HOUGANG SECONDARY SCHOOL
PRELIMINARY EXAMINATION / 2022
SECONDARY FOUR (EXPRESS)

CANDIDATE NAME: ______________________ CLASS: ________

CENTRE NUMBER: S _ _ _ _ INDEX NUMBER: _ _ _ _

ADDITIONAL MATHEMATICS 4049/01

Paper 1

Friday 19 August 2022

2 hours 15mins

Candidates answer on the Question Paper

Instructions to students:

- Write your name, index number and class clearly in the spaces at the top of this page.
- Write in dark blue or black pen on spaces provided.
- You may use an HB pencil for any diagrams or graphs.
- Do not use staples, paper clips, glue or correction fluid.
- Answer **all** the questions in this paper.
- The use of an approved scientific calculator is expected, where appropriate.
- Give non-exact numerical answers correct to 3 significant figures, or one decimal place in case of angles in degrees, unless a different level of accuracy is specified in the question.
- You are reminded of the need for clear presentation in your answers.

Information for pupils

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total mark for this paper is 90.

Calculator Model: ____________________

/ 90

The Question Paper consists of 20 printed pages (including this cover page)

Mathematical Formulae

1. ALGEBRA

Quadratic Equation :

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem :

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n$$

where n is appositive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\dots\dots\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\cos ec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta ABC = \frac{1}{2} ab \sin C$$

1 The line $2y = 3x - 7$ meets the curve $y = x^2 + 3x - 8$ at two points A and B.
Find the distance between A and B. [4]

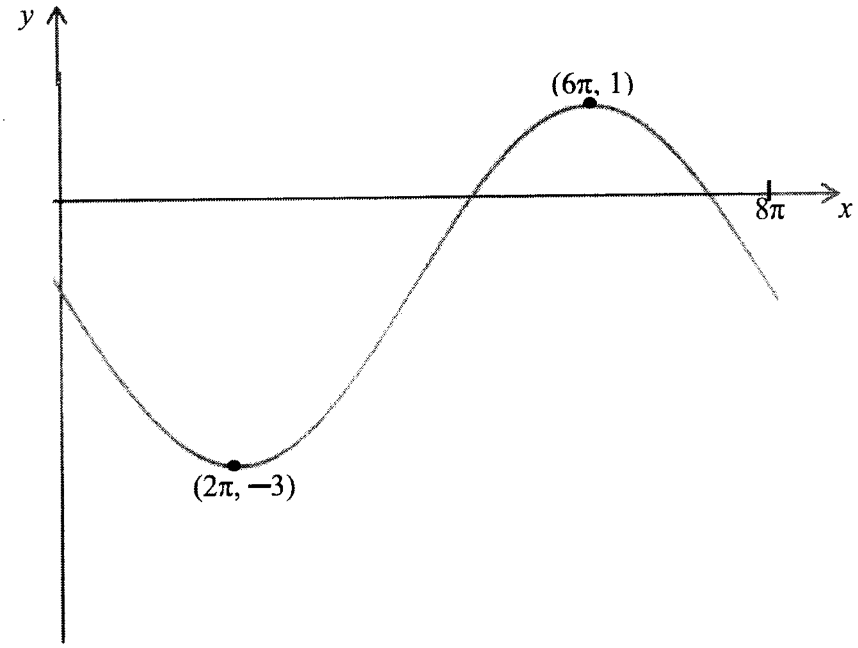
2 Express $\dfrac{7x+8}{(2x+1)(x-1)^2}$ in partial fractions. [5]

3 **Without using the calculator**, find the values of the integer a and b such that

$$\frac{4-\sqrt{3}}{a+b\sqrt{3}}=\frac{2+\sqrt{3}}{4-\sqrt{3}}.$$ [3]

4 The line of symmetry of a quadratic curve is $x=-2$ and the curve lies above the x-axis for all x. Given that the point $(-1,4)$ lies on the curve, find a possible equation of the curve in the form $y=a(x-h)^2+k$ where a, h, and k are integers. [4]

5



The diagram shows the curve $y = p\sin\frac{x}{q} + r$ for $0 \le x \le 8\pi$ radians. The curve has a minimum point at $(2\pi, -3)$ and a maximum point $(6\pi, 1)$.

(a) Show that $r = -1$. [1]

(b) Find the values of p and q. [2]

(c) Hence write down the equation of the curve. [1]

6 The function f is defined for all real values of x and is such that $f''(x) = 6x + 2$.

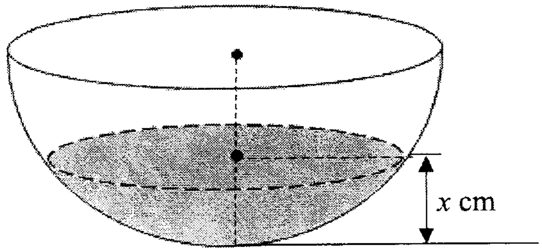
The gradient to the curve $y = f(x)$ at the point $(-1, 10)$ is 11.

(a) Find an expression for $f'(x)$. [3]

(b) Hence find the equation of the curve $y = f(x)$. [2]

(c) Determine whether the curve $y = f(x)$ have stationary point(s).
Explain with clear working. [3]

7



When the hemispherical bowl above contains water to a depth of x cm, the volume, V cm^3, of the water is given by $V = \frac{1}{3}\pi x^2(18 - x)$. The bowl is initially empty. After water has been poured into the bowl at a constant rate for 9 seconds, the depth of water is 4.5 cm.

(a) Find the constant rate of change of volume in terms of π. [3]

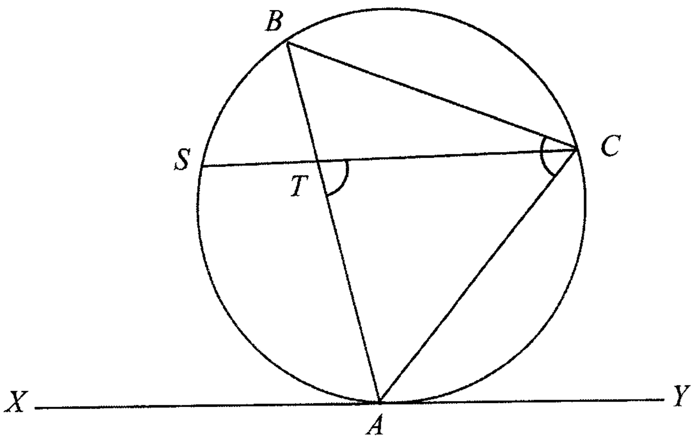
(b) Find the rate at which the water level is rising when the depth is 4.5cm. [4]

8 **(a)** Factorise $\sin^3 x + \cos^3 x$ completely. [1]

(b) Show that $\dfrac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = 1 - \dfrac{1}{2}\sin 2x$. [3]

(c) Hence solve the equation $\dfrac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = \dfrac{5}{4}$ for $0^\circ \le x \le 360^\circ$. [4]

9



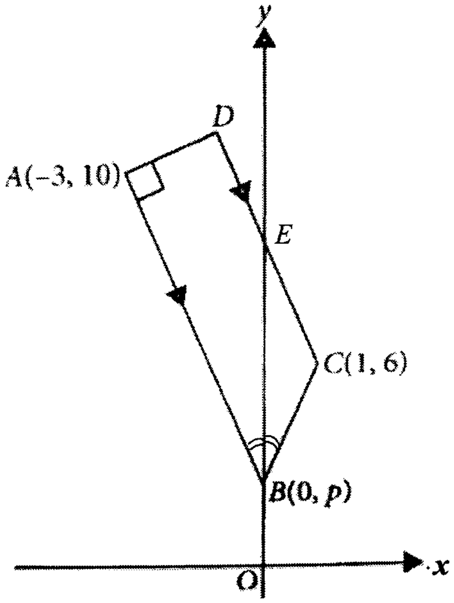
The diagram shows a point A on the circle and XAY is a tangent to the circle. Points S, B and C lie on the circle. The chords AB and SC intersect at T and angle ACB = angle ATC.

(a) Prove that triangles ABC and ACT are similar. [2]

(b) Show that $AC^2 - AT^2 = AT \times TB$. [3]

(c) Determine, with working, whether the lines *SC* and *XY* are parallel. [3]

10



The diagram shows a trapezium with vertices $A(-3,10)$, $B(0,p)$, $C(1,6)$ and D. The sides AB and DC are parallel and the angle BAD is a right angle. Angle ABE is equal to angle CBE.

(a) Express the gradients of lines AB and CB in terms of p and hence, or otherwise, show that $p = 4$. [3]

(b) Show that the equation of line AD is $y = \frac{1}{2}x + \frac{23}{2}$.

Hence find the coordinates of the point D. [5]

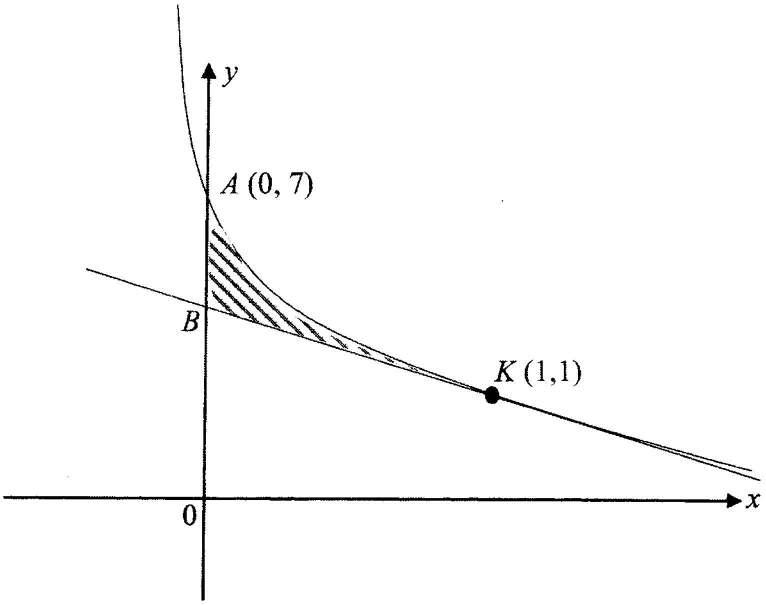
(c) Find the area of the trapezium *ABCD*. [2]

11 **(a)** The polynomial $P(x) = 5x^3 + ax^2 - x + b$, where a and b are constants is exactly divisible by $x^2 + 4x + 3$. Show that the value of $a = 16$ and find the value of b. [4]

(b) Hence solve the equation $P(x)=0$. [3]

(c) Using a suitable substitution and your answers in **(b)**, solve the equation
$5x^6+ax^4-x^2+b=0$. [2]

12



The diagram shows part of the curve $y=\dfrac{7}{6x+1}$ intersecting the y-axis at $A(0,7)$

The tangent to the curve at the point $K(1\,,1)$ intersects the y-axis at B.

(a) Find the coordinates of B. [5]

(b) Find the area of the shaded region bounded by the curve, the tangent KB and the y-axis. [5]

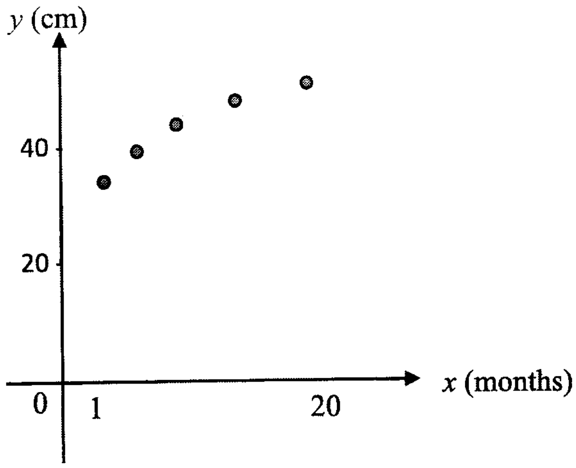
13 **(a)** Solve the equation $(\ln x)^2 + \dfrac{2}{\log_x e} = 3$. [4]

(b) It is given that $\lg p - \lg 2q = \lg(p+2q)$.

(i) Express p in terms of q [2]

(ii) State the range of values of p and explain clearly why $0 < q < \frac{1}{2}$. [2]

(c) Mrs Tan decides to track the relationship between the age, x (in months) of her newborn baby girl and the circumference of her baby girl's head, y (in centimetres).
After plotting the data collected for 1^{st}, 6^{th}, 12^{th}, 18^{th} and 20^{th} month, the following graph was obtained.



Determine, with a reason, which of the 2 equations below is suitable to model the data plotted in the above diagram.

(A) $y = ae^{bx}$ (Exponential function)
(B) $y = a\ln x + b$ (Logarithmic function) [2]

End of paper

HOUGANG SECONDARY SCHOOL
PRELIMINARY EXAMINATION / 2022
SECONDARY FOUR (EXPRESS)

CANDIDATE NAME:		CLASS:	
CENTRE NUMBER:	S	INDEX NUMBER:	

ADDITIONAL MATHEMATICS 4049 / 02

Paper 2

Tuesday 23 August 2022
2 hours 15 minutes

Candidates answer on the Question Paper

Instructions to students:
- Write your name, centre number, index number and class in the spaces at the top of this page.
- Write in dark blue or black pen on spaces provided.
- You may use a HB pencil for any diagrams or graphs.
- Do not use staples, paper clips, glue or correction fluid.
- Answer **all** the questions in this paper.
- Give non-exact numerical answers correct to 3 significant figures, or one decimal place in case of angles in degrees, unless a different level of accuracy is specified in the question.
- The use of an approved scientific calculator is expected, where appropriate.
- You are reminded of the need for clear presentation in your answers.

Information for pupils
- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total mark for this paper is 90.

Calculator Model: ____________________

/ 90

This question paper consists of 17 printed pages (including this cover page).

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is appositive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots\ldots\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\cos ec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 **(a)** Sketch the graph of $y = e^x + 2$. [2]

(b) Solve the equation $3 - e^{-x} = 2e^x$. [4]

[Turn over

2 The cubic polynomial f(x) is such that the coefficient of x^3 is -1 and the roots of $f(x) = 0$ are 1, k and k^2. It is given that f(x) has a remainder of -7 when divided by $x - 2$.

(i) Show that $k^3 - 2k^2 - 2k - 3 = 0$. [3]

(ii) Hence find a value for k and explain that there are no other real values of k which satisfy this equation. [6]

3 **(i)** Given that $y=(x+2)\sqrt{x-1}$, show that $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{kx}{2\sqrt{x-1}}$ where k is constant. [4]

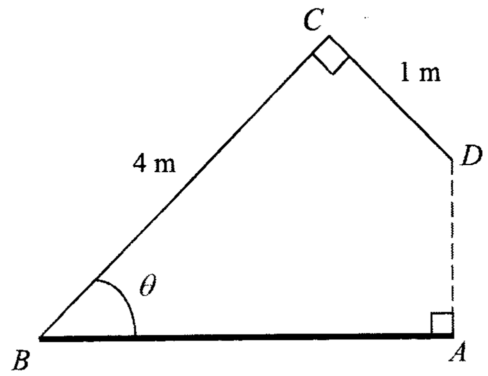
Hence

(ii) find the rate of change of x when $x=2$, given that y is changing at a constant rate of 2 units per second, [2]

(iii) evaluate $\int_2^5 \frac{x}{\sqrt{x-1}}\,\mathrm{d}x$. [3]

[Turn over

4



The diagram above shows the side view of a bus stop shelter BCD such that $BC = 4$ m, $CD = 1$ m, angle $BCD = 90°$ and angle $CBA = \theta$. AB is a concrete pavement under the shelter such that DA is perpendicular to AB.

(i) Show that $AB = 4\cos\theta + \sin\theta$. [2]

(ii) Express AB in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(iii) State the maximum value of *AB* and find the corresponding value of θ when *AB* is maximum. [2]

(iv) Find the value of θ when $AB = 3$ m. [2]

[Turn over

5 **(a)** A curve has the equation $y = 2x^2 - 6x + c$, where c is a constant.
Find the value of c for which the line $y + 2x = 8$ is a tangent to the curve. [3]

(b) Represent the solution set of $3(x^2 - 5) > x - 1$ on the number line. [3]

(c) Find the greatest value of integer p for which $-2x^2 + x - p$ has real roots for all real values of x. [3]

6 **(i)** Expand and simplify $\left(\frac{1}{2}-2x\right)^5$ in ascending powers of x, up to the first 4 terms. [2]

(ii) Hence find the value of a if the coefficient of x^2 in the expansion of

$\left(1+ax+3x^2\right)\left(\frac{1}{2}-2x\right)^5$ is $\frac{13}{2}$. [4]

(iii) Using the answer from part **(i)**, evaluate $(0.47)^5$ correct to 5 decimal places. [3]

[Turn over

7 The table shows experimental values of two variables, x and y.

x	0.5	1.0	1.5	2.0
y	15.9	19.1	23.4	30.2

It is known that x and y are related by the equation $y = 10 + Ab^x$.

(i) On **Pg 11**, draw the graph of $\lg(y-10)$ against x. [2]

(ii) Use your graph in **(i)** to estimate the value of A and of b. [4]

(ii) By drawing a suitable line on your graph, solve the equation $Ab^x = 10^{2x}$. [3]

[Turn over

8 A particle P moves in a straight line so that, t seconds after passing through a fixed point O, its velocity v m/s, is given by $v = 3t^2 + kt + 18$, where k is a constant. When $t = 1$, the acceleration of the particle is -9 m/s^2.

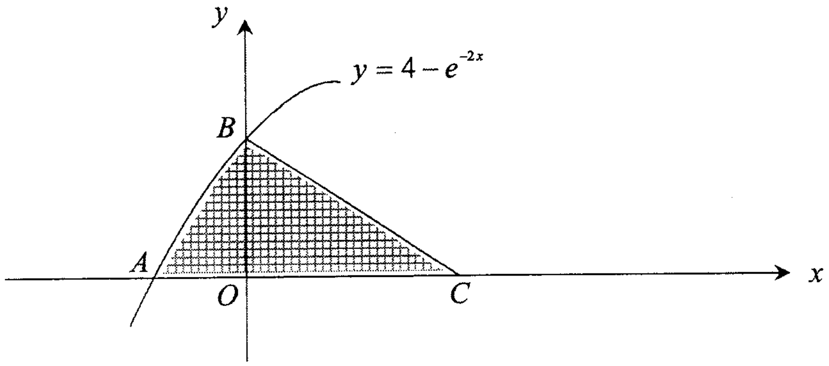
(i) Show that $k = -15$. [2]

(ii) Find the values of t for which particle P is instantaneously at rest. [2]

(iii) Find the total distance travelled by P in the first 3 seconds after passing through point O. [4]

[Turn over

9



The diagram shows part of the curve $y = 4 - e^{-2x}$ which crosses the axes at A and at B.

(i) Find the coordinates of A and of B. [2]

The normal to the curve at B meets the x-axis at C.

(ii) Find the coordinates of C. [4]

(iii) Find the area of the shaded region. [5]

10 A circle, C, has equation $x^2+y^2-6x+4y-12=0$.

(i) Find the radius and the coordinates of the centre of C. [2]

The equation of the normal to the circle at the point A is $3y=m-4x$.

(ii) Find the value of the constant m. [2]

The tangent to the circle at A cuts at the positive y-axis.

(iii) Use your answer in **(ii)** to show that A is (0, 2). [4]

(iv) B is a point on the circle. Given that the equation of tangent to the circle at B is parallel to the equation of the tangent to the circle at A, find the equation of tangent to the circle at B. [3]

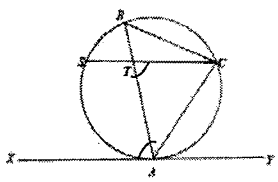
End of Paper

2022 Prelim A Maths Paper 1 Marking Scheme

Qn	Solution	Marks	Remarks
1	$y=x^2+3x-8$ —— (1) $y=\frac{3}{2}x-\frac{7}{2}$ —— (2) (1) = (2) $x^2+3x-8=\frac{3}{2}x-\frac{7}{2}$ $2x^2+3x-9=0$ $(2x-3)(x+3)=0$ $x=\frac{3}{2}$ or $x=-3$ $y=-\frac{5}{4}$ or $y=-8$ Coord of A and B are $\left(\frac{3}{2}, -\frac{5}{4}\right)$ & $(-3,-8)$. Distance btw A and B $=\sqrt{(1.5+3)^2+(-1.25+8)^2}$ $=8.11$ units	 [M1] [M1] [M1] [A1]	 Accept exact surd $\frac{\sqrt{1053}}{4}$
2	$\frac{7x+8}{(2x+1)(x-1)^2}=\frac{A}{2x+1}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$ $7x+8=A(x-1)^2+B(2x+1)(x-1)+C(2x+1)$ Let $x=1$, $15=3C$, $C=5$ Let $x=-\frac{1}{2}$, $\frac{9}{2}=A\left(-\frac{3}{2}\right)^2$, $A=2$ Let $x=0$, $8=2(-1)^2+B(1)(-1)+5(1)$, $B=-1$ $\frac{7x+8}{(2x+1)(x-1)^2}=\frac{2}{2x+1}-\frac{1}{x-1}+\frac{5}{(x-1)^2}$	[M1] [M1] [A3]	

3	$a+b\sqrt{3}=\dfrac{\left(4-\sqrt{3}\right)^2}{2+\sqrt{3}}$ $=\dfrac{19-8\sqrt{3}}{2+\sqrt{3}}\times\dfrac{2-\sqrt{3}}{2-\sqrt{3}}$ $=\dfrac{38-19\sqrt{3}-16\sqrt{3}+24}{2^2-3}$ $=62-35\sqrt{3}$ $a=62,\quad b=-35$	[M1] [M1] [A1]	Accept alternative method $\left(a+b\sqrt{3}\right)\left(2+\sqrt{3}\right)=\left(4-\sqrt{3}\right)^2$
4	$y=a\left(x+2\right)^2+k$ At $(-1,4)$, $4=a(-1+2)^2+k$ $a+k=4$ Since $a>0$ & $k>0$ as curve lies above x-axis, let $a=1$, then $k=3$ A possible equation for the curve is $y=\left(x+2\right)^2+3$	[M1] [M1] [M1] [A1]	(Any other suitable equation satisfying the relevant conditions accepted)
5(a)	$r=\dfrac{\max+\min}{2}=\dfrac{1+(-3)}{2}=-1$ (shown)	[B1]	
(b)	Period of curve $=\dfrac{2\pi}{\frac{1}{q}}=8\pi,\quad q=4$ Amplitude $=\dfrac{\max-\min}{2}=\dfrac{1-(-3)}{2}=2,\ p=-2$	[B1] [B1]	
(c)	Equation of the curve is $y=-2\sin\dfrac{x}{4}-1$	[B1]	
6(a)	$f'(x)=\int(6x+2)\,dx$ $=3x^2+2x+c$ At $x=-1$, $f'(x)=11$ $11=3(-1)^2+2(-1)+c$ $c=10$ $f'(x)=3x^2+2x+10$	[M1] [M1] [A1]	

(b)	$f(x)=\int(3x^2+2x+10)\,dx$ $=x^3+x^2+10x+d$ At $(-1,10)$, $10=(-1)^3+(-1)^2+10(-1)+d$ $d=20$ $f(x)=x^3+x^2+10x+20$	[M1] [A1]	
(c)	For $y=f(x)$ to have stationary points, set $f'(x)=0$. $3x^2+2x+10=0$ $b^2-4ac=2^2-4(3)(10)=-116<0$ $f'(x)=0$ has no real solution, ie no stationary points.	[M1] [M1] [A1]	Solve equation to show no real roots accepted No marks if no real roots is not mentioned
7(a)	$x=4.5,\quad V=\frac{1}{3}\pi(4.5)^2(18-4.5)$ $=\frac{729}{8}\pi$ $\frac{dV}{dt}=\frac{\frac{729}{8}\pi}{9}=\frac{81}{8}\pi$ cm/s (or $10\frac{1}{8}\pi$ cm/s)	[M1] [M1.A1]	Accept 10.125π *or* 31.8 cm/s
(b)	$V=\frac{\pi}{3}(18x^2-x^3)$ $\frac{dV}{dx}=\frac{\pi}{3}(36x-3x^2)$ *At* $x=4.5$, $\frac{dV}{dx}=\frac{\pi}{3}(36(4.5)-3(4.5)^2)$ $=\frac{135}{4}\pi$ Using $\frac{dV}{dt}=\frac{dV}{dx}\cdot\frac{dx}{dt}$ $\frac{dx}{dt}=\frac{\frac{81}{8}\pi}{\frac{135}{4}\pi}=0.3$ The water level is rising at a rate of 0.3 cm/s	[M1] [M1] [M1. A1]	

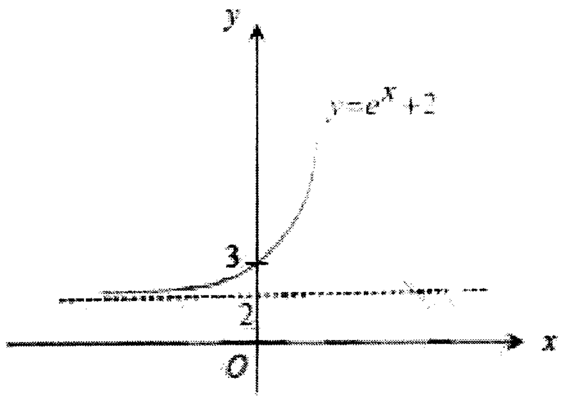
8(a)	$\sin^3 x + \cos^3 x$ $= (\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x)$	[B1]	
(b)	$LHS = \dfrac{\sin^3 x + \cos^3 x}{\sin x + \cos x}$ $= \dfrac{(\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x)}{\sin x + \cos x}$ $= \sin^2 x - \frac{1}{2}(2\sin x \cos x) + \cos^2 x$ $= 1 - \frac{1}{2}\sin 2x = RHS$	[M1] [A1, A1]	No mark awarded if student do not show $\sin x \cos x = \frac{1}{2}(2\sin x \cos x)$
(c)	$1 - \frac{1}{2}\sin 2x = \frac{5}{4}$ $\frac{1}{2}\sin 2x = -\frac{1}{4}$ $\sin 2x = -\frac{1}{2}$ Basic angle, $\alpha = 30^o$ $2x = 210^o, 330^o, 570^o, 690^o$ $x = 105^o, 165^o, 285^o, 345^o$	[M1] [M1] [M1] [A1]	
9(a)	$\angle ACB = \angle ATC$ (given) $\angle BAC = \angle CAT$ (common) $\therefore \Delta ABC \,\&\, \Delta ACT$ are similar (AA Test)	[M1] [A1]	Either statement
(b)	Since $\Delta ABC \,\&\, \Delta ACT$ are similar , $\dfrac{AB}{AC} = \dfrac{AC}{AT}$ $AC^2 = (AB)(AT)$ $= (AT + TB)(AT)$ $= AT^2 + AT \times TB$ $AC^2 - AT^2 = AT \times TB$ (shown)	[M1] [M1] [A1]	
(c)	$\angle BAX = \angle ACB$ (alt. segment thm) $\& \angle ACB = \angle ATC$ (given) $\therefore \angle BAX = \angle ATC$ By the alternate angle property, SC and XY are parallel.	[M1] [M1] [A1]	

10(a)	$m_{AB}=\frac{p-10}{3}$ $m_{CB}=\frac{6-p}{1}$ Since $\angle ABO=\angle CBO, m_{AB}=-m_{CB}$ $\frac{p-10}{3}=-(6-p)$ $p-10=-18+3p$ $-2p=-8$ $p=4$ (shown)	[M1] [M1] [A1]	Accept method involving tan θ
(b)	$m_{AB}=-2$ $m_{AD}=\frac{1}{2}$ Equation of line *AD* is $y-10=\frac{1}{2}(x+3)$ $y=\frac{1}{2}x+\frac{23}{2}$ (shown) $CD \mathbin{//} AB,\ m_{CD}=-2$ Equation of line *CD* is $y-6=-2(x-1)$ $y=-2x+8$ At *D*, $\frac{1}{2}x+\frac{23}{2}=-2x+8$ $\frac{5}{2}x=-\frac{7}{2}$ $x=-\frac{7}{5}, y=-2\left(-\frac{7}{5}\right)+8=\frac{54}{5}$ Coord of *D* is $\left(-\frac{7}{5},\frac{54}{5}\right)$	[M1] [A1] [M1] [M1] [A1]	Accept (-1.4, 10.8)

(c)	Area of $ABCD$ $=\frac{1}{2}\begin{vmatrix} 0 & 1 & -1.4 & -3 & 0 \\ 4 & 6 & 10.8 & 10 & 4 \end{vmatrix}$ $==\frac{1}{2}\left[(0+10.8-14-12)-(4-8.4-32.4)\right]$ $=10.8$ units^2	[M1] [A1]	
11(a)	$x^2+4x+3=(x+3)(x+1)$ By Factor Theorem, $P(-3)=0$ & $P(-1)=0$ $5(-3)^3+a(-3)^2+3+b=0$ $9a+b=132$ ----(1) $5(-1)^3+a(-1)^2+1+b=0$ $a+b=4$ ----(2) (1)−(2): $8a=128$ $a=16,\ b=-12$	[M1] [M1] [M1] [A1]	*overall minus 1 mark if any expression is not set to 0
(b)	$5x^3+16x^2-x-12=(x+3)(x+1)(px+q)$ By comparison, $p=5$ & $q=-4$ $P(x)=0$ $(x+3)(x+1)(5x-4)=0$ $x=-3$ or -1 or $\frac{4}{5}$	[M1] [M1] [A1]	
(c)	$5(x^2)^3+a(x^2)^2-x^2+b=0$ ---- (1) Let $u=x^2$, (1) becomes $5u^3+au^2-u+b=0$ From (b), $u=-3(rej)$ *or* $-1(rej)$ *or* $\frac{4}{5}$ $\therefore x^2=\frac{4}{5}$ $x=\frac{2}{\sqrt{5}}$ or $-\frac{2}{\sqrt{5}}$	[M1] [A1]	Accept $\pm\frac{2\sqrt{5}}{5}$

12(a)	$y=\frac{7}{6x+1}=7(6x+1)^{-1}$ $\frac{dy}{dx}=-7(6x+1)^{-2}(6)$ $=-\frac{42}{(6x+1)^2}$ At K, $x=1$, $\frac{dy}{dx}=-\frac{42}{7^2}=-\frac{6}{7}$ Equation of tangent at K is $y-1=-\frac{6}{7}(x-1)$ $y=-\frac{6}{7}x+\frac{13}{7}$ Coord of B is $\left(0,\frac{13}{7}\right)$	[M1] [M1] [M1] [M1] [A1]	
(b)	Area of the shaded region = Area under curve – Area of trapezium $=\int_0^1 \frac{7}{6x+1}dx-\frac{1}{2}(1)\left(1+\frac{13}{7}\right)$ $=7\left[\frac{1}{6}\ln(6x+1)\right]_0^1-\frac{10}{7}$ $=\frac{7}{6}(\ln 7-\ln 1)-\frac{10}{7}$ $=0.842$ units^2	[M1,,M1] [M1] [M1] [A1]	
13(a)	$(\ln x)^2+\frac{2}{\frac{\ln e}{\ln x}}=3$ $(\ln x)^2+2\ln x-3=0$ Let $u=\ln x$ $u^2+2u-3=0$ $(u+3)(u-1)=0$ $u=-3$ or $u=1$ $\ln x=-3$ or $\ln x=1$ $x=e^{-3}=\frac{1}{e^3}$ or $x=e$	[M1] [M1] [M1] [A1]	Accept $x=0.0498$ or $x=2.72$

(b)	$\lg\left(\frac{p}{2q}\right)=\lg(p+2q)$ $\frac{p}{2q} = p+2q$ (i) $p = 2pq+4q^2$ $p(1-2q)=4q^2$ $p=\frac{4q^2}{1-2q}$	[M1] [A1]	
	(ii) Range of p is $p>0$ $\therefore \frac{4q^2}{1-2q}>0$ Since $q>0$ for lg $2q$ to be defined, $4q^2>0$ & $1-2q>0$ $q<\frac{1}{2}$ $\therefore 0<q<\frac{1}{2}$ (shown)	[B1] [B1]	
(c)	By observing the shape of the curve, a logarithmic function, ie equation (B) $y=a\ln x+b$ is a suitable model since the rate of growth of the head circumference gets much slower when the baby gets older over the months.	[B1] [B1]	-for correct model -for correct reasoning

Marking Scheme for A Maths Paper 2			
Qn	**Working**	**Marks**	**Remarks**
1(a)		B1 B1	Shape of the curve y-intercept and horizontal asymptote
(b)	$3 - e^{-x} = 2e^x$ $3 - \frac{1}{e^x} = 2e^x$ $3e^x - 1 = 2e^{2x}$ $2e^{2x} - 3e^x + 1 = 0$ Let $y = e^x$ $2y^2 - 3y + 1 = 0$ $(2y-1)(y-1) = 0$ $y = \frac{1}{2}$ or $y = 1$ $e^x = \frac{1}{2}$ or $e^x = 1$ $x = \ln\frac{1}{2}$ or $x = \ln 1$ $x = -0.693$ or $x = 0$	M1 M1 M1 A1	

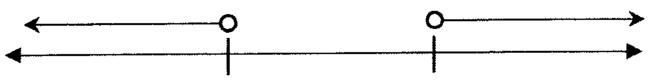
2(i)	Since roots of f(x) = 0 are 1, k and k^2 $f(x)=-(x-1)(x-k)(x-k^2)$ $f(2)=-7$ $-(2-1)(2-k)(2-k^2)=-7$ $(2-k)(2-k^2)=7$ $4-2k^2-2k+k^3=7$ $k^3+2k^2+2k-3=0$ (shown)	M1 M1 A1	
(ii)	Let $g(k)=k^3-2k^2-2k-3$ $g(3)=3^3-2(3)^2-2(3)-3$ $=0$ Since g(3) = 0, $k-3$ is a factor of g(k). $k^3-2k^2-2k-3=0$ $(k-3)(k^2+k+1)=0$ $k^2+k+1=0$ $k=3$ or $b^2-4ac=1^2-4(1)(1)$ $=-3<0$ (no real roots) Therefore $k^3-2k^2-2k-3=0$ has only 1 real root.	M1 M1 M1, M1 M1 B1	

[Turn over

3(i)	$y=(x+2)\sqrt{x-1}$		
	$\frac{dy}{dx}=(x+2)[\frac{1}{2}(x-1)^{-\frac{1}{2}}]+\sqrt{x-1}(1)$	M1	
	$=(x-1)^{-\frac{1}{2}}[\frac{1}{2}x+1+x-1]$	M1	
	$=(x-1)^{-\frac{1}{2}}(\frac{3}{2}x)$	M1	
	$=\frac{3x}{2\sqrt{x-1}}$	A1	
(ii)	By Chain Rule,		
	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$		
	$2=\frac{3x}{2\sqrt{x-1}}\times\frac{dx}{dt}$	M1	
	$\frac{dx}{dt}=2\div\frac{3(2)}{2\sqrt{2-1}}$		
	$=\frac{2}{3}$ units/s	A1	
(iii)	$\int_2^5\frac{x}{\sqrt{x-1}}\,dx=\frac{2}{3}\int_2^5\frac{3x}{2\sqrt{x-1}}\,dx$	M1	
	$=\frac{2}{3}[(x+2)\sqrt{x-1}]_2^5$		
	$=\frac{2}{3}[7\sqrt{4}-4\sqrt{1}]$	M1	
	$=\frac{2}{3}(10)$		
	$=6\frac{2}{3}$	A1	

4(i)	Let E be the point on AB such that BE is perpendicular to CE. F is the point on CE such that CF is perpendicular to FD. $\cos\theta = \frac{BE}{4}$ $BE = 4\cos\theta$ $\sin\theta = \frac{FD}{1}$ $FD = \sin\theta$ $AB = BE + FD$ $= 4\cos\theta + \sin\theta$ (shown)	M1 for either one correct A1	
(ii)	$AB = \sqrt{4^2+1^2}\cos\left(\theta - \tan^{-1}\frac{1}{4}\right)$ $= \sqrt{17}\cos(\theta - 14.036°)$ $= \sqrt{17}\cos(\theta - 14.0°)$	M1 for finding R, M1 for finding α A1	
(iii)	Max $AB = \sqrt{17}$ m For AB to be maximum, the value of $\cos(\theta - 14.036°)$ must be 1. $\cos(\theta - 14.036°) = 1$ $\theta - 14.036° = 0°$ $\theta = 14.036°$ $= 14.0°$ (1 d.p.)	B1 B1	
(iv)	$3 = \sqrt{17}\cos(\theta - 14.036°)$ $\cos(\theta - 14.036°) = \frac{3}{\sqrt{17}}$ $\theta - 14.036° = 43.313°$ $\theta = 43.313° + 14.036°$ $= 57.349°$ $= 57.3°$ (1 d.p.)		

[Turn over

5(a)	$y = 2x^2 - 6x + c$ --------- (1) $y + 2x = 8$ --------------- (2) Equating (1) & (2): $2x^2 - 6x + c = 8 - 2x$ $2x^2 - 4x + c - 8 = 0$ The line is a tangent to the curve, $b^2 - 4ac = 0$ $(-4)^2 - 4(2)(c-8) = 0$ $16 - 8c + 64 = 0$ $c = 10$	 M1 M1 A1	
(b)	$3(x^2 - 5) > x - 1$ $3x^2 - 15 - x + 1 > 0$ $3x^2 - x - 14 > 0$ $(3x-7)(x+2) > 0$ $x < -2$ or $x > 2\frac{1}{3}$ 	 M1 A1 A1	
(c)	$-2x^2 + x - p$ $a = -2,\ b = 1,\ c = -p$ For real roots, $b^2 - 4ac \geq 0$ $1 - 4(-2)(-p) \geq 0$ $1 - 8p \geq 0$ $p \leq \frac{1}{8}$ Greatest value of integer $p = 0$	 M1 M1 A1	

6(i)	$\left(\frac{1}{2}-2x\right)^5$ $=\left(\frac{1}{2}\right)^5+\binom{5}{1}\left(\frac{1}{2}\right)^4(-2x)^1+\binom{5}{2}\left(\frac{1}{2}\right)^3(-2x)^2+\binom{5}{3}\left(\frac{1}{2}\right)^2(-2x)^3+$ $=\frac{1}{32}-\frac{5x}{8}+5x^2-20x^3+$	M1 A1	
(ii)	$\left(1+ax+3x^2\right)\left(\frac{1}{2}-2x\right)^5$ $=\left(1+ax+3x^2\right)\left(\frac{1}{32}-\frac{5x}{8}+5x^2-20x^3+...\right)$ Considering x^2, $5x^2-\frac{5}{8}ax^2+\frac{3}{32}x^2=\frac{13}{2}x^2$ $5-\frac{5}{8}a+\frac{3}{32}=\frac{13}{2}$ $a=-2\frac{1}{4}$	M1 M1 M1 A1	
(iii)	$(0.47)^5=\left(\frac{1}{2}-2x\right)^5$ $\frac{1}{2}-2x=0.47$ $x=0.015$ $(0.47)^5=\frac{1}{32}-\frac{5(0.015)}{8}+5(0.015)^2-20(0.015)^3+...$ ≈ 0.02293	M1 M1 A1	

[Turn over

<table>
<tr>
<td>7(i)</td>
<td>$y = 10 + Ab^x$
$y - 10 = Ab^x$
Take log to the base 10 on both sides,
$\lg(y-10) = \lg b(x) + \lg A$
Plot $\lg(y-10)$ against x to obtain a straight line.
<table>
<tr><td>x</td><td>0.5</td><td>1.0</td><td>1.5</td><td>2.0</td></tr>
<tr><td>y</td><td>15.9</td><td>19.1</td><td>23.4</td><td>30.2</td></tr>
<tr><td>$y - 10$</td><td>5.9</td><td>9.1</td><td>13.4</td><td>20.2</td></tr>
<tr><td>$\lg(y-10)$</td><td>0.77</td><td>0.96</td><td>1.13</td><td>1.31</td></tr>
</table>
</td>
<td>M1, G1</td>
<td>1 mark for drawing the straight line</td>
</tr>
<tr>
<td>(ii)</td>
<td>$\lg A =$ vertical intercept of the graph
$= 0.60$ (Accept ± 0.02)
$A = 10^{0.60}$
≈ 3.98

$\lg b =$ Gradient of the graph
$= \frac{1.31 - 0.60}{2 - 0}$
$= 0.355$ (Accept ± 0.02)
$b = 10^{0.355}$
≈ 2.26</td>
<td>M1

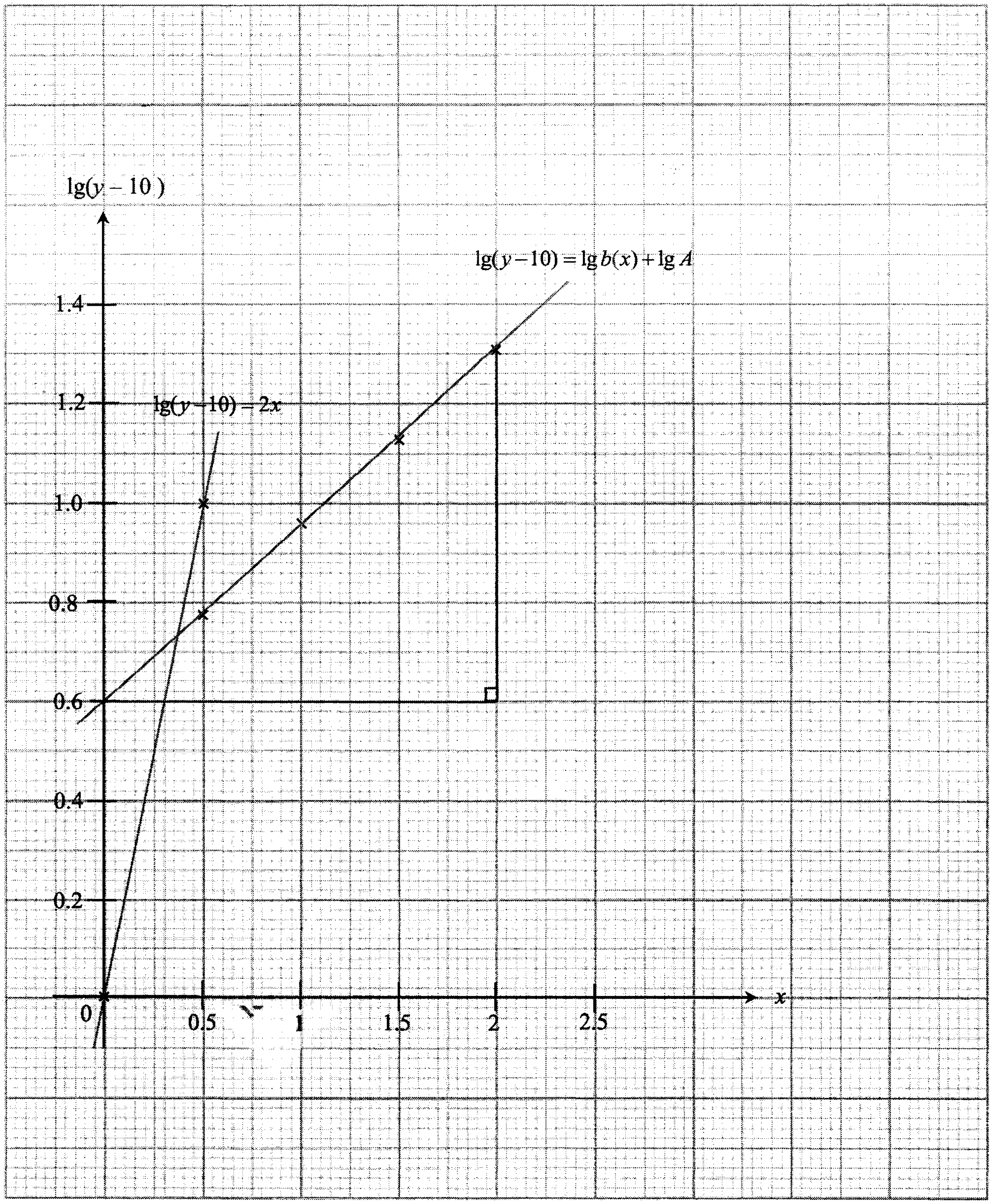
A1

M1

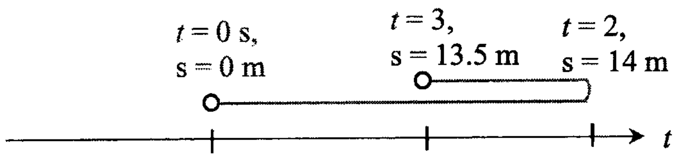
A1</td>
<td></td>
</tr>
<tr>
<td>(iii)</td>
<td>$Ab^x = 10^{2x}$
$\lg A + x \lg b = 2x$
$\therefore$ draw $\lg(y-10) = 2x$

From the graph, $x \approx 0.375$ (Accept ± 0.01)</td>
<td>
M1
M1

A1</td>
<td></td>
</tr>
</table>



[Turn over

8(i)	$v = 3t^2 + kt + 18$ $a = \dfrac{dv}{dt}$ $= 6t + k$ When $t = 1$, $6(1) + k = -9$ $k = -15$ (shown)	M1 A1	
(ii)	$3t^2 - 15t + 18 = 0$ $t^2 - 5t + 6 = 0$ $(t-2)(t-3) = 0$ $t = 2$ or $t = 3$	M1 A1 for both answers	
(iii)	$s = \int v \, dt$ $= \int (3t^2 - 15t + 18) \, dt$ $= t^3 - \dfrac{15}{2}t^2 + 18t + C$ When $t = 0, s = 0, C = 0$ $\therefore s = t^3 - \dfrac{15}{2}t^2 + 18t$ when $t = 2$ s, $s = 14$ m when $t = 3$ s, $s = 13.5$ m  Distance travelled in first 3 s = 14 + (14 – 13.5) = 14.5 m	 M1 M1 M1 A1	

9(i)	when $y=0$, $4-e^{-2x}=0$ $e^{-2x}=4$ $-2x=\ln 4$ $x=-\ln 2$ $\therefore A(-\ln 2,0)$ when $x=0$, $y=4-e^{0}$ $=3$ $\therefore B(0,3)$	B1 B1	
(ii)	$y=4-e^{-2x}$ $\frac{dy}{dx}=-(-2)e^{-2x}$ $=2e^{-2x}$ when $x=0$, $\frac{dy}{dx}=2e^{0}$ $=2$ Gradient of $BC=-\frac{1}{2}$ Let the coordinates of C be (x, y). $\frac{3-0}{0-c}=-\frac{1}{2}$ $c=6$ $\therefore C(6,0)$	M1 M1 M1 A1	
(iii)	Area of shaded region $=\int_{-\ln 2}^{0}(4-e^{-2x})\ dx+\frac{1}{2}(6)(3)$ $=\left[4x+\frac{1}{2}e^{-2x}\right]_{-\ln 2}^{0}+9$ $=\left(0+\frac{1}{2}\right)-\left(-4\ln 2+\frac{1}{2}e^{2\ln 2}\right)+9$ $=7\frac{1}{2}+4\ln 2$ ≈ 10.3 units^2	M1, M1 M1 M1 A1	

[Turn over

10(i)	$(x-3)^2+(y+2)^2=12+9+4$ $(x-3)^2+(y+2)^2=25$ $C(3,-2)$ Radius $=\sqrt{25}$ $=5$ units	B1 B1	
(ii)	Normal of a circle passes through centre of a circle. $3(-2)=m-4(3)$ $m=6$	M1 A1	
(iii)	$3y=6-4x$ $y=-\frac{4}{3}x+2$ ---------------- (1) $(x-3)^2+(y+2)^2=25$ ---------------- (2) $(x-3)^2+(-\frac{4}{3}x+2+2)^2=25$ $(x-3)^2+(4-\frac{4}{3}x)^2=25$ $x^2-6x+9+(\frac{16}{9}x^2-\frac{32}{3}x+16)=25$ $\frac{25}{9}x^2-\frac{50}{3}x=0$ $25x^2-150x=0$ $25x(x-6)=0$ $x=6$ (rejected) or $x=0$ when $x=0$, $y=-\frac{4}{3}(0)+2$ $y=2$ $\therefore A(0,2)$ (shown)	M1 M1 A1 A1	
(iv)	$(3,-2)$ is midpoint of AB and the required coordinates B be $B(e,f)$ $(\frac{0+e}{2},\frac{2+f}{2})=(3,-2)$ $B(6,-6)$ Equation of tangent at B: $y+6=\frac{3}{4}(x-6)$ $y=\frac{3}{4}x-\frac{21}{2}$	M1 M1 A1	