O

GAN ENG SENG SCHOOL
Preliminary Examination 2022

CANDIDATE NAME	

CLASS | | | **INDEX NUMBER** | | |

ADDITIONAL MATHEMATICS
4049/01

Paper 1

24 August 2022
2 hours 15 minutes

Sec 4 Express/ 5 Normal (Academic)

Candidates answer on the Question Paper

No Additional Materials are required

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 90.

	For Examiner's Use
Total	/ 90

This paper consists of **22** printed pages (including the cover page).

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\cdots+\binom{n}{r}a^{n-r}b^r+\cdots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!r!}=\frac{n(n-1)...(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A$$

$$cosec^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}ab\sin C$$

1. A square pyramid has a base area of $\left(2\sqrt{5}+7\right)\text{cm}^2$ and a volume of $\left(34\sqrt{5}+32\right)\text{cm}^3$.
Find the height of the pyramid in the form $p+q\sqrt{5}$, where p and q are integers. [3]

2. The gradient function of a curve is $3\cos 4x - \sec^2 3x$. Given that the curve passes through the point $\left(\frac{\pi}{12}, \frac{\sqrt{3}}{4}-\frac{1}{3}\right)$, find the equation of the curve. [4]

3. A cannon ball fired from a cannon that is mounted on a platform follows a parabolic trajectory that can be modelled by the function $y=-0.01x^2+3.5x+1.3$.
x m is the horizontal distance travelled by the cannon ball and y m is the height of the cannon ball above the ground.

(a) State the height of the platform. [1]

(b) (i) Express the function in the form $a(x-h)^2+k$, where a, h and k are constants. [3]

(ii) Hence, determine the highest height that a target can be located above the ground that is able to be hit by the cannon. [1]

4. Given that $y=\dfrac{\ln(x+3)^3}{3x+9}$ for $x>-3$, find the set of values of x for which y is an increasing function of x. Give your answer in terms of e. [4]

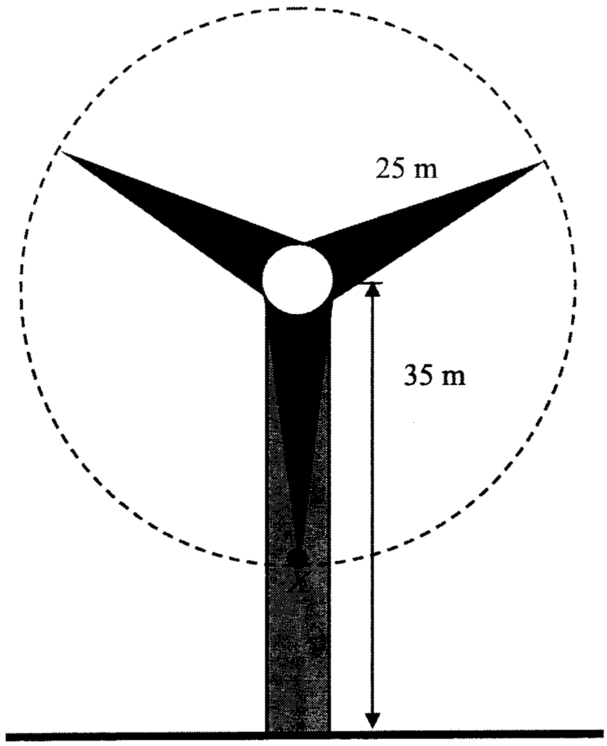
5. Express $\dfrac{5x^3-4x^2+15x-21}{(x-1)(x^2+4)}$ in partial fractions. [6]

6. The polynomial f(x) is such that the coefficient of x^4 is 2. The roots of the equation $f(x)=0$ are $\frac{3}{2}$ and -1. f(x) has a remainder of -8 when divided by $x-1$ and a remainder of 28 when divided by $x+2$.

(a) Find an expression for f(x). [4]

(b) Show with clear working the number of real roots for $f(x) = 0$. [2]

7. The diagram shows a wind turbine with blade 25 m in length. The centre of the wind turbine is 35 m from the ground. The height, h m, of the tip of a particular blade above the ground t seconds after leaving X can be modelled by $h = a\cos bt + c$, where k is a constant.
The blades of the wind turbine rotate at a speed of 1 revolution for every 3π seconds.

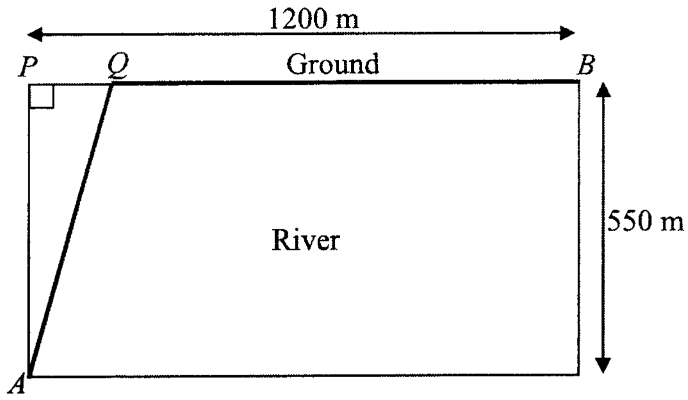


(a) Find the values of a, b and c. [3]

(b) Hence sketch the graph of $h = a\cos bt + c$ for $0 < t < 6\pi$. [2]

(c) Find how long it would take for the blade to first be 42 m above the ground after leaving X. [3]

8. Engineers need to lay pipes to connect two cities A and B that are separated by a river of width 550 metres as shown in the following diagram. They plan to lay the pipes under the river from A to Q and then under the ground from Q to B. The cost of laying the pipes under the river is eight times the cost of laying the pipes under the ground.
Angle $APQ = 90^\circ$, $PQ = x$ and $PB = 1200$ m.



(a) Given that the cost, in dollars per metre, of laying the pipes under the ground is k, show that the total cost C, in dollars, of laying the pipes from A to B is given by $C = 8k\sqrt{302500 + x^2} + (1200 - x)k$. [2]

(b) (i) Find $\frac{dC}{dx}$. [2]

(ii) Hence find the value of x for which the total cost is a minimum, justifying that this value is a minimum. [4]

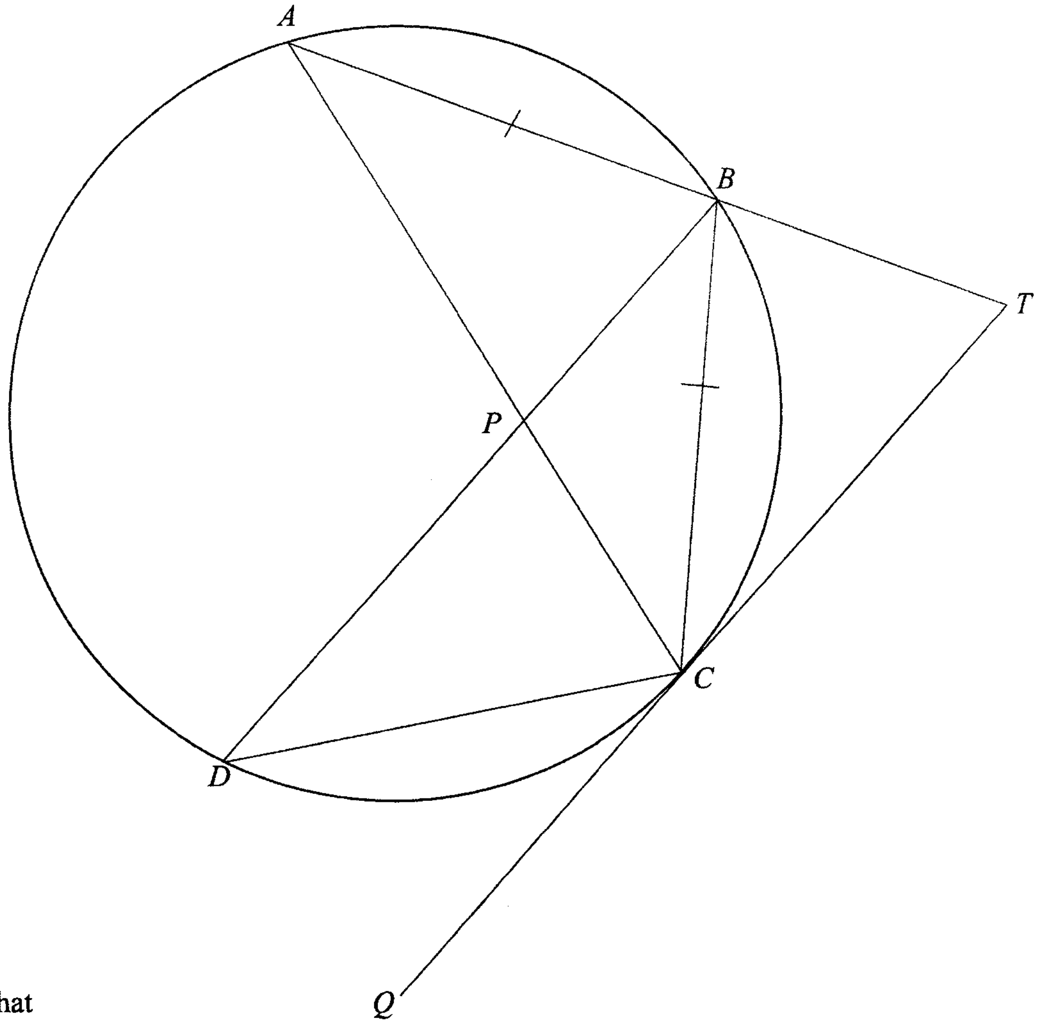
(c) Find the minimum total cost in terms of k. [1]

9. (a) Prove the identity $\dfrac{\sin 2A + \cos A}{1 - \cos 2A + \sin A} = \cot A$ [3]

(b) Hence, solve, for $-\pi \leq x \leq \pi$, the equation

$\dfrac{1-\cos 4x+\sin 2x}{\sin 4x+\cos 2x}=5-2\sec^2 2x$, giving your answers correct to two decimal places. [5]

10. In the diagram, A, B, C and D lie on a circle. QT is a tangent to the circle at C. AT is a straight line that intersects the circle at B. Chords AC and BD intersect at P and $AB = BC$.



Show that

(a) the line BC bisects angle ACT. [2]

(b) triangle ATC is similar to triangle CTB. [3]

(c) angle BCT + angle BTC = $180^\circ - 2\times$ angle PDC. [2]

11. (a) Find the solution of the equation
$\ln 3^{5x+1} = \ln 9^{x+5} + \log_2 16^{1-2x}$,
Expressing your answer in terms of $\ln 3$. [3]

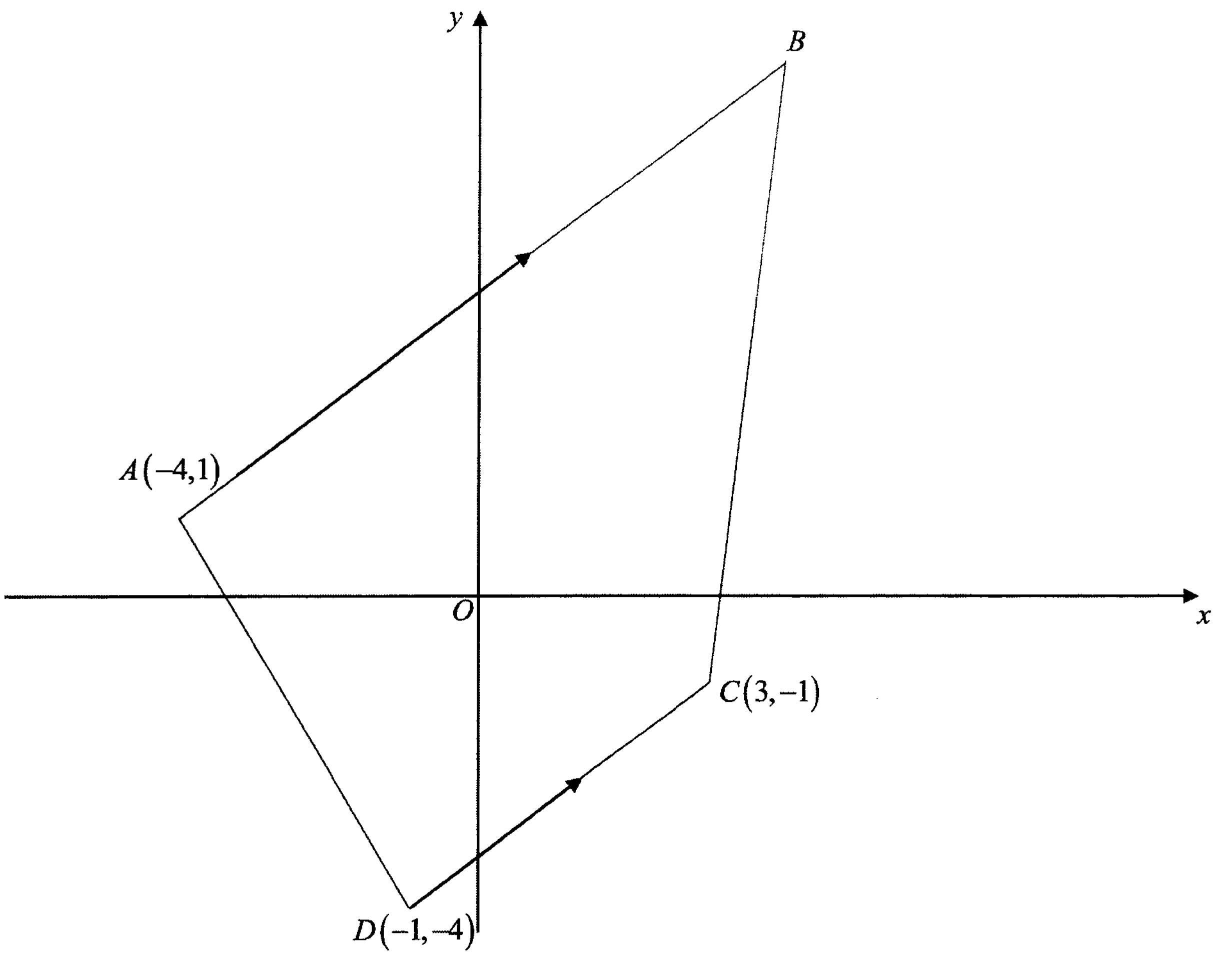
(b) Solve $\log_4(x-3) + \log_2(2x+1)^{\frac{1}{2}} = 1$. [3]

12. A bicycle cylindrical inner tube has a fixed length of 200 cm and radius r cm. The radius r increases as the inner tube is pumped up. Air is being pumped into the inner tube so that the volume of air in the tube increases at a constant rate of $30\ \text{cm}^3\text{s}^{-1}$.

(a) Find the radius of the inner tube after one minute. [2]

(b) Find the rate at which the radius of the inner tube is increasing when $r = 2$ cm. [3]

13. In the diagram, AB is parallel to DC and the coordinates of A, C and D are $(-4,1)$, $(3,-1)$ and $(-1,-4)$ respectively. The gradient of line OB is $\frac{7}{4}$.



Find

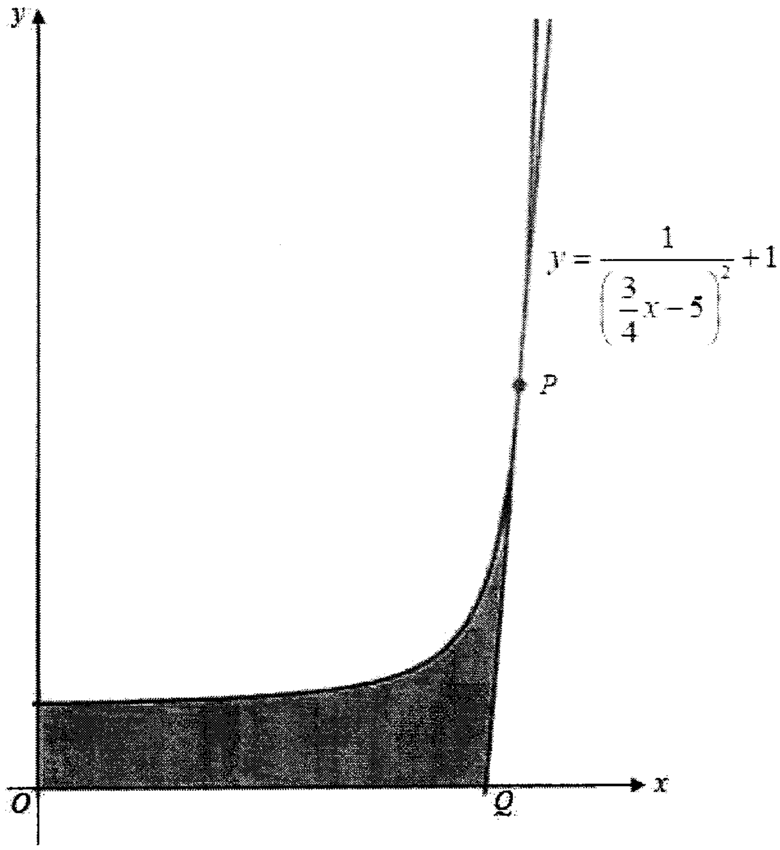
(a) the coordinates of B, [3]

(b) the area of the quadrilateral *ABCD*, [2]

(c) the coordinates of the point *P* on the line $x = 2.5$ which is equidistant from *C* and *D*. [2]

(d) the coordinates *C'* which is a point on *DC* extended such that *ABC'D* is a parallelogram. [2]

14.



The diagram shows part of the curve $y = \frac{1}{\left(\frac{3}{4}x-5\right)^2}+1$. P is the point on the curve where $x = 6$. The tangent to the curve at the point P meets the x-axis at Q.

(a) Find the coordinates of Q. [5]

(b) Find the exact area of the shaded region bounded by the curve, the tangent *PQ* and the *x*-coordinate axis. [5]

GAN ENG SENG SCHOOL
Preliminary Examination 2022

CANDIDATE NAME

CLASS

INDEX NUMBER

ADDITIONAL MATHEMATICS
Paper 2
Sec 4 Express

4049/02

29 August 2022
2 hours 15 min

Candidates answer on the Question Paper.
No Additional Materials are required.

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Mathematical Formulae

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

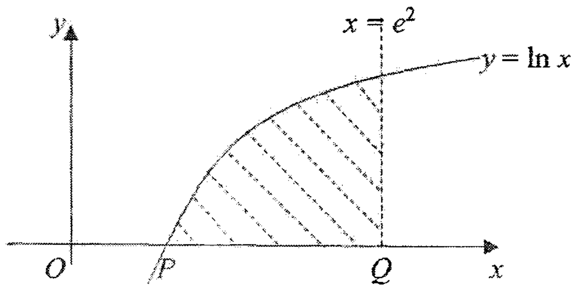
$$\Delta = \frac{1}{2} ab \sin C$$

1. Show that the equation $9^{x+1} - 5(3^{x+1}) - 10$ has only one solution and find its value correct to two decimal places. [6]

2 Solve the equation $2x^3 - 7x + 2 = 0$, leaving non-rational roots in the form $a \pm b\sqrt{2}$, where a and b are rational numbers. [5]

3 A particle moves from rest at A and comes to rest at B. Its speed, in m/s, when travelling from A to B is given by the equation $v = 10t - \frac{1}{2}t^2$, where t is the time in seconds starting from A.
Show that the particle has a speed of 5 m/s or more for $6\sqrt{10}$ s. [4]

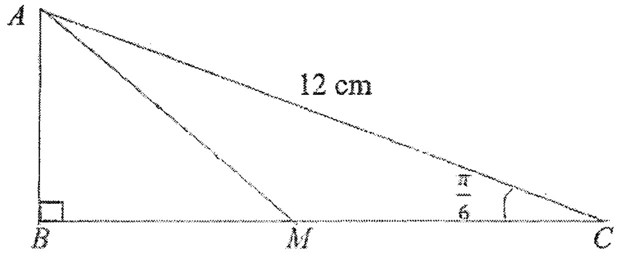
4



The curve $y = \ln x$ cuts the x-axis at P. The area, A units^2, is enclosed by the curve, the x-axis and the line $x = e^2$.

Explain why $e^2 - 1 < \int_1^{e^2} y\,dx < 2(e^2 - 1)$. [4]

5 In ΔABC, $AC = 12$ cm, $\angle ABC$ is a right angle, $\angle ACB = \frac{\pi}{6}$ radians and M is the mid-point of BC. Without the use of a calculator, find the value of the integer k, such that $\angle CAM = \sin^{-1}\left(\frac{\sqrt{k}}{14}\right)$. [5]



6 (a) (i) In the binomial expansion of $\left(x+\frac{1}{ax^2}\right)^8$ where a is a positive integer, the coefficient of x^2 and $\frac{1}{x}$ are equal. Find the value of a. [4]

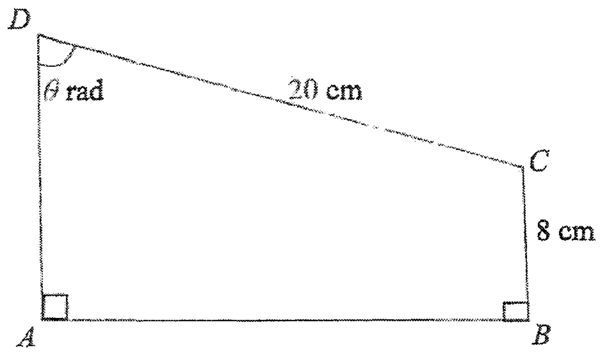
(ii) With the value of a found in part (i), show that there is no term independent of x in the expansion of $\left(x-\frac{1}{x^2}\right)\left(x+\frac{1}{ax^2}\right)^8$. [3]

6 (b) Calculate the term independent of x in the binomial expansion of $\left(x-\frac{1}{2x^5}\right)^{18}$. [3]

7 (i) Find the values of p for which the line $y = x + 1$ is a tangent to the curve $y = x^2 + (2p + 3)x + p + 4$. [4]

(ii) With the values of p found in part (i), find the coordinates of the points where the line meets the curve. [4]

8



The figure shows a piece of cardboard in the shape of a trapezium in which $\angle BAD$ and $\angle ABC$ are right angles. $CD = 20$ cm, $BC = 8$ cm and $\angle ADC = \theta$ radians.

(i) Show that the perimeter, P cm, is given by $P = 20\cos\theta + 20\sin\theta + 36$. [2]

(ii) Express P in the form $R\cos(\theta - \alpha) + k$, where α is acute and k is a constant. [3]

(iii) If θ, can vary, find the maximum value of P and the corresponding value of θ. [2]

(iv) Find the value of θ when the perimeter of the cardboard is 60 cm. [2]

9 (i) Show that $\frac{d}{dx}\left[x(3x-2)^{\frac{4}{3}}\right]=(7x-2)(3x-2)^{\frac{1}{3}}$ [3]

(ii) Hence evaluate $\int x(3x-2)^{\frac{1}{3}}\,dx$. [4]

(iii) Find the value of $\int_{-\frac{1}{2}}^{\frac{2}{3}} x(3x-2)^{\frac{1}{3}}\,dx$ and explain what the result implies about the curve $y = x(3x-2)^{\frac{1}{3}}$. [3]

10 A sports car driven along a straight road passes a traffic junction A at p m/s. A little later, it passes a second traffic junction B with a speed of 40 m/s. Between A and B, the speed of the car, v m/s. is given by $v = 5e^{0.05t} + 10$ where t is the time in seconds after passing A.

(i) State the value of p. [1]

(ii) Show that the time taken to travel from A to B is $20\ln 6$ seconds. [3]

(iii) Calculate the distance AB. [3]

(iv) Find the acceleration of the car when $t = 30$ s. [2]

11 Points $A(8, 1)$ and $B(1, 2)$ lie on a circle whose centre is C. The line $4y = 3x - 20$ is tangent to the circle at A.

(i) Find the equation of the normal to the circle at A. [2]

(ii) Find the equation of the circle. [6]

(iii) Explain why the coordinate axes are tangents to the circle. [1]

12 The table shows experimental values of x and y.

x	2	3	4	5	6
y	11.8	16.2	23.5	35.5	55.2

It is known that x and y are related by the equation $y = Ae^{kx} + 5$, where A and k are constants.

(i) Explain how a straight line graph may be drawn to estimate the values of A and of k. [2]

(ii) Draw a straight line graph to obtain estimated values of A and k. [5]

(iii) Use your graph to find the value of x when $y = 30$. [1]

(iv) By drawing a suitable straight line on the same axes, solve the equation $Ae^{kx} = 30(2^{-x})$ [3]

END OF PAPER

Marking Scheme
2022 Sec 4 Express PRELIM AM Paper 1

			Marker's Comments
1	$\text{Volume} = \frac{1}{3}\times\text{Base Area}\times\text{height}$ $34\sqrt{5}+32=\frac{1}{3}\times\left(2\sqrt{5}+7\right)\times\text{height}$ $\text{height}=\frac{3\left(34\sqrt{5}+32\right)}{2\sqrt{5}+7}\times\frac{2\sqrt{5}-7}{2\sqrt{5}-7}$ ---M1 $=\frac{3\left(68(5)-238\sqrt{5}+64\sqrt{5}-224\right)}{4(5)-49}$ ---M1 $=\frac{3\left(116-174\sqrt{5}\right)}{-29}$ $\frac{348-522\sqrt{5}}{-29}$ $=-12+18\sqrt{5}$ ---A1 ~~74√5~~	**Note: To award up to 2 method marks for correct methods shown.** **3 marks**	* Read qn Carefully: Qn wants to find height! Not p & q!
2	4 3 3 4 4 3 $\frac{3}{4}\times\left(\frac{\sqrt{3}}{2}\right)-\frac{1}{3}\times(1)+c=\frac{\sqrt{3}}{4}-\frac{1}{3}$ $c=\frac{\sqrt{3}}{4}-\frac{1}{3}-\frac{3\sqrt{3}}{8}+\frac{1}{3}$ $=-\frac{\sqrt{3}}{8}$ Equation of curve is $y=\frac{3}{4}\sin 4x-\frac{1}{3}\tan 3x-\frac{\sqrt{3}}{8}$ ---A1	**4 marks**	

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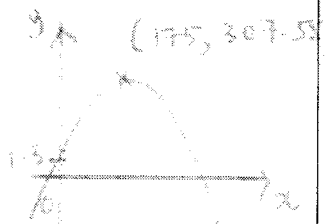
PRELIM 2022 S4EXP AM1 OLP

3(a)	1.3 m – – – B1 (Subst. $x=0$)	1 mark	
(b)(i)	$y=-0.01x^2+3.5x+1.3$ $=-0.01\left[x^2-350x\right]+1.3$ – – – M1 $=-0.01\left[\left(x-\frac{350}{2}\right)^2-\left(\frac{350}{2}\right)^2\right]+1.3$ – – – M1 $=-0.01\left[(x-175)^2-306.25\right]+1.3$ $=-0.01(x-175)^2+307.55$ – – – A1 	3 marks	
(ii)	307.55 m – – – B1 (max pt!!)	1 mark	5/5
4	$y=\frac{\ln(x+3)^3}{3x+9}$ $=\frac{3\ln(x+3)}{3x+9}$ $=\frac{\ln(x+3)}{x+3}$ (ALWAYS simplify first where possible) $\frac{dy}{dx}=\frac{(\;\;+3)\frac{d}{dx}\ln(x+3)-\ln(x+3)\frac{d}{dx}(x+3)}{(x+3)^2}$ $=\frac{(x+3)\times\frac{1}{x+3}-\ln(x+3)(1)}{(x+3)^2}$ – – – M1 (all terms correct) OR $\frac{dy}{dx}=\frac{(3x+9)\left(\frac{3}{x+3}\right)-\left[\ln(x+3)^3\right](3)}{(3x+9)^2}$ $=\frac{3(x+3)\left(\frac{3}{x+3}\right)-3\ln(x+3)^3}{(3x+9)^2}$ or $\frac{9-3\ln(x+3)^3}{(3x+9)^2}$ Or $=\frac{9-9\ln(x+3)}{9(x+3)^2}$ $=\frac{1-\ln(x+3)}{(x+3)^2}$ For y given … inc fn, $\frac{dy}{dx}>0$ For $\frac{dy}{dx}>0$, $1-\ln(x+3)>0$ – – – M1 $-\ln(x+3)>-1$ $\ln(x+3)<1$ $(x+3)<e$ $x<e-3$ $\therefore -3<x<e-3$ – – – A1 take note that $x>-3$ is given in qn & include it in the ans!!	4 marks	

5	$$\require{enclose}\begin{array}{r} 5 \\ x^3-x^2+4x-4\enclose{longdiv}{5x^3-4x^2+15x-21} \\ -(5x^3-5x^2+20x-20)---\text{M1} \\ \hline x^2-5x-1 \end{array}$$ * $\dfrac{5x^3-4x^2+15x-21}{x^3-x^2+4x-4}$ ← expand! $(x-1)(x^2+4)$ $=5+\dfrac{x^2-5x-1}{(x^2+4)(x-1)}$ $\dfrac{ax+b}{x^2+4}$ $(ax+b)(x-1)+c(x^2+4)=x^2-5x-1---\text{M1}$ Sub $x=1$ $5c=1-5-1$ $c=-1$ // Sub $x=0$ $-b+4c=-1$ $-b+4(-1)=-1$ $-b-4=-1$ $-b=3$ $b=-3$ // Compare coefficient of x^2: $a+c=1$ $a-1=1$ $a=2$ // $a=2, b=-3, c=-1---\text{A2,1,0}$ $\dfrac{5x^3-4x^2+15x-21}{x^3-x^2+4x-4}$ $=5+\dfrac{2x-3}{x^2+4}-\dfrac{1}{x-1}---\text{A1}$	**Note:** **To assign 0 mark if long division is not done.** ⇒	$\dfrac{5x^3+}{x^3+}$ * Long division is a must!!

6(a)	$f(x) = 2(x-\frac{3}{2})(x+1)(x^2+bx+c)$ ---M1 $f(1) = -8$ $2(1-\frac{3}{2})(1+1)(1^2+b(1)+c) = -8$ $-2(1+b+c) = -8$ $(1+b+c) = 4$ $b+c = 3$ --- (1) --- A1 $f(-2) = 28$ $2(-2-\frac{3}{2})(-2+1)((-2)^2+b(-2)+c) = 28$ $7(4-2b+c) = 28$ $4-2b+c = 4$ $-2b+c = 0$ --- (2) --- A1 (1) − (2) $(b+c)-(-2b+c) = 3$ $3b = 3$ $b = 1$ $c = 2$ $f(x) = 2(x-\frac{3}{2})(x+1)(x^2+x+2)$ --- A1	4 marks	
(b)	$\frac{3}{2}$ and $x = -1$	2 marks	

* important to form f(x) first using its factors!!

(a) $f(x) = (2x-3)(x+1)(x^2+bx+c)$

OR $f(1) = -8$

$(-1)(2)(1+b+c) = -8$

$1+b+c = 4$

$b+c = 3$ —(1)

$f(-2) = 28$

$(-7)(-1)(4-2b+c) = 28$ —(2)

Subst (2) into (1):

$\therefore f(x) = (2x-3)(x+1)(x^2+x+2)$

$2x^2-x-3$

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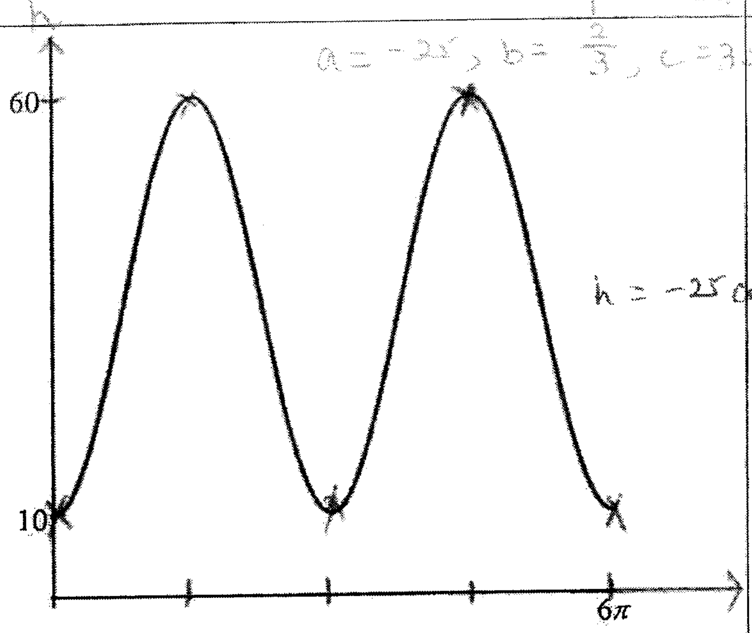
PRELIM 2022 S4EXP AM1_OLP

7(a)	$h=-25\cos\frac{2}{3}t+35$ $\frac{2\pi}{b}=3\pi$ $b=\frac{2}{3}$ ----B1 $a=-25$ ---B1 $c=35$ ---B1 • Centre (middle) is 35m ⇒ axis of curve is 35 ⇒ c = 35 • blade is 25m long ⇒ amplitude is 25 ⇒ a = −25 because height from ground is then drop • period = $\frac{2\pi}{b}$ = 3π | b = $\frac{2\pi}{3\pi}$ = $\frac{2}{3}$	3 marks	
(b)	a = −25, b = $\frac{2}{3}$, c = 35  h = −25 cos $\frac{2}{3}$ t + 35 Points labelled clearly on the h axis – 10 and 60 and t axis - 3π and 6π. (B1)	2 marks	* mark is crosses key points for an accurate curve!
(c)	$-25\cos\frac{2}{3}t+35=42$ $-25\cos\frac{2}{3}t=7$ $\cos\frac{2}{3}t=-\frac{7}{25}$ basic angle $\frac{2}{3}t=\cos^{-1}\left(\frac{7}{25}\right)$ ---M1 $=1.287$ rad $\frac{2}{3}t=\pi-1.287$ ---M1 $t=\frac{3}{2}(\pi-1.287)$ $t=2.78$ seconds ---A1 one 4 * first be 42m above	Note: To award 0 mark if the equation is incorrect 3 marks	

8(a)	Length of $QA = \sqrt{x^2 + 550^2}$ $= \sqrt{x^2 + 302500}$ ---B1 Cost of laying under river $= 8k\sqrt{x^2 + 302500}$ Cost of laying under ground $= k \times (1200 - x)$ ---B1 $\therefore C = 8k\sqrt{x^2 + 302500} + k(1200 - x)$	2 marks	
(b)(i)	$\frac{dC}{dx} = 8k \times \frac{1}{2}\left(x^2 + 302500\right)^{-\frac{1}{2}}(2x) - k$ $= \frac{8kx}{\sqrt{x^2 + 302500}}$ * k is a constant! No need to use product rule! $\frac{d}{dx}(kx) = k$	2 marks	
(ii)	Let $\frac{dC}{dx} = 0$ $\frac{8kx}{\sqrt{x^2 + 302500}} - k = 0$ $\frac{8kx}{\sqrt{x^2 + 302500}} = k$ $\sqrt{x^2 + 302500} = 8x$ $x^2 + 302500 = 64x^2$ ---M1 (correct sq) $63x^2 = 302500$ $x^2 = \frac{302500}{63}$ / 4801.5873 $= 69.2935$ $x = 69.3$ ---A1		
	Method 1: using 2nd derivative test $\frac{dC}{dx} = \frac{8kx}{\sqrt{x^2 + 302500}} - k$ $\frac{d^2C}{dx^2} = \frac{\left(\sqrt{x^2 + 302500} \times 8k\right) - 8kx\left(\frac{1}{2}\right)\left(x^2 + 302500\right)^{-\frac{1}{2}}(2x)}{\left(\sqrt{x^2 + 302500}\right)^2}$ $= \frac{(x^2 + 302500) \times 8k - 8kx^2}{\left(x^2 + 302500\right)^{\frac{3}{2}}}$ ---**M1** $= \frac{2420000k}{\left(x^2 + 302500\right)^{\frac{3}{2}}}$ **At** $x = 69.3$, $\frac{d^2C}{dx^2} = \frac{2420000k}{\left(69.3^2 + 302500\right)^{\frac{3}{2}}}$ $= 0.0142k > 0$ --- **A1** Since $\frac{d^2C}{dx^2} > 0$, total cost is minimum at $x = 69.3$		

(ii)

Method 2 : 1st derivative test

x	68	69.3	~~80~~ 70
$\frac{dC}{dx}$	$\frac{8k(68)}{\sqrt{68^2+302500}}-k$ $\approx 0.0153k-k$ $-0.9847k<0$	0	$\frac{8k(70)}{\sqrt{70^2+302500}}-k$ $\approx 1.01k-k$ $0.01k>0$
Shape	\	—	/

M1 for attempt to find gradient on either side of $x=69.2$ and A1 for 2 correct values of $\frac{dC}{dx}$.

Note: **To award 0 mark if gradient values are found but only a sketch of the shape is produced**

4 marks

(c)

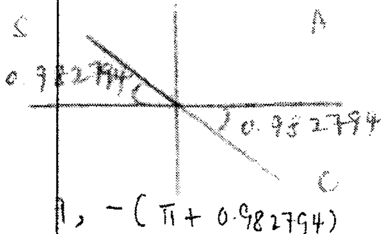
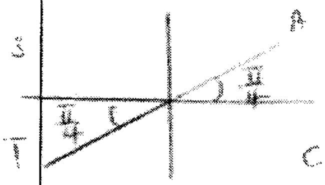
$$C=8k\sqrt{69.2934^2+302500}+(1200-69.2934)k$$
$$=4434.783k+1130.7066k$$
$$=5565.4869k$$
$$=5570k\text{ --- B1}$$

1 mark

9(a)

$$\text{LHS}=\frac{\sin 2A+\cos A}{1-\cos 2A+\sin A}$$
$$=\frac{2\sin A\cos A+\cos A}{1-(1-2\sin^2 A)+\sin A}\text{ --- M1,M1}$$
$$=\frac{\cos A(2\sin A+1)}{2\sin^2 A+\sin A}$$
$$=\frac{\cos A(2\sin A+1)}{\sin A(1+2\sin A)}\text{ --- A1}$$
$$=\cot A$$

3 marks

(b)	$\dfrac{1-\cos 4x+\sin 2x}{\sin 4x+\cos 2x}=5-2\sec^2 2x$ $\tan 2x=5-2\sec^2 2x$ $\tan 2x=5-2\left(1+\tan^2 2x\right)$ $2\tan^2 2x+\tan 2x-3=0$ ----M1 $\tan 2x=\dfrac{-1\pm\sqrt{(1)^2-4(2)(-3)}}{2(2)}$ $=\dfrac{-1\pm\sqrt{25}}{4}$ $=\dfrac{-1\pm 5}{4}$ ---M1 $=-\dfrac{3}{2},1$ x $x=1.08,2.65,-0.49,-2.06$ $\tan 2x=1$ $2x=\dfrac{\pi}{4},\pi+\dfrac{\pi}{4},-\left(\pi-\dfrac{\pi}{4}\right.\quad\left.\pi-\dfrac{\pi}{4}\right)$ $x=0.39,1.96,-2.36,-0.785$ A1 for correct positive angles, A1 for 2 correct negative angles eac	5 marks	 
10(a)	S $\angle CAB=\angle BCA$ (Base angles of isosceles triangle) $\therefore \angle BCT=\angle BCA$ (BC bisects $\angle ACT$)		
(b)		3 marks	

(c)	$\angle BCT+\angle BTC+\angle CBT=180^\circ$ $\angle BCT+\angle BTC=180^\circ-\angle CBT$ --- B1 From (b) $\angle CBT=\angle ACT$ $=2\times\angle BCA$ $=2\times\angle BAC$ $\angle BAC=\angle PDC$ (Angles in the same segment) --- B1 $\therefore \angle BCT+\angle BTC=180^\circ-2\times\angle PDC$	**2 marks**	
11(a)	$\ln 3^{5x+1}=\ln 9^{x+5}+\log_2 16^{1-2x}$ $(5x+1)\ln 3=\ln 3^{2(x+5)}+\log_2\left(2^4\right)^{1-2x}$ $(5x+1)\ln 3=2(x+5)\ln 3+4(1-2x)$ --- M1 $5x\ln 3+\ln 3=2x\ln 3+10\ln 3+4-8x$ --- M1 (expansion) $3x\ln 3+8x=9\ln 3+4$ $x(3\ln 3+8)=9\ln 3+4$ $x=\frac{9\ln 3+4}{3\ln 3+8}$ --- A1	$\log_a\left(\frac{x}{y}\right)=\log_a x-\log_a y$ **3 marks**	
(b)	$\log_4(x-3)+\frac{1}{2}\log_2(2x+1)=1$ $\frac{\log_2(x-3)}{\log_2 4}+\frac{1}{2}\log_2(2x+1)=1$ --- <Change base - M1> $\frac{\log_2(x-3)}{2}+\frac{1}{2}\log_2(2x+1)=1$ $\log_2(x-3)+\log_2(2x+1)=2$ $\log_2(x-3)(2x+1)=2$ --- M1 $(x-3)(2x+1)=2^2$ $2x^2+x-6x-3-4=0$ $2x^2-5x-7=0$ $(2x-7)(x+1)=0$ $x=\frac{7}{2},-1\text{(reject)}$ --- A1	**Note: Reject to be shown to be given 1 answer mark** **3 marks**	

OR change to base 10

$$\frac{\lg(x-3)}{\lg 4}+\frac{\frac{1}{2}\lg(2x+1)}{\lg 2}=1$$

$$\frac{\lg(x-3)}{\lg 4}+\frac{\lg(2x+1)}{2\lg 2}=1$$

$$\lg(x-3)(2x+1)=\lg 4$$

$$(x-3)(2x+1)=4$$

↓

OR

$$\frac{\log_4(x-3)+\log_4(2x+1)^{\frac{1}{2}}}{\log_4 2}=1$$

$$\log_4(x-3)+\frac{\log_4(2x+1)^{\frac{1}{2}}}{\frac{1}{2}}=1$$

$$\log_4(x-3)(2x+1)=1$$

↓

12(a)	At $t = 60$ Volume $= 30 \times 60$ $= 1800 \text{ cm}^3$ $\pi \times r^2 \times 200 = 1800$ ---M1 $r = \sqrt{\frac{9}{\pi}} = 1.69257$ $= 1.69$ cm ---A1	**2 marks**	
(b)	$V = \pi r^2 h$ $= \pi r^2 (200)$ $= 200\pi r^2$ $\frac{dV}{dt} = 30$ $\frac{dV}{dr} = 400\pi r$ ---M1 $\frac{dV}{dr} \times \frac{dr}{dt} = \frac{dV}{dt}$ $400\pi r \times \frac{dr}{dt} = 30$ ---M1 $\frac{dr}{dt} = \frac{30}{400\pi r}$ At $r = 2$ cm, $\frac{dr}{dt} = \frac{30}{400\pi(2)} = \frac{3}{80\pi}$ $= 0.0119$ cm/s ---A1	**3 marks**	
13(a)	Since gradient of $OB = \frac{7}{4}$ $B(x, \frac{7}{4}x)$ ---M1 Gradient of AB = Gradient of DC $A(-4, 1)$ $C(3, -1)$ & $D(-1, -4)$ $\frac{\frac{7}{4}x - 1}{x + 4} = \frac{-1-(-4)}{3-(-1)}$ $\frac{\frac{7}{4}x - 1}{x + 4} = \frac{3}{4}$ ---M1 $7x - 4 = 3x + 12$ $4x = 16$ $x = 4$ $\therefore B(4, 7)$ ---A1	**3 marks**	

wrong concept:
gradient of OB $= \frac{7}{4}$
Subst O(0, 0) & B(x, y)
$\frac{y}{x} = \frac{7}{4}$
$\therefore y = 7$ & $x = 4$

But $\frac{y}{x} = \frac{7}{4} = \frac{14}{8} = \frac{21}{12} \ldots$
Cannot use ratio here!

must be in
* anti-clockwise direction

b)	Area of quadrilateral $ABCD$ $=\frac{1}{2}\begin{vmatrix}-4 & -1 & 3 & 4 & -4\\ 1 & -4 & -1 & 7 & 1\end{vmatrix}$ ---M1 $=\frac{1}{2}\left((16+1+21+4)-(-1-12-4-28)\right)$ $=\frac{1}{2}(42+45)$ $=43.5\text{ units}^2$ ---A1	2 marks	
13(c) Mtd 1		2 marks	
13(d)	$C'(4+3,7-5)$---M1 Fastest mtd. $C'(7,2)$---A1 or B1,B1	2 marks	

13(c) Mtd 2 D(−1, −4) & C(3, −1)

Midpoint of DC = $\left(\frac{-1+3}{2}, \frac{-4-1}{2}\right)$

$= \left(1, -\frac{5}{2}\right)$

Gradient of ⊥ bisector of DC is $-\frac{4}{3}$.

$y + \frac{5}{2} = -\frac{4}{3}(x-1)$ M1

$y = -\frac{4}{3}x - \frac{7}{6}$

Subst. $x = 2.5$,

$y = -\frac{4}{3}(2.5) - \frac{7}{6}$

∴ ※ A1

13(d) Mtd 2 midpoint of BD

$= \left(\frac{4-1}{2}, \frac{7-4}{2}\right)$

$= \left(\frac{3}{2}, \frac{3}{2}\right)$

Let C be (x, y)

midpoint of AC'

$= \left(\frac{-4+x}{2}, \frac{1+y}{2}\right)$

$\frac{-4+x}{2} = \frac{3}{2}$, $\frac{1+y}{2} = \frac{3}{2}$

$-4+x = 3$, $1+y = 3$

$x = 7$ $y = 2$

∴ C(7, 2)

Mtd 3

eqn of BC' is $y = -\frac{5}{3}x + \frac{41}{3}$ –①

eqn of OC' is $y = \frac{3}{4}x - \frac{13}{4}$ –②

① = ②

130

14(a)	$y=\left(\frac{3}{4}x-5\right)^{-2}+1$ $\frac{dy}{dx}=-2\left(\frac{3}{4}x-5\right)^{-3}\left(\frac{3}{4}\right)$ $=-\frac{3}{2}\left(\frac{3}{4}x-5\right)^{-3}$ ---M1 At $x=6$, $\frac{dy}{dx}=-\frac{3}{2}\left(\frac{3}{4}(6)-5\right)^{-3}$ $=-\frac{3}{2}\left(-\frac{1}{2}\right)^{-3}$ $=12$ ---A1 At $x=6$, $y=\left(\frac{3}{4}(6)-5\right)^{-2}+1$ $=\left(-\frac{1}{2}\right)^{-2}+1$ $=5$ ---B1 Equation of PQ: $y=12x+c$ Sub $(6,5)$ $5=12(6)+c$ ---M1 $c=-67$ $y=12x-67$ At Q, $y=0$ $x=\frac{67}{12}$ $\therefore Q(\frac{67}{12},0)$ ---A1	5 marks	

Shaded area $=\int_0^6\left[\left(\frac{3}{4}x-5\right)^{-2}+1\right]dx-\frac{1}{2}\left(6-\frac{67}{12}\right)(5)$ ---M1,M1 $=\left[\frac{\left(\frac{3}{4}x-5\right)^{-1}}{-1\left(\frac{3}{4}\right)}+x\right]_0^6-\frac{25}{24}$ ---<Correct integration - M1> $=\frac{\left(\frac{3}{4}(6)-5\right)^{-1}}{-1\left(\frac{3}{4}\right)}+6-\frac{\left(\frac{3}{4}(0)-5\right)^{-1}}{-1\left(\frac{3}{4}\right)}-\frac{25}{24}$ $=\frac{26}{3}-\frac{4}{15}-\frac{25}{24}$ ---M1 $=7\frac{43}{120}$ units^2 ---A1	**5 marks**	

$\left[\frac{1}{\left(\frac{3}{4}x-5\right)}\right.$

GAN ENG SENG SCHOOL
Preliminary Examination 2022

CANDIDATE NAME: Marking Scheme

CLASS:

INDEX NUMBER:

ADDITIONAL MATHEMATICS
Paper 2

Sec 4 Express

4049/02

29 August 2022
2 hours 15 min

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 90.

	For Examiner's Use
Total	90

This paper consists of **20** printed pages including the cover page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1. Show that the equation $9^{x+1} - 5(3^{x+1}) = 10$ has only one solution and find its value correct to two decimal places. [6]

Solution

$9^{x+1} - 5(3^{x+1}) = 10$

$9^x \times 9 - 5(3^x \times 3) = 10$

$3^{2x} \times 9 - 15(3^x) - 10 = 0$ M1

Let $u = 3^x$

$9u^2 - 15u - 10 = 0$

M1

M1

M1

B1

A1

6/6

OR

$3^{2(x+1)} - 5(3^{x+1}) = 10$

$(3^{x+1})^2 - 5(3^{x+1}) = 10$

Let $u = 3^{x+1}$

$u^2 - 5u - 10 = 0$

$u = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(-10)}}{2(1)}$

$= \frac{5 \pm \sqrt{65}}{2}$

$= 6.53$

2 Solve the equation $2x^3 - 7x + 2 = 0$, leaving non-rational roots in the form $a \pm b\sqrt{2}$, where a and b are rational numbers. [5]

Solution

Let $f(x) = 2x^3 - 7x + 2$

Try $f(-2) = 2(-2)^3 - 7(-2) + 2$

$= -16 + 14 + 2$

$= 0$

Therefore $x + 2$ is a factor. M1 M1

By inspection $2x^3 - 7x + 2 = (x + 2)(2x^2 + ax + 1)$, where a is a constant.

Compare terms in x: $-7x = 2ax + x$

$-7 = 2a + 1$

$2a = -8$

$a = -4$ M1 M1

mtd 2 long division

$2x - 4x + 1$

Therefore $2x^3 - 7x + 2 = 0$

$(x + 2)(2x^2 - 4x + 1) = 0$

Either $x = -2$ or $2x^2 - 4x + 1 = 0$

If $2x^2 - 4x + 1 = 0$

$x = \dfrac{4 \pm \sqrt{(-4)^2 - 4 \times 2 \times 1}}{4}$ M1

$x = \dfrac{4 \pm \sqrt{8}}{4}$

$x = 1 \pm \dfrac{1}{2}\sqrt{2}$

Ans: $x = -2$ or $x = 1 \pm \dfrac{1}{2}\sqrt{2}$ A1, 1

A1 A1

3) A particle moves from rest at A and comes to rest at B. Its speed, in m/s, when travelling from A to B is given by the equation $v = 10t - \frac{1}{2}t^2$, where t is the time in seconds starting from A.
Show that the particle has a speed of 5 m/s or more for $6\sqrt{10}$ s. [4]

$v \geq 5$

<u>Solution</u>

$v \geq 5$

$10t - \frac{1}{2}t^2 \geq 5$

$20t - t^2 - 10 \geq 0$

$t^2 - 20t + 10 \leq 0$ m1 $\quad -\frac{1}{2}t^2 + 10t - 5 = 0$ M1

If $t^2 - 20t + 10 = 0$

$t = \dfrac{20 \pm \sqrt{(-20)^2 - 4 \times 1 \times (-10)}}{2}$ m1

OR

$t = \dfrac{-10 \pm \sqrt{(10^2) - 4(-\frac{1}{2})(-5)}}{2(-\frac{1}{2})}$ M1

$t = \dfrac{20 \pm \sqrt{360}}{2}$

$t = \dfrac{20 \pm 6\sqrt{10}}{2}$ $\quad = \dfrac{-10 \pm \sqrt{90}}{-1}$

$t = 10 \pm 3\sqrt{10}$

$t_1 = 10 - 3\sqrt{10}$ and $t_2 = 10 + 3\sqrt{10}$ s m1 $\quad = \dfrac{-10 \pm 3\sqrt{10}}{-1}$ M1

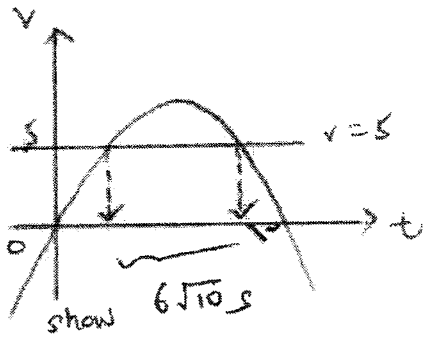
Interval of time is $10 + 3\sqrt{10} - (10 - 3\sqrt{10})$ s

$= 6\sqrt{10}$ s A1 A1

$= \dfrac{-10 + 3\sqrt{10}}{-1}$ or $\dfrac{-10 - 3\sqrt{10}}{-1}$

$= 10 - 3\sqrt{10}$ $\quad = 10 + 3\sqrt{10}$

$V = -\frac{1}{2}t^2 + 10t$



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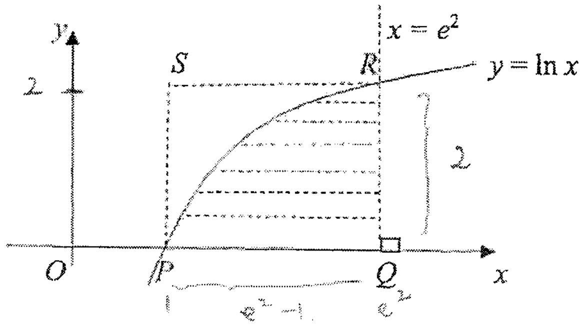


The curve $y = \ln x$ cuts the x-axis at P. The area, A units, is enclosed by the curve, the x-axis and the line $x = e^2$.

Explain why $e^2 - 1 < \int_1^{e^2} y\,dx < 2(e^2 - 1)$. [4]

Solution

If $y = \ln x$ meets $y = 0$,

$\ln x = 0$,

$x = 1$

Therefore P is (1, 0). m1 M1

Draw QR parallel to the y-axis through $(e^2, 0)$ to meet the curve at R.
Then draw rectangle $PQRS$.

If $x = e^2$, $y = \ln e^2 = 2$

Therefore $Q = (e^2, 0)$ and R is at $(e^2, 2)$ m1 M1

$$\text{Area of } \Delta PQR < \int_1^{e^2} y\,dx < \text{ Area of } PQRS$$ M1

$$\frac{1}{2}(e^2 - 1) \times 2 < \int_1^{e^2} y\,dx < 2(e^2 - 1)$$ m1

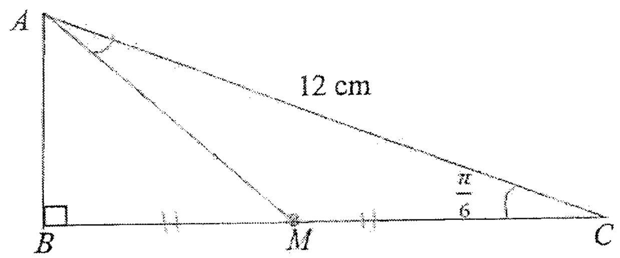
$$e^2 - 1 < \int_1^{e^2} y\,dx < 2(e^2 - 1)$$ A1 A1

* DO NOT

$\int \ln x \, dx = \frac{1}{x} + c$

Wrong concept !!

5. In ΔABC, $AC = 12$ cm, $\angle ABC$ is a right angle, $\angle ACB = \frac{\pi}{6}$ radians and M is the mid-point of BC. Without the use of a calculator, find the value of the integer k, such that $\angle CAM = \sin^{-1}\left(\frac{\sqrt{k}}{14}\right)$.



Solution [5]

$\sin\left(\frac{\pi}{6}\right) = \frac{AB}{12}$

$AB = 12\sin\left(\frac{\pi}{6}\right) = 6$ cm M1

$\cos\left(\frac{\pi}{6}\right) = \frac{BC}{12}$

$BC = 12\cos\left(\frac{\pi}{6}\right)$

$BC = 12 \times \frac{\sqrt{3}}{2} = 6\sqrt{3}$ cm

(M is midpt) $MC = BM = 3\sqrt{3}$ cm M1

$AM^2 = AB^2 + BM^2$

$= 6^2 + (3\sqrt{3})^2$

$= 36 + 27 = 63$

$AM = 3\sqrt{7}$ cm M1

OR

$AM^2 = AC^2 + MC^2 - 2(AC)(MC)\cos\frac{\pi}{6}$

$= 12^2 + \left(6\cos\frac{\pi}{6}\right)^2 - 2(12)\left(6\cos\frac{\pi}{6}\right)\cos\frac{\pi}{6}$

$= 63$

By the Sine Rule, $\frac{\sin\angle CAM}{MC} = \frac{\sin\left(\frac{\pi}{6}\right)}{AM}$

$\frac{\sin\angle CAM}{\frac{1}{2}\sqrt{108}} = \frac{\sin\left(\frac{\pi}{6}\right)}{\sqrt{63}}$

$\frac{\sin\angle CAM}{3\sqrt{3}} = \frac{\frac{1}{2}}{3\sqrt{7}}$ M1

$\sin\angle CAM = \frac{\frac{1}{2}\times 3\sqrt{3}}{3\sqrt{7}}$

$\sin\angle CAM = \frac{\sqrt{3}}{2\sqrt{7}} = \frac{\sqrt{21}}{14}$

$\angle CAM = \sin^{-1}\left(\frac{\sqrt{21}}{14}\right)$

Therefore $k = 21$ Ans A1

$T_{r+1} = \binom{n}{r} a^{n-r} b^{r}$

6 (a) (i) In the binomial expansion of $\left(x+\frac{1}{ax^2}\right)^8$ where a is a positive integer, the coefficient of x^2 and $\frac{1}{x}$ are equal. Find the value of a. [4]

Solution

$$\left(x+\frac{1}{ax^2}\right)^8 = x^8 + \binom{8}{1}(x^7)\left(\frac{1}{ax^2}\right) + \binom{8}{2}(x^6)\left(\frac{1}{ax^2}\right)^2 + \binom{8}{3}(x^5)\left(\frac{1}{ax^2}\right)^3 + \ldots\ldots$$

M1

$$= x^8 + \frac{8}{a}x^5 + 28x^6 \times \frac{1}{a^2x^4} + 56x^5 \times \frac{1}{a^3x^6} + \ldots\ldots\ldots\ldots$$

$$= x^8 + \frac{8}{a}x^5 + \frac{28}{a^2}x^2 + \frac{56}{a^3}\left(\frac{1}{x}\right) + \ldots\ldots\ldots\ldots$$ M1

$$\frac{28}{a^2} = \frac{56}{a^3}$$ M1 M1

$28a^3 = 56a^2$ * do not cancel a !!

$a^3 - 2a^2 = 0$

$a^2(a-2) = 0$

Since $a \neq 0, a = 2$ Ans A1 A1

(ii) With the value of a found in part (i), show that there is no term independent of x in the expansion of $\left(x-\frac{1}{x^2}\right)\left(x+\frac{1}{ax^2}\right)^8$. [3]

Solution

$$\left(x-\frac{1}{x^2}\right)\left(x+\frac{1}{ax^2}\right)^8 = \left(x-\frac{1}{x^2}\right)\left(x+\frac{1}{2x^2}\right)^8$$

$$\left(x-\frac{1}{x^2}\right)\left(x+\frac{1}{2x^2}\right) = \left(x-\frac{1}{x^2}\right)\left(x^8 + \frac{8x^5}{2} + \frac{28x^2}{2^2} + \frac{56}{2^3}\left(\frac{1}{x}\right) + \ldots\right)$$ M1

$$= \left(x-\frac{1}{x^2}\right)\left(x^8 + 4x^5 + 7x^2 + \frac{7}{x} + \ldots\ldots\right)$$

Term independent of x is $\left(x \times \frac{7}{x}\right) - \left(\frac{1}{x^2} \times 7x^2\right) + \ldots$ M1

$= 7 - 7$

$= 0$

Therefore, there is no term independent of x. A1

(b) Calculate the term independent of x in the binomial expansion of $\left(x-\frac{1}{2x^5}\right)^{18}$. [3]

Solution

General term $= \binom{18}{r}(x^{18-r})\left(-\frac{1}{2x^5}\right)^r$

$= \binom{18}{r}(x^{18-r})\frac{(-1)^r}{2^r(x^{5r})}$ $= \binom{18}{r}(x^{18-6r})\frac{(-1)^r}{2^r}$ M1

$= \binom{18}{r}(x^{18-6r})\left(-\frac{1}{2}\right)^r$

For term independent of x, $x^{18-r} = x^{5r}$ $= \binom{18}{r}(x^{18-6r})(-2^{-1})^r$

$18 - r = 5r$

$6r = 18$

$r = 3$ M1

Term independent of $x = \binom{18}{3}\times\frac{(-1)^3}{2^3}$

$= -102$ Ans A1

OR

$\left(x-\frac{1}{2x^5}\right)^{18}$

$= x^{18} + \binom{18}{1}(x^{17})\left(-\frac{1}{2x^5}\right)^1 + \binom{18}{2}(x^{16})\left(-\frac{1}{2x^5}\right)^2 + \binom{18}{3}(x^{15})\left(-\frac{1}{2x^5}\right)^3 + \ldots$

Term independent of $x = \binom{18}{3}(x^{15})\left(-\frac{1}{2x^5}\right)^3$

$= 816\left(-\frac{1}{8}\right)$

$= -102$.

7 (i) Find the values of p for which the line $y = x + 1$ is a tangent to the curve $y = x^2 + (2p + 3)x + p + 4$. [4]

Solution

If $y = x^2 + (2p+3)x + p + 4$ meets $y = x + 1$

$x^2 + (2p+3)x + p + 4 = x + 1$

$x^2 + (2p+3)x + p + 4 - x - 1 = 0$

$x^2 + (2p+2)x + p + 3 = 0$ M1

For equal roots, discriminant = 0

$(2p+2)^2 - 4(1)(p+3) = 0$ M1

$4p^2 + 4p - 8 = 0$

$p^2 + p - 2 = 0$

$(p+2)(p-1) = 0$

$p = -2$ or $p = 1$ Ans A1, A1

(ii) With the values of p found in part (i), find the coordinates of the points where the line meets the curve. [4]

Solution

If $p = 1$, $x^2 + (2\times1+2)x + 1 + 3 = 0$

$x^2 + 4x + 4 = 0$

$(x+2)^2 = 0$

$x = -2$ M1

If $x = -2$, $y = -2 + 1 = -1$

Therefore, one point is $(-2, -1)$. A1

If $p = -2$, $x^2 + (2(-2)+2)x + (-2) + 3 = 0$

$x^2 - 2x + 1 = 0$

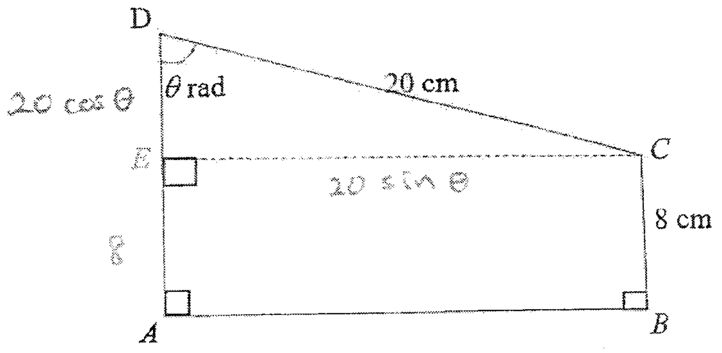
$(x-1)^2 = 0$

$x = 1$ M1

If $x = 1$, $y = 1 + 1 = 2$.

Therefore, the other point is $(1, 2)$ A1

8



The figure shows a piece of cardboard in the shape of a trapezium in which $\angle BAD$ and $\angle ABC$ are right angles. $CD = 20$ cm, $BC = 8$ cm and $\angle ADC = \theta$ radians.

(i) Show that the perimeter, P cm, is given by $P = 20\cos\theta + 20\sin\theta + 36$. [2]

Solution
Draw CE perpendicular to AD

In Δ CDE, $\cos\theta = \dfrac{DE}{20}$

$DE = 20\cos\theta$ cm

$\sin\theta = \dfrac{CE}{20}$ M1 M1

$CE = 20\sin\theta$

Therefore $P = AB + BC + CD + AD$

$= 20\sin\theta + 8 + 20 + 20\cos\theta + 8$

$P = 20\cos\theta + 20\sin\theta + 36$ Shown A1

(ii) Express P in the form $R\cos(\theta - \alpha) + k$, where α is acute. [3]

Solution

$R^2 = 20^2 + 20^2$

$= 800$

$R = \sqrt{800}$

$= 20\sqrt{2}$ m1 M1

$\tan\alpha = \dfrac{20}{20}$

$\alpha = \dfrac{\pi}{4}$ m1 M1

Therefore $P = 20\sqrt{2}\cos\left(\theta - \dfrac{\pi}{4}\right) + 36$ Answer A1

$P = 20\sqrt{2}\cos(\theta - 0.785) + 36$

(iii) If θ, can vary, find the maximum value of P and the corresponding value of θ. [2]

Solution

Maximum value of P is when $\cos\left(\theta - \frac{\pi}{4}\right) = 1$

Therefore $P = 20\sqrt{2} \times 1 + 36$

64.3 / $P = 20\sqrt{2} + 36$ Ans simplest & exact value B1

As θ is acute, $\theta - \frac{\pi}{4} = 0$

$\theta = \frac{\pi}{4}$ 0.785 rad B1

(iv) [2]

Solution

If $P = 60$ cm

$20\sqrt{2}\cos\left(\theta - \frac{\pi}{4}\right) + 36 = 60$

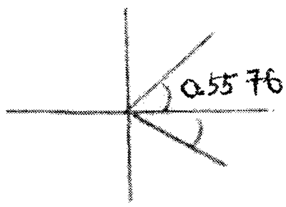
$20\sqrt{2}\cos\left(\theta - \frac{\pi}{4}\right) = 24$

$\cos\left(\theta - \frac{\pi}{4}\right) = \frac{24}{20\sqrt{2}}$ M1

$0 < \theta < \frac{\pi}{2}$

$-\frac{\pi}{4} < \theta - \frac{\pi}{4} < \frac{\pi}{2} - \frac{\pi}{4}$

$-0.785 < \theta - \frac{\pi}{4} < 0.785$



Basic angle = 0.557600 rad (6 s.f.)

$\theta - \frac{\pi}{4} = 0.557600$ or $\theta - \frac{\pi}{4} = -0.557600$

$\theta = \frac{\pi}{4} + 0.5576$ $\theta = \frac{\pi}{4} - 0.5576$

$\theta = 1.34$ radian (3 s.f.) Answer $= 0.228$ rad A1

$\therefore \theta = 1.34$ rad, 0.228 rad (A1)

Show ⇒ show every step!

9 (i) Show that $\frac{d}{dx}\left(x(3x-2)^{\frac{4}{3}}\right)=(7x-2)(3x-2)^{\frac{1}{3}}$ [3]

Solution

$$\frac{d}{dx}\left\{x(3x-2)^{\frac{4}{3}}\right\}=(3x-2)^{\frac{4}{3}}\frac{d}{dx}(x)+x\frac{d}{dx}(3x-2)^{\frac{4}{3}}$$

$$=(3x-2)^{\frac{4}{3}}\times 1+x\times\frac{4}{3}(3x-2)^{\frac{1}{3}}\times 3$$ M1

$$=(3x-2)^{\frac{1}{3}}[3x-2+4x]$$ M1

$$=(3x-2)^{\frac{1}{3}}(7x-2)\quad \text{Proven}$$ A1

(ii) Hence evaluate $\int x(3x-2)^{\frac{1}{3}}dx$. [4]

Solution

$\int(3x-2)^{\frac{1}{3}}(7x-2)dx=x(3x-2)^{\frac{4}{3}}+c_1$ where c_1 is a constant M1

$\int 7x(3x-2)^{\frac{1}{3}}dx-\int 2(3x-2)^{\frac{1}{3}}dx=x(3x-2)^{\frac{4}{3}}+c_1$ M1

$\int 7x(3x-2)^{\frac{1}{3}}dx=\int 2(3x-2)^{\frac{1}{3}}dx+x(3x-2)^{\frac{4}{3}}+c_1$

$\int 7x(3x-2)^{\frac{1}{3}}dx=\frac{2(3x-2)^{\frac{4}{3}}}{\frac{4}{3}\times 3}+c_2+x(3x-2)^{\frac{4}{3}}+c_1$ where c_2 is a constant M1 *

$\int 7x(3x-2)^{\frac{1}{3}}dx=\frac{(3x-2)^{\frac{4}{3}}}{2}+c_2+x(3x-2)^{\frac{4}{3}}+c_1$

$\int x(3x-2)^{\frac{1}{3}}dx=\frac{1}{14}(3x-2)^{\frac{4}{3}}+\frac{1}{7}x(3x-2)^{\frac{4}{3}}+c_1+c_2$ * Factorise

$\int x(3x-2)^{\frac{1}{3}}dx=\frac{1}{14}(3x-2)^{\frac{4}{3}}(1+2x)+c$ where c is a constant. Ans A1

(iii) Find the value of $\int_{-\frac{1}{2}}^{\frac{2}{3}} x(3x-2)^{\frac{1}{3}}dx$ and explain what the result implies about the curve $y = x(3x-2)^{\frac{1}{3}}$. [3]

Solution

$\int_{-\frac{1}{2}}^{\frac{2}{3}} x(3x-2)^{\frac{1}{3}}dx = \frac{1}{14}\left[(3x-2)^{\frac{4}{3}}(2x+1)\right]_{-\frac{1}{2}}^{\frac{2}{3}}$ * must show subst^n of x values

$= \frac{1}{14}\left[\left(3\times\frac{2}{3}-2\right)^{\frac{4}{3}}\left(2\times\frac{2}{3}+1\right)-\left(3\left(-\frac{1}{2}\right)-2\right)^{\frac{4}{3}}\left(2\left(-\frac{1}{2}\right)+1\right)\right]$ M1

$= 0$ A1

The area enclosed by the curve, ~~and~~ and the x-axis ~~from~~ $x=-\frac{1}{2}$ to $x=0$ is equal to the area enclosed by the curve and the x-axis from $x=0$ to $x=\frac{2}{3}$. B1

& below

10 A sports car driven along a straight road passes a traffic junction A at p m/s. A little later, it passes a second traffic junction B with a speed of 40 m/s. Between A and B, the speed of the car, v m/s. is given by $v = 5e^{0.05t} + 10$ where t is the time in seconds after passing A.

(i) State the value of p. [1]

Answer

When $t = 0$, $p = 5e^0 + 10$

$p = 5 + 10$

$= 15$ Ans B1

(ii) Show that the time taken to travel from A to B is $20\ln 6$ seconds. [3]

Solution

$5e^{0.05t} + 10 = 40$

$5e^{0.05t} = 30$

$e^{0.05t} = 6$ m1 M1

$\ln e^{0.05t} = \ln 6$

$0.05t = \ln 6$ m1

~~$t = \frac{\ln 6}{0.05}$~~ $\frac{1}{20}t = \ln 6$ M1

$t = 20\ln 6$

$t = 20\ln 6$

Time taken to travel from A to B = $20\ln 6$ seconds A1

(iii) Calculate the distance AB. [3]

Solution

Displacement, $s = \int_0^{20\ln 6} v\,dt$

$= \int_0^{20\ln 6} (5e^{0.05t} + 10)dt$ m1 M1

$= \left[\frac{5e^{0.05t}}{0.05} + 10t\right]_0^{20\ln 6}$ m1 M1

$= \left[100e^{0.05t} + 10t\right]_0^{20\ln 6}$

$= 100e^{0.05\times 20\ln 6} + 10\times 20\ln 6 - 100e^0$

$= 600 + 358.35 - 100$

$= 835.35$ m

$= 858$ m (3 s.f.) A1 A1

Mtd 2

$s = 100e^{0.05t} + 10t + c$

When $t = 0$, $s = 0$,

$0 = 100 + c$

$c = -100$

$\therefore s = 100e^{0.05t} + 10t - 100$.

At $t = 20\ln 6$,

$s = 858$

(iv) Find the acceleration of the car when $t = 30$ s. [2]

Solution

Acceleration, $a = \dfrac{dv}{dt}$

$$a = \frac{d}{dt}(5e^{0.05t} + 10) \qquad \text{M1}$$

$$= 5 \times 0.05e^{0.05t} \quad = 0.25e^{0.05t}$$

When the time is 30 s, $a = 5 \times 0.05e^{0.05 \times 30}$

$$= 1.12 \text{ m/s}^2 \qquad \text{A1}$$

11 Points $A(8, 1)$ and $B(1, 2)$ lie on a circle whose centre is C. The line $4y = 3x - 20$ is tangent to the circle at A.

(i) Find the equation of the normal to the circle at A. [2]

Solution

Equation of tangent is $y = \frac{3}{4}x - 5$.

Gradient of normal at A is $-\frac{4}{3}$. m1 M1

Equation of normal $y - 1 = -\frac{4}{3}(x - 8)$

$3y - 3 = -4x + 32$

$3y + 4x = 35$ $y = -\frac{4}{3}x + \frac{35}{3}$ A1 $11\frac{2}{3}$

(ii) Find the equation of the circle. [6]

Solution

Gradient of $AB = \frac{1-2}{8-1} = -\frac{1}{7}$

Gradient of perpendicular bisector of AB =7. M1 M1

Mid-point of $AB = \left(\frac{8+1}{2}, \frac{2+1}{2}\right)$

$= \left(\frac{9}{2}, \frac{3}{2}\right)$

Equation of perpendicular bisector of AB is

$y - \frac{3}{2} = 7\left(x - \frac{9}{2}\right)$

$y - \frac{3}{2} = 7x - \frac{63}{2}$ ★ ⊥ bisector of chord AB cuts through centre.

$y = 7x - 30$ M1 M1

If $y = 7x - 30$ meets $3y + 4x = 35$

$3(7x - 30) + 4x = 35$

$21x - 90 + 4x = 35$

$25x = 125$

$x = 5$ M1 M1

If $x = 5$, $y = 7 \times 5 - 30 = 5$

Therefore centre of circle $C(5, 5)$. M1 M1

$r^2 = (8-5)^2 + (1-5)^2$

$r^2 = 3^2 + 4^2$

$r^2 = 25$ M1 M1

Equation of circle is $(x - 5)^2 + (y - 5)^2 = 25$. A1 A1

$x^2 - 10x + y^2 - 10y + 25 = 0$

(iii) Explain why the coordinate axes are tangents to the circle. [1]

Answer
Since the centre is at (5, 5), the centre is 5 units from each axis and the radius of the circle is 5 units. Therefore, the coordinate axes are tangents to the circle. B1

12 The table shows experimental values of x and y.

x	2	3	4	5	6
y	11.8	16.2	23.5	35.5	55.2

It is known that x and y are related by the equation $y = Ae^{kx} + 5$, where A and k are constants.

(i) Explain how a straight line graph may be drawn to estimate the values of A and of k. [2]

Solution
$y = Ae^{kx} + 5$
$y - 5 = Ae^{kx}$
$\ln(y-5) = \ln(Ae^{kx})$ M1
$\ln(y-5) = \ln A + kx$
Plot $\ln(y-5)$ against x. A1

(ii) Draw a straight line graph to obtain estimated values of A and k. [5]

Solution

x	2	3	4	5	6
$\ln(y-5)$	1.92	2.42	2.92	3.42	3.92

M1

Answers

From the graph, $\ln A \approx 0.95$

$A \approx 2.59$ B1

$k \approx \frac{3-1.5}{4.1-1.1}$

$k \approx 0.5$ B1

(iii) Use your graph to find the value of x when $y = 30$. [1]

When $y = 30$, $\ln(30-5) = 3.22$

$x \approx 4.6$ Ans Accept 4.6 – 4.7) B1

003

(iv) By drawing a suitable straight line on the same axes, solve the equation [3]

$Ac^{kx} - 30(2^{-x})$ Given $y = Ae^{kx} + 5$

Solution

$Ae^{kx} + 5 = 30(2^{-x}) + 5$

$y = 30(2^{-x}) + 5$

$y - 5 = 30(2^{-x})$

$\ln(y-5) = \ln 30 + \ln(3^{-x})$

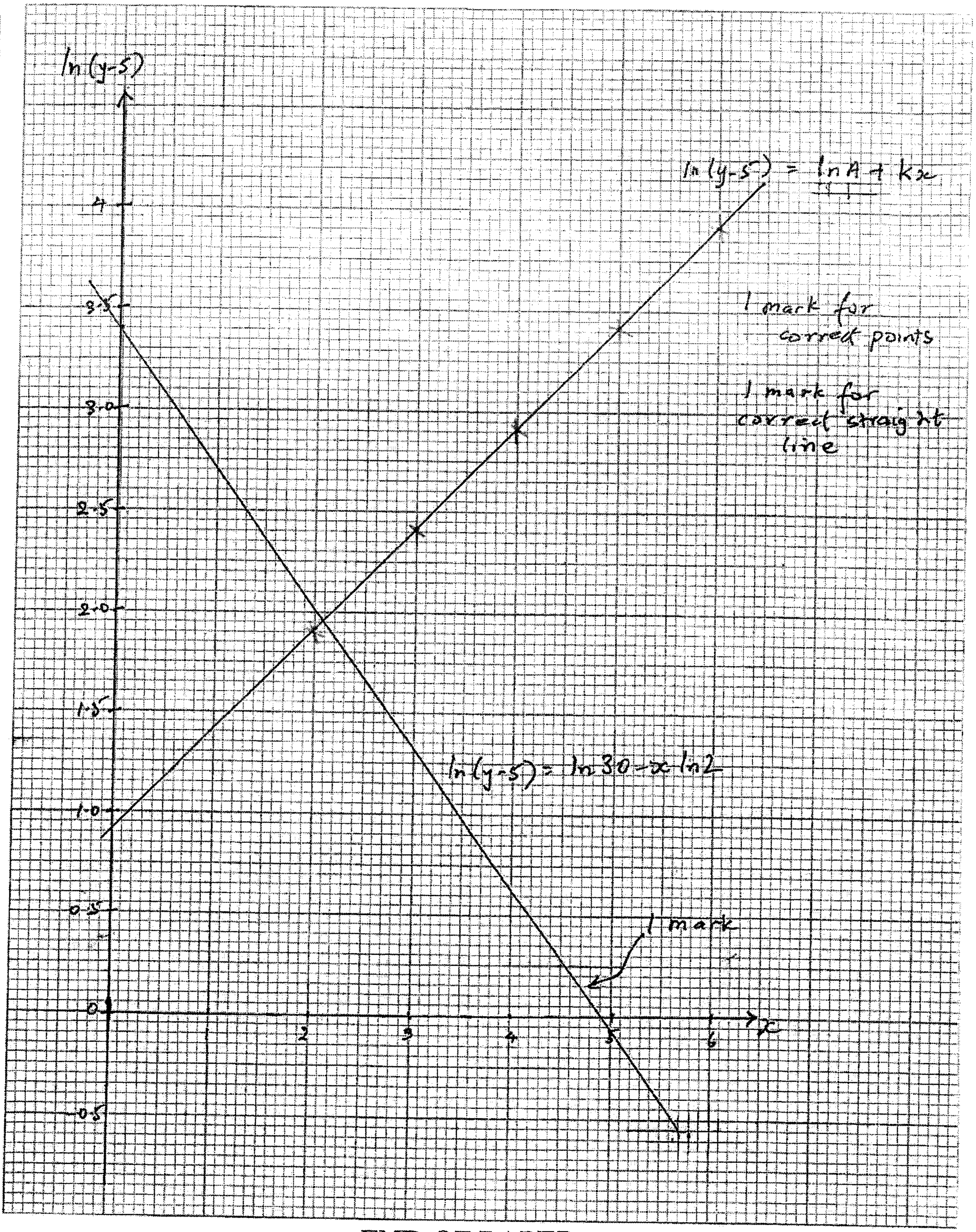
$\ln(y-5) = \ln 30 - x\ln 2$

Draw $\ln(y-5) = \ln 30 - x\ln 2$ M1 M1

x	0	5	1		
$\ln(y-5)$	3.4	– 0.06	2.71		1.32

From the graph, $x \approx 2.05$ Accept (2.0 to 2.1) A1

11/11


ln(y-5)
ln(y-5) = lnA + kx
1 mark for correct points
1 mark for correct straight line
ln(y-5) = ln 30 - x ln2
1 mark
x
4
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
-0.5
1
2
3
4
5
6

END OF PAPER