-t}\ \mathrm{d}t$ $=\frac{2}{15}e^{3t}\ +\ 6e^{\frac{1}{2}-t}\ +\ c$ *When* $t=0$, $s=0$. $\Rightarrow\ 0=\frac{2}{15}+6e^{\frac{1}{2}}+c$ $\Rightarrow\ c=-\left(\frac{2}{15}+6e^{\frac{1}{2}}\right)$ *When* $t=\frac{1}{8}(1+2\ln 15)\approx 0.802013$ sec, *Displacement* $OA=-4.111031$ *Distance* $OA=4.11\ m$	**M1** **M1** **M1** **A1**
12(iv)	*When* $t=1$, $s=-3.71\ m$ *When* $t=2$, $s=45.1\ m$ During the 2nd second, the displacement changes from negative to positive. Hence, the particle will pass through point O again during the 2nd second.	**M1 for finding correct displacement at either t = 1 or t = 2** **A1 for correct explanation**

[Turn over

2022 4E5N Prelim Amath Paper 2 MS

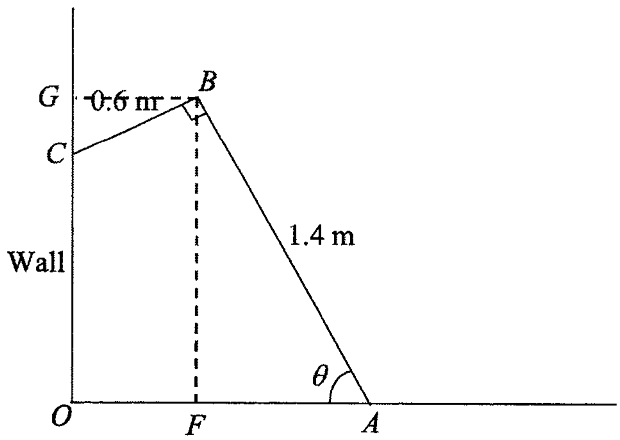
Qn	Solution	Marking Scheme
1	$5+8x-2x^2=-2\left(x^2-4x\right)+5$ $=-2\left[\left(x-2\right)^2-4\right]+5$ $=-2\left(x-2\right)^2+13$ t.p. is (2 , 13)	M1 for factorizing −2 M1 for +4 and −4 or equivalent A1 A1
2	$\frac{4x^2+5x-11}{\left(x-2\right)\left(x^2+1\right)}=\frac{A}{5\left(x-2\right)}+\frac{Bx+C}{5\left(x^2+1\right)}$ $4x^2+5x-11=A\left(x^2+1\right)+\left(Bx+C\right)\left(x-2\right)$ Let $x=2$: $15=5A$ $A=3$ Let $x=0$: $-11=3-2C$ $C=7$ Let $x=1$: $-2=6+(B+7)(-1)$ $B=1$ $\frac{4x^2+5x-11}{\left(x-2\right)\left(x^2+1\right)}=\frac{3}{x-2}+\frac{x+7}{x^2+1}$	M1 for "getting rid" of denominator M1 for substituting values or correct expansion B1 B1 (can give ecf) B1 (can give ecf) A1 (no ecf)
3i	$OB=OD$ $OE=OE$ (common side) $\angle OBE=\angle ODE$ $(=90)$ $BE=DE$ (Pythagoras' Thm) $\Delta OBE\equiv\Delta ODE$ (SSS or RHS)	B1 B1 B1 A1 must include SSS or RHS
3ii	from (i) $\angle BOE=\angle DOE$ $\angle BAF=\frac{1}{2}\angle BOD$ (angle at centre = $2\times$angle at circumference) $=\angle BOE$ $\angle ABF=\angle OBF$ (common angle) ΔOBE is similar to ΔABF (AA) $\frac{BE}{BF}=\frac{OB}{AB}$ $\frac{BE}{BF}=\frac{1}{2}$ $BE=\frac{1}{2}BF$ There E is the midpoint of BF } →	B1 M1 (must have reason) M1 for conclusion A1 for either

<table>
<tr><td>4i</td><td>

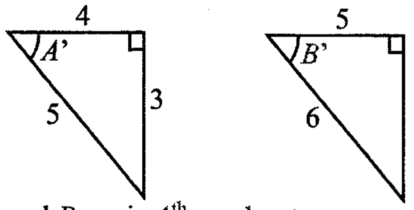
lg N	2.298	3.099	3.899	4.699	5.499

See pdf file for graph</td><td>B1 values of lg N
M1 for scale of axes
P1 points plotted correctly and line passes through all points</td></tr>
<tr><td>4ii</td><td>$N = AB^t$
$\lg N = \lg\left(AB^t\right)$
$\lg N = (\lg B)t + \lg A$
$\lg A = 1.5$ (allow 1.4 to 1.6)
$A = 31.6$ (allow 25.1 to 39.8)
$\lg B = \frac{5.5-1.5}{5-0}$
$= \frac{4}{5}$ (allow 0.7 to 0.9)
$B = 6.31$ (allow 5.01 to 7.94)</td><td>M1 addition or power law
A1
M1
A1</td></tr>
<tr><td>5i</td><td>$Grad_{AM} = \frac{3-6}{3-2}$
$= -3$
$Grad_{BC} = \frac{1}{3}$
equation of BC:
$y - 3 = \frac{1}{3}(x-3)$
$y = \frac{1}{3}x + 2$</td><td>B1
M1 for using the formula of perpendicular lines
A1</td></tr>
<tr><td>5ii</td><td>$\frac{1}{3}x + 2 = 2x + 3$
$\frac{5}{3}x = -1$
$x = -\frac{3}{5}$
$y = \frac{9}{5}$
$B(-\frac{3}{5}, \frac{9}{5})$</td><td>M1
A1</td></tr>
</table>

5iii	Let C be (x, y) $\frac{x-\frac{3}{5}}{2}=3$ or $\frac{y+\frac{9}{5}}{2}=3$ $x=\frac{33}{5}$ $y=\frac{21}{5}$ $C\left(\frac{33}{5},\frac{21}{5}\right)$ or $\left(6\frac{3}{5},4\frac{1}{5}\right)$ or $(6.6, 4.2)$	M1 for either equation A1
5iv	Area of $\Delta OAM=\frac{1}{2}\begin{vmatrix}0 & 3 & 2 & 0\\0 & 3 & 6 & 0\end{vmatrix}$ $=\frac{1}{2}\lvert 18-6\rvert$ $=6 \text{ units}^2$	 M1 A1
6ai	Centre $(-3, 5)$ $r=\sqrt{(-3-0)^2+(5-1)^2}$ $=5$	B1 M1 A1
6aii	Since the y-coordinate of centre = 5 and radius = 5, the ***x*-axis must be tangent** to the circle but it is **not touching** the circle at all	 A1 A1
6bi	$L(7, 10)$ $r=\sqrt{7^2+10^2-113}$ $=6$	B1 M1 A1
6bii	$LM=\sqrt{(7-2)^2+(10-2)^2}$ $=\sqrt{89}$ $\angle LMN=\sin^{-1}\left(\frac{6}{\sqrt{89}}\right)$ $=39.5$	M1 M1 A1
7a	$y=\ln\left[\frac{(x-1)^{\frac{1}{2}}}{x+2}\right]$ $=\ln(x-1)^{\frac{1}{2}}-\ln(x+2)$ $=\frac{1}{2}\ln(x-1)-\ln(x+2)$ $\frac{dy}{dx}=\frac{1}{2(x-1)}-\frac{1}{x+2}$	 M1 for "minus" law M1 for power law A1

7bi	$y=\dfrac{x}{(3x+2)^2}$ $\dfrac{dy}{dx}=\dfrac{(3x+2)^2-x(2)(3x+2)(3)}{(3x+2)^4}$ $=\dfrac{(3x+2)[(3x+2)-6x]}{(3x+2)^4}$ $=\dfrac{2-3x}{(3x+2)^3}$	M1 for quotient rule M1 for chain rule M1 for factorizing A1
7bii	$\int_0^1 \dfrac{2-3x}{(3x+2)^3}\,dx=\left[\dfrac{x}{(3x+2)^2}\right]_0^1$ $\int_0^1 \dfrac{2}{(3x+2)^3}+\dfrac{-3x}{(3x+2)^3}\,dx=\dfrac{1}{25}$ $\int_0^1 \dfrac{-3x}{(3x+2)^3}\,dx=\dfrac{1}{25}-\int_0^1 2(3x+2)^{-3}\,dx$ $=\dfrac{1}{25}-\left[-\dfrac{(3x+2)^{-2}}{3}\right]_0^1$ $=\dfrac{1}{25}+\dfrac{1}{3}\left[\dfrac{1}{25}-\dfrac{1}{4}\right]$ $=-\dfrac{3}{100}$	M1 M1 for splitting the integral B1 for $\dfrac{1}{25}$ A1
8i		

	$BF = 1.4\sin\theta$ $BG = 0.6\cos\theta$ } → $OC = 1.4\sin\theta - 0.6\cos\theta$	M1 (either) A1
8ii	$R\sin(\theta-\alpha) = R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$ $R\cos\alpha = 1.4$ $R\sin\alpha = 0.6$ $R = \sqrt{1.4^2 + 0.6^2}$ $= \sqrt{2.32}$ or $\frac{\sqrt{58}}{5}$ $\alpha = \tan^{-1}\frac{0.6}{1.4}$ $= 23.19$ $OC = \sqrt{2.32}\sin(\theta - 23.19)$	M1 A1 M1 (or $\tan\alpha = \frac{0.6}{1.4}$) A1
8iii	$\sqrt{2.32}\sin(\theta - 23.19) = 1$ $\sin(\theta - 23.19) = \frac{1}{\sqrt{2.32}}$ basic angle $= 41.03$ $\theta - 23.19 = 41.03$ $\theta = 64.2$	M1 for basic angle A1
8iv	Min $OC = 0$ AC is horizontal (or equivalent) $\theta - 23.19 = 0$ $\theta = 23.2$	B1 A1
9ai	$LHS = \sin 2\theta - \tan\theta\cos 2\theta$ $= 2\sin\theta\cos\theta - \tan\theta(2\cos^2\theta - 1)$ $= 2\sin\theta\cos\theta - 2\sin\theta\cos\theta + \tan\theta$ $= \tan\theta$ $= RHS$	M1 for double angle formula M1 for simplification A1

9aii	$\tan\theta = \frac{3}{\tan\theta}$ $\tan^2\theta = 3$ $\tan\theta = \pm\sqrt{3}$ $\alpha = \tan^{-1}\sqrt{3}$ $= \frac{\pi}{3}$ $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ or $1.04, 2.09, 4.19, 5.24$	M1 M1 method to find basic angle M1 showed signs to find solutions in correct quadrants A1 for all four solutions
9bi	 *A* and *B* are in 4th quadrant $\cos(A-B) = \cos A\cos B + \sin A\sin B$ $= \left(\frac{4}{5}\right)\left(\frac{5}{6}\right) + \left(-\frac{3}{5}\right)\left(-\frac{\sqrt{11}}{6}\right)$ $= \frac{2}{3} + \frac{\sqrt{11}}{10}$ or $\frac{20+3\sqrt{11}}{30}$	B1 for either diagram or evidence of finding the missing side B1 substituting the correct values A1
9bii	$\cos B = 1 - 2\sin^2\frac{B}{2}$ $\sin^2\frac{B}{2} = \frac{1}{2}(1-\cos B)$ $= \frac{1}{12}$ $\sin\frac{B}{2} = \frac{1}{\sqrt{12}}$	M1 for changing the subject of formula A1, no marks if $-\frac{1}{\sqrt{12}}$ is not rejected.
10	Find *A*: $\sqrt{2x-1} = 0$ $x = \frac{1}{2}$ $\left(\frac{1}{2}, 0\right)$	M1 B1

	Find B:	
	$y=(2x-1)^{\frac{1}{2}}$	
	$\frac{dy}{dx}=(2x-1)^{-\frac{1}{2}}$	M1 for differentiation
	$(2x-1)^{-\frac{1}{2}}=\frac{1}{3}$	M1 for equating dydx to gradient
	$2x-1=3$	
	$x=5$	
	$y=3$	
	$(5,3)$	A1
	Gradient of normal $=-3$	B1
	Let C be $(a\ ,0)$	
	$\frac{0-3}{a-5}=-3$	M1
	$a-5=1$	
	$a=6$	
	$(6,0)$	A1
	Shaded Area $=\int_{\frac{1}{2}}^{5}(2x-1)^{\frac{1}{2}}dx+\frac{1}{2}(1)(3)$	M1 for correct limits M1 for forming the expression for shaded area M1 for integration
	$=\frac{1}{3}\left[(2x-1)^{\frac{3}{2}}\right]_{\frac{1}{2}}^{5}+\frac{3}{2}$	
	$=\frac{1}{3}[27-0]+\frac{3}{2}$	
	$=10.5$	A1

(i) By plotting lg y against x, draw a straight line to represent the above data. [3]