ZHONGHUA SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2021
SECONDARY 4E/4N/5N

Candidate's Name	Class	Register Number

MATHEMATICS
PAPER 1

4048/01
27 August 2021
2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **80**.

For Examiner's Use		
Marks Obtained		**80**
Marks Deducted		
Final Total		**/80**

This question paper consists of **20** printed pages (including this cover page)

[Turn over]

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2(b)(c)\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions

1 Arrange the numbers in ascending order.

$$3\pi,\ \sqrt{101},\ 1.7^4,\ 11.1$$

Answer ....., ..., ..., ... [1]

2 **(a)** Given that $7^x \times 2^{2x} = \frac{1}{28}$, find x.

Answer $x =$ ..... [2]

(b) Express as a single fraction in its simplest form $\frac{7}{2x-1} + \frac{5}{2-4x}$.

Answer ..... [2]

3 Adam took a loan from a bank for 8 years. The bank charges an interest rate of 2.3% per annum that is compounded monthly. At the end of 8 years, Adam owed the bank a total of $31,168.95.
Calculate the sum of money he borrowed, correct your answer to the nearest dollar.

Answer $ ..... [2]

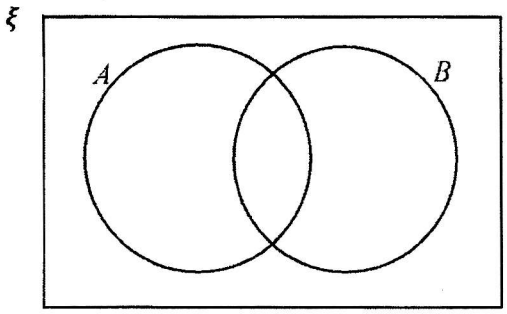
4 **(a)** Express $x^2 - \frac{5}{2}x + \frac{3}{2}$ in the form $(x+a)^2 + b$.

Answer ..... [2]

(b) Hence, solve the equation $2x^2 - 5x + 3 = 0$.

Answer $x =$ ..... or ..... [3]

5 **(a)** In the Venn diagram below, shade the region that represents $(A \cup B)'$. [1]



(b) $\xi = \{ \text{integers } x : 1 \leq x \leq 21 \}$

$A = \{\text{odd numbers}\}$

$B = \{\text{divisible by 3}\}$

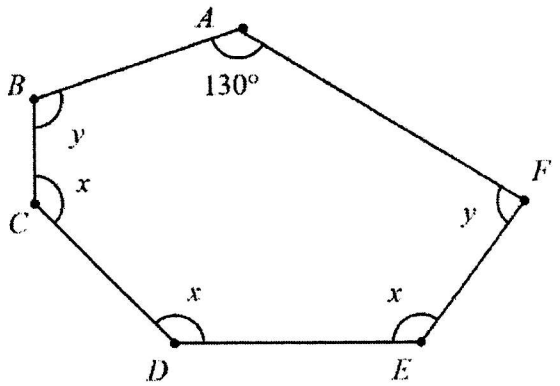
(i) List the elements of set B.

Answer $B = \{$ ..... $\}$ [1]

(ii) Write down the set notation in terms of A and B that represents numbers which are multiples of 6.

Answer ..... [2]

6 The diagram shows a hexagon $ABCDEF$.
Three angles are x and two angles are y and the last angle is 130°.



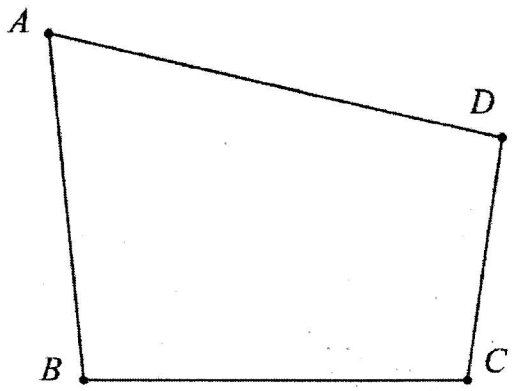
(a) Write down and simplify an equation in terms of x and y.

Answer ..... [2]

(b) If $x = 135°$, calculate the value of y.

Answer $y =$ ..... [1]

7 The diagram shows a plot of land *ABCD*. It is drawn to scale of 1 cm to 3 m.



(a) On the diagram,

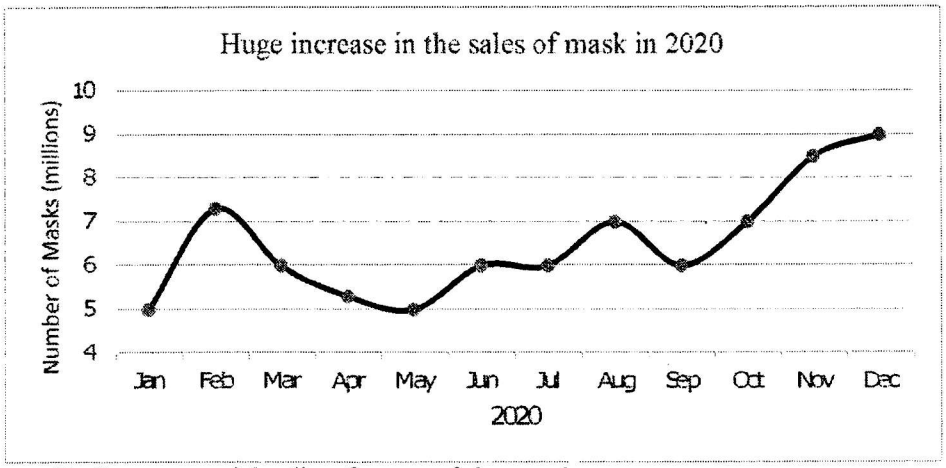
(i) construct a perpendicular bisector of *AB*, [1]

(ii) construct a angle bisector of angle *ABC*. [1]

(b) A tree is to be planted nearer to *B* than to *A*, nearer to *BC* than to *AB*. [2]
Construct a circle with radius 1 cm within the plot of land *ABCD* representing the area in which the tree could be planted.

[Turn over]

8 **(a)** The graph shows the monthly sales of mask at the end of 2020.



(i) State one misleading feature of the graph.

---

[1]

(ii) Explain how this feature affects the reader's interpretation of the graph.

---

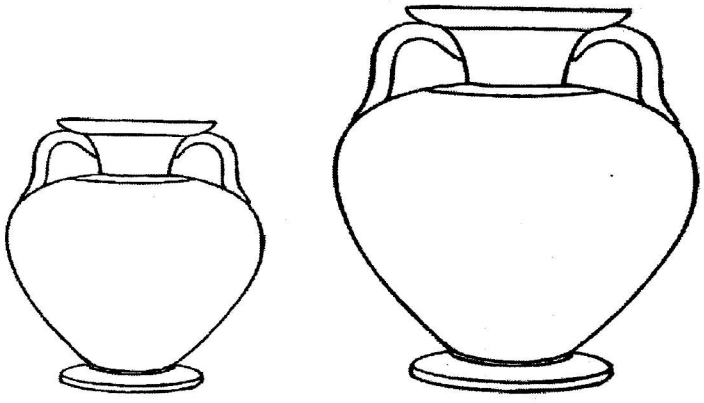
[1]

(b) Solve the simultaneous equations.

$$3x - y = 11$$
$$5x + 3y = 9$$

Answer $x =$ ..... $y =$ ..... [2]

9 The ratio of the area of the bases of two geometrically similar vases is 9 : 25.
The height of the larger vase is 20 cm more than the height of the smaller vase.



(a) Find the height of the smaller vase.

Answer ..... cm [2]

(b) Sunny thinks that it will take at least 2 full small vases to completely fill up the big vase with water. Determine if what Sunny says is correct.

--- [2]

10 The table shows the time taken for 20 delivery riders to deliver food for a restaurant.

Time range (mins)	Frequency
$0 < t \leq 10$	3
$10 < t \leq 20$	10
$20 < t \leq 30$	5
$30 < t \leq 40$	2

(a) Calculate an estimate for

(i) the mean time taken,

Answer ..... mins [1]

(ii) the standard deviation of the time taken.

Answer ..... mins [1]

(b) It was discovered that one of the timing was incorrectly recorded as 15 mins instead of 39 mins. **Without any calculations,** explain how the standard deviation has been affected by this error.

_____

_____

_____ [1]

11 (a) Use prime factors to explain why 1925 is not a perfect square.

---

[2]

(b) The number $1925k$ is a perfect cube. Find the smallest positive integer value of k.

Answer $k =$ ..... [1]

12 A map is drawn to a scale of 1 : 60 000.

(a) This scale can be expressed as 1 cm represents n km.
Find n.

Answer $n =$ ..... [1]

(b) The distance between a seaport and an airport on the map is 45 cm.
Find the actual distance, in kilometres, between the seaport and the airport.

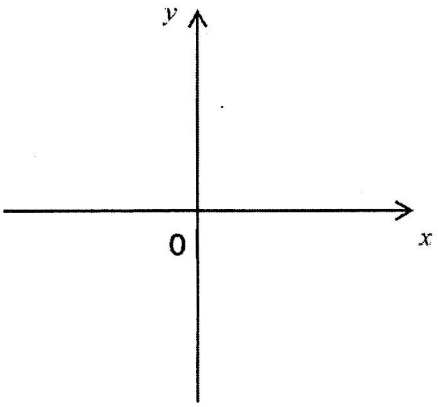
Answer .....km [1]

(c) A bus depot has an actual area of 9 km^2.
Find the area, in square centimetres, of the bus depot on the map.

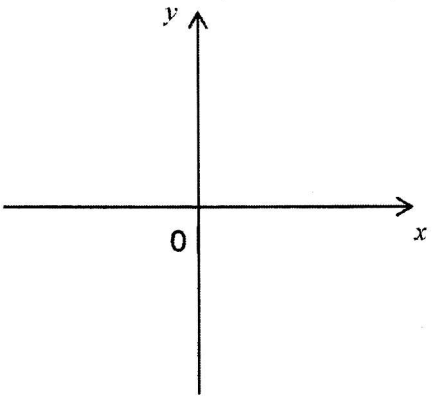
Answer ..... cm^2 [1]

13 On the axes provided, sketch the graph of: [4]

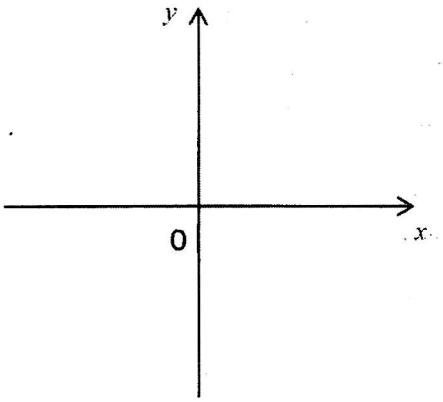
(a) $y = -x^3$



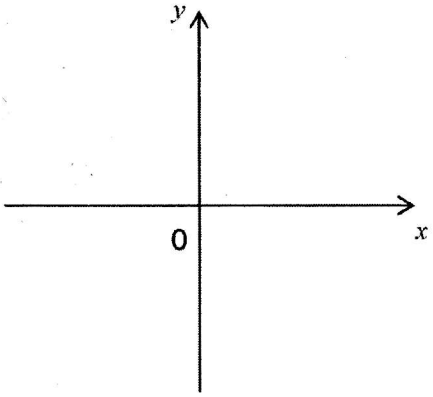
(b) $y = \frac{2}{x}$



(c) $y = -x^2 + 4$



(d) $x = 3$



14 An online store sells keyboards and tablets from Brand *A*, Brand *B* and Brand *C*.
In one month, the store sold 135 keyboards and 209 tablets; Brand *A* contributed 56 keyboards and 85 tablets, Brand *B* contributed 79 keyboards and 124 tablets and none of Brand *C* items are sold.

(a) Represent this information in a 3×2 matrix **P.**

Answer $\mathbf{P} =$ $\begin{pmatrix} & \\ & \\ & \end{pmatrix}$ [1]

(b) A keyboard is sold at \$110 and a tablet is sold at \$420. This information can be represented by the matrix $\mathbf{M} = \begin{pmatrix} 110 \\ 420 \end{pmatrix}$. Evaluate the matrix **PM**.

Answer **PM**= $\begin{pmatrix} & \\ & \\ & \end{pmatrix}$ [2]

(c) State what the elements of **PM** represent.

_____

_____

_____ [1]

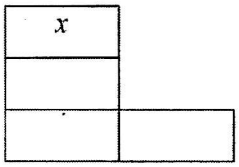
15 The diagram shows a number table with a 'L' frame on it. The number at the top gives the L-number. The L-number of the 'L' shown below is L-13.
The sum of numbers in L-13 is 13+23+33+34 = 103.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60

(a) Find the sum of the numbers in L-37.

Answer ..... [1]

(b) (i) Fill in the diagram below, in terms of x, the numbers in L-x. [1]



(ii) What is the sum of the numbers in L-x? Give your answer in terms of x.

Answer ..... [1]

(iii) Which L-number has a sum of 427?

Answer ..... [1]

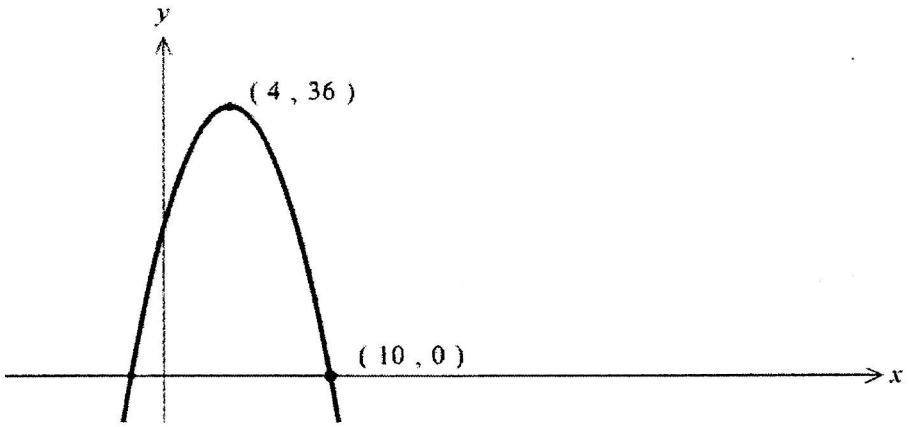
(c) Explain why the sum of numbers in L-x is always odd.

---

[1]

16 The diagram shows the graph of $y = a(x+b)(x+c)$.

(a) The curve has a maximum point at (4, 36) and x-intercept at (10, 0).
Find the values of a, b and c.



Answer $a =$ ..... [1]

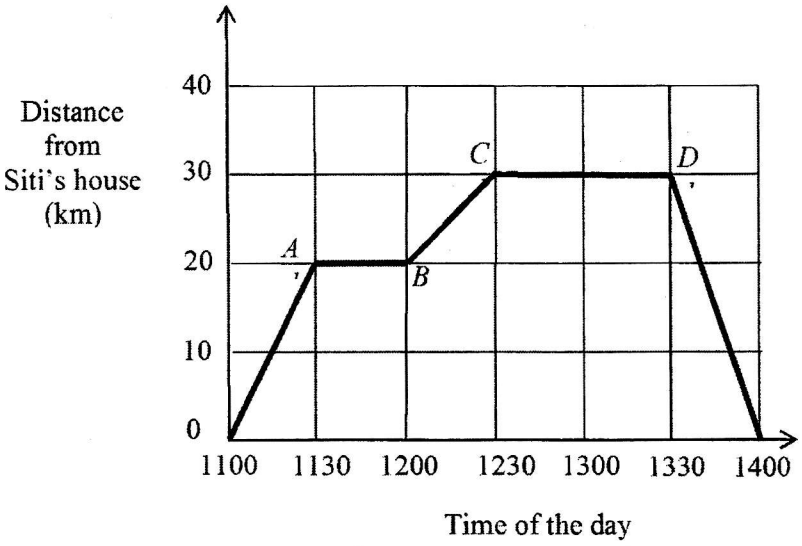
$b =$ ..... [1]

$c =$ ..... [1]

(b) State the coordinates of the y-intercept for the graph.

Answer (..... , ...) [1]

17 Siti drives from her home to the shopping centre and back.
On the way to the shopping centre, she stops to pick up her friend Ah Bee from her house.
The travel graph shows her journey.



(a) How far is the shopping centre from Ah Bee's house?

Answer ..... km [1]

(b) What does the line *AB* represent ?

---

[1]

(c) How long did Siti spend at the shopping centre ?

Answer ..... h [1]

(d) Find the speed from point *B* to *C* ?

Answer ..... km/h [2]

[Turn over]

18 A bag contains five identical cards, numbered 1, 2, 3, 4 and 5. Two cards are drawn at random, one after the other, **with replacement**. The sum of the number shown on the two cards drawn are calculated.

(a) Draw the possibility diagram to show the sum of the draw. [1]

(b) Find, as a fraction in its simplest form, the probability that the sum of the two numbers is

(i) an even number,

Answer ..... [1]

(ii) a prime number.

Answer ..... [1]

(c) A student wanted to find the probability that the sum is either an even number or a prime number. To do so, he added the answer in b(i) and b(ii).
Explain why this is incorrect and give the correct answer.

---

[2]

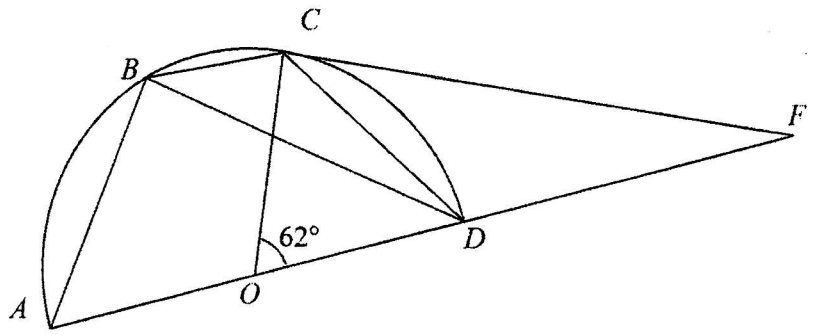
19 (a) A is directly proportional to the cube root of B.
It is given that $A = 136$ for a certain value of B.
Find the value of A when this value of B is decreased by 87.5%.

Answer $A =$ ..... [3]

(b) 10 girl scouts can build 5 tree houses in 56 days.
How long would it take 8 girl scouts to build 1 tree house?

Answer ..... days [3]

20



The diagram shows a semi-circle $ABCD$ with centre at O. CF is a tangent to the semi-circle at point C. Angle $COD = 62°$.

(a) Complete these statements.

(i) Angle CBD = ... because .....

..... [2]

(ii) Angle ABD = ... because .....

..... [2]

(b) Find

(i) angle FCO,

Answer ..... [1]

(ii) angle CDA,

Answer ..... [1]

(iii) angle DCF,

Answer ..... [1]

--- End of Paper ---

ANS

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$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

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$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2(b)(c)\cos A$$

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$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions

1 Arrange the numbers in ascending order.

$$3\pi,\ \sqrt{101},\ 1.7^4,\ 11.1$$

Answer 1.7^4, 3π, $\sqrt{101}$, 11.1 [1]

2 **(a)** Given that $7^x \times 2^{2x} = \frac{1}{28}$, find x.

$$7^x \times 2^{2x} = \frac{1}{7 \times 4}$$
$$= 7^{-1} \times 2^{-2}$$

Answer $x = -1$ [2]

(b) Express as a single fraction in its simplest form $\frac{7}{2x-1} + \frac{5}{2-4x}$.

$$\frac{7}{2x-1} + \frac{5}{2-4x} = \frac{7}{2x-1} - \frac{5}{2(2x-1)}$$
$$= \frac{7(2)-5}{2(2x-1)}$$
$$= \frac{9}{2(2x-1)}$$

Answer $\frac{9}{2(2x-1)}$ [2]

3 Adam took a loan from a bank for 8 years. The bank charges an interest rate of 2.3% per annum that is compounded monthly. At the end of 8 years, Adam owed the bank a total of $31,168.95.
Calculate the sum of money he borrowed, correct your answer to the nearest dollar.

$$31168.95 = P(1+\frac{2.3}{12\times100})^{8\times12}$$
$$=1.2018P$$
$$P=25935.2$$

Answer $ 25935 [2]

4 **(a)** Express $x^2-\frac{5}{2}x+\frac{3}{2}$ in the form $(x+a)^2+b$.

$$x^2-\frac{5}{2}x+\frac{3}{2}=x^2-\frac{5}{2}x+\left(-\frac{5}{4}\right)^2-\left(-\frac{5}{4}\right)^2+\frac{3}{2}$$
$$=(x-\frac{5}{4})^2-\frac{1}{16}$$

$(x-\frac{5}{4})^2-\frac{1}{16}$

Answer ..... [2]

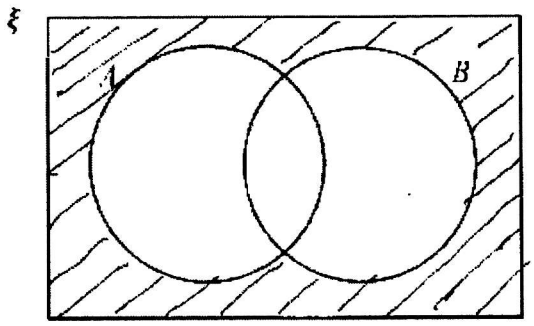
(b) Hence, solve the equation $2x^2-5x+3=0$.

$$2x^2-5x+3=0$$
$$x^2-\frac{5}{2}x+\frac{3}{2}=0$$
$$(x-\frac{5}{4})^2-\frac{1}{16}=0$$
$$(x-\frac{5}{4})^2=\frac{1}{16}$$
$$x-\frac{5}{4}=\pm\sqrt{\frac{1}{16}}$$

$$x=\frac{3}{2} \text{ or } x=1$$

Answer $x=$ 1.5 or 1 [3]

5 (a) In the Venn diagram below, shade the region that represents $(A \cup B)'$. [1]



(b) $\xi = \{ \text{integers } x : 1 \leq x \leq 21 \}$

$A = \{\text{odd numbers}\}$

$B = \{\text{divisible by 3}\}$

(i) List the elements of set B.

Answer $B = \{ \quad 3,6,9,12,15,18,21 \quad \}$ [1]

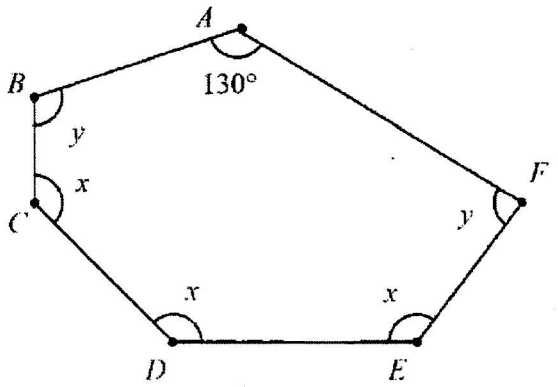
(ii) Write down the set notation in terms of A and B that represents numbers which are multiples of 6.

$A' = \text{even numbers}$

$A' \cap B = \text{multiples of 6}$

Answer $A' \cap B$ [2]

6 The diagram shows a hexagon *ABCDEF*.
Three angles are x and two angles are y and the last angle is 130°.



(a) Write down and simplify an equation in terms of x and y.

Sum of angles $= (6-2)\times 180 = 3x+2y+130$

$3x+2y = 720-130$

$3x+2y = 590$

Answer ..... [2]

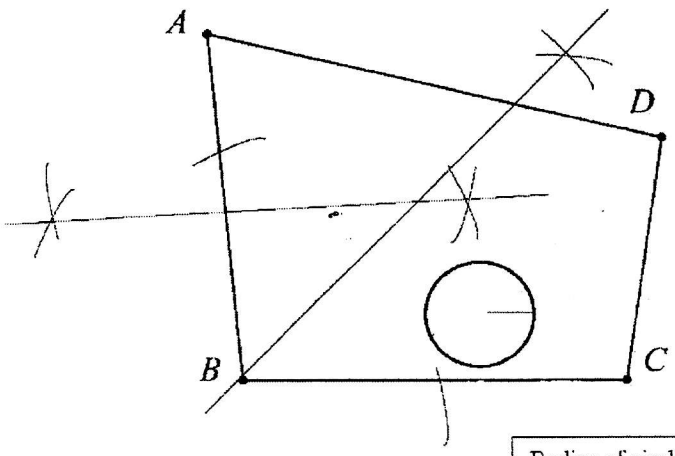
(b) If $x = 135°$, calculate the value of y.

$3(135)+2y = 590$
$2y = 590-405$

92.5

Answer $y =$ ..... [1]

7 The diagram shows a plot of land $ABCD$. It is drawn to scale of 1 cm to 3 m.



Radius of circle 1 cm B1

Circle within the correct region B1

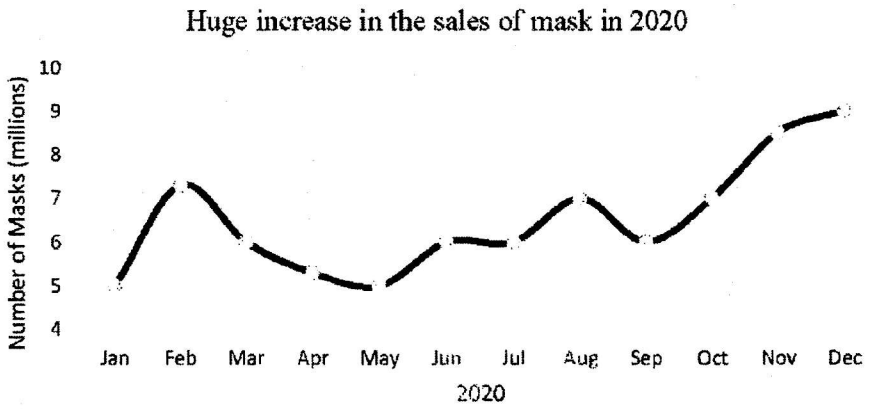
(a) On the diagram,

(i) construct a perpendicular bisector of AB, [1]

(ii) construct a angle bisector of angle ABC. [1]

(b) A tree is to be planted nearer to B than to A, nearer to BC than to AB. [2]
Construct a circle with radius 1 cm within the plot of land $ABCD$ representing the area in which the tree could be planted.

8 **(a)** The graph shows the monthly sales of mask at the end of 2020.



(i) State one misleading feature of the graph.

The vertical axis did not start from 0

[1]

(ii) Explain how this feature affects the reader's interpretation of the graph.

This feature will amplify the difference in the number of masks sold between the months.

[1]

(b) Solve the simultaneous equations.

$$3x - y = 11 \quad \text{----(1)}$$
$$5x + 3y = 9$$

(1)×3 $\quad 9x - 3y = 33 \quad$ ———(3)

$14x = 42$

$x = 3$

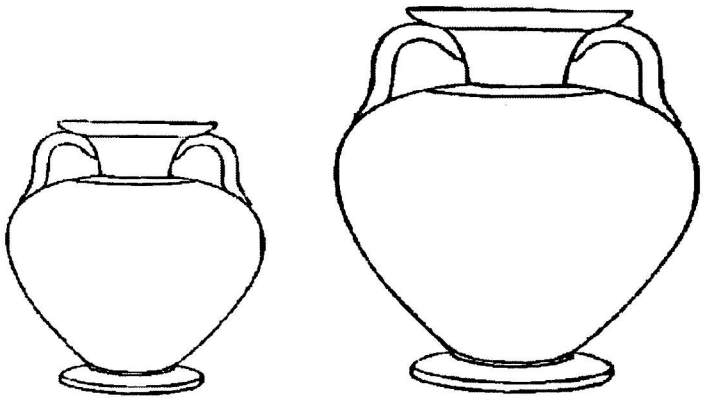
Sub $x = 3$ into (1)

$9 - y = 11$

$y = -2$

Answer $x =$ 3 $y =$ −2 [2]

9 The ratio of the area of the bases of two geometrically similar vases is 9 : 25.
The height of the larger vase is 20 cm more than the height of the smaller vase.



(a) Find the height of the smaller vase.

Let the height of small vase be x

$$\text{Height ratio} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\frac{x}{x+20} = \frac{3}{5}$$
$$5x = 3x + 60$$
$$2x = 60$$
$$x = 30$$

Answer 30 cm [2]

(b) Sunny thinks that it will take at least 2 full small vases to completely fill up the big vase with water. Determine if what Sunny says is correct.

$$\text{Volume ratio} = \left(\frac{3}{5}\right)^3 = \frac{27}{125}$$

$$\text{Volume ratio} = \frac{125}{27} = 4.63 \text{ times of small vase}$$

Sunny is correct as it will take at least 5 small vase to completely filled up the big vase.

[2]

10 The table shows the time taken for 20 delivery riders to deliver food for a restaurant.

Time range (mins)	Frequency
$0 < t \leq 10$	3
$10 < t \leq 20$	10
$20 < t \leq 30$	5
$30 < t \leq 40$	2

(a) Calculate an estimate for

(i) the mean time taken,

Answer 18 mins [1]

(ii) the standard deviation of the time taken.

Answer 8.43 mins [1]

(b) It was discovered that one of the timing was incorrectly recorded as 15 mins instead of 39 mins. **Without any calculations,** explain how the standard deviation has been affected by this error.

The standard deviation will be larger as 39 min is further away from the mean.

[1]

11 (a) Use prime factors to explain why 1925 is not a perfect square.

$1925 = 5^2 \times 7 \times 11$

Prime factors of 1925 are $5^2 \times 7 \times 11$.

Since 7 and 11 are not square numbers, 1925 is not a perfect squares

[2]

(b) The number $1925k$ is a perfect cube. Find the smallest positive integer value of k.

$k = 5 \times 7^2 \times 11^2$

$k = 29645$

Answer $k =$ 29645 [1]

12 A map is drawn to a scale of 1 : 60 000.

(a) This scale can be expressed as 1 cm represents n km.
Find n.

Answer $n =$ 0.6 [1]

(b) The distance between a seaport and an airport on the map is 45 cm.
Find the actual distance, in kilometres, between the seaport and the airport.

Answer 27 km [1]

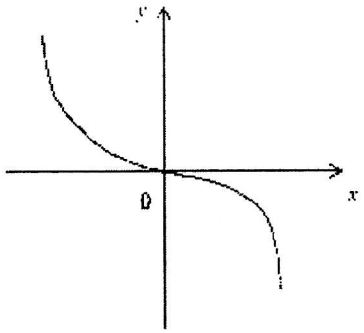
(c) A bus depot has an actual area of 9 km^2.
Find the area, in square centimetres, of the bus depot on the map.

$1^2 cm^2 : 0.6^2 km^2$
$1\ cm^2 : 0.36\ km^2$
$\frac{1}{0.36} \times 9 :\ 9\ km^2$
$= 25 cm^2$

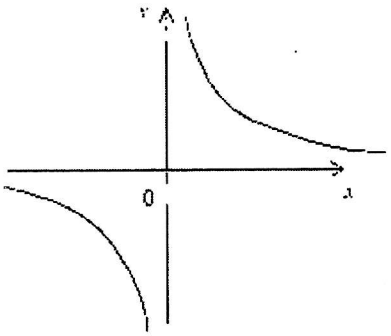
Answer 25 cm^2 [1]

13 On the axes provided, sketch the graph of: [4]

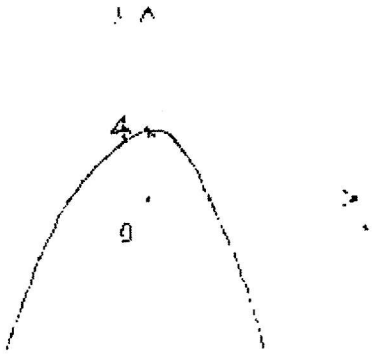
(a) $y = -x^3$



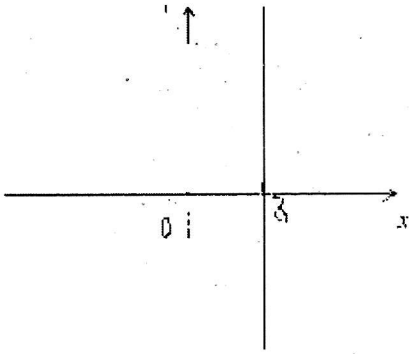
(b) $y = \frac{2}{x}$



(c) $y = -x^2 + 4$



(d) $x = 3$



14 An online store sells keyboards and tablets from Brand *A*, Brand *B* and Brand *C*.
In one month, the store sold 135 keyboards and 209 tablets; Brand *A* contributed 56 keyboards and 85 tablets, Brand *B* contributed 79 keyboards and 124 tablets and none of Brand *C* items are sold.

(a) Represent this information in a 3 × 2 matrix **P**.

Answer $\mathbf{P} = \begin{pmatrix} 56 & 85 \\ 79 & 124 \\ 0 & 0 \end{pmatrix}$ [1]

(b) A keyboard is sold at \$110 and a tablet is sold at \$420. This information can be represented by the matrix $\mathbf{M} = \begin{pmatrix} 110 \\ 420 \end{pmatrix}$. Evaluate the matrix **PM**.

$$PM = \begin{pmatrix} 56 & 85 \\ 79 & 124 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 110 \\ 420 \end{pmatrix}$$

$$= \begin{pmatrix} 6160 + 35700 \\ 8690 + 52080 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 41860 \\ 60770 \\ 0 \end{pmatrix}$$

Answer $\mathbf{PM} = \begin{pmatrix} 41860 \\ 60770 \\ 0 \end{pmatrix}$ [2]

(c) State what the elements of **PM** represent.

PM represents the revenue of Brand A, Brand B and Brand C respectively.

[1]

15 The diagram shows a number table with a 'L' frame on it. The number at the top gives the L-number. The L-number of the 'L' shown below is L-13.

The sum of numbers in L-13 is 13+23+33+34 = 103.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60

(a) Find the sum of the numbers in L-37.

Answer 199 ..... [1]

(b) (i) Fill in the diagram below, in terms of x, the numbers in L-x. [1]

x	
$10+x$	
$20+x$	$21+x$

(ii) What is the sum of the numbers in L-x? Give your answer in terms of x.

$L-x = x+10+x+20+x+21+x$

$= 4x+51$

Answer $4x+51$ ..... [1]

(iii) Which L-number has a sum of 427?

$4x+51 = 427$

$4x = 376$

$x = 94$

Answer L_94 ..... [1]

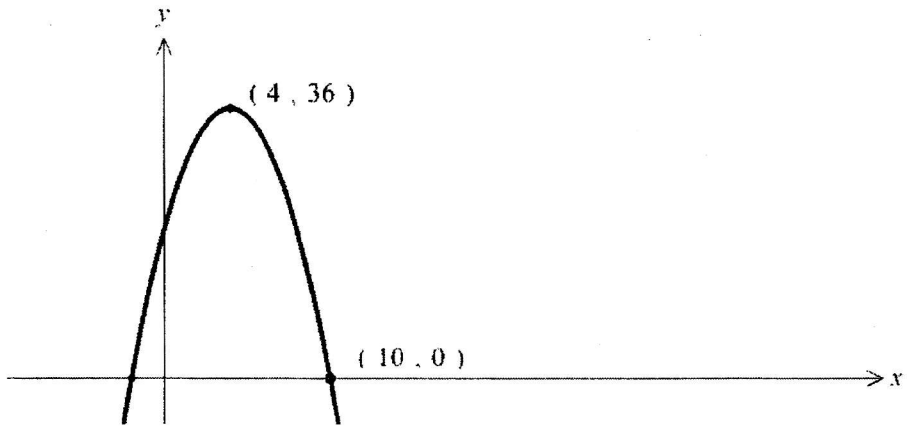
(c) Explain why the sum of numbers in L-x is always odd.

The sum of L-frame is $4x+51$, since $4x$ is always even, and 51 is odd, the sum $4x+51$ is always odd. [1]

16 The diagram shows the graph of $y = a(x+b)(x+c)$.

(a) The curve has a maximum point at (4, 36) and x-intercept at (10, 0).

Find the values of a, b and c.



$y = -1(x-10)(x+2)$

Answer $a =$ ..... −1 ..... [1]

$b =$ ..... −10 ..... [1]

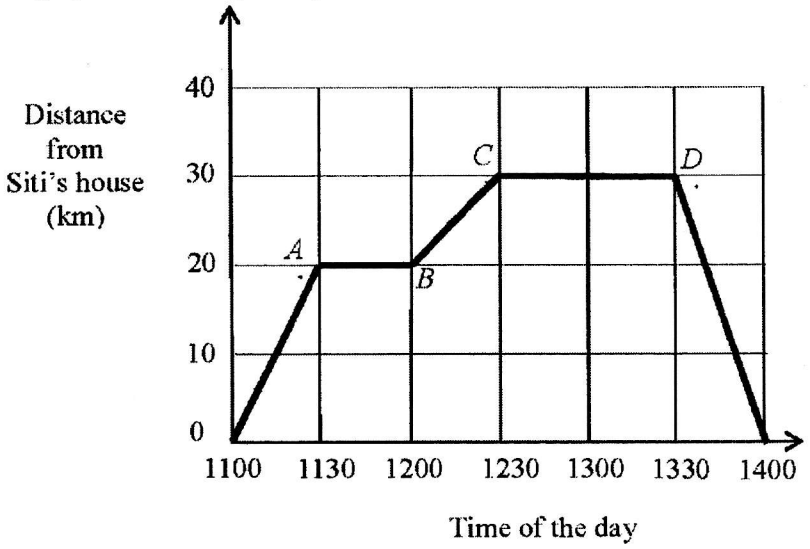
$c =$ ..... 2 ..... [1]

(b) State the coordinates of the y-intercept for the graph.

Answer (..... 0 ..... , ... 20 .....) [1]

17 Siti drives from her home to the shopping centre and back.
On the way to the shopping centre, she stops to pick up her friend Ah Bee from her house.

The travel graph shows her journey.



(a) How far is the shopping centre from Ah Bee's house?

Answer 10 ..... km [1]

(b) What does the line *AB* represent ?

Line AB represents the time Siti spent waiting at Ah Bee's house

[1]

(c) How long did Siti spend at the shopping centre ?

Answer 1 ..... h [1]

(d) Find the speed from point *B* to *C* ?

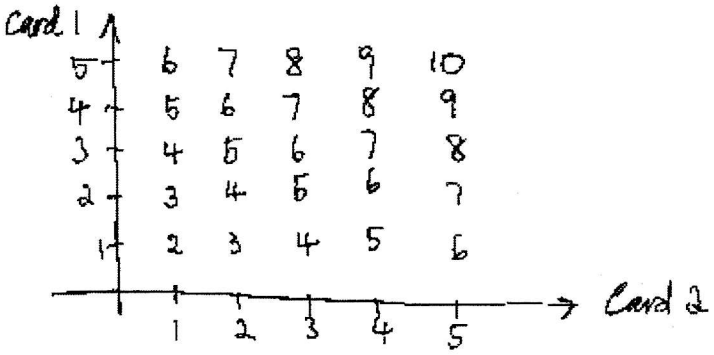
$$\text{Speed} = \frac{10km}{0.5h}$$

$$= 20km/h$$

Answer 20 ..... km/h [2]

18 A bag contains five identical cards, numbered 1, 2, 3, 4 and 5. Two cards are drawn at random, one after the other, **with replacement**. The sum of the number shown on the two cards drawn are calculated.

(a) Draw the possibility diagram to show the sum of the draw. [1]



(b) Find, as a fraction in its simplest form, the probability that the sum of the two numbers is

(i) an even number,

Answer $\frac{13}{25}$ [1]

(ii) a prime number.

Answer $\frac{11}{25}$ [1]

(c) A student wanted to find the probability that the sum is either an even number or a prime number. To do so, he added the answer in b(i) and b(ii).
Explain why this is incorrect and give the correct answer.

$P(\text{sum is even or odd}) = \frac{13}{25} + \frac{11}{25}$

$\frac{24}{25}$

$P(\text{sum is even or odd}) = \frac{23}{25}$

As even and odd prime numbers are not mutually exclusive

$P(A \text{ or } B) \neq P(A) + P(B)$ [2]

Or '2' is found in both A and B.

19 (a) A is directly proportional to the cube root of B.
It is given that $A = 136$ for a certain value of B.
Find the value of A when this value of B is decreased by 87.5%.

$$A = kB^{\frac{1}{3}}$$
$$A = k\left(\frac{1}{8}B\right)^{\frac{1}{3}}$$
$$A = k\frac{1}{2}B^{\frac{1}{3}}$$
$$A = \frac{1}{2}kB^{\frac{1}{3}} = \frac{1}{2}A$$
$$A = \frac{1}{2}\times 136$$
$$A = 68$$

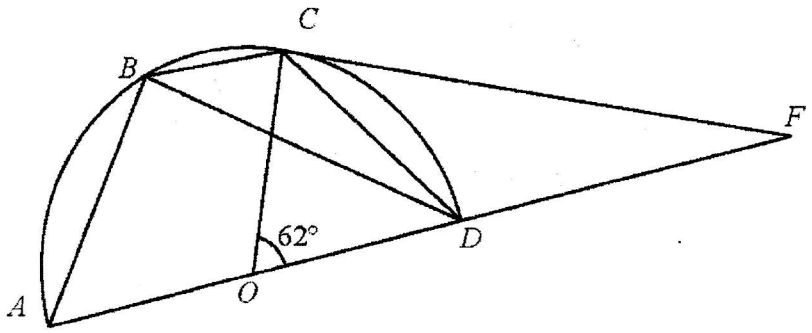
Answer $A =$ 68 ..... [3]

(b) 10 girl scouts can build 5 tree houses in 56 days.
How long would it take 8 girl scouts to build 1 tree house?

5 tree house takes 10 x 56 = 560 man-days
1 tree house takes 112 man -days
8 x days = 112
days = 14

Answer 14 ..... days [3]

20



The diagram shows a semi-circle $ABCD$ with centre at O. CF is a tangent to the semi-circle at point C. Angle $COD = 62°$.

(a) Complete these statements.

(i) Angle CBD = ...31°..... because .....

Angle at centre = 2 x angle at circumference

..... [2]

(ii) Angle ABD = ...90°..... because .....

Angle in semicircle

..... [2]

(b) Find

(i) angle FCO,

$\angle FCO = 90°$ (tangent perpendicular to radius)

Answer .....90°..... [1]

(ii) angle CDA,

$\angle CDA = \frac{1}{2}(180° - 62°)$ (Base angles of isosceles triangles)

$\angle CDA = 59°$

Answer .....59°..... [1]

(iii) angle DCF,

$\angle DCF = 90° - 59°$ (Base angles of isosceles triangles)

$\angle DCF = 31°$

Answer .....31°..... [1]

ZHONGHUA SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2021
SECONDARY 4E/4N/5N

Candidate's Name	Class	Register Number

MATHEMATICS
PAPER 2

4048/02
31 Aug 2021
2 hours and 30 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **100**.

For Examiner's Use		
Marks Obtained		**100**
Marks Deducted		
Final Total		**/100**

This question paper consists of **21** printed pages (including this cover page)

[Turn Over

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1 **(a)** Simplify $\dfrac{8a^2-50b^2}{2a^2+7ab-30b^2}$.

Answer ..... [3]

(b) Solve the equation $\dfrac{1}{3y-2}=\dfrac{y}{9y^2-12y+4}-1$, giving your answers correct to 2 decimal places.

Answer $y=$..... or ..... [4]

1 (c) Simplify $\frac{p^5}{3s^2} \div \frac{(2p)^2}{2sp^{-2}}$.

Answer ..... [2]

1 (d) Solve the inequality $\frac{2}{3}x+1<\frac{1}{5}(8x+1)\leq 2x-10$.

Answer ..... [4]

2 (a) The frequency and cumulative frequency tables below show the results from a survey of 200 students on the number of hours they spend on social media on a particular day.

Time (hours)	Frequency
$0 < x \leq 1$	32
$1 < x \leq 2$	a
$2 < x \leq 3$	35
$3 < x \leq 4$	39
$4 < x \leq 5$	b
$5 < x \leq 6$	20

Time (hours)	Cumulative frequency
$x \leq 1$	32
$x \leq 2$	$a+32$
$x \leq 3$	$3a-19$
$x \leq 4$	$2b+87$
$x \leq 5$	180
$x \leq 6$	200

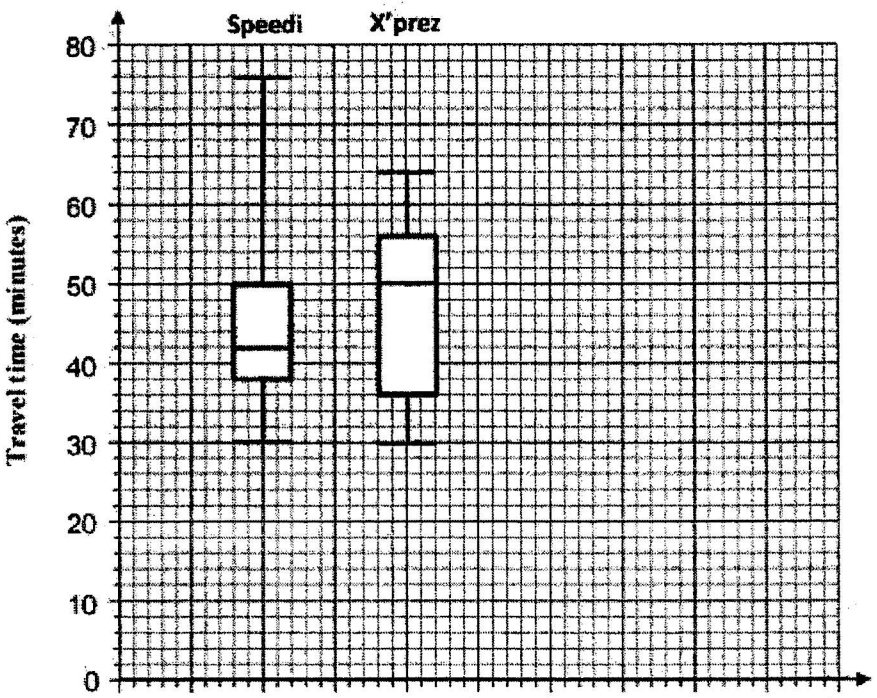
(i) Find the value of a and b.

Answer $a =$..... $b =$ ..... [3]

(ii) Hence, find the number of students who spend more than 4 hours on social media.

Answer .....[1]

2 **(b)** Two transport companies, Speedi and X'prez, provide coach services from Singapore to Johor Bahru, and the coaches leave from the same coach park in Singapore. The following box and whisker diagrams provide the data for the travel time, in minutes, of 80 coaches from each company for the journey from Singapore to Johor Bahru.



(i) Explain which company provides a more time efficient coach service, stating your reasons with supporting evidence from 2 different statistics.

Answer ..... provides a more time efficient service. [3]

.....

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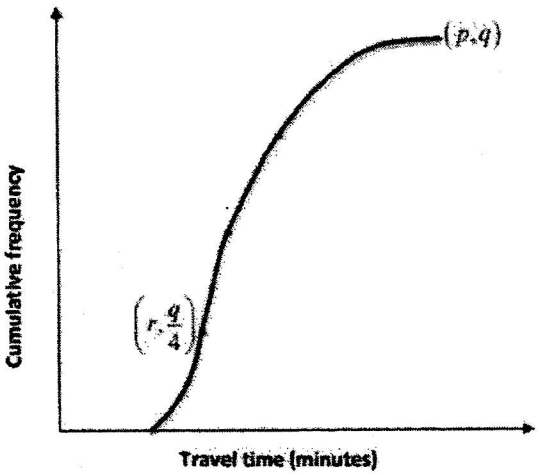
2 (b) (iii) The data for the travel time of coaches from Speedi company can also be represented by the cumulative frequency curve shown below.



State the exact values of p, q and r.

Answer p =..... [1]

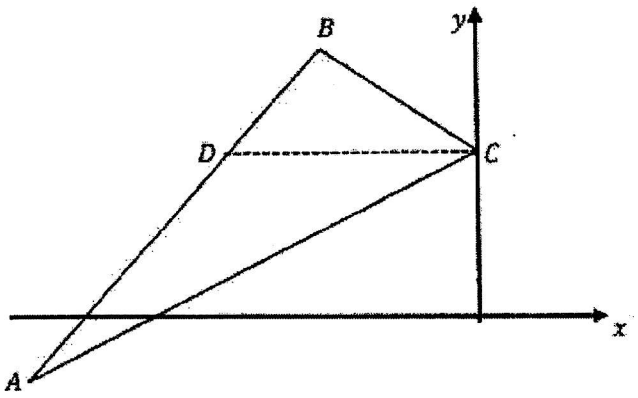
q =..... [1]

r =..... [1]

(iii) Given that (60,74) is a point on the cumulative frequency curve, find the probability that a Speedi coach will take more than an hour to reach Johor Bahru

Answer ..... [2]

3 The diagram shows a triangle ABC, where C lies on the y-axis.
The equations of the line AB and BC are $y = 2x + 18$ and $3y + 5x = 21$ respectively.
D is a point on AB such that DC is parallel to the x-axis.



(a) Find the coordinates of B.

Answer B (..... , ...) [3]

(b) Find the coordinates of C and D.

Answer C (..... , ...)

Answer D (..... , ...) [4]

(c) Find the area of triangle BCD.

Answer .....units^2 [2]

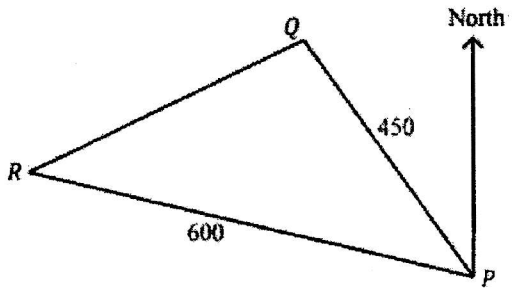
(d) Given that the ratio of lengths $AB : AD = 14 : 9$, state the ratio of area of triangle BCD : area of triangle ABC.

Answer ..... , ... [1]

(e) A point E lies on the x-axis such that $ABCE$ is a trapezium.
Find the coordinates of E.

Answer E (..... , ...) [2]

4 In the diagram, P, Q and R are three points on horizontal ground.



Q is on a bearing of 330° from P and $PQ = 450$ m.
R is on a bearing of 290° from P and $PR = 600$ m.

(a) Calculate the bearing of P from R.

Answer .....[1]

(b) Calculate the distance QR, correct to 1 decimal place.

Answer .....m [3]

(c) A drone, D, is hovering directly above Q. A man, standing at P, looks up at the drone and observes that the angle of elevation of D is 14°.

(i) Calculate the height of the drone, D, above Q, correct to 1 decimal place.

Answer .....m [2]

(ii) Give a possible reason why the actual height of the drone, D, above the point Q could be 1.7 m more than the answer found in c(i).

Answer .....

..... [1]

(d) S is a point along PR and is directly south of Q. Another drone, E, takes off from P and flies directly towards S at a speed of 12 m/s. Its flight path is parallel to the ground.

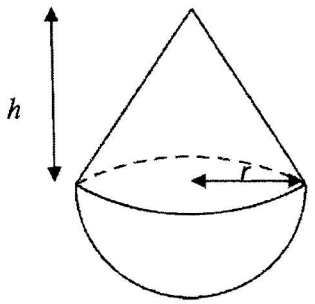
(i) Calculate the distance drone E flew from P to S, giving your answer correct to 1 decimal place.

Answer .....m [3]

(ii) Hence, find the time taken for drone E to reach S.

Answer ..... s [2]

5 A glass perfume bottle is made up of a hemisphere and a cone as shown below. A manufacturer wants to keep the volume of the bottle constant at 36π cm^3. Assume that the thickness of the glass is negligible.



(a) If the volume of the cone is equal to the volume of the hemisphere, show that the radius of the hemisphere is 3 cm. Hence, find the height of the cone.

Answer .....[3]

(b) The manufacturer wants to vary the radius, r, and the height, h, of the bottle while keeping the volume constant at 36π cm3.

(i) Write down, but do not simplify, an equation that must be satisfied by r and h.

Answer .....[1]

(b) (ii) Hence, show that $h = \dfrac{108 - 2r^3}{r^2}$

Answer [1]

(c) The manufacturer decided to produce bottles with a radius of 3.2 cm and a volume of 36π cm^3.

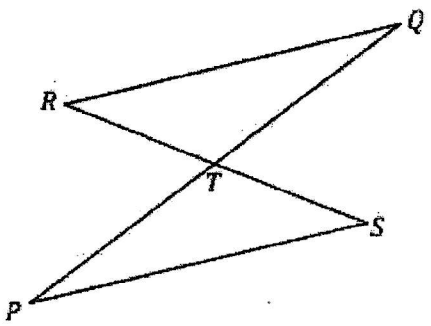
(i) Find the value of h, correct to 1 decimal place.

Answer $h =$ ..... cm [2]

(ii) Hence, find the total surface area of the bottle.

Answer ..... cm^2 [4]

6 (a)



The straight lines PQ and RS bisect each other at T. $PQ = 26.2$ cm and $RS = 18.6$ cm.

(i) Using the dimensions given, show that $\frac{\text{Area of triangle } PST}{\text{Area of triangle } QRT} = 1$.

Answer : [2]

(ii) Stating your reasons clearly, explain why PS is parallel to RQ.

Answer : [2]

.....

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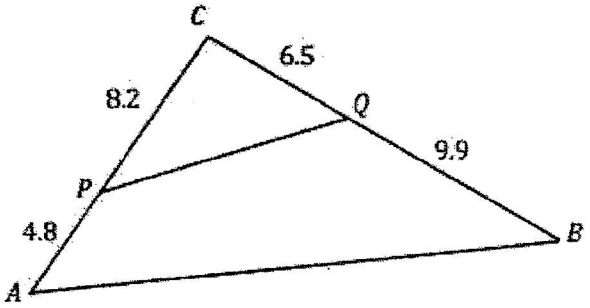
6 (b)



ABC is a triangle and P and Q lie on AC and BC respectively such that $AP = 4.8$ cm, $PC = 8.2$ cm, $BQ = 9.9$ cm and $QC = 6.5$ cm.

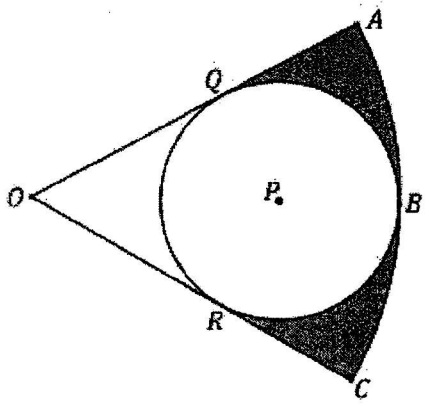
(i) Show that triangle ABC is similar to triangle QPC.

Answer : [4]

(ii) Hence, find the area of triangle QPC, given that the area of triangle ABC is 102.8 cm^2.

Answer .....cm^2 [2]

7



In the diagram, $OABC$ is a sector of a circle with centre at O, radius 12 cm. A circle BQR, with centre at P, is inscribed in the sector. Angle $AOC = \pi$ radians.

(a) Find the area of the sector $OABC$, leaving your answer in terms of π.

Answer .....cm^2 [2]

(b) Show that the radius of the circle BQR is 4 cm.

Answer : [3]

(c) Find the area of the shaded region.

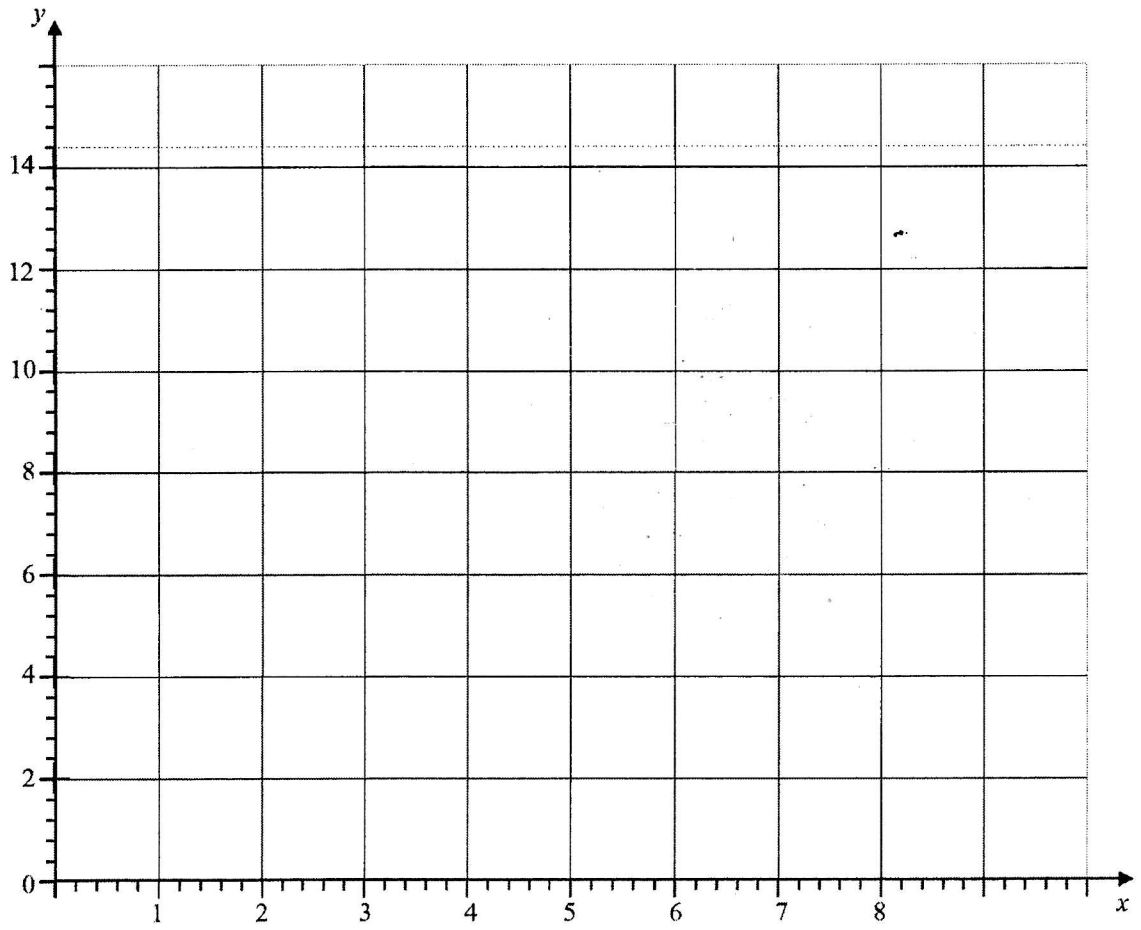
Answer .....cm^2 [6]

8 **(a)** Complete the table of values of $y = \frac{x^2}{5} + \frac{8}{x} - 3$.

Give your answer correct to 1 decimal place. [1]

x	0.5	1	1.5	2	2.5	3	4	5	6	7	8
y	13.1	5.2	2.8		1.5	1.5	2.2	3.6	5.5	7.9	10.8

(b) On the grid, draw the graph of $y = \frac{x^2}{5} + \frac{8}{x} - 3$ for $0.5 \leq x \leq 8$. [3]



(c) By drawing a tangent, find the gradient of the curve at the point where $x = 1.5$.

Answer ..... [2]

8 **(d)** Use your graph to find the minimum value of $\frac{x^2}{5}+\frac{8}{x}$ for values of x in the range $0.5 \leq x \leq 8$.

Answer ..... [2]

(e) By drawing a suitable straight line on the grid in part **(b)**, use your graph to find the solutions to the equation $\frac{x^2}{5}+\frac{8}{x}+x=12$ in the range $0.5 \leq x \leq 8$.

Answer $x =$..... or ..... [3]

(f) The equation $\frac{x^2}{5}+\frac{8}{x}+x=k$, where k is a positive integer, does not have any solutions for $0.5 \leq x \leq 8$. State the value of k.

Answer $k =$ ..... [1]

9 Mr Chan, his wife, 2 sons and his 80 year old mother live together in a HDB flat. His mother goes to the day care centre for the elderly. In order to qualify for government subsidies for the day care services, means-testing is used to calculate the amount of subsidies that will be allocated.

For means-testing, the annual income of all working family members living in the household is taken into consideration. The annual income for each working member of the household includes bonuses. The monthly household income per person is then used to determine the amount of subsidy allocated.

$$\text{monthly household income per person} = \frac{\text{total annual income of all household members}}{\text{number of household members} \times 12}$$

Eligibility for Subsidies on Day Care Source: www.ntuchealth.sg

Criteria for monthly household income per person	Subsidy levels
\$800 and below	80%
\$801 - \$1,200	75%
\$1,201 - \$1,900	60%
\$1,901 - \$2,000	50%
\$2,001 - \$2,800	30%
\$2,801 and above	0%

In January 2021, Mr Chan applied for subsidies for his mother. Only Mr Chan and his wife were working in 2020.

Mr Chan earned a monthly income of \$6000 and his company paid him a bonus of 1.2 months' pay in 2020.

Mrs Chan earned a weekly income of \$800. For every full 10 weeks of work, she will receive an additional 5% of the 10 weeks' income. She worked for 46 weeks in 2020.

(a) Calculate the total annual household income of Mr Chan and his wife in 2020.

Answer \$ ..... [2]

(b) Calculate the monthly household income per person of Mr Chan's family.

Answer $ ..... [1]

(c) Hence, determine the level of subsidy that the Chan family will qualify for.

Answer $ ..... [1]

(d) Mr Chan's older son started working on 1 March 2021. If the Chan family will not qualify for any subsidies when they re-apply again in January 2022, calculate the minimum possible monthly income of Mr Chan's son, given that he did not receive any annual bonus in 2021.

Assume that Mr and Mrs Chan's annual income remained unchanged.
Give your answer correct to the nearest $.

Answer $ ..... [3]

END OF PAPER

ANS

ZHONGHUA SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2021
SECONDARY 4E/4N/5N

Candidate's Name	Class	Register Number

MATHEMATICS
Paper 2

4048/02
31 Aug 2021
2 hours 30 minutes

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **100**.

For Examiner's Use
100

Setter: Mrs See YN
Vetted by: Mr Poh WB

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle ABC = $\frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area = $\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f}-\left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1 **(a)** Simplify $\dfrac{8a^2-50b^2}{2a^2+7ab-30b^2}$.

$\dfrac{2(4a^2-25b^2)}{(2a-5b)(a+6b)}$ or $\dfrac{(2a-5b)(4a+10b)}{(2a-5b)(a+6b)}$

$= \dfrac{2(2a-5b)(2a+5b)}{(2a-5b)(a+6b)}$ $= \dfrac{4a+10b}{a+6b}$

$= \dfrac{2(2a+5b)}{(a+6b)}$

Factors of $8a^2-50b^2$ – M1

Factors $(2a-5b)(a+6b)$ – M1

$\dfrac{4a+10b}{a+6b}$ $\dfrac{2(2a+5b)}{(a+6b)}$ A1

Answer..... [3]

(b) Solve the equation $\dfrac{1}{3y-2} = \dfrac{y}{9y^2-12y+4} - 1$, giving your answers correct to 2 decimal places.

$\dfrac{1}{3y-2} = \dfrac{y}{(3y-2)(3y-2)} -$ Factors $(3y-2)^2$ M1

multiply all terms by $(3y-2)(3y-2)$

$3y-2 = y - (3y-2)(3y-2)$

$3y-2 = y - 9y^2+12y-4$

$9y^2-10y+2=0$ R.E. seen M1

$y = \dfrac{-(-10) \pm \sqrt{(-10)^2-4(9)(2)}}{2(9)}$ Quadratic formula with values substituted. M1

$= \dfrac{10 \pm \sqrt{28}}{18}$

$= 0.26158$ or 0.84952

Both correct A1

Answer $y =$ 0.26 or 0.85 [4]

1 (c) Simplify $\frac{p^5}{3s^2} \div \frac{(2p)^2}{2sp^{-2}}$.

$\frac{p^5}{3s^2} \times \frac{2sp^{-2}}{4p^2}$ invert fraction correctly (M1)

$= \frac{2sp^3}{6s^2p^2}$

$= \frac{p}{6s}$ (A1)

Answer $\frac{p}{6s}$ [2]

(d) Solve the inequality $\frac{2}{3}x+2 < \frac{1}{5}(8x+1) \leq 2x-10$.

$\frac{2}{3}x + 2 < \frac{1}{5}(8x+1)$

multiply by 15

$10x + 30 < 24x + 3$

$-14x < -27$

$x > \frac{27}{14}$

$x > 1\frac{13}{14}$

$\frac{1}{5}(8x+1) \leq 2x-10$

multiply by 5

$8x + 1 \leq 10x - 50$

$-2x \leq -51$

$x \geq \frac{51}{2}$

$x \geq 25\frac{1}{2}$

split into 2 inequalities (M1)

simplification (M1)

either answer correct (A1)

(A1)

Answer $x \geq 25\frac{1}{2}$ [4]

2 **(a)** The frequency and cumulative frequency tables below show the results from a survey of 200 students on the number of hours they spend on social media on a particular day.

Time (hours)	Frequency
$0 < x \leq 1$	32
$1 < x \leq 2$	a
$2 < x \leq 3$	35
$3 < x \leq 4$	39
$4 < x \leq 5$	b
$5 < x \leq 6$	20

Time (hours)	Cumulative frequency
$x \leq 1$	32
$x \leq 2$	$a + 32$
$x \leq 3$	$3a - 19$
$x \leq 4$	$2b + 87$
$x \leq 5$	180
$x \leq 6$	200

(i) Find the value of a and b.

$a + b = 200 - (32 + 35 + 39 + 20)$

$= 74$ (M1)

when $x \leq 3$, $32 + a + 25 = 3a - 19$

$2a = 86$

$a = 43$

subs $a = 43$ into $a + b = 74$

$\therefore b = 31$

(A1) (A1)

Answer $a =$ 43 $b =$ 31 [3]

(ii) Hence, find the number of students who spend more than 4 hours on social media.

$b + 20 = 31 + 20$

$= 51$

or

$200 - (2b + 87)$

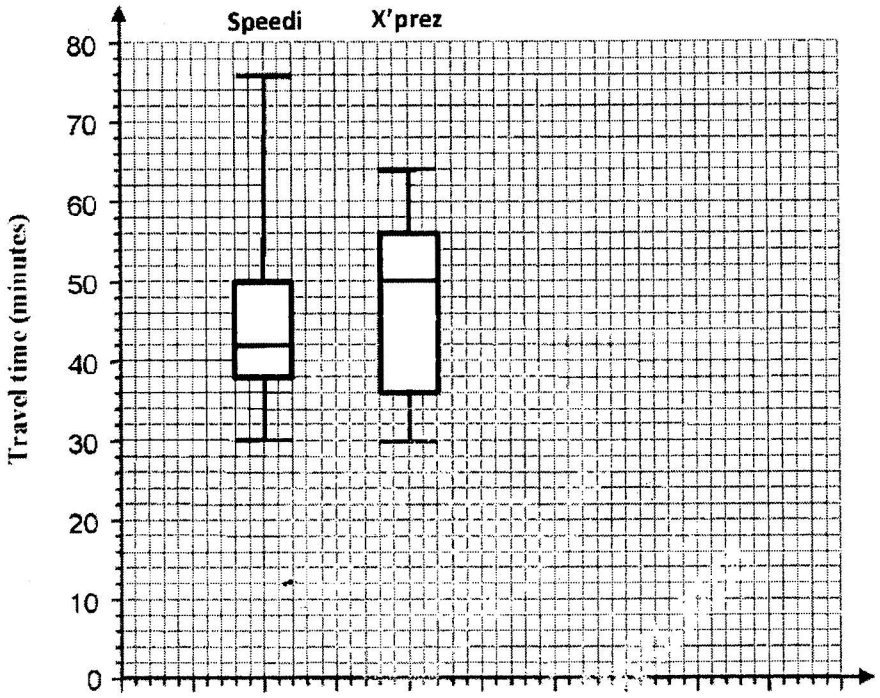
$= 200 - (62 + 87)$

$= 51$

(A1)✓

Answer 51 [1]

2 (b) Two transport companies, Speedi and X'prez, provide coach services from Singapore to Johor Bahru, and the coaches leave from the same coach park in Singapore. The following box and whisker diagrams provide the data for the travel time, in minutes, of 80 coaches from each company for the journey from Singapore to Johor Bahru.



(i) Explain which company provides a more time efficient coach service, stating your reasons with supporting evidence from 2 different statistics.

Answer [3]

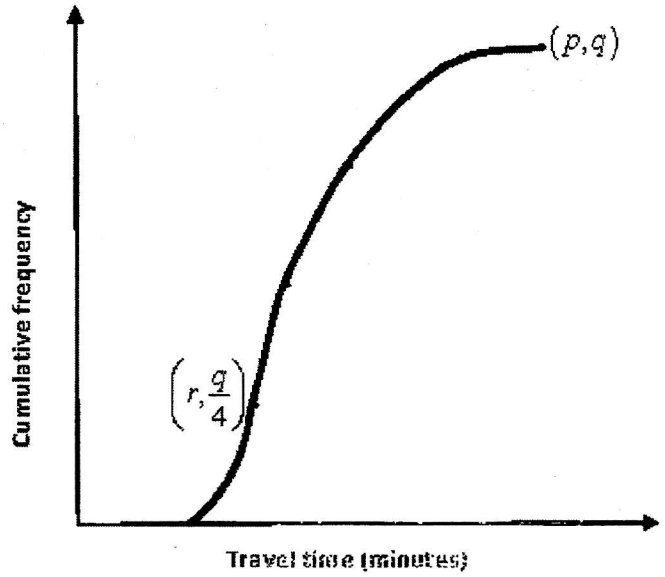
Speedi provides a more time efficient service. (B1)

The median time for Speedi (42 mins) is less than X'prez (50 mins). (B1)

Moreover, the interquartile range for Speedi (12 mins) as compared to X'prez (20 mins) shows that Speedi's coaches are more consistent in their travel time than X'prez. (B1)

other acceptable reasons

2 (b) (ii) The data for the travel time of coaches from Speedi company can also be represented by the cumulative frequency curve shown below.



State the exact values of p, q and r.

Answer p = 76 (B1) [1]

q = 80 (B1) [1]

r = 38 (B1) [1]

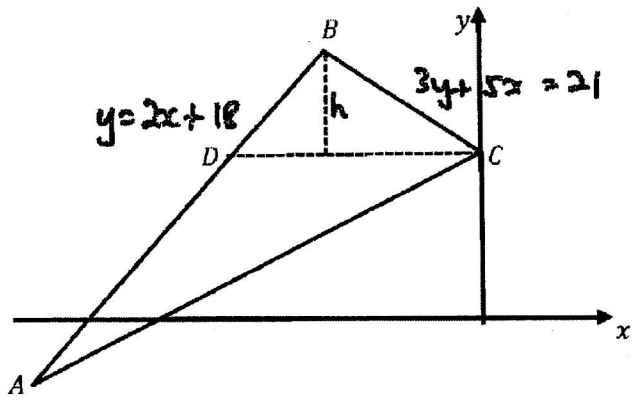
(iii) Given that (60,74) is a point on the cumulative frequency curve, find the probability that a Speedi coach will take more than an hour to reach Johor Bahru.

74 coaches took less than 60 mins. } either (M1)
6 coaches took more than 60 mins }

(A1)✓

Answer $\frac{6}{80}$ or $\frac{3}{40}$ or 0.075 [2]

3 The diagram shows a triangle ABC, where C lies on the y-axis. The equations of the lines AB and BC are $y = 2x + 18$ and $3y + 5x = 21$ respectively. D is a point on AB such that DC is parallel to the x-axis.



(a) Find the coordinates of B.

Subs $y = 2x+18$ into $3y + 5x = 21$

$3(2x + 18) + 5x = 21$ — subst^n or elimination method (M1)

$11x + 54 = 21$

$x = -3$ (A1) for either x or y correct.

$\therefore y = 2(-3) + 18$

$= 12$

(A1)

Answer $B($ ..-3.. , ..12.. $)$ [3]

(b) Find the coordinates of C and D.

At C, $x = 0$ (B1)

$\therefore 3y + 5(0) = 21$

$y = 7$

At D $y = 7$, (B1)

$\therefore 7 = 2(x) + 18$

$x = \frac{7-18}{2}$

$= -\frac{11}{2}$

(A1)

Answer $C($ ..0.. , ..7.. $)$

$D($ ..$-\frac{11}{2}$.. , ..7.. $)$ [4]

(A1)

3 **(c)** Find the area of triangle BCD.

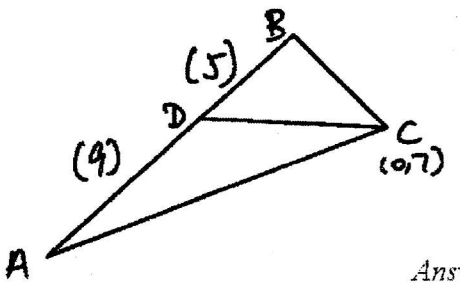
area BCD $= \frac{1}{2}(CD)(ht)$
$= \frac{1}{2}(\frac{11}{2})(12-7)$ either base or height correct in the formula (M1)
$= 13.75$

zero mark for shortcut method.

(A1)

Answer 13.75 units^2 [2]

(d) Given that the ratio of the lengths $AB : AD = 14 : 9$, state the ratio of area of triangle BCD : area of triangle ABC.



(B1)

Answer 5 : 14 [1]

(e) A point E lies on the x-axis such that $ABCE$ is a trapezium. Find the coordinates of E.

For ABCE to be a trapezium, CE must be parallel to AB. Let the coords of E be $(x, 0)$

grad of AB $= 2$
grad of CE $= \frac{7-0}{0-x}$
grad AB = grad CE soi (M1)

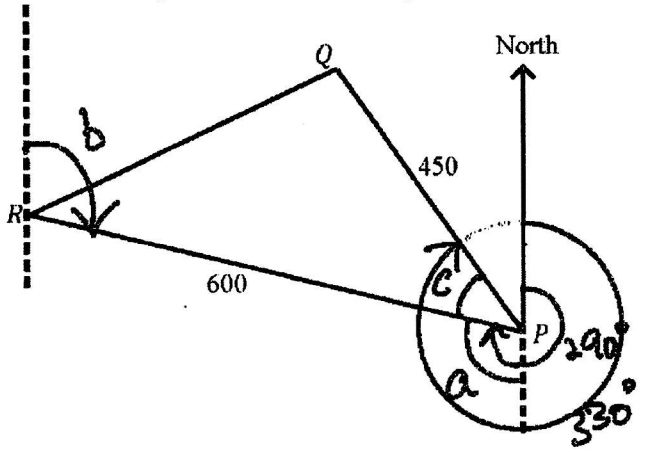
$\frac{7}{-x} = 2$

$x = -\frac{7}{2} = -3\frac{1}{2}$

(A1)

Answer $E(-3\frac{1}{2}, 0)$ [2]

4 In the diagram, P, Q and R are three points on horizontal ground.



Q is on a bearing of 330° from P and $PQ = 450$ m.
R is on a bearing of 290° from P and $PR = 600$ m.

(a) Calculate the bearing of P from R.

bearing of P from R = 70° (alternate ∠s)
= 290° − 180°
= 110°

B1

Answer 110° [1]

(b) Calculate the distance QR, correct to 1 decimal place.

∠C = 330° − 290° = 40° M1
$QR^2 = 450^2 + 600^2 - 2(450)(600)\cos 40°$ M1 √
$QR = \sqrt{148836.0007}$
= 385.792

A1

Answer 385.8 m [3]

4 (c) A drone, D, is hovering directly above Q. A man, standing at P, looks up at the drone and observes that the angle of elevation of D is 14°.

(i) Calculate the height of the drone, D, above Q, correct to 1 decimal place.

$\tan 14° = \frac{DQ}{450}$ (M1)

$DQ = 450 \tan 14°$

$= 112.197$

(A1)

Answer 112.2 m [2]

(ii) Give a possible reason why the actual height of the drone, D, above the point Q could be 1.7 m more than the answer found in c(i).

Answer The angle of elevation is observed from a man whose height is 1.7m (B1) [1]

(d) S is a point along PR and is directly south of Q. Another drone, E, takes off from P and flies directly towards S at a speed of 12 m/s. Its flight path is parallel to the ground.

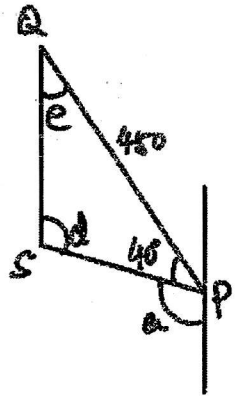
(i) Calculate the distance drone E flew from P to S, giving your answer correct to 1 decimal place.

$\angle d = \angle a$ (alternate $\angle$s)

$= 110°$

$\angle e = 180° - 110° - 30°$ $\angle e$ correct (M1)√

$= 30°$



$\frac{PS}{\sin 30°} = \frac{450}{\sin 110°}$ (M1)√

$PS = \frac{450}{\sin 110°} \times \sin 30°$

$= 239.439$

(A1)

Answer 239.4 m [3]

(ii) Hence, find the time taken for drone E to reach S.

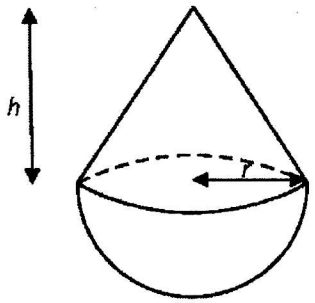
$\text{Time} = \frac{239.439}{12}$ (M1)√

$= 19.953$

(A1)

Answer 20.0 s [2]

5 A glass perfume bottle is made up of a hemisphere and a cone as shown below. A manufacturer wants to keep the volume of the bottle constant at 36π cm^3. Assume that the thickness of the glass is negligible.



(a) If the volume of the cone is equal to the volume of the hemisphere, show that the radius of the hemisphere is 3 cm. Hence, find the height of the cone.

Vol of hemisphere = $\frac{1}{2} \times \frac{4}{3}\pi r^3$

$\frac{2}{3}\pi r^3 = 18\pi$ (A1) Vol = 18π

$r^3 = \frac{18 \times 3}{2}$

$r = \sqrt[3]{27} = 3$ (AG)

$\frac{1}{3}\pi r^2 h = 18\pi$ (M1) Vol = 18π

$h = \frac{18 \times 3}{3} = 6$

6 (A1)

Answer ..... cm [3]

(b) The manufacturer wants to vary the radius, r, and the height, h, of the bottle while keeping the volume constant at 36π cm^3.

(i) Write down, but do not simplify, an equation that must be satisfied by r and h.

Answer: $\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h = 36\pi$ (B1) [1]

5 (b) (ii) Hence, show that $h = \frac{108 - 2r^3}{r^2}$

Answer:

$\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h = 36\pi$

multiply by $\frac{3}{\pi}$. Simplification steps must be seen (M1)

$2r^3 + r^2 h = 108$

$h = \frac{108 - 2r^3}{r^2}$ (AG)

[1]

(c) The manufacturer decided to produce bottles with a radius of 3.2 cm and a volume of 36π cm^3.

(i) Find the value of h, correct to 1 decimal place.

$h = \frac{108 - 2(3.2)^3}{(3.2)^2}$ substitution into formula (M1)

$= 4.146875$

(A1)

Answer $h =$ 4.1 cm [2]

(ii) Hence, find the total surface area of the bottle.

let l be the slant length of cone.

$l^2 = r^2 + h^2$

$= 3.2^2 + 4.146875^2$ (M1)✓

$= 27.43657$

$l = 5.23799$

Total surface area $= 2\pi(3.2)^2 + \pi(3.2)(5.23799)$ (M1) + (M1)

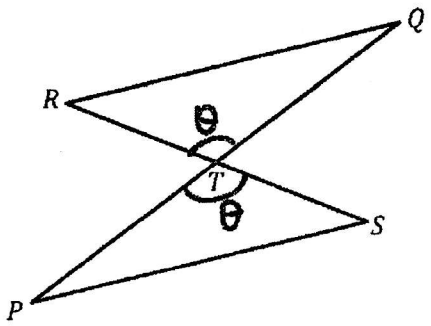
$= 116.9978$

$= 117\ \text{cm}^2$ (2 s.f.)

(A1)

Answer 117 cm^2 [4]

6 (a)



The straight lines PQ and RS bisect each other at T. $PQ = 26.2$ cm and $RS = 18.6$ cm.

(i) Using the dimensions given, show that $\frac{area\ of\ triangle\ PST}{area\ of\ triangle\ QRT} = 1$

Answer: [2]

$\angle PTS = \angle QTR = \theta$ (vertically opp $\angle$s) (B1)

$\frac{\text{area } \Delta PST}{\text{area } \Delta QRT} = \frac{\frac{1}{2}(13.1)(9.3)\sin\theta}{\frac{1}{2}(13.1)(9.3)\sin\theta}$ (B1)

$= 1$

(ii) Stating your reasons clearly, explain why PS is parallel to RQ.

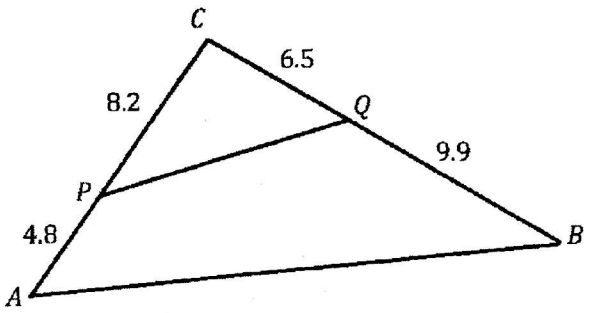
Answer: [2]

ΔPST is congruent to ΔQRT as 2 pairs of corresponding sides and their included angles are equal. (B1)

$\therefore \angle PST = \angle QRT$ (alternate $\angle$s) (B1)

and PS is parallel RQ

6 (b)



ABC is a triangle and P and Q lie on AC and BC respectively such that $AP = 4.8$ cm, $PC = 8.2$ cm, $BQ = 9.9$ cm and $QC = 6.5$ cm.

(i) Show that triangle ABC is similar to triangle QPC.

Answer

[4]

$\frac{AC}{QC} = \frac{4.8 + 8.2}{6.5} = \frac{13}{6.5} = 2$ (M1)

$\frac{BC}{PC} = \frac{9.9 + 6.5}{8.2} = \frac{16.4}{8.2} = 2$ (M1)

$\angle ACB = \angle QCP$ (common $\angle$) (M1)

Since 2 pairs of corresponding sides are in the same ratio and their included angles are equal, (B1)

$\therefore \Delta ABC$ is similar to ΔQPC.

Award 2 out of 4, if pt two (M1) steps not clearly written.

(ii) Hence, find the area of triangle QPC, given that the area of triangle ABC is 102.8 cm^2.

$\frac{\text{area } \Delta QPC}{\text{area } \Delta ABC} = \left(\frac{l_1}{l_2}\right)^2 = \left(\frac{1}{2}\right)^2$ (M1)√

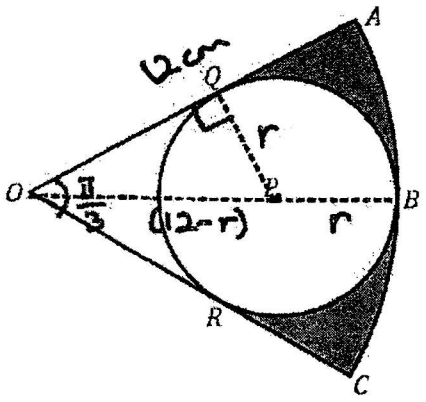
$\therefore$ area $\Delta QPC = \frac{1}{4} \times 102.8$

$= 25.7$

(A1)

Answer 25.7 cm^2 [2]

7



In the diagram, $OABC$ is a sector of a circle with centre at O, radius 12 cm. A circle BQR, with centre at P, is inscribed in the sector. Angle $AOC = \frac{\pi}{3}$ radians.

(a) Find the area of the sector $OABC$, leaving your answer in terms of π.

area of sector $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}(12)^2\left(\frac{\pi}{3}\right)$ (M1)

$= 24\pi$

(A1)

Answer 24π cm^2 [2]

(b) Show that the radius of the circle BQR is 4 cm.

Answer: [3]

In the ... $\triangle OPQ$,

$\angle POQ = \frac{1}{2}\left(\frac{\pi}{3}\right)$

$= \frac{\pi}{6}$ (M1)

$\sin\left(\frac{\pi}{6}\right) = \frac{r}{12-r}$ (M1)

$\frac{1}{2} = \frac{r}{12-r}$ Simplification seen (M1)

$2r = 12 - r$

$r = 4$ (AG)

7 (c) Find the area of the shaded region.

In rt $\angle$ ΔOPQ, $OQ = \sqrt{(12-4)^2 - 4^2}$

$= \sqrt{48}$ (6.92820) (M1)

area OQPR = 2 × area ΔOPQ

$= 2 \times \frac{1}{2}(PQ)(OQ)$

$= 4\sqrt{48}$ (27.7128) (M1)

obtuse $\angle QPR = 2\pi - \frac{\pi}{3} - \frac{\pi}{2} - \frac{\pi}{2}$ ($\angle$ sum of quadrilateral)

$= \frac{2\pi}{3}$ (120°) (M1)

reflex $\angle QPR = 2\pi - \frac{2\pi}{3}$

$= \frac{4\pi}{3}$ (240°)

area of major sector PQR $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}(4)^2\left(\frac{4\pi}{3}\right)$ (M1)

$= \frac{32\pi}{3}$ (33.5103)

$\therefore$ area of shaded region $= 24\pi - \frac{32\pi}{3} - 4\sqrt{48}$ (M1)

$= 14.175$

(A1)

Answer 14.2 cm^2 [6]

8 **(a)** Complete the table of values for $y = \frac{x^2}{5} + \frac{8}{x} - 3$.

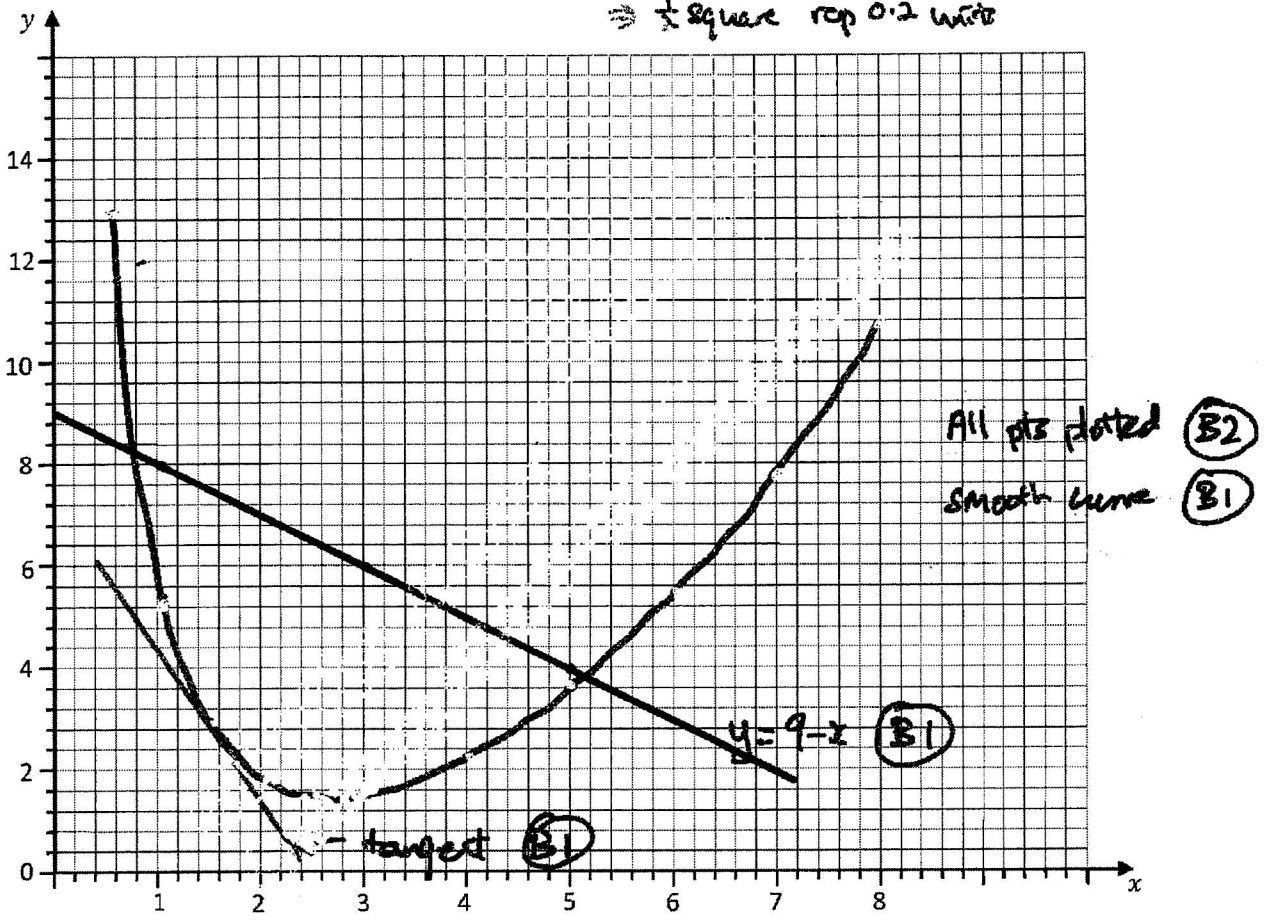
Give your answer correct to one decimal place. [1]

x	0.5	1	1.5	2	2.5	3	4	5	6	7	8
y	13.1	5.2	2.8	1.8	1.5	1.5	2.2	3.6	5.5	7.9	10.8

B1

(b) On the grid, draw the graph of $y = \frac{x^2}{5} + \frac{8}{x} - 3$ for $0.5 \leq x \leq 8$. [3]

y-axis: 5 squares rep 2 units ⇒ 1 square rep 0.4 units.
⇒ $\frac{1}{2}$ square rep 0.2 units



All pts plotted B2
Smooth curve B1
y = 9 − x B1
tangent B1

(c) By drawing a tangent, find the gradient of the curve at the point where $x = 1.5$.

(1.5, 2.8) cannot be used to calculate gradient.

Answer −2.96 A1 [2]

accept only decimals

8 **(d)** Use your graph to find the minimum value of $\frac{x^2}{5}+\frac{8}{x}$ for values of x in the range $0.5 \leq x \leq 8$.

min of $\frac{x^2}{5}+\frac{8}{x}-3 = 1.4$ (M1) (read off from their graph)

$\therefore$ min of $\frac{x^2}{5}+\frac{8}{x} = 4.4$

Answer 4.4 (A1) [2]

(e) By drawing a suitable straight line on the grid in part **(b)**, use your graph to find the solutions to the equation $\frac{x^2}{5}+\frac{8}{x}+x=12$ in the range $0.5 \leq x \leq 8$.

$\frac{x^2}{5}+\frac{8}{x}+x = 12$

$\frac{x^2}{5}+\frac{8}{x}-3 = 12-x-3$

Suitable line is $y = 9-x$ (M1)

(A1) both

Answer $x =$ 0.72 or 5.15 [3]

(f) The equation $\frac{x^2}{5}+\frac{8}{x}-3=k$, where k is a positive integer, does not have any solutions for $0.5 \leq x \leq 8$. State the value of k.

(B1)

Answer $k =$ 1 [1]

9 Mr Chan, his wife, 2 sons and his 80 year old mother live together in a HDB flat. His mother goes to the day care centre for the elderly. In order to qualify for government subsidies for the day care services, means-testing is used to calculate the amount of subsidies that will be allocated.

For means-testing, the annual income of **all** working family members living in the household is taken into consideration. The annual income for each working member of the household includes bonuses. The **monthly household income per person** is then used to determine the amount of subsidy allocated.

$$\textbf{monthly household income per person} = \frac{\text{total annual income of all household members}}{\text{number of household members} \times 12}$$

Eligibility for Subsidies on Day Care Source: www.ntuchealth.sg

Criteria for monthly household income per person	Subsidy levels
\$800 and below	80%
\$801 - \$1,200	75%
\$1,201 - \$1,900	60%
\$1,901 - \$2,000	50%
\$2,001 - \$2,800	30%
\$2,801 and above	0%

In January 2021, Mr Chan applied for subsidies for his mother. Only Mr Chan and his wife were working in 2020.
Mr Chan earned a monthly income of \$6000 and his company paid him a bonus of 1.2 months' pay in 2020.
Mrs Chan earned a weekly income of \$800. For every full 10 weeks of work, she will receive an additional 5% of the 10 weeks' income. She worked for 46 weeks in 2020.

(a) Calculate the total annual household income of Mr Chan and his wife in 2020.

Total annual household income

$= \boxed{(6000 \times 12) + (1.2 \times 6000)} + \boxed{(800 \times 46) + 4 \times \frac{5}{100} \times (800 \times 10)}$

$= (72000 + 7200) + (36800 + 1600)$

$= 79200 + 38400$

$= 117\,600$

either Mr Chan or Mrs Chan income correct (M1)

(A1)

Answer \$ 117 600 [2]

(b) Calculate the monthly household income per person of Mr Chan's family.

$\frac{\$117\,600}{12 \times 5}$

A1 ✓

Answer S..... 1960 ..... [1]

(c) Hence, determine the level of subsidy that the Chan family will qualify for.

A1 ✓

Answer..... 50% ..... [1]

(d) Mr Chan's older son started working on 1 March 2021. If the Chan family will not qualify for any subsidies when they re-apply again in January 2022, calculate the minimum possible monthly income of Mr Chan's son, given that he did not receive any annual bonus in 2021.
Assume that Mr and Mrs Chan's annual income remained unchanged.
Give your answer correct to the nearest $.

For zero subsidies,

min. monthly household income per person = $2801

∴ annual total household income = $2801 × 5 × 12

= $168 060 M1

Son's income for 2021 = $168 060 − 117 600

= $50 460 M1

He worked for 10 months,

∴ his minimum monthly income

$= \frac{\$50\,460}{10} = \$5046$

Alternatively:

$\frac{117\,600 + 10x}{5 \times 12} \geq 2801$

A1

Answer S..... 5046 ..... [3]

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