Candidate Name: ______________________ Class [ ] Index No. [ ]

FUHUA SECONDARY SCHOOL

Secondary Four Express / Five Normal (Academic)

Preliminary Examination 2022

4E/5N

Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary
Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary
Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary
Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary Fuhua Secondary

ADDITIONAL MATHEMATICS 4049/01

Paper 1

DATE: **19 Aug 2022**
TIME: **1030 – 1245**
DURATION: 2 hours 15 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to 3 significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is **90**.

PARENT'S SIGNATURE	FOR EXAMINER'S USE		
	Units		/ 90
	Statements/Accuracy		
	Presentation		

Setter: Miss Ho Ying Yee Vetter: Mr Sun Daojun

This question paper consists of 23 printed pages and 1 blank page, including this page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \tfrac{1}{2} bc \sin A$$

1 A cylinder has a radius of $\left(\sqrt{2}-1\right)$ cm and a volume of $\left(12+3\sqrt{2}\right)\pi$ cm^3.

Find, **without using a calculator**, the exact value of its height, h cm, in the form $a+b\sqrt{2}$, where a and b are integers. [3]

[Turn over

2 Suppose that x and y are non-zero real numbers such that $\frac{x}{3} = y^2$ and $\frac{x}{9} = 9y$.

Find the value of $x - y$. [3]

3 Express $y = -2x^2 - 12x + 1$ in the form $y = a - 2(x+b)^2$ and hence state the maximum value of y, and its corresponding value of x. [4]

[Turn over

4 A curve is such that $\frac{dy}{dx} = \frac{2}{x^2} + \frac{3}{(7-2x)}$ and $P(3, 1)$ is a point on the curve.

Find the equation of the curve. [4]

5 Express $\dfrac{2x-1}{x^2(x+1)}$ in partial fractions. [6]

[Turn over

6 The function f is defined by $f(x) = 3x^3 - 5cx^2 + kc^2x + 4c^3$, where c and k are non-zero constants. $f(x)$ leaves a remainder of $-32c^3$ when divided by $x + 2c$.

(a) Find the value of k. [2]

(b) Using the value of k, determine whether $x^2 - cx - 2c^2$ is a factor of $f(x)$. Justify your answer. [3]

7 The function f is defined by $f(x) = 4\cos ax + b$ for $0 \leq x \leq \pi$, where a and b are constants. The period of f is $\frac{2\pi}{3}$ and the function has a maximum value of 2.

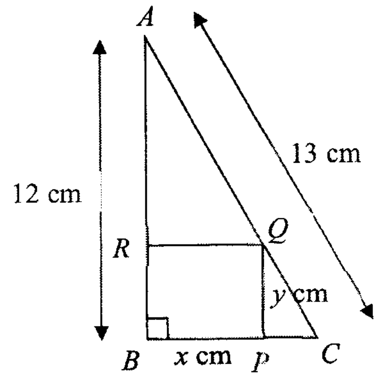
(i) State the amplitude of f. [1]

(ii) Write down the value of a and of b. [2]

(iii) Sketch the graph of $y = f(x)$ for $0 \leq x \leq \pi$, indicating clearly the x-coordinates, in terms of π, of the points where the graph crosses the x-axis. [4]

[Turn over

8 In ΔABC, $\angle ABC = 90°$, $AB = 12$ cm and $AC = 13$ cm. The rectangle $BPQR$ is such that its vertices P, Q and R lie on BC, CA and AB respectively.



It is given that $BP = x$ cm and $PQ = y$ cm.

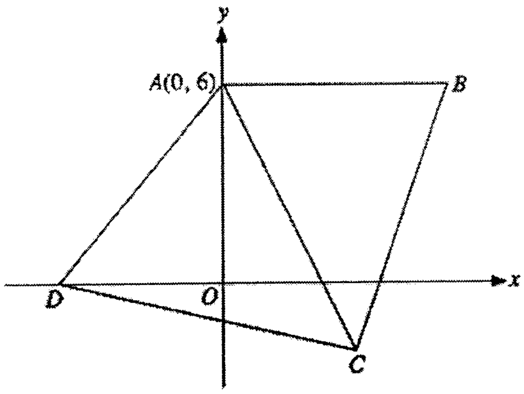
(a) Show that $y = \dfrac{60 - 12x}{5}$. [2]

(b) Show that the area, A cm^2, of the rectangle $BPQR$ is given by $A = 12x - \dfrac{12x^2}{5}$. [1]

8 (c) Given that x can vary, find the stationary value of A and determine its nature. [4]

[Turn over

9 The diagram shows a quadrilateral $ABCD$ in which A is (0, 6) and AB is parallel to the x-axis. D is a point on the x-axis such that the equation of DC is $x+5y=-6$. AC is perpendicular to the line $2y-x=7$.



(a) Find,

(i) the equation of AC, [2]

(ii) the coordinates of C. [2]

9 **(b)** Given that the area of ΔACD is 1.5 times that of ΔABC, find the coordinates of B. [3]

(c) Showing your working clearly, explain whether $ABCD$ is a kite. [2]

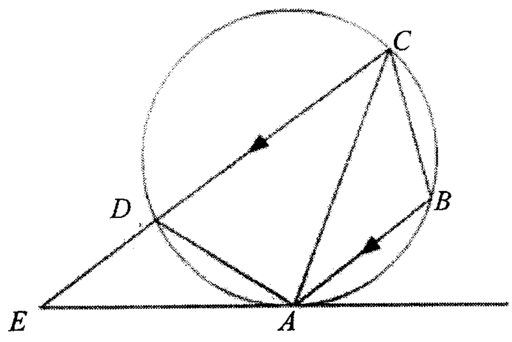
[Turn over

10 **(a)** Prove the identity $\dfrac{1-\sin^4 x}{\sin^2 x} = \cot^2 x + \cos^2 x$. [4]

10 **(b)** Hence solve the equation $\cot^2 2x + \cos^2 2x = 0$ for $0° < x < 180°$. [4]

[Turn over

11 The diagram shows a quadrilateral *ABCD* whose vertices lie on the circumference of a circle. The point *E* lies on the extended line *CD* such that *AE* is a tangent to the circle at *A*. *CD* and *AB* are parallel lines.



(a) Explain why angle *CBA* = angle *EDA*. [2]

(b) Show that triangles *DAE* and *BAC* are similar. [2]

11 (c) Given that $AD = DE$, explain why the line AC bisects the angle BCD. [3]

[Turn over

12 **(a)** Given that $2^{x-2} \times 3^{x+2} = 6^{2x}$, show that $x = \dfrac{\lg 9 - \lg 4}{\lg 6}$ [4]

12 **(b)** Solve $\log_9 y - 2 = \log_3 2y$. [4]

[Turn over

13 **(a)** Water leaks from a container at a rate of 150 cm^3/s. The volume, V cm^3, of the water in the container, when the height of water is h cm, is given by $V = 10\pi + \frac{4\pi h^3}{9}$. When $V = 334\pi$ cm^3, find the

(i) value of h, [2]

(ii) rate of change of h at this instant, correct to 3 significant figures. [3]

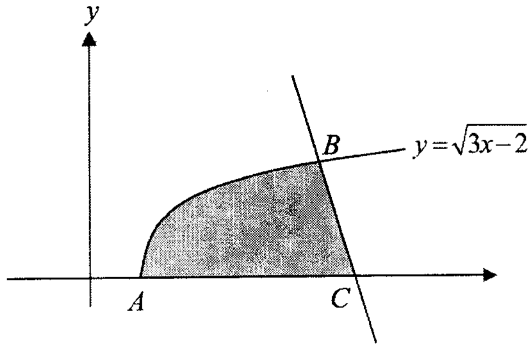
13 **(b)** The stopping distance, s m, of a car moving at v km/h can be modelled by the formula $\frac{s}{v}=\frac{1}{6}+\frac{v}{50}$.

(i) Find the rate at which s is changing with respect to v when $v=60$. [3]

(ii) Explain the meaning of your answer to part **(i)**. [1]

[Turn over

14



The diagram shows part of the curve $y=\sqrt{3x-2}$. The normal to the curve at B meets the x-axis at C. Given that the x-coordinate of B is 9, find

(a) the coordinates of A and of C, [5]

14 **(b)** the area of the shaded region. [5]

~ End of Paper ~

Candidate Name:		Class	Index No.

FUHUA SECONDARY SCHOOL

Secondary Four Express/ Five Normal (A)

4E/5N

Preliminary Examination 2022

ADDITIONAL MATHEMATICS

4049/02

Paper 2

DATE	**26 August 2022**
TIME	**0800 – 1015**
DURATION	**2 hours 15 minutes**

Candidates answer on the Question paper.
Additional Materials: Nil

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90**.

PARENT'S SIGNATURE	FOR EXAMINER'S USE		
	Units		**/ 90**
	Statements/Accuracy		
	Presentation		

This question paper consists of 19 printed pages and 1 blank pages.

Setter: Mr Liu Yaozhong   Vetter: Mr Sun Daojun

[Turn Over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$cosec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 By using an appropriate substitution, solve the equation $e^{\frac{1}{2}x} = 2 + 24e^{-\frac{1}{2}x}$. [5]

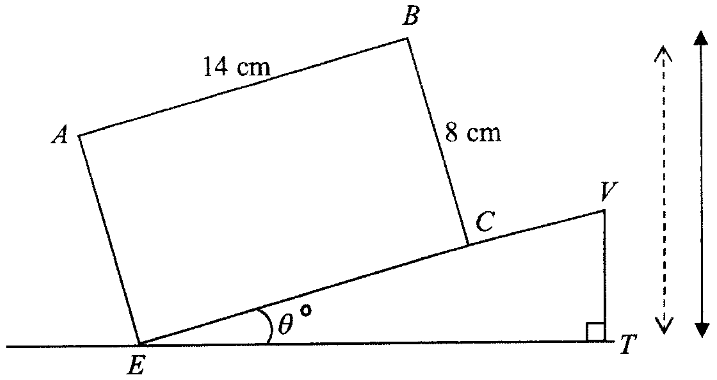
2 **(a)** Integrate $\left(\frac{6}{e^{3-x}}\right)^2$ with respect to x. [3]

(b) Explain, with working, why the equation $2x^3+3x^2+2x+8=0$ has only 1 real root. Hence find this root. [5]

3 **(a)** Given that $y = x^2\sqrt{2x-1}$, show that $\frac{dy}{dx} = \frac{x(5x-2)}{\sqrt{2x-1}}$. Determine whether $y = x^2\sqrt{2x-1}$ is always an increasing function. [5]

(b) Hence evaluate $\int_1^5 \frac{5x^2-2x+1}{\sqrt{2x-1}}\,dx$. [4]

4 The diagram shows the front view of a rectangular block $ABCE$, with dimensions 14cm by 8 cm, placed on a ramp, VE, tilted at an acute angle of $\theta°$ and $\angle VTE = 90°$. The ramp is placed on a horizontal surface ET and d is the perpendicular distance from B to ET.



(a) Show that $d = 14\sin\theta + 8\cos\theta$. [2]

(b) Express d in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° \le \alpha \le 90°$. [3]

(c) State the maximum value of d and find the corresponding value of θ. [2]

(d) Find the smallest value θ such that $d = 13$. [2]

5 **(a)** Explain by completing the square that $x^2 - 2x + 3$ is always positive for all real values of x. [2]

(b) Hence, find the range of values of p if the inequality $\dfrac{3x^2 + px + 3}{x^2 - 2x + 3} > 0$ is satisfied for all real values of x. [4]

6 (a) In the binomial expansion of $\left(x+\frac{k}{x}\right)^5$, where k is a positive integer, the coefficients of $\frac{1}{x^3}$ and $\frac{1}{x}$ are the same. Find the value of k. [5]

(b) Without expanding all the terms, explain why there is no constant term in the expansion of $\left(1-3x^2\right)\left(x+\frac{k}{x}\right)^5$. [3]

7 **(a)** The variables x and y, are related by the equation $\frac{3y^2}{k}+\frac{2x}{h}=6$, where h and k are constants. When the graph of y^2 against x is plotted, a straight line is obtained. Given that the intercept on the y^2 axis is 8 and that the gradient of the line is -6, calculate the value of h and of k. [3]

(b) A glass of hot liquid is put on a table to cool. The temperature of the liquid, T°C, after n minutes is given by $T=25+pe^{-kn}$, where p and k are constants. The table below shows the measured values of T and n.

n (minutes)	10	20	30	40	50
T(°C)	79.6	58.1	45.1	37.2	32.4

(i) Using the grid on page 11, plot $\ln(T-25)$ against n and draw a straight line graph. [3]

Use your graph to estimate

(ii) value of p and of k, [4]

(iii) the temperature of the liquid after 35 minutes, [2]

(iv) the number of minutes it takes for the temperature of the liquid to drop by 75% of its initial value. [2]

8 The velocity, $v\,\text{ms}^{-1}$, of a particle, moving in a straight line, t seconds after motion has begun, is given by $v = 6t^2 + kt + 12$, where k is a constant. The particle passes a fixed point O with an acceleration of $-6\,\text{ms}^{-2}$ when $t = 1$.

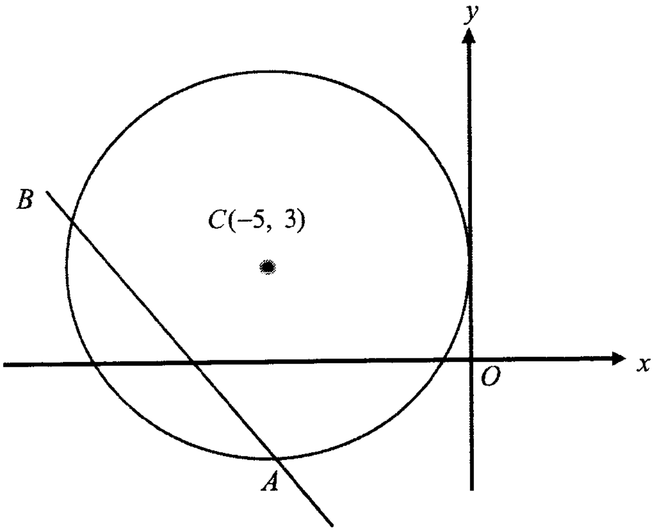
(a) Show that $k = -18$. [2]

(b) Hence, find

(i) the minimum velocity achieved by the particle, [3]

(ii) the total distance travelled during the first 4 seconds. [5]

9



(a) The coordinates of the centre of a circle C_1 is $C(-5, 3)$. If the y-axis is a tangent of the circle, find the equation of the circle. [1]

A straight line cuts the circle at points A and B such that AB is parallel to $y + 3x - 11 = 0$ and that AC is parallel to the y-axis.

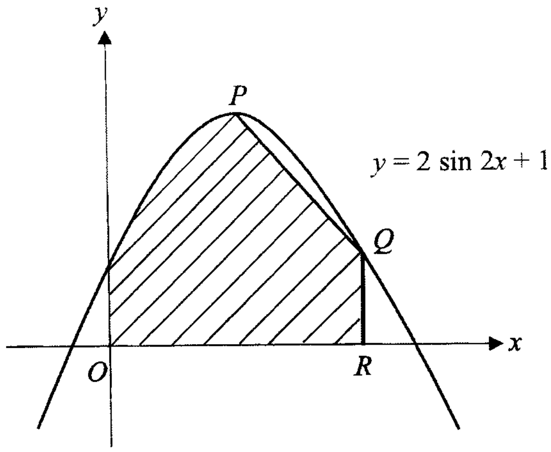
(b) Find

(i) the equation of the line AB and the equation of the perpendicular bisector of AB, [5]

(ii) hence, find the coordinates of B. [3]

(c) Circle C_2 is obtained by reflecting circle C_1 in the line AB. Find the equation of circle C_2. [3]

10 The diagram shows part of the curve $y = 2\sin 2x + 1$. P is the maximum point of the curve and Q is the point on the curve at which the gradient of the tangent is -4 .



(a) Find the coordinates of P and of Q. [5]

(b) Find the area of the shaded region bounded by the curve, the axes, line PQ and vertical line QR. Leave your answer in the exact form. [4]

-End of Paper-

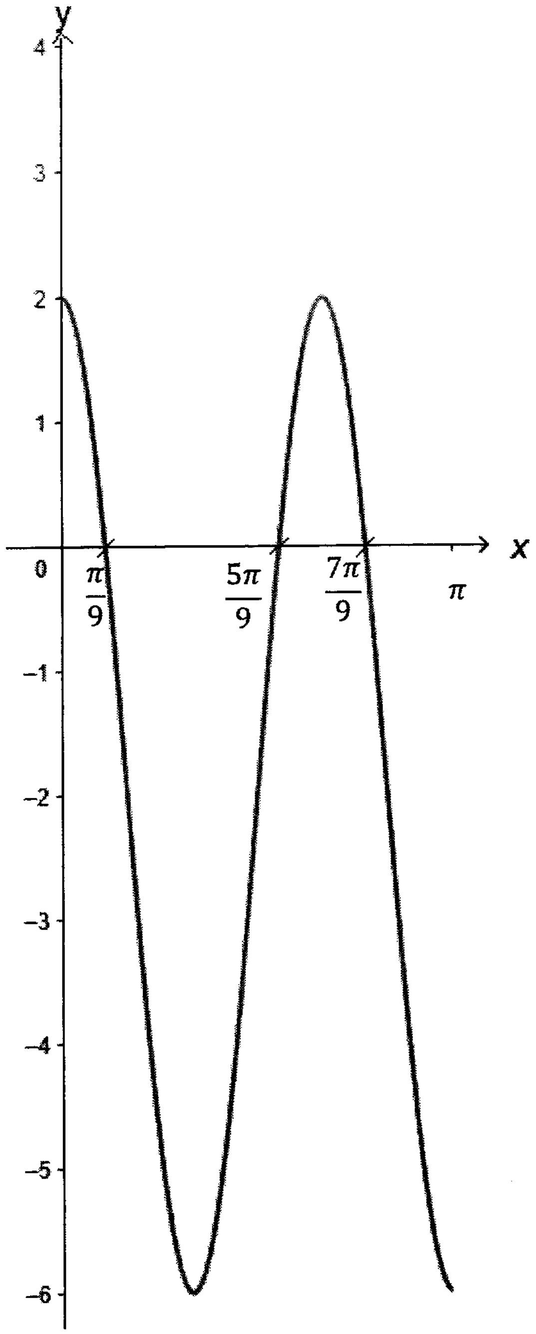
FHSS Prelim AM P1 2022 Marking Scheme

Qn	Solution	Mark allocation	Remarks
1	Volume of a cylinder, $V=\pi r^2 h$		
	$\left(12+3\sqrt{2}\right)\pi=\pi\left(\sqrt{2}-1\right)^2 h$		
	$h=\dfrac{12+3\sqrt{2}}{\left(\sqrt{2}-1\right)^2}$	M1	
	$h=\dfrac{12+3\sqrt{2}}{3-2\sqrt{2}}$		
	$h=\dfrac{\left(12+3\sqrt{2}\right)\left(3+2\sqrt{2}\right)}{\left(3-2\sqrt{2}\right)\left(3+2\sqrt{2}\right)}$	M1	
	$h=\dfrac{\left(48+33\sqrt{2}\right)}{9-8}$		
	$h=48+33\sqrt{2}$	A1	
2	$\frac{x}{3}=y^2$ —(1)		
	$\frac{x}{9}=9y$ ---(2)		
	From (2), $x=81y$		
	From (1), $x=3y^2$		
	$3y^2=81y$	M1	
	$3y^2-81y=0$		
	$3y(y-27)=0$		
	$y=0$ (reject) or $y=27$	M1	
	$x=2187$		
	$x-y=2187-27$		
	$=2160$	A1	

3	$y=-2x^2-12x+1$ $=-2\left(x^2+6x-\frac{1}{2}\right)$ $=-2\left[x^2+6x+\left(\frac{6}{2}\right)^2-\left(\frac{6}{2}\right)^2-\frac{1}{2}\right]$ $=-2\left[(x+3)^2-9-\frac{1}{2}\right]$ $=-2\left[(x+3)^2-9\frac{1}{2}\right]$ $=19-2(x+3)^2$ Maximum value of y is 19. The maximum value occurs when $x=-3$.	M1 M1 A1 A1	
4	$y=\int\frac{2}{x^2}+\frac{3}{(7-2x)}dx$ $y=-\frac{2}{x}+\frac{3}{-2}\ln(7-2x)+c$ Substitute $P(3,\ 1)$, $1=-\frac{2}{3}+\frac{3}{-2}\ln(1)+c$ $c=\frac{5}{3}$ Equation of curve is $y=-\frac{2}{x}-\frac{3}{2}\ln(7-2x)+\frac{5}{3}$	M2 M1 A1	
5	$\frac{2x-1}{x^2(x+1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{(x+1)}$ $=\frac{Ax(x+1)+B(x+1)+Cx^2}{x^2(x+1)}$ $=\frac{Ax^2+Cx^2+Ax+Bx+B}{x^2(x+1)}$	M1 M1 M1	

	Comparing coefficient of: $x^0 : -1 = B$ $x^1 :$ $2 = A + B$ $2 + 1 = A$ $3 = A$ $x^2 :$ $0 = A + C$ $0 = 3 + C$ $-3 = C$ $\therefore \frac{2x-1}{x^2(x+1)} = \frac{3}{x} - \frac{1}{x^2} - \frac{3}{(x+1)}$ Alternative method, $\frac{2x-1}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x+1)}$ $= \frac{Ax(x+1) + B(x+1) + Cx^2}{x^2(x+1)}$ $2x-1 = Ax(x+1) + B(x+1) + Cx^2$ Let $x = -1$, $2(-1) - 1 = C(-1)^2$ $C = -3$ Let $x = 0$, $-1 = B(1)$ $B = -1$ Let $x = 1$, $2 - 1 = A(1)(2) + (-1)(2) + (-3)(1)^2$ $A = 3$	M1 M1 A1 M1 M1 M1 M1 M1	

	$\therefore \frac{2x-1}{x^2(x+1)} = \frac{3}{x} - \frac{1}{x^2} - \frac{3}{(x+1)}$	A1	
6a	$f(-2c) = -32c^3$ $3(-2c)^3 - 5c(-2c)^2 + kc^2(-2c) + 4c^3 = -32c^3$ $-24c^3 - 20c^3 - 2kc^3 + 4c^3 = -32c^3$ $-2kc^3 = 8c^3$ $k = -4$	M1 A1	Allow long division method
6b	$f(x) = 3x^3 - 5cx^2 - 4c^2x + 4c^3$ $x^2 - cx - 2c^2 \overline{)3x^3 - 5cx^2 - 4c^2x + 4c^3}$ quotient $3x - 2c$ $\underline{3x^3 - 3cx^2 - 6c^2x}$ $-2cx^2 + 2c^2x + 4c^3$ $\underline{-2cx^2 + 2c^2x + 4c^3}$ Since remainder is 0, $x^2 - cx - 2c^2$ is a factor of $f(x)$. <u>Alternative method.</u> $x^2 - cx - 2c^2 = (x+c)(x-2c)$ $f(-1) = 0$ $f(2c) = 0$	M1 M1 A1 M1 M1	
7i	Amplitude = 4	B1	
7ii	$\frac{2\pi}{3} = \frac{2\pi}{a}$ $a = 3$ $b = -2$	 B1 B1	

7iii		Shape -1 Intercept -1 Min, max -1 Period -1	

8a	In right-angled ΔABC, by Pythagoras' Theorem, $AC^2 = AB^2 + BC^2$ $13^2 = 12^2 + BC^2$ $BC^2 = 13^2 - 12^2$ $= 25$ $BC = \pm 5$ Since $BC>0$, $BC=5$. By similar triangles ΔABC and ΔQPC, $\frac{y}{12} = \frac{5-x}{5}$ $y = \frac{12(5-x)}{5}$ $= \frac{60-12x}{5}$ (shown)	M1 A1	
8b	$A = xy$ $= x\left(\frac{60-12x}{5}\right)$ $= x\left(12 - \frac{12x}{5}\right)$ $= 12x - \frac{12x^2}{5}$ (shown)	B1	
8c	$A = 12x - \frac{12x^2}{5}$ $\frac{dA}{dx} = 12 - \frac{24x}{5}$ For stationary values, $\frac{dA}{dx} = 0$. $12 - \frac{24x}{5} = 0$ $60 - 24x = 0$ $24x = 60$ $x = 2.5$	M1 M1	

	When $x=2.5$, $A=12(2.5)-\frac{12(2.5)^2}{5}$ $=15$ $\frac{dA}{dx}=12-\frac{24x}{5}$ $\frac{d^2A}{dx^2}=-\frac{24}{5}<0$ By the second derivative test, A is a maximum. Hence, the stationary value of A is 15 cm^2 and it is a maximum.	A1 A1	
9ai	$m_{AC}=-2$ Equation of AC is $y=-2x+6$	M1 A1	
9aii	$y=-2x+6$ ---- (1) $x+5y=-6$ --- (2) Sub (1) into (2) $x+5(-2x+6)=-6$ $-9x=-36$ $x=4$ $y=-2$ $C(4,-2)$	M1 A1	

9b	$D=(-6,0)$ Area of $\Delta ACD=\frac{1}{2}\begin{vmatrix}0 & -6 & 4 & 0\\ 6 & 0 & -2 & 6\end{vmatrix}$ $=\frac{1}{2}(12+24-(-36))$ $=36\text{units}^2$ Area of $\Delta ABC=\frac{36}{1.5}$ $=24\text{units}^2$ Let $B=(k,6)$ $\frac{1}{2}\begin{vmatrix}0 & 4 & k & 0\\ 6 & -2 & 6 & 6\end{vmatrix}=24$ $\frac{1}{2}(24+6k-24+2k)=24$ $k=6$ $B=(6,6)$ <u>Alternative method,</u> $\frac{1}{2}(AB)(8)=24$ $AB=6$ $B(6,6)$	M1 M1 A1 M1 M1 A1	
9c	$\lvert AB\rvert=6\text{units}$ $\lvert AD\rvert=\sqrt{72}\text{units}$ $\lvert CD\rvert=\sqrt{(4+6)^2+(-2)^2}$ $=\sqrt{104}\text{units}$ Since $\lvert AB\rvert\neq\lvert AD\rvert\neq\lvert CD\rvert$, $ABCD$ is not a kite.	 M1 A1	

10a	$\frac{1-\sin^4 x}{\sin^2 x} = \frac{1}{\sin^2 x} - \sin^2 x$ $= \operatorname{cosec}^2 x - (1-\cos^2 x)$ $= \cot^2 x + 1 - 1 + \cos^2 x$ $= \cot^2 x + \cos^2 x$ (Proven)	M1 M1 M1 A1	
10b	$\cot^2 2x + \cos^2 2x = 0$ $\frac{1-\sin^4 2x}{\sin^2 2x} = 0$ $1-\sin^4 2x = 0$ $\sin^4 2x = 1$ $\sin 2x = 1$ or $\sin 2x = -1$ basic angle = 90° $2x = 90°, 270°$ $x = 45°, 135°$	M1 M1 M1 A1	
11a	$\angle CBA = 180° - \angle ADC$ (opp. ∠s of cyclic *quadrilateral*) $\angle ADE = 180° - \angle ADC$ (adj. ∠s on a str. line) $\therefore \angle CBA = \angle ADE$	M1 A1	
11b	In triangles *DAE* and *BAC*, $\angle CBA = \angle ADE$ (shown in (a)) $\angle DAE = \angle ACD$ (tangent-chord theorem) $= \angle CAB$ (alt. ∠) Hence, triangles *BAE* and *DAC* are similar.	M1 A1	
11c	*AD* = *DE*, implying that triangles *DAE* and *BAC* are similar isosceles triangles. $\angle ACB = \angle CAB$ (alt. ∠s) $= \angle DCA$ Hence, the line *AC* bisects the angle *BCD*.	M1 M1 A1	

12a	$2^{x-2}\times 3^{x+2}=6^{2x}$ $\dfrac{2^x\times 3^x\times 3^2}{2^2}=6^{2x}$ $\dfrac{6^x\times 9}{4}=6^{2x}$ $6^x=\dfrac{9}{4}$ $x=\dfrac{\lg\left(\dfrac{9}{4}\right)}{\lg 6}$ $=\dfrac{\lg 9-\lg 4}{\lg 6}$ (shown)	M1 M1 M1 A1	
12b	$\dfrac{\log_3 y}{\log_3 9}-2=\log_3 2y$ $\dfrac{1}{2}\log_3 y-2=\log_3 2y$ $\dfrac{1}{2}\log_3 y-\log_3 2y=2$ $\log_3 y-2\log_3 2y=4$ $\log_3 y-\log_3(2y)^2=4$ $\log_3\dfrac{y}{4y^2}=4$ $\dfrac{1}{4y}=3^4$ (since y>0) $y=\dfrac{1}{324}$	M1 M1 M1 A1	
13ai	When $V=334\pi$ cm^3, $10\pi+\dfrac{4\pi h^3}{9}=334\pi$ $\dfrac{4\pi h^3}{9}=324\pi$ $h^3=729$ $h=9$ Hence, the value of h is 9.	M1 A1	

13aii	$V = 10\pi + \frac{4\pi h^3}{9}$ $\frac{dV}{dh} = 3\left(\frac{4\pi h^2}{9}\right)$ $= \frac{4\pi h^2}{3}$ Since $\frac{dV}{dt} = -150 \text{ cm}^3/\text{s}$, when h=9, $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ $-150 = \frac{4\pi(9)^2}{3} \times \frac{dh}{dt}$ $-150 = 108\pi \times \frac{dh}{dt}$ $\frac{dh}{dt} = -\frac{25}{18\pi}$ $= -0.442$ (correct to 3 sig. fig.) Hence, the rate of change of h is -0.442 cm/s.	M1 M1 A1	
13bi	$\frac{s}{v} = \frac{1}{6} + \frac{v}{50}$ $s = \frac{1}{6}v + \frac{v^2}{50}$ $\frac{ds}{dv} = \frac{1}{6} + \frac{2v}{50}$ $\left.\frac{ds}{dv}\right\|_{v=60} = \frac{1}{6} + \frac{2(60)}{50}$ $= \frac{77}{30} \frac{\text{m}}{\text{km/h}}$	M1 M1 A1	
13bii	$\frac{ds}{dv}$ is the rate of change of the stopping distance with respect to the speed of the car. When the car is travelling at 60 km/h, for every 1km/h increase in its speed, its stopping distance increases by approximately 2.57 m.	B1	
14a	To find the coordinates of A,		

	When $y = 0$, $0 = \sqrt{3x-2}$ $x = \dfrac{2}{3}$	M1	
	Therefore, $A(\dfrac{2}{3}, 0)$.	A1	
	Coordinates of $B(9, 5)$ To find the coordinates of C, $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{3x-2}\right) = \dfrac{3}{2\sqrt{3x-2}}$	M1	
	At $x = 9$, Grad. of tangent $= \dfrac{3}{10}$ Gradient of normal $= -\dfrac{10}{3}$		
	Let $C = (c, 0)$, $\dfrac{5-0}{9-c} = -\dfrac{10}{3}$ $15 = -90 + 10c$	M1	
	Therefore $C = (10.5 , 0)$.	A1	

14b	Area of shaded region $= \int_{\frac{2}{3}}^{9} \sqrt{3x-2}\,dx + \text{Area of } \Delta$	M1	
	$= \left[\dfrac{(3x-2)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)(3)} \right]_{\frac{2}{3}}^{9} + \left(\dfrac{1}{2} \times \dfrac{3}{2} \times 5 \right)$	M2	
	$= \left[\dfrac{25^{\frac{3}{2}}}{\frac{9}{2}} - \dfrac{0^{\frac{3}{2}}}{\frac{9}{2}} \right] + \dfrac{15}{4}$	M1	
	$\approx 31.5 \text{ units}^2$	A1	

1. $e^{\frac{1}{2}x} = 2 + 24e^{-\frac{1}{2}x}$

$e^{\frac{1}{2}x} = 2 + \frac{24}{e^{\frac{1}{2}x}}$

let $u = e^{\frac{1}{2}x}$

$u = 2 + \frac{24}{u}$ M1

$u^2 = 2u + 24$

$u^2 - 2u - 24 = 0$

$(u-6)(u+4) = 0$ M1

$u-6 = 0$ or $u+4=0$

$u = 6$ or $u=-4$ M1

$e^{\frac{1}{2}x} = 6$ or $e^{\frac{1}{2}x} = -4$

$\frac{1}{2}x = \ln 6$ A1 $\frac{1}{2}x = \ln -4$ (rej.)

$x = 2\ln 6$

A1

2a $$\int \left(\frac{6}{e^{3-x}}\right)^2 dx$$

$$= \int \frac{36}{e^{6-2x}}$$

$$= 36\int e^{2x-6} dx$$

$$= \frac{36e^{2x-6}}{2} + c$$

$$= 18e^{2x-6} +$$

2b let $p(x) = 2x^3 + 3x^2 + 2x + 8$

$p(-2) = 2(-2)^3 + 3(-2)^2 + 2(-2) + 8$ M1

$= 0$

$x+2$ is a factor

$$
\begin{array}{r|l}
 & 2x^2 - x + 4 \\
x+2 & 2x^3 + 3x^2 + 2x + 8 \\
 & -(2x^3 + 4x^2) \\
 & \quad -x^2 + 2x \\
 & \quad -(-x^2 - 2x) \\
 & \qquad 4x + 8 \\
 & \qquad -(4x + 8) \\
 & \qquad\quad 0
\end{array}
$$

M1

$P(x) = (x+2)(2x^2 - x + 4)$

$P(x) = 0$

$(x+2)(2x^2 - x + 4) = 0$

$x+2 = 0$

$x = -2$

A1

$2x^2 - x + 4 = 0$

$(-1)^2 - 4(2)(4) = -31 < 0$ M1

$b^2 - 4ac < 0$

Since $b^2 - 4ac < 0$, $2x^2 - x + 4 =$ has no real solutions

$\therefore P(x) = 0$ has only one real root A1

3a $y = x^2 \sqrt{2x-1}$

$u = x^2$ $v = (2x-1)^{\frac{1}{2}}$

$\frac{du}{dx} = 2x$ $\frac{dv}{dx} = \frac{1}{2}(2x-1)^{-\frac{1}{2}}(2)$

$= (2x-1)^{-\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{x^2}{\sqrt{2x-1}} + 2x\sqrt{2x-1}$$

$$= \frac{x^2 + 2x\sqrt{2x-1}(\sqrt{2x-1}}{\sqrt{2x-1}}$$

$$= \frac{x^2 + 2x(2x-1)}{\sqrt{2x-1}}$$

$$= \frac{x^2 + 4x^2 - 2x}{\sqrt{2x-1}}$$

$$= \frac{5x^2 - 2x}{\sqrt{2x-1}}$$

$$= \frac{x(5x-2)}{\sqrt{2x-1}}$$

$2x-1 > 0$

$x > \frac{1}{2}$ B1

$5x-2 > \frac{1}{2}$

$\therefore \quad 5x-2 > 0$

$x(5x-2) > 0$

$\frac{dy}{dx} > 0$

It is an increasing function.

3b

$$\int_1^5 \frac{5x^2 - 2x + 1}{\sqrt{2x-1}}\, dx$$

$$= \int_1^5 \frac{5x^2 - 2x}{\sqrt{2x-1}}\, dx + \int_1^5 \frac{1}{\sqrt{2x-1}}\, dx \quad \text{m1}$$

$$= \left(x^2 \sqrt{2x-1}\right)_1^5 + \left[\frac{(2x-1)^{\frac{1}{2}}}{\frac{1}{2}\times 2}\right]_1^5$$

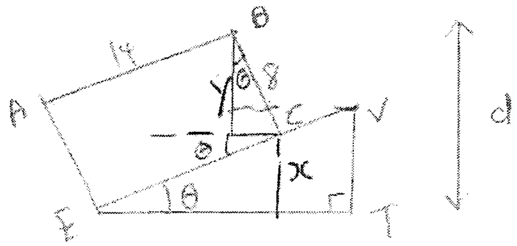
$$= \left[5^2\sqrt{2(5)-1} - 1^2\sqrt{2(1)-1}\right] + \left[(2x-1)^{\frac{1}{2}}\right]_1^5$$

$$= 74 + [2(5)-1]^{\frac{1}{2}} - [2(1)-1]^{\frac{1}{2}} \quad \text{m1}$$

$$= 74 + 2$$

$$= 76$$

4.a.



$\sin\theta = \frac{x}{14}$

$x = 14\sin\theta$ M1

$\cos\theta =$

$y = 8\cos$

$d = x + y$

$d = 14\sin\theta + 8\ \theta$. A1

180 - (90+90-θ)

4b.

$= \sqrt{14^2\ \ 8^2}$ M1

$= \sqrt{260}$

$= 2\sqrt{65}$

$\tan\alpha = \frac{8}{14}$ M1

$=$

$d =$ $+ 29.7°)$ A1

4c maximum of $d = 2\sqrt{65}$ B1

$= 16.1$

$(\theta + 29.7448°) = 90°$

corresponding $\theta = 60.2552°$

$= 60.3°$ (1dp) B1

4d. $13 = 2\sqrt{65} \sin(\theta + 29.7°)$

$\sin(\theta + 29.7448°) = \frac{13}{2\sqrt{65}}$ M1

$\theta + 29.7448° = 53.7288$

smallest value of $\theta = 23.9800$ (4dp)

$= 24.0$ (1dp) A1

5ai

$$x^2 - 2x + 3$$

$$= x^2 - 2x + \left(\frac{-2}{2}\right)^2 + 3 - \left(\frac{-2}{2}\right)^2 \quad M1$$

$$= (x-1)^2 + 2$$

Since $(x-1)^2 \geq 0$

$(x-1)^2 + 2 \geq 2$

$\therefore$ $x^2 - 2x + 3$ is always positive for all real values of x. B1

5b

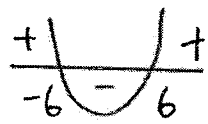
Since $x^2 - 2x + 3$ is always positive,

$3x^2 + px + 3 > 0$ M1

$\therefore\ b^2 - 4ac < 0$

$(p)^2 - 4(3)(3) < 0$ M1

$p^2 - 36 < 0$



$(p-6)(p+6) < 0$ M1

$-6 < p < 6$ A1

6a. $T_{r+1} = \binom{n}{r} a^{n-r} b^{r}$

$= \binom{5}{r} (x)^{5-r} \left(\frac{k}{x}\right)^{r}$ m1

$= \binom{5}{r} x^{5-r} k^{r} x^{-r}$

$= \binom{5}{r} k^{r} x^{5-2r}$

For $\frac{1}{x^3}$, $x^{5-2r} = x^{-3}$ m1

$5-2r = -3$

$2r = 8$

$r = 4$

coefficient of $\frac{1}{x^3} = \binom{5}{4} k^{4}$

$= 5k^{4}$

For $\frac{1}{x}$, $x^{5-2r} = x^{-1}$ m1

$5-2r = -1$

$2r = 6$

$r = 3$

coefficient of $\frac{1}{x} = \binom{5}{3} k^{3}$

$= 10k^{3}$

$5k^4 = 10k^3$

$5k^4 - 10k^3 = 0$ M1

$5k^3(k-2) = 0$

$k = 0$ or $k - 2 = 0$
(rej) $k = 2$ A1

6b. $T_{r+1} = \binom{5}{r} k^r x^{5-2r}$

For constant term, $5 - 2r = 0$

$2r$

$r \quad \frac{5}{2}$

coefficient of $\frac{1}{x^2}$ $5 - 2r = -2$ M1

$2r = 7$

$r = \frac{7}{2}$

Since r is not a positive integer,
there is no constant term in
the expansion of $(1-3x^2)\left(x + \frac{k}{x}\right)^5$

7a $\frac{3y^2}{k} + \frac{2x}{h} = 6$

$$\frac{3y^2}{k} = -\frac{2x}{h} + 6$$

$$3y^2 = \frac{-2k}{h}x + 6k$$

$$y^2 = \frac{-2k}{3h}x + 2k \quad \text{M1}$$

$Y = y^2$, $X = x$, $m = \frac{-2k}{3h}$, $c = 2k$

By Comparison

$2k = 8$

$k = 4$

$$-6 = \frac{-2(4)}{3h}$$

$$3h = \frac{4}{3}$$

$$h = \frac{4}{9}$$

$\therefore k = 4$, $h = \frac{4}{9}$

7bi

$$T = 25 + p.e^{-kn}$$

$$T - 25 = pe^{-kn}$$

$$\ln(T-25) = \ln pe^{-kn}$$

$$= \ln p + \ln e^{-kn}$$

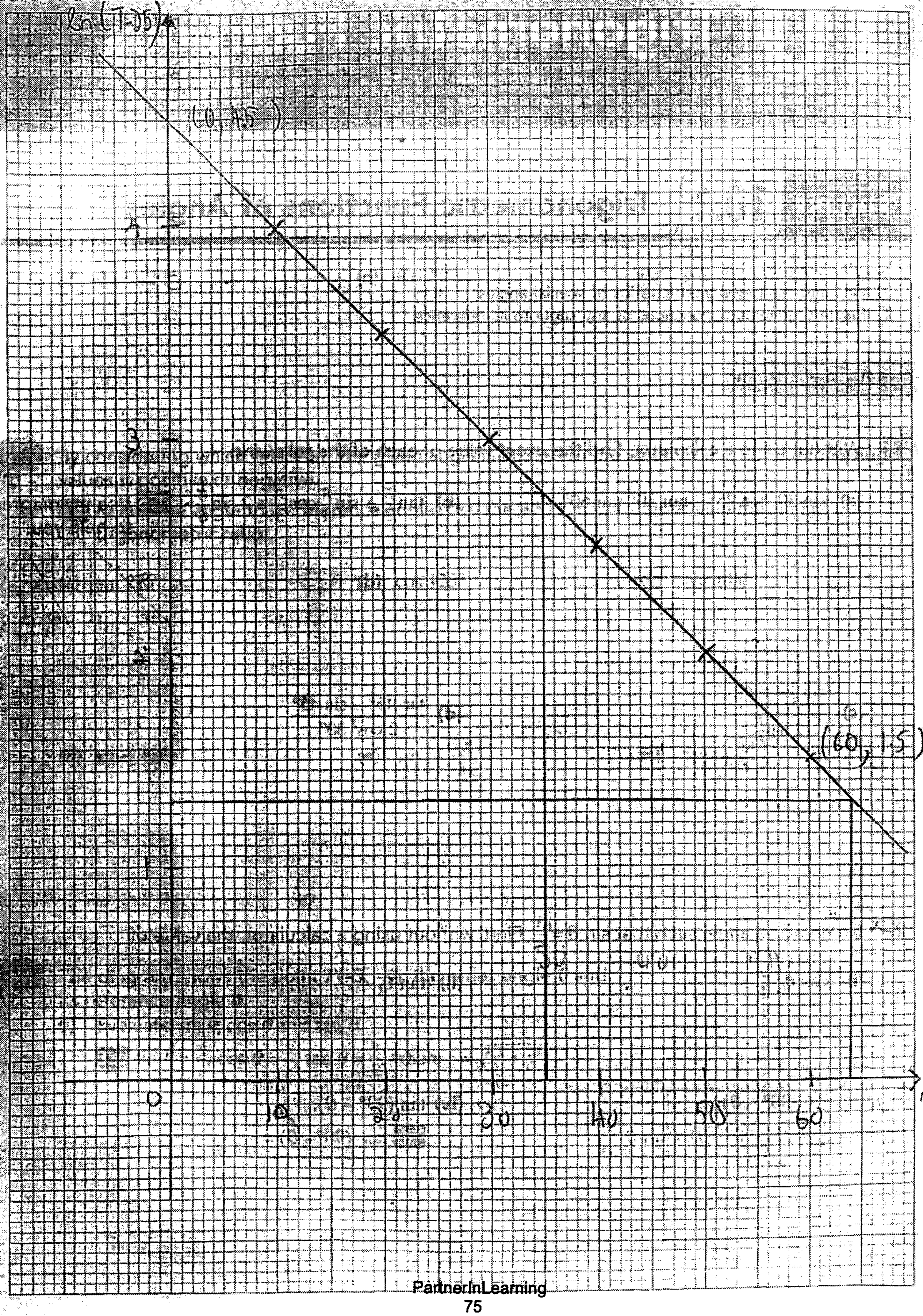
$$\ln(T-25) = -kn + \ln p$$

$Y = \ln(T-25)$, $X = n$, $m = -k$, $c = \ln p$

n	10	20	30	40	50
ln (T-25)	4.0	3.5	3.0	2.5	2.0

Plot ln (T-25) against n   B1

Qn 7bi



7bii

$$-k \approx \frac{1.5-4.5}{60} \quad M1$$

$$k \approx -0.05 \quad A1$$

$$\ln \approx 4.5$$

$$p \approx e^{4.5} \quad M1$$

$$\approx 90.017 \text{ (5s.f)}$$

$$\approx 90.0 \text{ (3s.f)} \quad A1$$

7biii $n = 35$

$$\ln(T-25) \approx 2.75 \quad M1$$

$$T \approx 40.642\,°C \text{ (5s.f)}$$

$$\approx 40.6\,°C \text{ (3s.f)} \quad A1$$

The temperarture is 40.6°C after 35 minutes.

7biv $T = 25 + 90e^{-0.05n}$

$n = 0$, $T = 115$

25% of $T \approx 28.75$ mi

$\ln(28.75 - 25) \approx$

At 1.3 $n \approx 64$ minutes

It takes 64 minutes for the temperature of the liquid to drop by 75% of its initial value.

9a. Equation of circle, C_1

$$(x+5)^2 + (y-3)^2 = 5^2$$

$$(x+5)^2 + (y-3)^2 =$$

9bi $y = -3x + 11$ $A(-5,-2)$

$y = -3x + c$

Sub $A(-5,-2)$ into eqn

$-2 = -3(-5) + c$

$c = -17$

$\therefore$

$y = -3x - 17$ A1

$m_1 \times m_2 = -1$

$-3 \times m_2 = -1$ m_1

$m_2 = \frac{1}{3}$

$y = \frac{1}{3}x + c$

Sub C $(-5, 3)$ into eqn

$3 = \frac{1}{3}(-5) + c$

$c = \frac{14}{3}$

Equation of perpendicular bisector

$y = \frac{1}{3}x + \frac{14}{3}$

9bii $y = -3x - 17$ — ①

$y = \frac{1}{3}x + \frac{14}{3}$ — ②

① = ② $-3x - 17 = \frac{1}{3}x + \frac{14}{3}$ M1

$3\frac{1}{3}x = -21\frac{2}{3}$

$x = -\frac{13}{2}$

Sub $x = -\frac{13}{2}$ ① : $y = -3\left(-\frac{13}{2}\right) - 17$

$= \frac{5}{2}$

$\therefore$ midpoint of AB $= \left(-\frac{13}{2}, \frac{5}{2}\right)$

let coordinates of B (x_1, y_1)

$\frac{x_1 + (-5)}{2} = -\frac{13}{2}$ $\frac{y_1 + (-2)}{2} = \frac{5}{2}$

$x_1 = -8$ $y_1 = 7$

$\therefore$ coordinates of B $(-8, 7)$

9c Let coordinates of centre of C_2 be (x_2, y_2)

$$\frac{x_2 + (-5)}{2} = -\frac{13}{2} \quad M1 \qquad \frac{y_2 + (3)}{2} = \frac{5}{2} \quad M1$$

$$x_2 = -8 \qquad y_2 = 2$$

coordinates of centre of C_2 $(-8, 2)$

Equation of circle, C_2

$$(x+8)^2 + (y-2)^2 = 5^2$$

$$(x+8)^2 + (y-2)^2 = 25 \quad A1$$

10

$y = 2\sin 2x + 1$

$\frac{dy}{dx} = 4\cos 2x$

$4\cos 2x = 0$

$\cos 2x = 0$

$2x = \frac{\pi}{2}$

$x = \frac{\pi}{4}$

$x = \frac{\pi}{4}$, $y = 2\sin 2\left(\frac{\pi}{4}\right) + 1$

$= 3$

coordinates of P $\left(\frac{\pi}{4}, 3\right)$

$\frac{d^2y}{dx^2} = -8\sin 2x$

At $x = \frac{\pi}{4}$

$\frac{d^2y}{dx^2} = -8\sin 2\left(\frac{\pi}{4}\right)$

$= -8 < 0$

$\therefore$ P is maximum point

$4\cos 2x = -4$

$\cos 2x = -1$

$2x = \pi$

$x =$

$y = 2\sin 2\left(\frac{\pi}{2}\right) + 1$

$= 1$

coordinates of Q $\left(\frac{\pi}{2}, 1\right)$

Area of shaded region

$$= \int_0^{\frac{\pi}{4}} (2\sin 2x + 1)\,dx + \left[\frac{1}{2} \times (3+1) \times \left(\frac{\pi}{2} - \frac{\pi}{4}\right)\right]$$ M1 M1

$$= \left(\frac{-2\cos 2x}{2} + x\right)_0^{\frac{\pi}{4}} + \frac{\pi}{2}$$ M1

$$= \left(-\cos 2x + x\right)_0^{\frac{\pi}{4}} + \frac{\pi}{2}$$

$$= -(-1) + \frac{\pi}{2}$$

$$= 1 + \frac{3\pi}{4}$$ A1