Name:	Index Number:	Class:

HUA YI SECONDARY SCHOOL

4E5N Preliminary Examination 2022 **4E5N**

ADDITIONAL MATHEMATICS

4049/01

Paper 1

31 August 2022

2 hours 15 minutes

Candidates answer on the Question Paper.

No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Name, Class and Index Number in the spaces provided at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 90.

For Examiner's Use
90

This document consists of **16** printed pages including the cover page.

[Turn Over

Mathematical Formula

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots.(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\tan^2 A + 1 = \sec^2 A$$

$$1 + \cot^2 A = \operatorname{cosec}^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab \sin C$$

1. A curve has the equation $y = x^2 + px - 4p + 9$, where p is a constant.

   (a) Find the range of values of p such that $x^2 + px - 4p + 9$ is always positive for all real values of x. [3]

   (b) In the case where $p = 4$, show that the line $y = 6x - 8$ is a tangent to the curve. [3]

2. **(a)** The function $f(x) = 2x^3 + ax^2 + bx - 6$ is divisible by $x+3$ and $f'(x)$ leaves a remainder of 1 when divided by $x-1$.

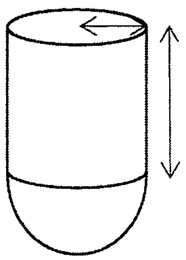
(i) Find the values of a and b. [3]

(ii) Using the integer values of a and b found in part **(i)**, factorise $f(x)$ completely. [3]

(iii) Hence, solve $2+3y-11y^2-6y^3=0$. [2]

(b) Fully factorize the expression $54x^6-16y^3$. [2]

3. A factory manufactures miniature containers that are made up of a closed circular cylinder and a hemisphere which have the same radius. The radius of the cylinder is r cm and its height is h cm. The volume of the container is 60π cm^3.



(a) Express h in terms of r. [2]

(b) The container is made up of two different types of thin metal sheets with negligible thickness. The cost of the cover and curved surface of the cylinder is 3 cents per cm^2 and that of the hemisphere is 4 cents per cm^2. The total cost of materials to make the container is C cents.

Show that $C = 7\pi r^2 + \dfrac{360\pi}{r}$. [3]

(c) Given that r can vary, find the value of r which gives a stationary value of C. [2]

(d) Find the nature of this stationary value and explain if the factory owner should continue to produce this container if he wants to keep the cost below \$5.60. [3]

4. **(a)** Solve the equation $2^{4p+1}+20(4^{p-1})=3$. [4]

(b) Explain why $2^{4p+1}+20(4^{p-1})=k$ has no solution if $k<-3\frac{1}{8}$. [3]

5. **(a)** Given that $\dfrac{(\log_y x)^2}{\log_x y} + 64 = 0$, express y in terms of x. [3]

(b) Solve the equation $\log_2 x + 3 = 2\log_2 (x-1)$. [4]

6. The height of water in a harbour at time t hours after midnight on a certain day is modelled by the formula $h = 4.8\sin k\pi t + 5.1$ where $0 \le t \le 24$.

(a) State the maximum and minimum levels of the tide. [2]

(b) A tide cycle starts at high tide and ends when it is high tide again. Given that there are 2 tide cycles in 24 hours, show that $k = \frac{1}{6}$. [1]

(c) Boats can come into the harbour when the tide is above 2 m. Find the range of times at which the boats can come into the harbour in the first 12 hour cycle. [4]

7. **(a)** **(i)** Prove the identity $\cot x - \sin 2x = \cot x \cos 2x$. [3]

(ii) Hence, solve the equation $4\cot x - 4\sin 2x = \cos 2x$ for $0 \le x \le \pi$. [4]

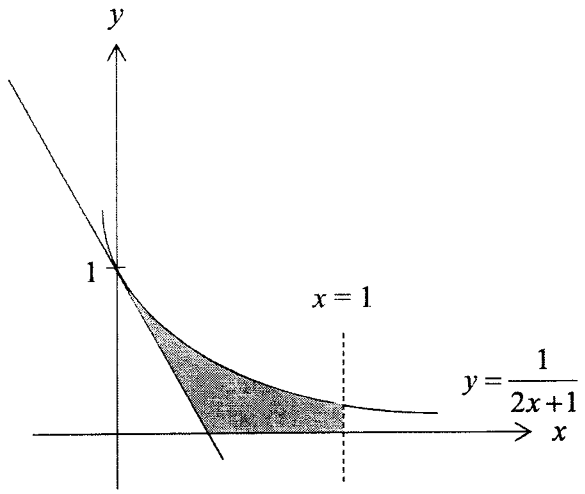
(b) A and B are acute angles such that $\sin(A+B) = \frac{6}{7}$ and $\sin B \cos A = \frac{2}{7}$.

Without calculating angles A and B, find $\frac{\tan A}{\tan B}$. [4]

8. **(a)** A particle moves along the curve $y=\frac{2}{\sqrt{3x+1}}$ in such a way that the y-coordinate of the particle is decreasing at a constant rate of 3 units per second. Find the y-coordinate of the particle at the instant when the x-coordinate is increasing at 0.125 units per second. [5]

(b) Show that $y=x\ln x^2$ is an increasing function when $x>\frac{1}{e}$. [2]

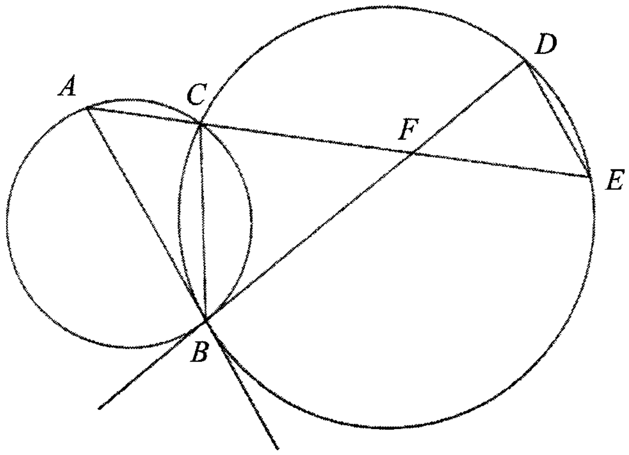
9. The diagram below shows the graph of $y = \frac{1}{2x+1}$ and the tangent to the curve where the curve intersects the y-axis.



(a) Find the equation of the tangent. [3]

(b) Calculate the area of the shaded region. [4]

10. In the diagram below, *BD* is a tangent to circle *ACB* and *AB* is a tangent to circle *BCDE*. *ACFE* is a straight line.

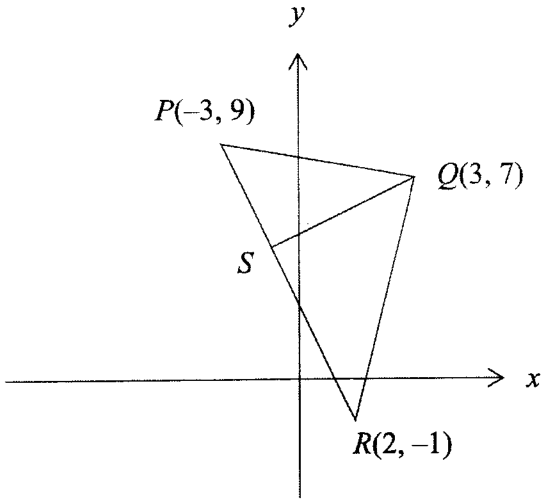


(a) Show that *AB* is parallel to *DE*. [3]

(b) Name two triangles that are similar to triangle *ABF*. [2]

(c) Show that $AF \times CF = BF^2$. [2]

11. The diagram below, which is not drawn to scale, shows a triangle PQR with vertices $P(-3,\ 9)$, $Q(3,\ 7)$ and $R(2,\ -1)$. The point S which lies on PR is the foot of the perpendicular line from Q.



(a) Find the equation of QS. [3]

(b) Find the equation of a line that is parallel to PR and passes through Q. [2]

(c) Find the area of triangle *PQR*. [2]

12. **(a)** Sketch the graph of $y = 3^x$. [1]

(b) In order to solve the equation $\log_9(5-x) = \frac{x}{2}$, a suitable straight line has to be drawn on the same set of axes as the graph of $y = 3^x$. Find the equation of the straight line. [2]

(c) State the number of solutions for the equation $\log_9(5-x) = \frac{x}{2}$. [1]

-End of Paper-

Name:	Index Number:	Class:

HUA YI SECONDARY SCHOOL

4E5N Preliminary Examination 2022 **4E5N**

ADDITIONAL MATHEMATICS

4049/02

Paper 2

14 September 2022

2 hours 15 minutes

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1. **(a)** It is given that a and b are positive constants such that $3b > a^2$. Explain why $y = x^2 + 2ax + 2a^2 - 3b$ has two x-intercepts for all real values of x. [3]

(b) The curve $y^2 + y - 3x = 9$ and the line $2x = 5 - y$ intersect at the points P and Q. Find the x-coordinate of P and Q. [3]

(c) An object is projected upwards from a mini launcher. The height, h m of the object above the ground at the time t seconds is given by
$h=-\frac{1}{5}t^2+4t+2$.

(i) Express $h=-\frac{1}{5}t^2+4t+2$ in the form $h=-a(t-b)^2+c$. [3]

(ii) Using your answer in **(i)**, explain whether the object can reach a height of 25 meters when projected from the mini laucher. [1]

2. A right circular cone has a vertical height of $\left(3-\sqrt{3}\right)$ cm and a slant height of l cm. The volume of the cone is $\left(10+2\sqrt{3}\right)\pi$ cm^3. **Without using a calculator,** express l^2 in the form $a+b\sqrt{3}$, where a and b are constants.

[Volume of cone = $\frac{1}{3}\pi r^2 h$]

[5]

3 **(a)** By considering the general term in the binomial expansion of $\left(x^2 - \frac{1}{3x}\right)^9$, explain why the powers of x are always divisible by 3. [3]

(b) Find the term independent of x in the expansion of $\left(x^2 - \frac{1}{3x}\right)^9 (1+3x^3)^2$. [5]

4. The points $A(5, 7)$ and $B(-11, 15)$ lie on a circle C. The centre of the circle O lies on the line $y = -2x - 3$.

(a) Find the equation of the perpendicular bisector of AB. [3]

(b) Find the equation of the circle. [4]

(c) Point D lies on the circle such that angle ABD is $90°$. Find the coordinates of D, show your working and explanation clearly. [2]

(d) Given that the circle intersects the line $x = k$ at two points, state the range of values of k. [1]

(e) The circle C_2 is a reflection of C in the y-axis. Find the equation of C_2. [1]

5. The equation of a curve is $y = \dfrac{1-\sin x}{\cos x}$.

(a) Show that $\dfrac{dy}{dx} = \tan x \sec x - \sec^2 x$. [2]

(b) Hence find $\int_0^{\frac{\pi}{4}} \tan x \sec x \, dx$, giving your answer in the exact value. [5]

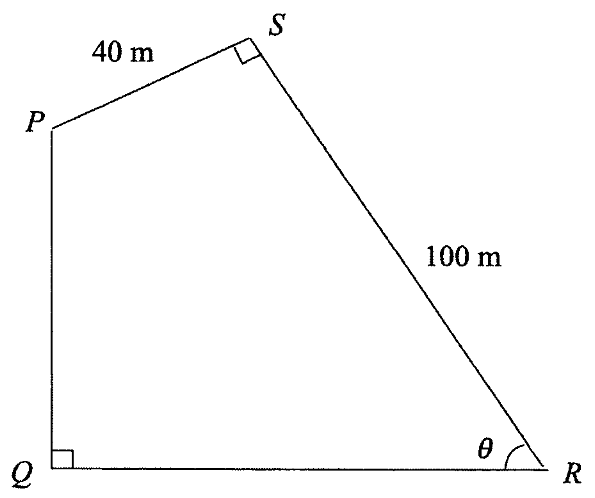
6. **(a)** Given that $\int_0^3 f(x)\,dx = \int_3^5 f(x)\,dx = 4$, find the value of k for which

$\int_0^5 3f(x)\,dx + \int_5^3 x - kf(x)\,dx = 8$. [4]

(b) **(i)** Express $\frac{-14x^2 + 14x - 3}{x(2x-1)^2}$ in partial fractions. [5]

(ii) Hence find $\int \frac{-14x^2+14x-3}{x(2x-1)^2}\,dx$. [3]

7. The diagram below shows a fenced field *PQRS*. It is given that $PS = 40$ m, $SR = 100$ m, both angles *PQR* and *PSR* are right angles and angle $QRS = \theta$, where θ is an acute angle in degrees.



(a) Show that the perimeter, *P* m, of the fence *PQRS* is given by $P = 140 + 140\sin\theta + 60\cos\theta$. [3]

(b) Express P in the form $140 + R\cos(\theta - \alpha)$, where $R > 0$ and $0° \leq \theta \leq 90°$. [3]

(c) Find the value of θ when $P = 250$. [2]

8. A curve is such that $\frac{d^2y}{dx^2} = 6e^{3x} - x$ and passes through the point $\left(2,\ \frac{2}{3}e^6\right)$.

(a) Given that $\frac{dy}{dx} = 5$ when $x = 0$, find the equation of the curve. [5]

(b) Find the equation of the tangent at the point $\left(2,\ \frac{2}{3}e^6\right)$ leaving your answer in the exact form. [3]

9. **(a)** Two variables x and y are related by the equation $y = ax^b$ where a and b are constants. When the graph of $\lg y$ against $\lg x$ is drawn, a straight line is obtained. Given that this line passes through the point (6, 7) and has a gradient of $\frac{1}{3}$, find the value of a and of b. [4]

(b) The table shows some experimental values of x and y which are known to be related by the equation $y = \frac{p}{x^2 + k}$. One of the values of y was recorded wrongly.

x	1.61	1.41	1.10	0.77	0.45
y	5.00	3.50	1.11	0.83	0.71

(i) On the grid on the next page, plot x^2 against $\frac{1}{y}$ and hence determine which value of y, in the table above, is the incorrect recording. [3]

(ii) Using the graph, estimate a value of y to replace the incorrect recoding of y found in part **(i)**. [2]

(iii) Use your graph to estimate the value of p and k. [3]

10. A particle X travels in a straight line so that t seconds after leaving a fixed point O, its velocity v m/s, is given by $v=\frac{1}{2}t^2-t-4$.

(a) Calculate the displacement of the particle X when the velocity is minimum. [4]

(b) Find the time when the particle turns. [2]

(c) Find the total distance travelled in the first 6 seconds. [3]

HUA YI SECONDARY SCHOOL

4E5N

Preliminary Examination 2022

4E5N

ADDITIONAL MATHEMATICS

Paper 1

MARKING SCHEME

1.	(a)		$p^2-4(1)(-4p+9)<0$ $p^2+16p-36<0$ $(p-2)(p+18)<0$ $-18<p<2$
	(b)		$x^2+4x-16+9=6x-8$ $x^2-2x+1=0$ $(x-1)^2=0$ $x=1$ Since there is only one solution, $y=6x-8$ is a tangent. Or use b^2-4ac.
2.	(a)	(i)	$f(-3)=2(-3)^3+a(-3)^2+b(-3)-6=0$ $-54+9a-3b-6=0$ $9a-3b=60$ $3a-b=20$ $f'(x)=6x^2+2ax+b$ $f'(1)=6(1)^2+2a(1)+b=1$ $2a+b=-5$ $5a=15$ $a=3$ $b=-11$
		(ii)	$2x^3+3x^2-11x-6=(x+3)(2x^2+px-2)$ $3p-2=-11$ $p=-3$ $2x^3+3x^2-11x-6=(x+3)(2x^2-3x-2)$ $2x^3+3x^2-11x-6=(x+3)(2x+1)(x-2)$

		(iii)	$2x^3+3x^2-11x-6=(x+3)(2x+1)(x-2)=0$ $x=-3,-\frac{1}{2},2$ $2+3y-11y^2-6y^3=0$ $\frac{1}{y}=-3,-\frac{1}{2},2$ $y=-\frac{1}{3},-2,\frac{1}{2}$
	(b)	$54x^6-16y^3$ $=2(27x^6-8y^3)$ $=2[(3x^2)^3-(2y)^3]$ $=2(3x^2-2y)(9x^4+6x^2y+4y^2)$	
3.	(a)	$V=\pi r^2h+\frac{2}{3}\pi r^3$ $60\pi=\pi r^2h+\frac{2}{3}\pi r^3$ $60\pi-\frac{2}{3}\pi r^3=\pi r^2h$ $h=\frac{60}{r^2}-\frac{2}{3}r$	
	(b)	$SA=2\pi rh+\pi r^2+2\pi r^2$ $C=3(2\pi rh+\pi r^2)+4(2\pi r^2)$ $=6\pi r\left(\frac{60}{r^2}-\frac{2}{3}r\right)+11\pi r^2$ $=\frac{360\pi}{r}-4\pi r^2+11\pi r^2$ $=\frac{360\pi}{r}+7\pi r^2$	
	(c)	$\frac{dC}{dr}=-\frac{360\pi}{r^2}+14\pi r=0$ $\frac{360\pi}{r^2}=14\pi r$ $r^3=\frac{360}{14}$ $r=2.9516$	

	(d)	When $r = 2.9516$, $C = \frac{360\pi}{2.9516} + 7\pi(2.9516)^2 = 574.76$ $\frac{d^2C}{dr^2} = \frac{720\pi}{r^3} + 14\pi > 0$ \$5.75 is a minimum value which is > \$5.60. He should not continue to make this product.
4.	**(a)**	$2^{4p+1} + 20(4^{p-1}) = 3$ $2(4^p)^2 + 5(4^p) = 3$ $2k^2 + 5k - 3 = 0$ $(2k-1)(k+3) = 0$ $k = \frac{1}{2}, -3$ $4^p = \frac{1}{2}$ or -3 (rej) $p = -\frac{1}{2}$
	(b)	$2u^2 + 5u - k = 0$ If no solution, $b^2 - 4ac < 0$ $25 + 8k < 0$ $k < -\frac{25}{8}$ OR $2^{4p+1} + 20(4^{p-1}) = k$ $2(4^p)^2 + 5(4^p) = k$ Since $4^p > 0$ for all p, $2(4^p)^2 + 5(4^p)$ is always positive for all p. Therefore, no solution for $k < -3\frac{1}{8}$.

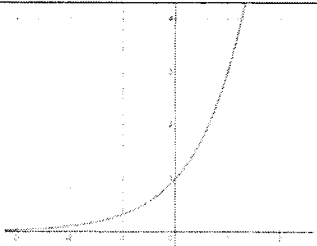
5	(a)	$\dfrac{(\log_y x)^2}{\log_x y}+64=0$ $\dfrac{(\log_y x)^2}{\dfrac{\log_x x}{\log_x y}}=-64$ $(\log_y x)^3=-64$ $\log_y x=-4$ $x=y^{-4}$ $y=x^{-\frac{1}{4}}$
	(b)	$2\log_2(x-1)-\log_2 x=3$ $\log_2\dfrac{(x-1)^2}{x}=3$ $\dfrac{x^2-2x+1}{x}=8$ $x^2-10x+1=0$ $x=0.101$ (rej) or 9.90

6.	(a)	Max = 9.9 m Min = 0.3 m
	(b)	$k\pi$ cycles in 2π hours 1 cycle in $\dfrac{2}{k}$ hours 2 cycles in $\dfrac{4}{k}$ hours $\dfrac{4}{k}=24$ $k=\dfrac{1}{6}$ Note: Do not accept $24k\pi=4\pi$ as working.

	(c)		$4.8\sin\frac{\pi t}{6}+5.1>2$ $4.8\sin\frac{\pi t}{6}>-3.1$ $\sin\frac{\pi t}{6}>-\frac{31}{48}$ Basic Angle $=0.702$ (Q3,4) $\frac{\pi t}{6}=\pi+0.702, 2\pi-0.702$ $t=7.34, 10.66$ $0<t<7.34, 10.66<t<12$
7.	(a)	(i)	$LHS=\frac{\cos x}{\sin x}-2\sin x\cos x$ $=\frac{\cos x-2\sin^2 x\cos x}{\sin x}$ $=\frac{\cos x(1-2\sin^2 x)}{\sin x}$ $=\cot x\cos 2x$
		(ii)	$4(\cot x-\sin 2x)=\cos 2x$ $4(\cot x\cos 2x)=\cos 2x$ $\cos 2x(4\cot x-1)=0$ $\cos 2x=0$ or $\cot x=\frac{1}{4}$ $\alpha=\pi$ (Q1,4) or $\tan x=4$ $2x=\frac{\pi}{2},\frac{3\pi}{2}$ or $\alpha=1.3258$ (Q1,3) $x=\frac{\pi}{4},\frac{3\pi}{4}$ or $x=1.33, 4.47$ (rej) Ans: $x=\frac{\pi}{4},\frac{3\pi}{4},1.33$
	(b)		$\sin A\cos B+\cos A\sin B=\frac{6}{7}$ $\sin A\cos B+\frac{2}{7}=\frac{6}{7}$ $\sin A\cos B=\frac{4}{7}$ $\frac{\sin A\cos B}{\cos A\sin B}=\frac{4}{7}\div\frac{2}{7}$ $\frac{\tan A}{\tan B}=2$

8.	**(a)**	$y=\frac{2}{\sqrt{3x+1}}=2(3x+1)^{-\frac{1}{2}}$ $\frac{dy}{dx}=-3(3x+1)^{-\frac{3}{2}}$ $\frac{dy}{dt}=\frac{dy}{dx}\cdot\frac{dx}{dt}$ $-3=-3(3x+1)^{-\frac{3}{2}}(0.125)$ $8=(3x+1)^{-\frac{3}{2}}$ $3x+1=\frac{1}{4}$ $x=-\frac{1}{4},\ y=4$	M1 M1 M1 M1 A1
	(b)	$y=2x\ln x$ $\frac{dy}{dx}=2x\left(\frac{1}{x}\right)+2\ln x=2+2\ln x$ For increasing functions, $1+\ln x>0$ $\ln x>-1$ $x>\frac{1}{e}$	M1 A1
9.	**(a)**	$y=(2x+1)^{-1}$ $\frac{dy}{dx}=-2(2x+1)^{-2}$ $x=0, \frac{dy}{dx}=-2$ Eqn of tangent: $y=-2x+c$ $1=-2(0)+c$ $c=1$ $y=-2x+1$	M1 M1 A1
	(b)	Where tangent intersects the x-axis, $y=0$ $0=-2x+1$ $x=\frac{1}{2}$ $Area=\int_0^1\frac{1}{2x+1}\,dx-\frac{1}{2}\left(\frac{1}{2}\right)1$ $=\left[\frac{1}{2}\ln(2x+1)\right]_0^1-\frac{1}{4}$ $=\frac{1}{2}\ln 3-\frac{1}{4}$	M1 M1 M1 A1

10.	**(a)**	Let $\angle FBC = x$, $\angle CAB = x$ (tangent chord theorem) $\angle FED = x$ (angles in same segment) By the converse of alternate angles, since $\angle CBA = \angle FED$, AB is parallel to DE.	M1 M1 A1
	(b)	ΔBCF and ΔEDF	B1 B1
	(b)	ΔABF is similar to ΔBCF. Therefore $\frac{AF}{BF} = \frac{BF}{CF}$ $AF \times CF = BF^2$ (shown)	M1 M1
11.	**(a)**	Grad $PR = \frac{10}{-5} = -2$ Grad $SQ = \frac{1}{2}$ Eqn SQ: $y = \frac{1}{2}x + c$ $7 = \frac{1}{2}(3) + c$ $c = \frac{11}{2}$ Eqn SQ: $y = \frac{1}{2}x + \frac{11}{2}$	M1 M1 A1
	(b)	Grad $PR = \frac{10}{-5} = -2$ Eqn: $y = -2x + c$ $7 = -2(3) + c$ $c = 13$ $y = -2x + 13$	M1 A1
	(c)	$Area\ PQR = \frac{1}{2}\begin{vmatrix} -3 & 2 & 3 & -3 \\ 9 & -1 & 7 & 9 \end{vmatrix}$ $= \frac{1}{2}[(3+14+27)-(18-3-21)]$ $= \frac{1}{2}(44-(-6))$ $= 25$	M1 A1

12	(a)		B1
	(b)	$\log_9(5-x)=\frac{x}{2}$ $(5-x)^2=3^{2x}$ $5-x=3^x$ Eqn straight line: $y=5-x$	 M1 A1
	(c)	1 solution	B1

HUA YI SECONDARY SCHOOL

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Preliminary Examination 2022

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ADDITIONAL MATHEMATICS

Paper 2

MARKING SCHEME

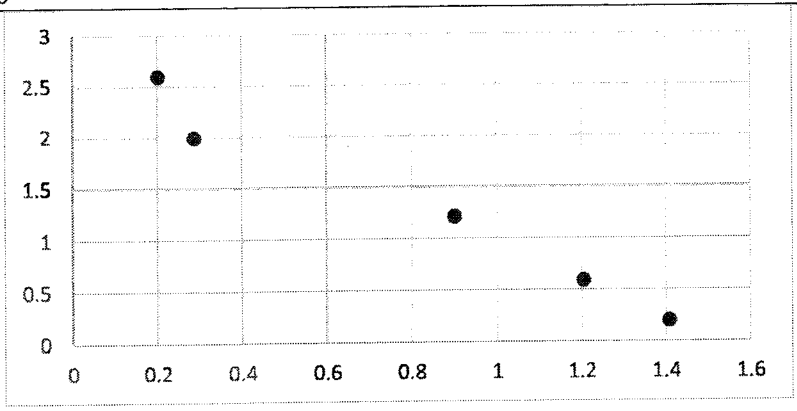
1	(a)		$b^2-4ac=(2a)^2-4(1)(2a^2-3b)$ $=4a^2-8a^2+12b$ $=12b-4a^2$ $=4(3b-a^2)$ Since $3b-a^2>0$, hence $b^2-ac>0$. Two real and distinct roots.
	(b)		Subst $y=5-2x$ into $y^2+y-3x=9$ $(5-2x)^2+5-2x-3x=9$ $25-20x+4x^2+5-5x=9$ $4x^2-25x+21=0$ $(4x-21)(x-1)=0$ $x=\frac{21}{4}$ or $x=1$
	(c)	(i)	$h=-\frac{1}{5}t^2+4t+2$ $=-\frac{1}{5}(t^2-20t-10)$ $=-\frac{1}{5}[(t-10)^2-100-10]$ $=-\frac{1}{5}[(t-10)^2-110]$ $=-\frac{1}{5}(t-10)^2+22$
		(ii)	Max height = 22 m
2			$(10+2\sqrt{3})\pi=\frac{1}{3}\pi r^2(3-\sqrt{3})$ $r^2=\frac{30+6\sqrt{3}}{3-\sqrt{3}}\times\frac{3+\sqrt{3}}{3+\sqrt{3}}$ $=\frac{90+30\sqrt{3}+18\sqrt{3}+18}{9-3}$ $=\frac{108+48\sqrt{3}}{6}=18+8\sqrt{3}$ $l^2=18+8\sqrt{3}+(3-\sqrt{3})^2$ $=18+8\sqrt{3}+9-6\sqrt{3}+3$ $=30+2\sqrt{3}$

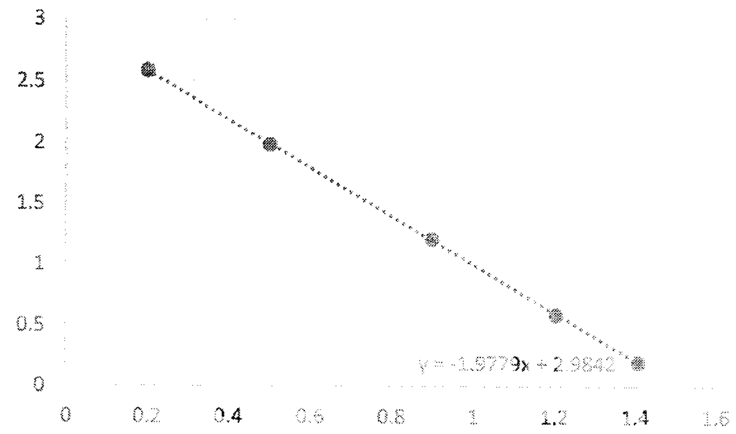
3	(a)	$\left(x^2-\frac{1}{3x}\right)^9$ $T_r=\binom{9}{r}\left(x^2\right)^{9-r}\left(-\frac{1}{3x}\right)^r$ Powers of $x=18-2r-r$ $=18-3r$ $=3(6-r)$ Multiple of 3
	(b)	$\left(x^2-\frac{1}{3x}\right)^9(1+6x^3+9x^6)$ Term with $x^{-6}, r=8 \to \frac{9}{3^8x^6}$ Term with $x^{-3}, r=7 \to -\frac{36}{3^7x^3}$ Term with $x^0, r=6 \to \frac{84}{3^6}$ Product $=\frac{84}{3^6}(1)-\frac{36}{3^7x^3}(6x^3)+\frac{9}{3^8x^6}(9x^6)=\frac{28}{243}-\frac{8}{81}+\frac{1}{81}=\frac{7}{243}$
4	(a)	$MidptAB=\left(\frac{5-11}{2},\frac{7+15}{2}\right)=(-3,11)$ $GradAB=\frac{15-7}{-11-5}=-\frac{1}{2}$ $GradPB=2$ $EqnPB: y=2x+c$ $11=2(-3)+c$ $c=17$ $Eqn: y=2x+17$
	(b)	$y=2x+17$------(1) $y=-2x-3$------(2) $2x+17=-2x-3$ $x=-5$ $y=7$ $C(-5,7)$ $Radius=\sqrt{(5+5)^2+(0)}=10$ $EqnCircle:(x+5)^2+(y-7)^2=100$

	(c)	AD is diameter $C(-5,7)$ is midpoint of AD $\left(\frac{5+x}{2},\frac{7+y}{2}\right)=(-5,7)$ $x=-15$ $y=7$ $D(-15,7)$
	(d)	$-15<k<5$
	(e)	$(x-5)^2+(y-7)^2=100$
5	**(a)**	$y=\frac{1-\sin x}{\cos x}$ $\frac{dy}{dx}=\frac{\cos x(-\cos x)-(1-\sin x)(-\sin x)}{\cos^2 x}$ $=\frac{-\cos^2 x+\sin x-\sin^2 x}{\cos^2 x}$ $=\frac{\sin x-1}{\cos^2 x}$ $=\tan x\sec x-\sec^2 x$
	(b)	$\int \tan x\sec x-\sec^2 x\,dx=\frac{1-\sin x}{\cos x}+c$ $\int \tan x\sec x\,dx-\int \sec^2 x\,dx=\frac{1-\sin x}{\cos x}+c$ $\int \tan x\sec x\,dx-\tan x=\frac{1-\sin x}{\cos x}+c$ $\int \tan x\sec x\,dx=\frac{1-\sin x}{\cos x}+\tan x+c$ $\int_0^{\frac{\pi}{4}} \tan x\sec x\,dx=\left[\frac{1-\sin x}{\cos x}+\tan x\right]_0^{\frac{\pi}{4}}$ $=\left[\frac{1-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}+1\right]-[1+0]$ $=\sqrt{2}-1+1-1=\sqrt{2}-1$

6	**(a)**		$\int_0^5 3f(x)\,dx+\int_5^3 x-kf(x)\,dx=8$ $3(8)-\int_3^5 x-kf(x)\,dx=8$ $24-\int_3^5 x\,dx+\int_3^5 kf(x)\,dx=8$ $24-\left[\frac{x^2}{2}\right]_3^5+k\int_3^5 f(x)\,dx=8$ $24-\left(\frac{25}{2}-\frac{9}{2}\right)+k(4)=8$ $4k=-8$ $k=-2$
	(b)	**(i)**	$\frac{-14x^2+14x-3}{x(2x-1)^2}=\frac{A}{x}+\frac{B}{2x-1}+\frac{C}{(2x-1)^2}$ $-14x^2+14x-3=A(2x-1)^2+Bx(2x-1)+Cx$ Let $x=0,-3=A$ Let $x=\frac{1}{2},-\frac{14}{4}+7-3=\frac{1}{2}C\to C=1$ Let $x=1,-14+14-3=A+B+C$ $-3=-2+B$ $B=-1$ $\frac{-14x^2+14x-3}{x(2x-1)^2}=-\frac{3}{x}-\frac{1}{2x-1}+\frac{1}{(2x-1)^2}$
		(ii)	$\int\frac{-14x^2+14x-3}{x(2x-1)^2}\,dx$ $=\int-\frac{3}{x}-\frac{1}{2x-1}+\frac{1}{(2x-1)^2}\,dx$ $=-3\ln x-\frac{1}{2}\ln(2x-1)+\frac{(2x-1)^{-1}}{2(-1)}+c$ $=-3\ln x-\frac{1}{2}\ln(2x-1)-\frac{1}{2(2x-1)}+c$
7	**(a)**		$PQ=100\sin\theta-40\cos\theta$ $QR=100\cos\theta+40\sin\theta$ $P=100+40+100\sin\theta-40\cos\theta+100\cos\theta+40\sin\theta$ $=140+140\sin\theta+60\cos\theta$

	(b)	$P = 140 + 140\sin\theta + 60\cos\theta$ $60\cos\theta + 140\sin\theta = R\cos(\theta - \alpha)$ $R = \sqrt{23200}$ $\alpha = 66.8°$ $P = 140 + \sqrt{23200}\cos(\theta - 66.8°)$
	(c)	$250 = 140 + \sqrt{23200}\cos(\theta - 66.8°)$ $\cos(\theta - 66.8°) = \dfrac{110}{\sqrt{23200}}$ Basic angle $= 43.764°$ (Q1, 4) $\theta - 66.8° = -43.764°$ or $43.764°$ $\theta = 23.0°$
8.	(a)	$\dfrac{d^2y}{dx^2} = 6e^{3x} - x$ $\dfrac{dy}{dx} = \dfrac{6e^{3x}}{3} - \dfrac{x^2}{2} + c$ $5 = 2 + c$ $c = 3$ $\dfrac{dy}{dx} = \dfrac{6e^{3x}}{3} - \dfrac{x^2}{2} + 3$ $y = \dfrac{2e^{3x}}{3} - \dfrac{x^3}{6} + 3x + c$ $\dfrac{2}{3}e^6 = \dfrac{2}{3}e^6 - \dfrac{4}{3} + 6 + c$ $c = -\dfrac{14}{3}$ $y = \dfrac{2e^{3x}}{3} - \dfrac{x^3}{6} + 3x - \dfrac{14}{3}$

	(b)	$\frac{dy}{dx}=2e^{3x}-\frac{x^2}{2}+3$ When $x=2$ $\frac{dy}{dx}=2e^6+1$ $y=(2e^6+1)x+c$ $\frac{2}{3}e^6=(2e^6+1)(2)+c$ $\frac{2}{3}e^6=4e^6+2+c$ $c=-\frac{10}{3}e^6-2$ $y=(2e^6+1)x-\frac{10}{3}e^6-2$
9.	(a)	$\lg y=\lg a+b\lg x$ $\lg y=b\lg x+\lg a$ $Y=\frac{1}{3}X+c$ $7=\frac{1}{3}(6)+c$ $c=5$ $b=\frac{1}{3}$ $\lg a=5$ $a=10^5$
	(b) (i)	 Axis M1, Points and line M1 Incorrect value: $y=3.5$ A1

		(ii)	 $\frac{1}{y}=\frac{1}{2}$ Correct value of $y=2$
		(iii)	$y=\frac{p}{x^2+k}$ $x^2y+ky=p$ $x^2y=-ky+p$ $x^2=\frac{p}{y}-k$ Gradient $=p=-2$ Vertical intercept $=-k=3$ $k=-3$
10	(a)		$v=\frac{1}{2}t^2-t-4$ $s=\frac{1}{6}t^3-\frac{1}{2}t^2-4t+c$ When $s=0, t=0$ therefore $c=0$ $s=\frac{1}{6}t^3-\frac{1}{2}t^2-4t$ At max velocity, $\frac{dv}{dt}=0$ $t-1=0$ $t=1$ Displacement $=\frac{1}{6}-\frac{1}{2}-4=-\frac{13}{3}$
	(b)		$t^2-2t-8=0$ $(t-4)(t+2)=0$ $t=4$ or -2

	(c)	$s=\frac{1}{6}t^3-\frac{1}{2}t^2-4t$ $t=0, s=0$ $t=4, s=-13\frac{1}{3}$ $t=6, s=-6$ Total distance travelled $=13\frac{1}{3}+7\frac{1}{3}=20\frac{2}{3}$ m

-End of Paper-