CANDIDATE NAME	
CLASS	4
CENTRE NUMBER	S
REGISTER NUMBER	
INDEX NUMBER	

MATHEMATICS
PAPER 1

4048/01

Friday	**20 August 2021**	**2 hours**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class, register number, centre number and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid/tape.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use

This document consists of **17** printed pages and **1** blank page.   [**Turn over**

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1 A number, when rounded off to one significant figure, is 300.
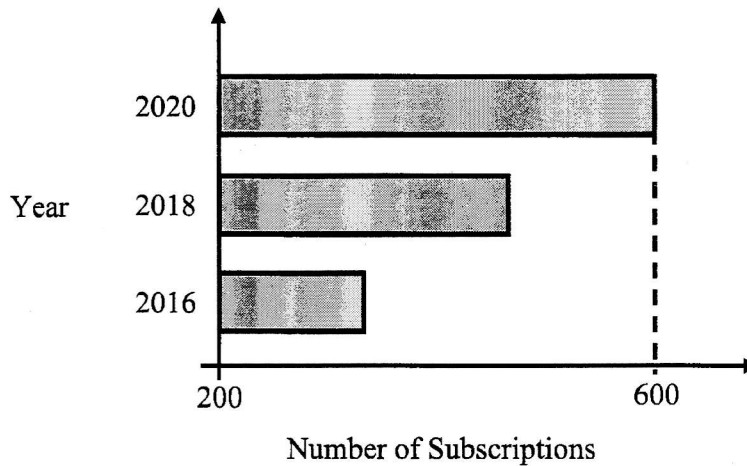Write down the smallest possible value of this number.

Answer .... [1]

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2 Write the following numbers in order of size, starting with the largest.

$$-\sqrt{6} \ , \ 24.5\% \ , \ \frac{245}{999} \ , \ -\frac{49}{20}$$

Answer ....,..................,...................,............... [1]
          largest   smallest

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3 The chart below shows the number of subscriptions to a local newsletter from 2016 to 2020.

(a) State one misleading feature of the chart.

....

....

.... [1]

(b) Explain how this feature affects the reader's interpretation of the chart.

....

....

.... [1]

[Turn over

4 **(a)** Solve the inequalities $3x < 4x + 5 \le 20$.

Answer .... [2]

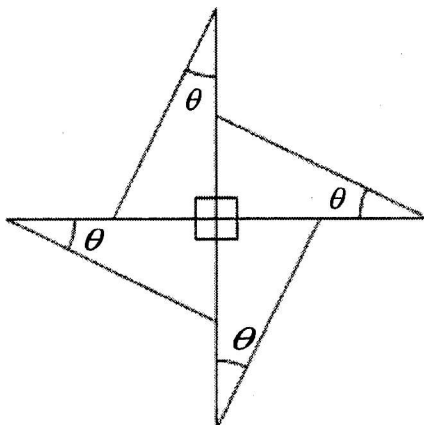**(b)** $W = \{$integers x: $3x < 4x + 5 \le 20\}$
A number is selected at random from the set W.
Find the probability that the number selected is a prime number.

Answer .... [1]

5 The figure below is made up of four congruent right-angle triangles.

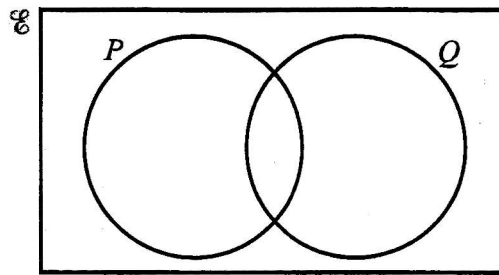It is given that the length of the longest side of the right-angle triangle is 15 cm and $\sin \theta = \dfrac{3}{5}$.

Without solving for θ, calculate the perimeter of the figure.

Answer .... cm [2]

6 **(a)** On the Venn diagram, shade the region which represents $(P' \cap Q)'$.

[1]

(b) $\mathscr{E} = \{s, i, n, g, a, p, o, r, e\}$
$A \cup B = \mathscr{E}$
$A \cap B = \{s, g, p\}$
$\{o, e\} \cap B = \varnothing$, $\{n, r\} \subset B$, $a \in A$, $i \notin B$

(i) List the elements in the set B.

Answer .... [1]

(ii) Write down the set $(A \cup B)'$.

Answer .... [1]

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7 1 microgram = 10^{-6} grams
1 milligram = 10^{-3} grams

(a) It is given that k micrograms = 1 milligram.
Find the integer value of k.

Answer $k =$ .... [1]

(b) It is recommended that a person consumes 750 micrograms of vitamin A daily.
Meg ate a baked sweet potato that contains 1.7 milligrams of vitamin A.
Calculate the amount of vitamin A that she has consumed in excess as a percentage of
the recommended daily intake.

Answer .... % [2]

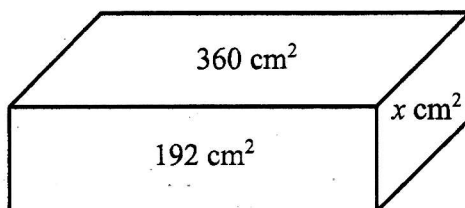**8**   When written as the product of their prime factors,

$$360 = 2^3 \times 3^2 \times 5,$$

$$192 = 2^6 \times 3.$$

(a) x is a number between 100 and 140.
The highest common factor (HCF) of x, 192 and 360 is 24.
Find the value of x.

Answer $x = $ .... [2]

**(b)**   The diagram below shows a rectangular block of wood with three of its faces marked x cm², 192 cm² and 360 cm².

The lengths, in cm, of the sides of the wood are whole numbers.
Find the dimensions of the block of wood.

Answer   .... cm × .... cm × .... cm   [1]

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9   The present ages of Nur and her sister are in the ratio 11 : 8.
14 years ago, the ratio of their ages is 5 : 3.
Calculate Nur's present age.

Answer   .... years old   [3]

10 The diagram shows part of a *n*-sided polygon, *ABCDE*…, where *AB* is parallel to *ED*.
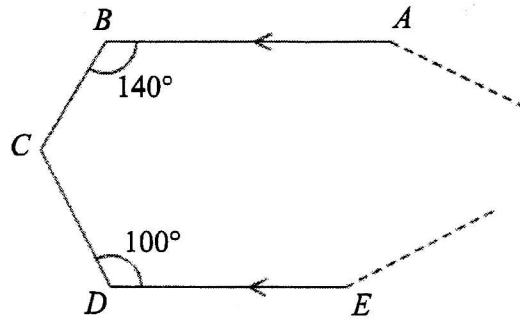
Angle *ABC* = 140° and angle *CDE* = 100°.
The exterior angles of the remaining parts of the polygon are 36° each.
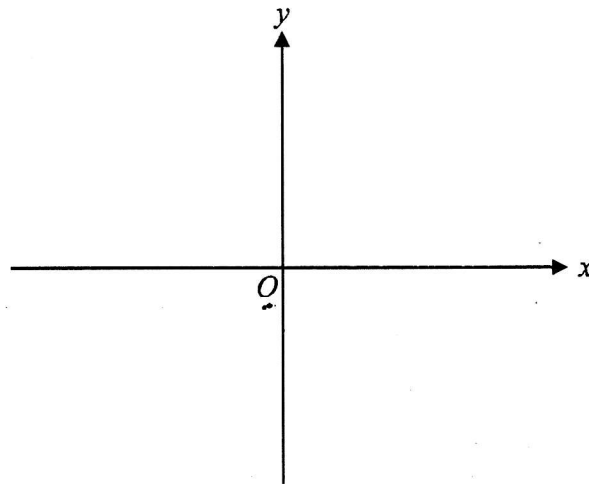Find the value of *n*.

Answer   *n* = ………………………….. [3]

11 The graph of $y=(p-x)(x-q)$ passes through the points $(0,-5)$ and $(5,0)$.

(a) State the value of p and of q, where $q > p$.

Answer $p = $ ....

$q = $ .... [2]

(b) Hence, sketch the graph of $y=(p-x)(x-q)$, indicating clearly the coordinates of the points where the graph crosses the axes and the coordinates of the turning point on the curve.

[2]

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12 **(a)** Factorise completely $6xy-9y^2-2x+3y$.

Answer .... [2]

(b) Hence, simplify $\dfrac{24xy-36y^2-8x+12y}{(1-3y)^2} \div \left(4x^2y^2-6xy^3\right)$.

Answer .... [3]

13 Lee made tea to sell over a particular weekend.
The matrix, **M**, shows the number of cups of tea he made.

$$\mathbf{M} = \begin{matrix} \text{Honey} & \text{Milk} \\ \begin{pmatrix} 60 & 80 \\ 50 & 70 \end{pmatrix} & \begin{matrix} \text{Saturday} \\ \text{Sunday} \end{matrix} \end{matrix}$$

He sold 85% of each type of tea on Saturday and 90% of each type of tea on Sunday.

(a) Evaluate the matrix $\mathbf{S} = \begin{pmatrix} 0.85 & 0.9 \end{pmatrix}\mathbf{M}$.

Answer **S** = ... [1]

(b) State what each element of matrix **S** represents.

....

....

.... [1]

Lee sold each cup of honey tea at a profit of n cents and each cup of milk tea at a loss of 20 cents.

(c) The matrix **V**, where $\mathbf{V} = \dfrac{1}{100}\mathbf{SF}$, represents the total profit, in dollars, he earned over the weekend.

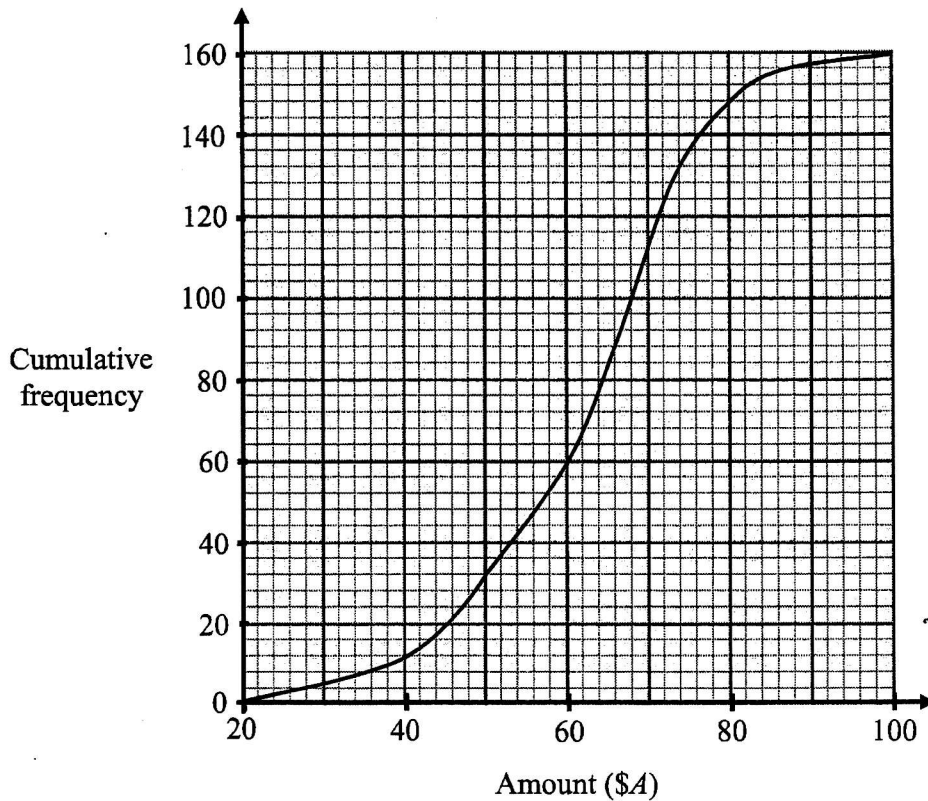**(i)** Write down the matrix **F**, in terms of n.

Answer **F** = ... [1]

(ii) Hence, find the value of n, given that he earned a total profit of $21.80.

Answer n = ... [2]

14 160 students were surveyed on the amount of pocket money they received in a week.
The cumulative frequency curve below shows the distribution of the amount of pocket money received.

(a) **(i)** Complete the grouped frequency table for the amount of pocket money received.

Amount (A)	$20 \leq A < 40$	$40 \leq A < 60$	$60 \leq A < 80$	$80 \leq A < 100$
Frequency				

[1]

(ii) State the modal interval.

Answer .... [1]

(b) Use the curve to estimate the 30th percentile.

Answer $ .... [1]

(c) Raj says,

"The range of pocket money received in a week is $100-\$20 = \$80$."

Explain why he is wrong.

....

....

.... [1]

(d) Steph says,

"80% of the students received at least $73 for pocket money in a week."

Is Steph correct?
Justify your answer with clear calculations.

....

....

.... [2]

---

15 **(a)** Given that $\dfrac{a+3b}{3} = 2(b-a)$, find the ratio $a : b$.

Answer .... : .... [2]

(b) Use factorisation to solve the equation.

$$(5c+1)(3-c) = 11$$

Answer $c = $ .... [3]

16 **(a)** Simplify $\dfrac{6^m + 1}{36^m - 1}$.

Answer .... [2]

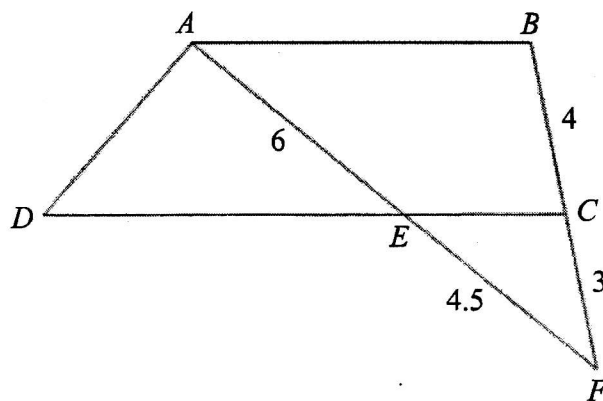**(b)** Given that $\sqrt{7^n} \div 49^{\frac{1}{n}} + 1 = 2\left(\dfrac{1}{14^0}\right)^n$, find the values of n.

Answer $n =$ .... [3]

17

The diagram shows a quadrilateral *ABCD*.
E is a point on *DC* such that *AE* and *BC* produced meet at *F*.
AE = 6 cm, *EF* = 4.5 cm, *CF* = 3 cm and *BC* = 4 cm.

(a) Prove that triangle *ABF* is similar to triangle *ECF*.

Answer

[2]

(b) Stating your reasons clearly, explain why *ABCD* is a trapezium.

....

....

....

....

....

.... [1]

M is the midpoint of *AE*.
N is a point on *DC* such that triangle *AMB* is congruent to triangle *EMN*.

(c) Find the ratio *EN* : *CN*.

Answer .... : .... [1]

18
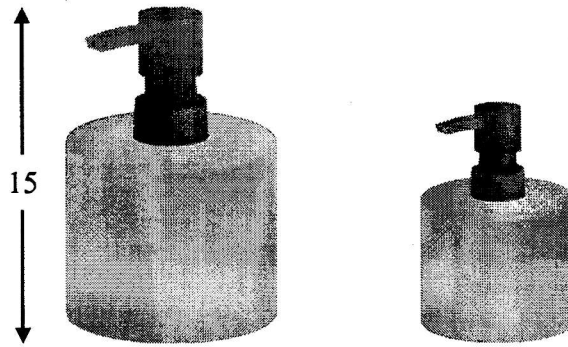
The diagram shows two geometrically similar empty bottles.
The larger bottle has a base area of 90 cm².
The smaller bottle has a base area of 62.5 cm².

(a) The height of the larger bottle is 15 cm.
Calculate the height of the smaller bottle.

Answer .... cm   [2]

Kim filled the empty bottles with homemade sanitiser up to their maximum capacities.
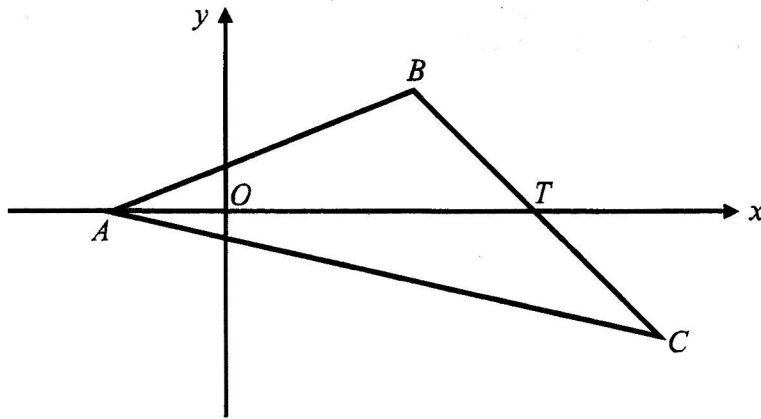She sells the larger bottle at $3.90 each and the smaller bottle at $2.90 each.

(b) Determine if the cost of the sanitiser is directly proportional to the capacity of the bottle.

Answer

....

....

....

.... [3]

19 In the diagram, ABC is a triangle.

A is a point on the x-axis such that the equation of the line AB is $5y - 6x = 12$.
BC intersects the x-axis at T such that $BC = 2BT$.
The equation of the line BC is $3x + y = 15$.
Calculate the area of triangle ABC.

Answer .... square units [5]

20 The diagram shows a triangular park *PQR* on horizontal ground.
R is due east of *P*.

Scale: 1 cm to 200 m

North

P

R

Q
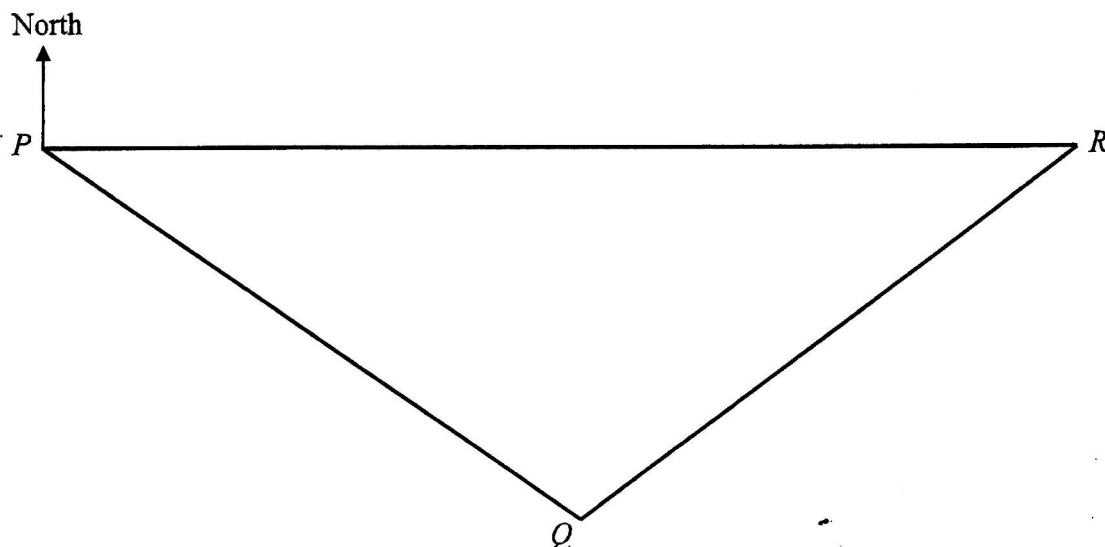
(a) Measure and write down the bearing of *R* from *Q*.

Answer ....° [1]

(b) Construct

(i) the perpendicular bisector of *PQ*, [1]

(ii) the bisector of angle *PRQ*. [1]

(c) A shelter is to be built in the park, nearer to *Q* than to *P* and nearer to *PR* than to *QR*.
Shade the region where the shelter is to be built. [1]

(d) Fyn jogged from point *P*, in a straight line, to point *R* at an average speed of 8 km/h.
Calculate the total time she took, giving your answer in minutes and seconds.

Answer .... min .... s [2]

21   The diagram shows the speed-time graph for a bus journey.
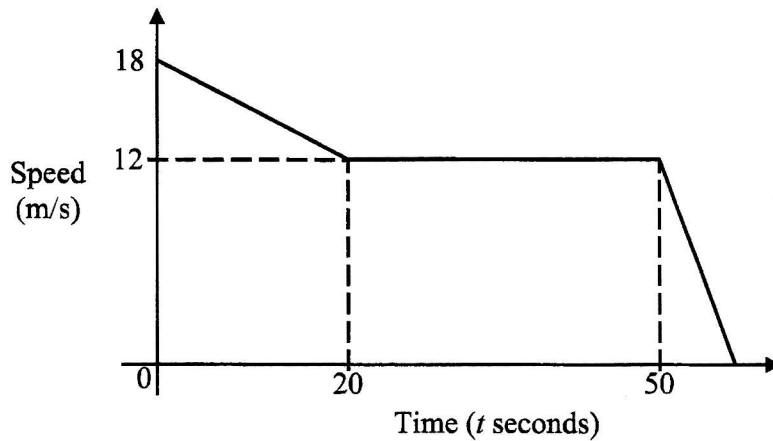
(a)   The bus decelerates to a stop at a constant rate of 1.5 m/s² for the last k seconds of its journey.
Calculate the value of k.

Answer $k =$ .... [1]

(b)   Find the speed of the bus at $t = 12$.

Answer   .... m/s   [2]
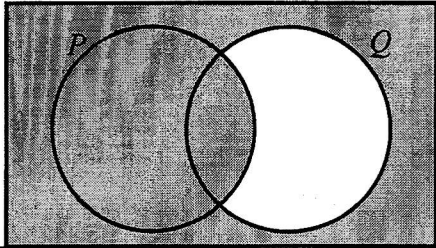
(c)   Calculate the distance travelled by the bus in the first 50 seconds.

Answer   .... m   [1]
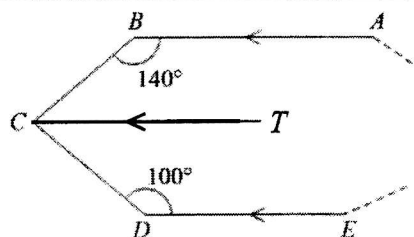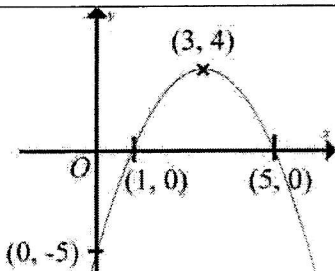
(d)   A car starts its journey at the same place and time as the bus.
It accelerates uniformly from rest until it overtakes the bus at $t = 50$.
Calculate the speed of the car at this instant.

Answer   .... m/s   [2]

P1 ANS   SCGS

2021 Sec 4OLP Mathematics Preliminary Examinations Paper 1 Marking Scheme

Qn	Answer	Marks	Remarks
	Total Marks: [80 Marks]		
1	250	B1	
2	$\dfrac{245}{999}$, 24.5%, $-\sqrt{6}$, $-\dfrac{49}{20}$	B1	
3a	• The vertical axis (or year-axis) is incomplete. The bars for 2017 and 2019 are not represented. • The horizontal axis (or number of subscriptions-axis) does not start from zero.	B1	Accept other valid features.
3b	• It will mislead the reader into thinking that the number of subscriptions is always increasing when there may be a decrease in subscriptions in the years 2017 or 2019. • It will mislead the reader into thinking that the number of subscriptions is tripled from 2016 to 2020, based on the length of the bars.	B1	Accept other valid reasons based on the feature in (a).
4a	$3x < 4x + 5 \le 20$ $3x < 4x + 5$ or $4x + 5 \le 20$ $-x < 5$ or $4x \le 15$ $x > -5$ or $x \le 3.75$ $\therefore -5 < x \le 3.75$	M1 A1	M1 for splitting inequalities and simplifying like terms.
4b	$W = \{-4, -3, -2, -1, 0, 1, 2, 3\}$ $\therefore \text{P(prime no.)} = \dfrac{2}{8} = \dfrac{1}{4}$	B1	
5	$\sin\theta = \dfrac{3}{5} = \dfrac{9}{15}$ i.e. Length of opp. side = 9 cm Length of adj. side $= \sqrt{15^2 - 9^2}$ $\qquad = 12$ cm $\therefore$ Perimeter of figure $= 4(15+3)$ $\qquad\qquad = 72$ cm	M1 A1	M1 for using sine ratio to obtain opposite side of right-angle tri.
6a		B1	
6bi	$B = \{s, g, p, n, r\}$	B1	
6bii	$\{\ \}$ or $\varnothing$	B1	
7a	$k = 1000$	B1	
7b	Additional percentage $= \dfrac{1.7 \times 10^{-3} - 750 \times 10^{-6}}{750 \times 10^{-6}} \times 100\%$ $\approx 126.666\cdot$ $= 127\%$ (3 s.f.)	M1 A1	

1

Qn	Answer	Marks	Remarks
8a	HCF = $24 = 2^3 \times 3$ $x = 2^3 \times 3 \times 5$ $\therefore x = 120$	B1 B1	
8b	8 cm × 15 cm × 24 cm	B1	Accept in any order
9	**Method 1** Let Nur's present age be $11x$ years old. $\dfrac{11x-14}{8x-14} = \dfrac{5}{3}$ $33x - 42 = 40x - 70$ $7x = 28$ $x = 4$ $\therefore$ Nur's present age $= 11(4) = 44$ years old	M1 A1 A1	M1 for forming equation.

	Method 2
	 Difference in units for present ages = 11 – 8 = 3
	 Difference in units 14 years ago = 5 – 3 = 2
	 LCM of 3 and 2 = 6 units
	 i.e Ratio of Nur : Sis : Difference
	        22 : 16 : 6 (Present)
	        15 : 9 : 6 (14 years ago)
	 $\therefore$ Nur's present age $= \dfrac{14}{7} \times 22$
	            $= 44$ years old
		M1
		
		 M1
		 A1
			M1 for showing units for difference in ages is the same.
			
			 M1

10	 $\angle BCD = \angle BCT + \angle TCD$ $\angle BCD = 40° + 80°$ $\angle BCD = 120°$ Total exterior angles of remaining polygon $= 360° - 40° - 80° - 60°$ $= 180°$ $\therefore n = \dfrac{180°}{36°} + 3$ $= 8$	 B1 s.o.i. M1 A1	 M1 for division to get remaining angles.
11a	$p = 1$, $q = 5$	B1, B1	B1 for both answers.
11b		 B2	B1 for open downwards graph with (0, -5) and (5, 0) labelled at the correct positions. B1 for (3, 4) and (1, 0) labelled at the correct positions.

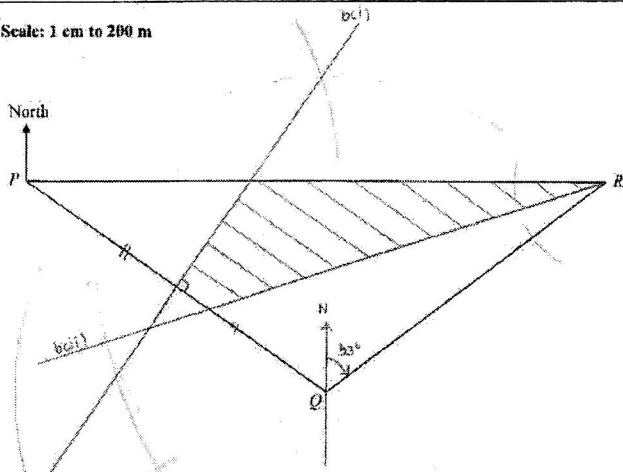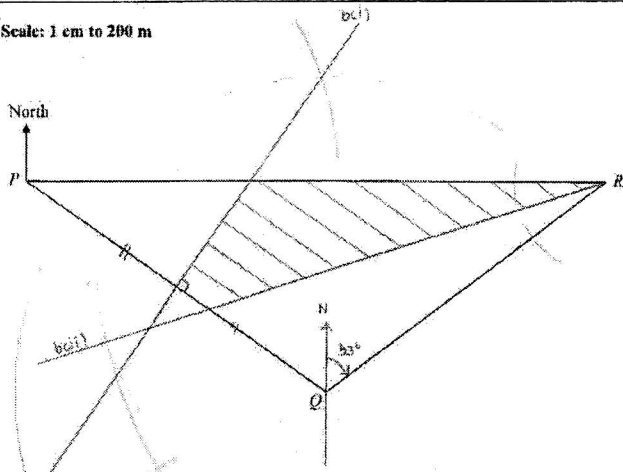| Qn	Answer	Marks	Remarks
12a	$6xy - 9y^2 - 2x + 3y$ $= 3y(2x - 3y) - (2x - 3y)$ $= (2x - 3y)(3y - 1)$	M1 A1	M1 for factorising after grouping (up to one mistake).
12b	$\dfrac{24xy - 36y^2 - 8x + 12y}{(1-3y)^2} \div \left(4x^2y^2 - 6xy^3\right)$ $= \dfrac{4(2x-3y)(3y-1)}{(3y-1)^2} \times \dfrac{1}{2xy^2(2x-3y)}$ $= \dfrac{2}{xy^2(3y-1)}$	M1,B1 A1	M1 for factorising 4 and replacing with answer in (a). B1 for $2xy^2(2x-3y)$. No need to expand denominator. If expanded wrongly, no A1.
13a	$\mathbf{S} = \begin{pmatrix} 96 & 131 \end{pmatrix}$	B1	
13b	**96** represents the **total number** of cups of **honey tea sold** over the **weekend**. **131** represents the **total number** of cups of **milk tea sold** over the **weekend**. OR **S** represents the **total number** of cups of **each type of tea sold** over the **weekend**.	B1	Accept: "weekend/both days"
13ci	$\mathbf{F} = \begin{pmatrix} n \\ -20 \end{pmatrix}$	B1	
13cii	$\dfrac{1}{100} \begin{pmatrix} 96 & 131 \end{pmatrix} \begin{pmatrix} n \\ -20 \end{pmatrix} = \begin{pmatrix} 21.8 \end{pmatrix}$ $\begin{pmatrix} 0.96n - 26.2 \end{pmatrix} = \begin{pmatrix} 21.8 \end{pmatrix}$ $0.96n = 48$ $\therefore n = 50$	M1 A1	M1 for multiplication of matrices resulting in 1×1 matrix.
14ai	12, 48, 88, 12	B1	
14aii	$60 \le A < 80$	B1	
14b	$56	B1	
14c	Since the highest amount of pocket money received is a value smaller than $100 based on the grouped frequency table, he is wrong as he **assumed the highest value is exactly $100**. OR Since the **exact amount** of pocket money received by each student is **not known**, he is wrong to **assume the lowest and highest amount of money received is $20 and $100 respectively**.	B1	
14d	**Method 1** 80% of students $= 0.8 \times 160 = 128$ students. Since the top 128 students received at least $50, then Steph is not correct.	M1 A1	

Qn	Answer	Marks	Remarks
14d	**Method 2** Percentage who received at least $73 $= \dfrac{160-128}{160} \times 100\%$ $= 20\%$ Since the percentage of students who received at least $73 is 20%, then Steph is not correct.	M1 A1	
15a	$\dfrac{a+3b}{3} = 2(b-a)$ $a+3b = 6b-6a$ $7a = 3b$ $\dfrac{a}{b} = \dfrac{3}{7}$ $\therefore a:b = 3:7$	M1 A1	M1 for simplifying like terms.
15b	$(5c+1)(3-c) = 11$ $15c - 5c^2 + 3 - c = 11$ $5c^2 - 14c + 8 = 0$ $(5c-4)(c-2) = 0$ $c = \dfrac{4}{5}$ or $c = 2$	B1 o.e. M1 A1	No marks awarded for Q15b if show $5c+1=11$ or $3-c=11$. A1 for both answers.
16a	$\dfrac{6^m+1}{36^m-1} = \dfrac{6^m+1}{(6^m-1)(6^m+1)}$ $= \dfrac{1}{6^m-1}$	M1 A1	M1 for applying difference of two squares.
16b	$\sqrt{7^n} \div 49^{\frac{1}{n}} + 1 = 2\left(\dfrac{1}{14^0}\right)^n$ $7^{\frac{n}{2}} \div 7^{\frac{2}{n}} + 1 = 2(1)$ $7^{\frac{n}{2}-\frac{2}{n}} = 7^0$ $\dfrac{n}{2} - \dfrac{2}{n} = 0$ $n^2 = 4$ $\therefore n = 2$ or $n = -2$	B1 M1 A1	B1 for any two: $7^{\frac{n}{2}}$, $7^{\frac{2}{n}}$ or 1^n (or 1) seen. M1 for applying laws of indices and comparing index.
17a	$\dfrac{AF}{EF} = \dfrac{10.5}{4.5} = \dfrac{7}{3}$, $\dfrac{BF}{CF} = \dfrac{7}{3}$ $\angle AFB = \angle EFC$ (common angle) $\therefore \triangle ABF$ is similar to $\triangle ECF$.	B1 B1	B1 for checking ratios of both pairs of corresponding sides. Minus 1 mark overall if conclusion missing.

Qn	Answer	Marks	Remarks
17b	Since **$\triangle ABF$ is similar to $\triangle ECF$**, then $\angle ABF = \angle ECF$. By the property of corresponding angles in parallel lines, **AB is parallel to EC** (or DC). Since there exists a pair of opposite and parallel lines, **$ABCD$ is a trapezium.**	B1	
17c	7 : 10	B1	
18a	$\therefore$ Height of smaller bottle $= 15 \times \sqrt{\dfrac{62.5}{90}}$ $= 12.5$ cm	M1 A1	M1 for taking square root of ratio of areas to obtain ratio of sides.
18b	**Method 1** Ratio of vol. of small to large bottle $= \left(\sqrt{\dfrac{62.5}{90}} \right)^3$ $= \dfrac{125}{216}$ Cost per unit vol. of large bottle $= 3.9 \div 216$ $\qquad = \$0.018$ (2 s.f.) Cost per unit vol. of small bottle $= 2.9 \div 125$ $\qquad = \$0.023$ (2 s.f.) Since the **cost per unit volume** for each type of bottle **is not the same**, then the cost of the sanitiser is **not directly proportional** to the quantity of sanitiser.	M1 M1 A1	M1 for taking cube of ratio of sides. M1 for showing mode of comparison clearly. Either cost vs common volume Or common cost vs volume. A1 for <u>not</u> directly proportional **and** reasoning (only if comparison is valid).
	Method 2 Ratio of vol. of small to large bottle $= \left(\sqrt{\dfrac{62.5}{90}} \right)^3$ $= \dfrac{125}{216}$ Ratio of cost of small to large bottle $= \dfrac{29}{39}$ Since the **ratio of the volume of the small bottle to the large bottle is not the same as the ratio of their cost**, then the cost of the sanitiser is **not directly proportional** to the capacity of the bottle.	M1 M1 A1	M1 for taking cube of ratio of sides. M1 for ratio of cost. A1 for <u>not</u> directly proportional **and** reasoning (only if comparison is valid).

Qn	Answer	Marks	Remarks
19	$5y - 6x = 12 \cdots\cdots(1)$ $3x + y = 15$ $y = -3x + 15 \cdots\cdots(2)$ Sub. (2) into (1): $5(-3x+15) - 6x = 12$ $\qquad\qquad -21x = -63$ $\qquad\qquad x = 3$, i.e. $y = -3(3) + 15 = 6$ Coordinates of B are $(3, 6)$. Sub. $y = 0$ into (1) and $y = 0$ into (2): $\qquad x = -2$ and $x = 5$ Coordinates of A and T are $(-2, 0)$ and $(5, 0)$ respectively. Since $BC = 2BT$, coordinates of C are $(7, -6)$. $\therefore \Delta ABC = \dfrac{1}{2}(7)(12) = 42$ units^2 OR $\therefore \Delta ABC = (9)(12) - \dfrac{1}{2}(4)(12) - \dfrac{1}{2}(5)(6) - \dfrac{1}{2}(9)(6)$ $\qquad\qquad = 42$ units^2	M1 A1 M1 √B1 A1	M1 for solving equations by elimination/substitution. A1 for both x and y correct. M1 for finding either A or T. B1 for coordinates of C. Allow e.c.f. based on student's coordinates of B. If B1 not seen, award working for calculation of area – M1 and accurate answer – A1.
20a	Bearing of R from $Q = 053.5°$	B1	Accept: 053°, 053.5°, 054°
20b		B1 B1	Perpendicular bisector of PQ. Bisector of angle PRQ.
20c		B1	Shaded area.
20d	$\therefore$ Total time $= \dfrac{(14.2 \times 200) \div 1000}{8}$ $\qquad\qquad = 0.355$ hours $\qquad\qquad = 21$ min 18 s	M1 A1	M1 for dividing distance by speed (units for distance must be the same). Accept: 14.2 cm (21min 18 s), 14.25 cm (21 min 23 s), 14.3 cm (21 min 27 s).

Qn	Answer	Marks	Remarks
21a	$\dfrac{12}{k} = 1.5$ $\therefore k = 8$	 B1	
21b	Let the speed of the bus at $t = 12$ be v m/s. $\dfrac{v-12}{12-20} = \dfrac{18-12}{0-20}$ $v - 12 = 2.4$ $v = 14.4$ $\therefore$ Speed of bus is 14.4 m/s.	 M1 A1	M1 for using gradient/equation of line method to find speed.
21c	$\therefore$ Distance travelled $= \dfrac{1}{2}(20)(6) + (12)(50)$ $= 660$ m	 B1	
21d	Let the speed of the car at $t = 50$ be w m/s. $\dfrac{1}{2}(50)(w) = 660$ $w = 26.4$ $\therefore$ Speed of car is 26.4 m/s.	 M1 A1	M1 for attempt to form equation showing distance travelled by car is equal to distance travelled by bus.

SINGAPORE CHINESE GIRLS' SCHOOL
PRELIMINARY EXAMINATION 2021
SECONDARY FOUR
O-LEVEL PROGRAMME

CANDIDATE NAME					

CLASS	4				REGISTER NUMBER		
CENTRE NUMBER	S				INDEX NUMBER			

MATHEMATICS   4048/02
PAPER 2

Friday   **27 August 2021**   **2 hours 30 minutes**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class, register number, centre number and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid/tape.

Answer **all** questions.
If working is needed for any question, it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use

This document consists of **25** printed pages and **1** blank page.

[Turn Over

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P(1+\frac{r}{100})^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 **(a)** Simplify $\sqrt{\dfrac{16}{b}} \div \left(a^2 b\right)^{-3}$.

Answer .... [2]

(b) $x = \dfrac{3}{\sqrt{yz^2}} + 1$

(i) Evaluate x when $y = 9$ and $z = -2$.

Answer $x =$ .... [1]

(ii) Express z in terms of x and y.

Answer $z =$ .... [2]

(c) Solve $\dfrac{6}{m+4} - 5 = \dfrac{7}{2m-3}$.

Answer   *m* = ...   [4]

2 **(a)** A map is drawn to a scale of 1 : 250 000.

(i) Two lighthouses are 382 km apart.
Calculate the distance, in cm, between the two lighthouses on the map.

Answer .... cm [1]

(ii) A lake is represented by an area of 6.4 cm^2 on the map.
Calculate the actual area, in km^2, of the lake.

Answer .... km^2 [2]

(b) A shopkeeper makes a profit of 25% if he sells a teapot for $ 81.25.
During a sale, the shopkeeper decides to reduce the selling price by d %.
Find the value of d if the shopkeeper does not make a profit or a loss.

Answer $d =$ .... [3]

(c) In 2020, a precious stone was valued at Laotian Kip (₭) 90 million.

(i) The exchange rate between Singapore Dollars ($) and Laotian Kip (₭) was
$ 1 = ₭ 6542.
Calculate the value, in Singapore Dollars, of the precious stone.

Answer $ .... [2]

(ii) The value of the precious stone increased by 0.8% every year since 2000.
Calculate the value of the precious stone in 2000 in Laotian Kip (₭).
Leave your answer, correct to 4 significant figures, in standard form.

Answer ₭ .... [2]

3 (a) (i) Express $12 - 5x + x^2$ in the form $c + (d + x)^2$.

Answer .... [2]

(ii) Hence, explain why the equation $12 - 5x + x^2 = h$ does not have solutions for some values of h.

Answer ....

....

....

....

.... [1]

(b) The times, to the nearest minute, taken by two groups of students, Group A and Group B, to complete a puzzle were recorded.
The results are shown in the stem-and-leaf diagram.

			Group A						Group B						
						9	2								
				5	4	2	3	5	7	8					
	9	8	7	4	2	0	4	2	5	6	7	8	9	9	
8	8	6	5	3	1	1	5	4	4	6	7	7	8		
			8	6	4	3	6	1	2	3					
							7								
							8								
							9	5	6						

Key (Group A)
2 | 3 means 32 minutes

Key (Group B)
4 | 2 means 42 minutes

(i) Write down the median time for Group A.

Answer .... minutes [1]

(ii) Find the interquartile range of Group B.

Answer ………………...……… minutes   [1]

(iii) Calculate the standard deviation of Group B.

Answer ………………...……… minutes   [1]

(iv) Would the interquartile range or standard deviation be a more appropriate representation of the spread of times for Group B? Explain your answer.

Answer ………………………………………………………………………………

………………………………………………………………………………

………………………………………………………………………………

…………………………………………………………………………… [1]

(v) Make two comments comparing the times taken by Group A and Group B students to complete the puzzle.

Answer ………………………………………………………………………………

………………………………………………………………………………

………………………………………………………………………………

………………………………………………………………………………

………………………………………………………………………………

………………………………………………………………………………

………………………………………………………………………………

…………………………………………………………………………… [2]

4 **(a)** In the diagram, which is not drawn accurately, O is the centre of the circle passing through P, Q, R, S and T.
The diameter of the circle, PR, intersects QT at U.
Angle $TQR = 50°$ and angle $QRP = 26°$.
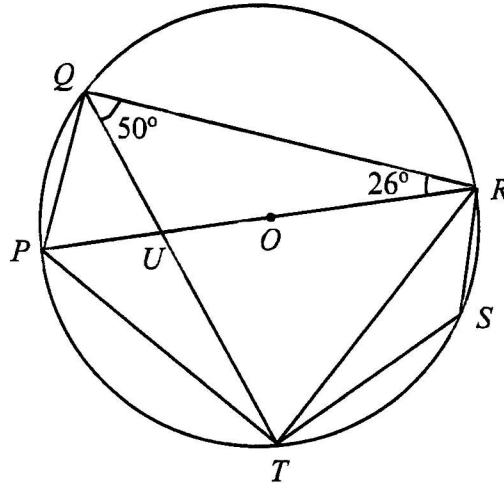
(i) Find, giving reasons for each answer,

(a) angle PTQ,

Answer ....° [1]

(b) angle RST,

Answer ....° [1]

(c) angle PQT,

Answer ....° [1]

(d) angle PUQ.

Answer ....° [1]

(ii) A point V lies between P and R such that angle PVQ is smaller than $52°$.
On the diagram, mark and label clearly a possible position for the point V. [1]

(b) The diagram shows the cross-section of a cylindrical tunnel of length 300 m.
A level surface, *HK*, is laid inside the tunnel to carry vehicles and the space below *HK* is filled with rubble.

The centre of the circle is *O* and reflex angle $HOK = \dfrac{4\pi}{3}$ radians.

The perimeter of the major sector *HOK* is 49.5 m.
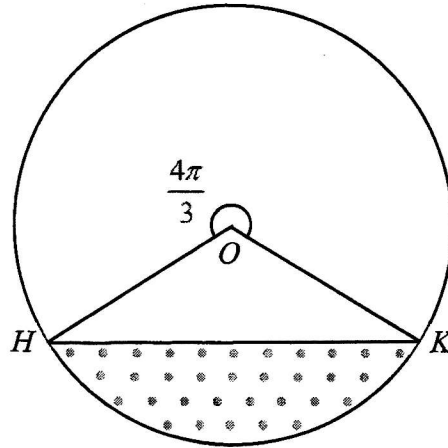
(i) Calculate the radius of the tunnel.

Answer .... m [2]

(ii) Calculate the volume of rubble in the tunnel.

Answer .... m³ [4]

5   The variables x and y are connected by the equation $y = 2 + 3x - \dfrac{1}{5}x^3$.

Some corresponding values of x and y are given in the table below.

x	-4	-3	-2	-1	0	1	2	3	4
y	2.8	-1.6	-2.4	-0.8	2	4.8	6.4	5.6	1.2

(a)   On the grid opposite, draw the graph of $y = 2 + 3x - \dfrac{1}{5}x^3$ for $-4 \le x \le 4$.   [3]

(b)   Use your graph to write down an equality in x to describe the range of values of x where the gradient of the graph is positive.

Answer .... [1]

(c)   (i)   On the same axes, draw the graph of $2y - x - 6 = 0$ for $-4 \le x \le 4$.   [2]

(ii)   Show that the points of intersection of the line and the curve give the solutions of the equation $2x^3 - 25x + 10 = 0$.

Answer

[2]

(iii)   Use your graphs to solve the equation $2x^3 - 25x + 10 = 0$.

Answer $x = $ .... or .... or .... [2]
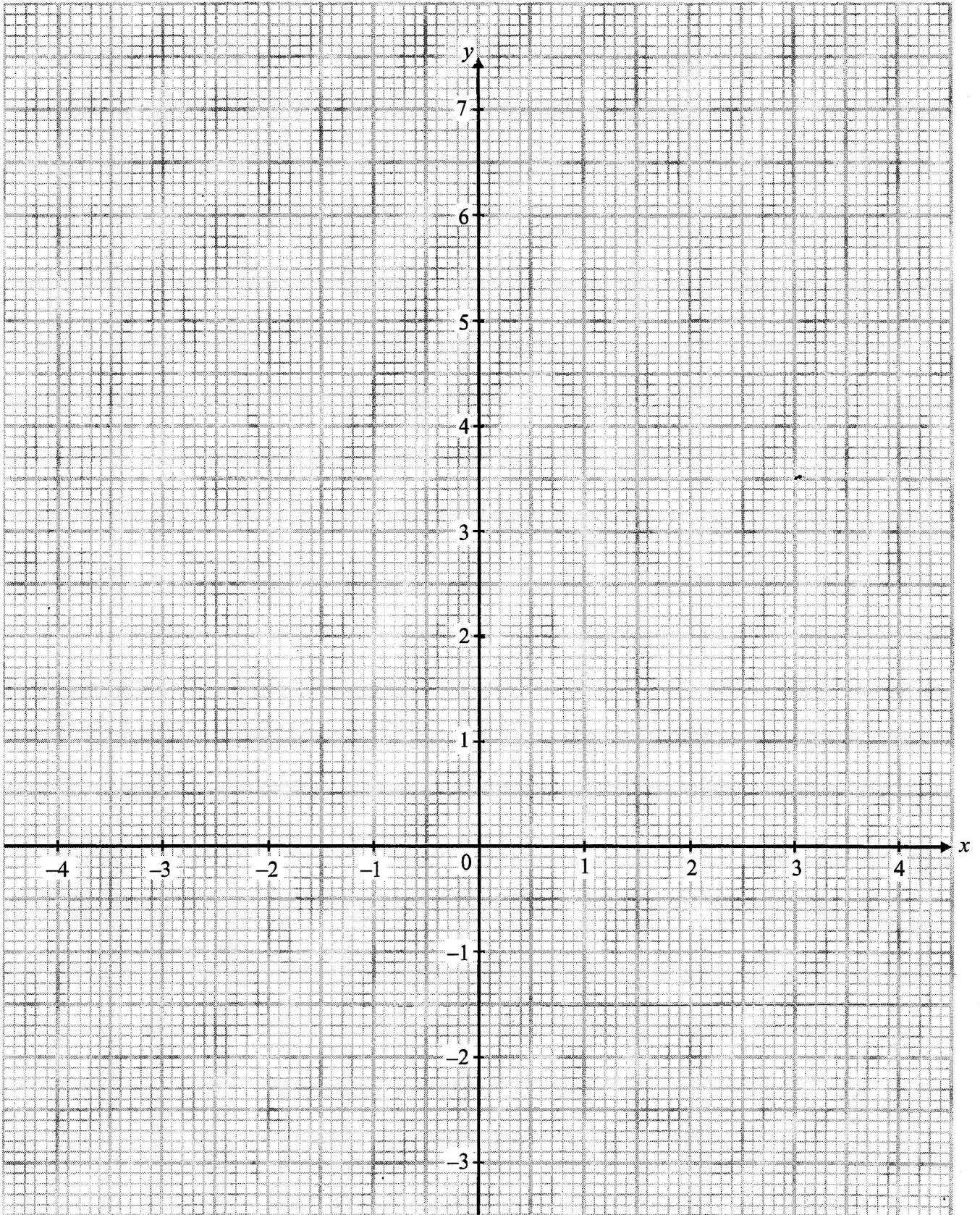
6 In the diagram, *AB* is a vertical wall.

A beam, *CD*, rests with one end, *C*, on horizontal ground. It is held in place by two cables, *BC* and *BD*.

AB = 11 m, *CD* = 8 m, *BC* = 12.7 m and angle *BDC* = 75°.

A weight, *P*, hangs on a rope of length 5 m from *D*.

Q is a point on *AC* produced such that it is vertically below *D* and *P*.
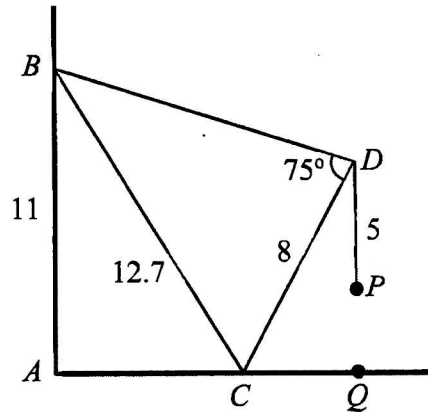
(a) Calculate angle *CBD*.

Answer ....° [2]

(b) Show that angle *DCQ*, correct to 1 decimal place, is 52.5°.

Answer

[2]

(c) Calculate *CP*.

Answer .... m [3]

(d) Calculate the area of triangle *BCD*.

Answer .... m² [1]

An additional cable is to be added to hold up the beam.
The cable will be secured to the point *B*.

(e) Calculate the shortest possible length of the cable that is to be added.

Answer .... m [2]

7   **(a)**   A cylindrical wax pellet has a height of 18 mm.
The volume of the pellet is 3000 mm³.

Calculate the radius of the pellet.

Answer   .... mm   [2]

(b)   Some wax pellets were melted and made into a pyramidal candle as shown.
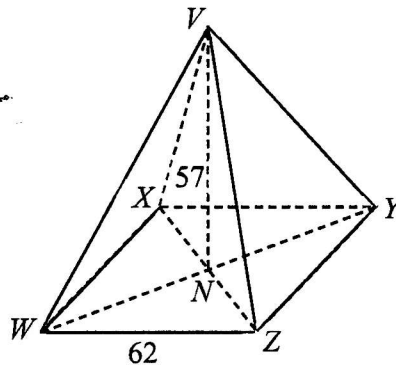
The candle has a horizontal square base *WXYZ*.
XZ meets *WY* at *N* such that *V* is vertically above *N*.
WZ = 62 mm and *VN* = 57 mm.

(i)   Determine the minimum number of wax pellets needed to make the candle.

Answer   .... [3]

All the surfaces of the candle are to be coated with a layer of glitter.
It costs 0.06 cents to coat 10 mm² of the candle.

(ii) Calculate the cost, correct to the nearest cent, of coating the candle.

Answer .... cents [4]

8 (a) A series of diagrams made of dots and lines is shown below.

Diagram 1   Diagram 2   Diagram 3

Diagram	Number of dots along outer edges	Number of lines joining dots	Number of small right-angled triangles
1	3	3	1
2	6	9	4
3	9	18	9
$\vdots$	$\vdots$	$\vdots$	$\vdots$
n		q	

(i) Draw Diagram 4.

Answer

[1]

(ii) Diagram P has 387 dots along the outer edges. Write down the value of P.

Answer $P =$ .... [1]

(iii) Write down the number of small right-angled triangles in Diagram 24.

Answer   .... [1]

(iv) Express q in terms of n.

Answer $q =$ .... [1]

(b) A missile launcher is programmed to fire missiles at enemy aircraft until the aircraft is hit. Each time a missile is fired, the probability that it hits the target is 0.85 .

(i) Find the probability that

(a) the first two missiles miss the target,

Answer ………..………..………….. [1]

(b) the first missile misses the target and the second missile hits the target,

Answer ………..………..………….. [1]

(c) the target is hit in at most three attempts by the missile launcher.

Answer ………..………..………….. [2]

(ii) Find the probability, in terms of n, that the target is still not hit after n missiles are fired.

Answer ………..………..………….. [1]

(iii) Write down the probability, in terms of n, that the target is hit after n missiles are fired.

Answer ………..………..………….. [1]

[Turn Over

9 **(a)** *ABCDEF* is a wedge with a horizontal rectangular base *ABCD*.
Rectangle *AFED* is vertical.
$AF = DE = 10$ cm and $AB = 24$ cm.
The angle of elevation of *E* from *B* is 12°.
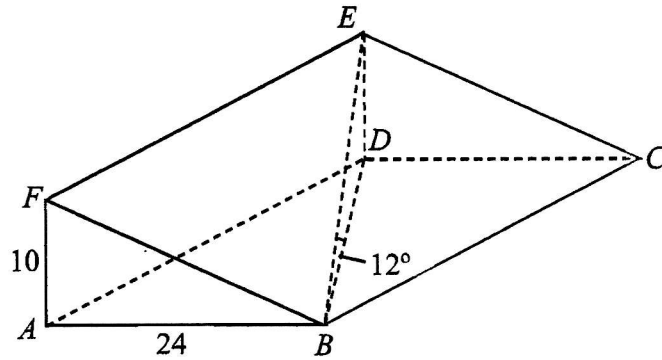
(i) Calculate *BD*.

Answer ……..………….…..…… cm   [2]

(ii) Calculate the volume of the wedge.

Answer ……..………….…..…… cm³   [3]

(b)
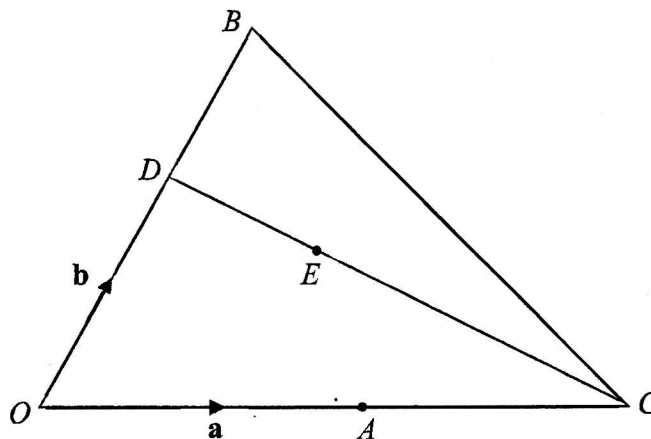
OBC is a triangle.
A is a point on *OC* such that $OA : AC = 5 : 4$.
D is a point *OB* such that $2OD = 3DB$.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(i) Express $\overrightarrow{OC}$ in terms of **a** and/or **b**, as simply as possible.

Answer .... [1]

(ii) Express $\overrightarrow{AB}$ in terms of **a** and/or **b**, as simply as possible.

Answer .... [1]

(iii) Express $\overrightarrow{CD}$ in terms of **a** and/or **b**, as simply as possible.

Answer .... [1]

It is given that $\overrightarrow{CE} = \dfrac{2}{3}\overrightarrow{CD}$.

(iv) Express $\overrightarrow{AE}$ in terms of **a** and/or **b**, as simply as possible.

Answer .... [1]

(v) Hence, write down two facts about *A*, *E* and *B*.

Answer ....

....

....

....

.... [2]

(vi) Find the ratio   area of triangle *ODE* : area of triangle *CEA*.

Answer .... : .... [1]

[Turn Over

10 On 30 December 2020, a nurse working at the National Centre for Infectious Diseases (NCID) became the first person in Singapore to receive a COVID-19 vaccination.

As of 14 June 2021, a total of 1 990 994 persons have completed the full vaccination regimen.

(a) Given that Singapore has a population of approximately 5 900 000 in June 2021, calculate the percentage of the population who have completed the full vaccination regimen in Singapore as of 14 June 2021. Give your answer to a reasonable degree of accuracy.

Answer .... % [2]

(b) Peter owns a restaurant.

During the COVID-19 Phase 2 (Heightened Alert), dining-in at his restaurant was restricted to group sizes of up to 2 persons starting 21 June 2021. This meant that a maximum number of 2 diners could be seated at the same table at any time.

In consideration of the Safety Management Measures which require a minimal distance between the diners, he decided to arrange the tables in his restaurant such that each table is placed in a circular area of radius 2 m starting 21 June 2021. The circular areas of adjacent tables should not overlap.

The restaurant has a rectangular seating area of 26 m by 9 m.

Write down the greatest number of diners that could be seated in the restaurant at any time on 21 June 2021.

Answer .... [1]

(c) The restaurant temporarily suspended operations in mid-May 2021 and resumed operations on 21 June 2021 with the following operating hours.

Operating Hours
11.00am to 10.30pm

NO Service Charge or Goods and Services Tax (GST)!
Food delivery service is unavailable.

Open for business every day of the week!

The costs involved in operating his restaurant in a typical week starting 21 June 2021 are given below.

Costs involved in operating restaurant in a typical week	
Salary for each cook (3 cooks are required)	$ 775
Salary for each staff member (excluding cooks) (6 staff members, excluding cooks, are required)	$ 450
Cost of ingredients for food	$ 6 000
Rental of restaurant	$ 3 000
General utilities	$ 2 000
Miscellaneous costs	$ 800

Based on his experience, Peter estimated that the restaurant would seat an average of 60 diners per day of operation and each diner would typically spend a median amount of $ 28. He also estimated that the restaurant would collect an average of $ 400 per day of operation from takeaway orders.

Peter knew that he would make a loss by operating his restaurant for 3 weeks starting from 21 June 2021 and his friend has agreed to give him a loan to help him tide over this period.

Suggest a suitable amount Peter should loan from his friend to cover his loss over these 3 weeks.
Justify any decisions you make and show your calculations clearly.

SINGAPORE CHINESE GIRLS' SCHOOL
PRELIMINARY EXAMINATION 2021
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MATHEMATICS   4048/02
PAPER 2

Friday   **27 August 2021**   **2 hours 30 minutes**

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$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

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$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 (a) Simplify $\sqrt{\dfrac{16}{b}} \div \left(a^2 b\right)^{-3}$.

$$\sqrt{\dfrac{16}{b}} \div \left(a^2 b\right)^{-3} = \dfrac{4}{b^{\frac{1}{2}}} \times a^6 b^3$$

$$= 4a^6 b^{\frac{5}{2}}$$

Answer $\quad 4a^6 b^{\frac{5}{2}}$ [2]

(b) $x = \dfrac{3}{\sqrt{yz^2}} + 1$

(i) Evaluate x when $y = 9$ and $z = -2$.

Answer $\quad x = 1.5$ [1]

(ii) Express z in terms of x and y.

$$\left(x - 1\right)^2 = \dfrac{9}{yz^2}$$

$$yz^2 = \dfrac{9}{\left(x-1\right)^2}$$

$$z = \pm \sqrt{\dfrac{9}{y\left(x-1\right)^2}}$$

Answer $\quad z = \pm \sqrt{\dfrac{9}{y\left(x-1\right)^2}}$ [2]

(c) Solve $\dfrac{6}{m+4} - 5 = \dfrac{7}{2m-3}$.

$$6(2m-3) - 7(m+4) = 5(m+4)(2m-3)$$
$$12m - 18 - 7m - 28 = 10m^2 + 25m - 60$$
$$5m^2 + 10m - 7 = 0$$
$$m = \dfrac{-10 \pm \sqrt{10^2 - 4(5)(-7)}}{2(5)}$$
$$= 0.549 \quad \text{or} \quad -2.55 \ (3\text{sf})$$

Answer $\quad m = 0.549 \quad \text{or} \quad -2.55$ [4]

[Turn Over

2 (a) A map is drawn to a scale of 1 : 250 000.

(i) Two lighthouses are 382 km apart.
Calculate the distance, in cm, between the two lighthouses on the map.

Answer   152.8 cm   [1]

(ii) A lake is represented by an area of 6.4 cm² on the map.
Calculate the actual area, in km², of the lake.

Actual area $= 6.4 \times (2.5)^2$
$\qquad\quad = 40 \text{ km}^2$

Answer   40 km²   [2]

(b) A shopkeeper makes a profit of 25% if he sells a teapot for $ 81.25.
During a sale, the shopkeeper decides to reduce the selling price by d %.
Find the value of d if the shopkeeper does not make a profit or a loss.

Cost price $= \dfrac{100}{125} \times 81.25$

$\qquad\qquad = \$65$

$\dfrac{100-d}{100} \times 81.25 = 65$

$\qquad\qquad d = 20$

Answer $d =$ 20   [3]

(c) In 2020, a precious stone was valued at Laotian Kip (₭) 90 million.

(i) The exchange rate between Singapore Dollars ($) and Laotian Kip (₭) was
$ 1 = ₭ 6542.
Calculate the value, in Singapore Dollars, of the precious stone.

Value $= \$ \left(\dfrac{90 \times 10^6}{6542} \right)$

$\qquad\quad = \$ 13757.26$ (nearest cent)

Answer $ 13757.26   [2]

(ii) The value of the precious stone increased by 0.8% every year since 2000.
Calculate the value of the precious stone in 2000 in Laotian Kip (₭).
Leave your answer, correct to 4 significant figures, in standard form.

Let ₭ P be the value in 2000.

$90 \times 10^6 = P \left(1 + \dfrac{0.8}{100} \right)^{20}$

$\qquad P = 7.674 \times 10^7$

Answer   ₭ 7.674×10^7 [2]

3 **(a)** **(i)** Express $12 - 5x + x^2$ in the form $c + (d + x)^2$.

$$\left(x - \frac{5}{2}\right)^2 + 12 - \frac{25}{4}$$

$$= 5\frac{3}{4} + \left(-\frac{5}{2} + x\right)^2$$

Answer $\quad 5\dfrac{3}{4} + \left(-\dfrac{5}{2} + x\right)^2$. $\qquad$ [2]

(ii) Hence, explain why the equation $12 - 5x + x^2 = h$ does not have solutions for some values of h.

Answer

The **minimum value** of $12 - 5x + x^2$ is $5\dfrac{3}{4}$. Hence the equation $12 - 5x + x^2 = h$

does not have solutions when $h < 5\dfrac{3}{4}$.

Or,

Since $\left(x - \dfrac{5}{2}\right)^2 \geq 0$, $\quad \left(x - \dfrac{5}{2}\right)^2 + 5\dfrac{3}{4} \geq 5\dfrac{3}{4}$. Hence the equation $12 - 5x + x^2 = h$ does

not have solutions when $h < 5\dfrac{3}{4}$. $\qquad$ [1]

(b) The times, to the nearest minute, taken by two groups of students, Group A and Group B, to complete a puzzle were recorded.
The results are shown in the stem-and-leaf diagram.

					Group A		Stem		Group B					
						9	2							
				5	4	2	3	5	7	8				
	9	8	7	4	2	0	4	2	5	6	7	8	9	9
8	8	6	5	3	1	1	5	4	4	6	7	7	8	
		8	6	4	3		6	1	2	3				
							7							
							8							
							9	5	6					

Key (Group A)   Key (Group B)

2 | 3 means 32 minutes   4 | 2 means 42 minutes

(i) Write down the median time for Group A.

Answer   51 minutes $\qquad$ [1]

[Turn Over

(ii) Find the interquartile range of Group B.

Answer   14 minutes   [1]

(iii) Calculate the standard deviation of Group B.

Answer   15.4 minutes   [1]

(iv) Would the interquartile range or standard deviation be a more appropriate representation of the spread of times for Group B? Explain your answer.

Answer

The **interquartile range** would be a more appropriate representation as there are **outliers** of 95 minutes and 96 minutes **in** the data for **Group B**. The interquartile range, unlike the standard deviation, is not sensitive to outliers.   [1]

(v) Make two comments comparing the times taken by Group A and Group B students to complete the puzzle.

Answer

Group A   **Group B**
median = 51 minutes   median = 54 minutes
interquartile range   interquartile range
= 17 minutes   = 14 minutes

As the median time for Group B is longer than that of Group A, the students in Group B generally took a longer time to complete the puzzle.

As the interquartile range of Group A is larger than that of Group B, the times taken by the students in Group A to complete the puzzle are generally more widely spread.
                                                                               [2]

4 **(a)** In the diagram, which is not drawn accurately, *O* is the centre of the circle passing through *P, Q, R, S* and *T*.
The diameter of the circle, *PR*, intersects *QT* at *U*.
Angle *TQR* = 50° and angle *QRP* = 26°.
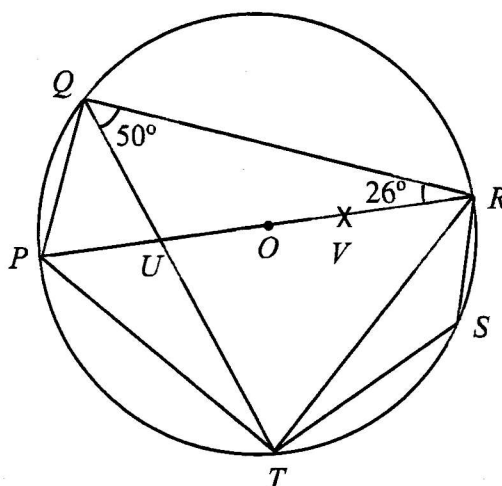
(i) Find, giving reasons for each answer,

(a) angle *PTQ*,

angle *PTQ* = 26° (angles in same segment)

Answer 26° [1]

(b) angle *RST*,

angle *RST* = 130° (angles in opposite segments)

Answer 130° [1]

(c) angle *PQT*,

angle *PQT* = 40° (right angle in semi-circle)

Answer 40° [1]

(d) angle *PUQ*.

angle *PUQ* = 76° (exterior angle of triangle)

Answer 76° [1]

(ii) A point *V* lies on *PR* such that angle *PVQ* is smaller than 52°.
On the diagram, mark and label clearly a possible position for the point *V*. [1]

[Turn Over

(b) The diagram shows the cross-section of a cylindrical tunnel of length 300 m.
A level surface, *HK*, is laid inside the tunnel to carry vehicles and the space below *HK* is filled with rubble.

The centre of the circle is *O* and reflex angle $HOK = \dfrac{4\pi}{3}$ radians.

The perimeter of the major sector *HOK* is 49.5 m.
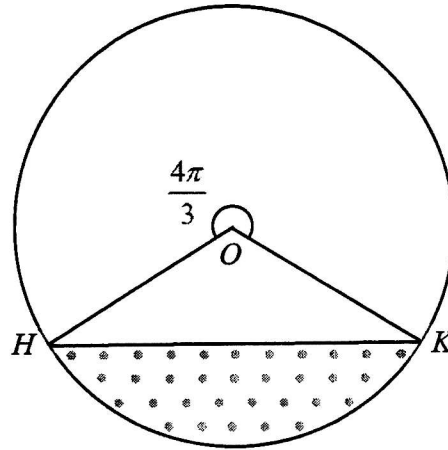
(i) Calculate the radius of the tunnel.

Let *r* m be radius.

$$2r + r\left(\frac{4\pi}{3}\right) = 49.5$$

$$r = 7.9983 \text{ m}$$
$$= 8.00 \text{ m (3sf)}$$

Answer   8.00 m   [2]

(ii) Calculate the volume of rubble in the tunnel.

Area of minor sector *HOK*

$$= \frac{1}{2}(7.9983)^2\left(\frac{2\pi}{3}\right)$$

Area of minor segment *HOK*

$$= \frac{1}{2}(7.9983)^2\left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)$$

$$\text{Volume} = \frac{1}{2}(7.9983)^2\left(\frac{2\pi}{3} - \sin\frac{2\pi}{3}\right)(300)$$

$$= 11800 \text{ m}^3 \text{ (3sf)}$$

Answer   11800 m³   [4]

5 The variables x and y are connected by the equation $y = 2 + 3x - \frac{1}{5}x^3$.

Some corresponding values of x and y are given in the table below.

x	-4	-3	-2	-1	0	1	2	3	4
y	2.8	-1.6	-2.4	-0.8	2	4.8	6.4	5.6	1.2

(a) On the grid opposite, draw the graph of $y = 2 + 3x - \frac{1}{5}x^3$ for $-4 \le x \le 4$. [3]
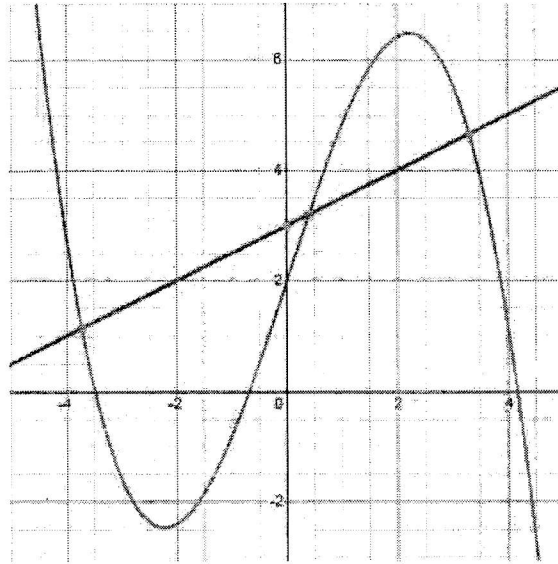
(b) Use your graph to write down an equality in x to describe the range of values of x where the gradient of the graph is positive.

$\qquad$ *Answer* $-2.25 < x < 2.25$ [1]
(accept $-2.4, -2.35, -2.3, -2.2, -2.15, -2.1$ and $2.1, 2.15, 2.2, 2.3, 2.35, 2.4$)

(c) **(i)** On the same axes, draw the graph of $2y - x - 6 = 0$ for $-4 \le x \le 4$. [2]

(ii) Show that the points of intersection of the line and the curve give the solutions of the equation $2x^3 - 25x + 10 = 0$.

Answer

$$2 + 3x - \frac{1}{5}x^3 = \frac{1}{2}x + 3$$
$$20 + 30x - 2x^3 = 5x + 30$$
$$2x^3 - 25x + 10 = 0$$

[2]

(iii) Use your graphs to solve the equation $2x^3 - 25x + 10 = 0$.

$\qquad$ *Answer* $x = -3.7, 0.4, 3.3$ [2]
(accept $-3.75, -3.65, 0.35, 0.45, 3.25, 3.35$)

[Turn Over

6 In the diagram, *AB* is a vertical wall.
A beam, *CD*, rests with one end, *C*, on horizontal ground. It is held in place by two cables, *BC* and *BD*.
AB = 11 m, *CD* = 8 m, *BC* = 12.7 m and angle *BDC* = 75°.
A weight, *P*, hangs on a rope of length 5 m from *D*.
Q is a point on *AC* produced such that it is vertically below *D* and *P*.
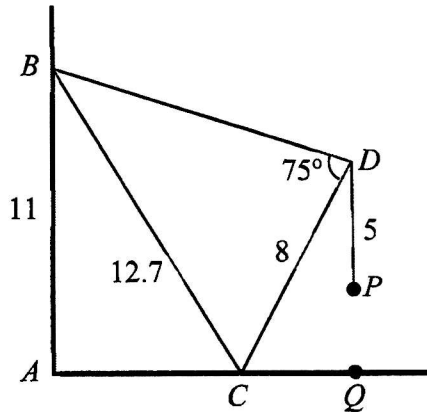
(a) Calculate angle *CBD*.

$$\frac{\sin \angle CBD}{8} = \frac{\sin 75^0}{12.7}$$
$$\angle CBD = 37.478^\circ$$
$$= 37.5^\circ \ (1\text{dp})$$

Answer 37.5°   [2]

(b) Show that angle *DCQ*, correct to 1 decimal place, is 52.5°.

Answer

$$\sin \angle ACB = \frac{11}{12.7}$$
$$\angle ACB = 60.013^\circ$$
Angle *DCQ* $= 180^\circ - (180^\circ - 75^\circ - 37.478^\circ)$
$$- \ 60.013^\circ$$
$$= 52.465^\circ$$
$$= 52.5^\circ \ (1\text{dp})$$

[2]

(c) Calculate *CP*.

$$\angle CDQ = 180^\circ - 90^\circ - 52.465$$
$$= 37.535^\circ$$
$$CP^2 = 5^2 + 8^2 - 2(5)(8)\cos 37.535^\circ$$
$$CP = 5.0558 \text{ m}$$
$$= 5.06 \text{ m (3sf)}$$

Answer 5.06 m   [3]

(d) Calculate the area of triangle *BCD*.

$$\text{Area} = \frac{1}{2}(12.7)(8)\sin 67.522°$$
$$= 46.940 \text{ m}^2$$
$$= 46.9 \text{ m}^2 \text{ (3sf)}$$

Answer   46.9 m²   [1]

An additional cable is to be added to hold up the beam.
The cable will be secured to the point *B*.

(e) Calculate the shortest possible length of the cable that is to be added.

Let *l* m be length of cable.
$$l = \frac{46.940}{\frac{1}{2}(8)}$$
$$= 11.7 \text{ m (3sf)}$$

Answer   11.7 m   [2]

[Turn Over

7 **(a)** A cylindrical wax pellet has a height of 18 mm.
The volume of the pellet is 3000 mm^3.

Calculate the radius of the pellet.

Let r mm be radius.
$$\pi r^2 (18) = 3000$$
$$r = 7.28 \text{ mm}$$

Answer   7.28 mm   [2]

(b) Some wax pellets were melted and made into a pyramidal candle as shown.
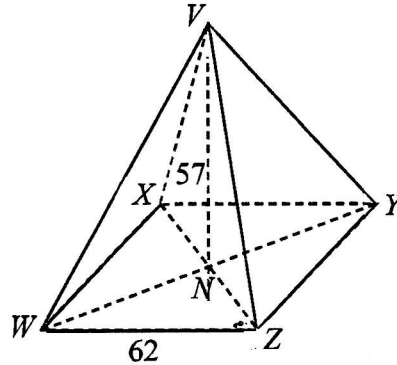
The candle has a horizontal square base *WXYZ*.
XZ meets *WY* at *N* such that *V* is vertically above *N*.
WZ = 62 mm and *VN* = 57 mm.

(i) Determine the minimum number of wax pellets needed to make the candle.

$$\text{Number of pellets} = \frac{\frac{1}{3}(62)^2 (57)}{3000}$$
$$= 24.345$$

Minimum number of pellets = 25

Answer   25   [3]

All the surfaces of the candle are to be coated with a layer of glitter.
It costs 0.06 cents to coat 10 mm^2 of the candle.

(ii) Calculate the cost, correct to the nearest cent, of coating the candle.

Let *M* be midpoint of *WZ*.
$$VM = \sqrt{31^2 + 57^2}$$
$$= \sqrt{4210}$$
$$\text{Cost} = \left[4\left(\frac{1}{2}\right)(62)\sqrt{4210} + 62^2 \right] \times \frac{0.06}{10}$$
$$= 71 \text{ cents (nearest cent)}$$

Answer   71 cents   [4]

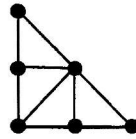**8** **(a)** A series of diagrams made of dots and lines is shown below.

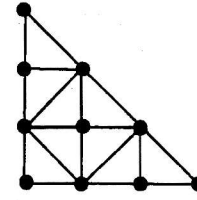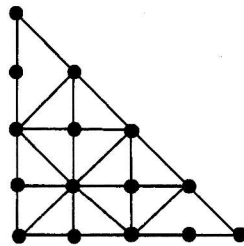Diagram 1   Diagram 2   Diagram 3

Diagram	Number of dots along outer edges	Number of lines joining dots	Number of small right-angled triangles
1	3	3	1
2	6	9	4
3	9	18	9
⋮	⋮	⋮	⋮
n		q	

(i) Draw Diagram 4.

Answer

[1]

(ii) Diagram P has 387 dots along the outer edges. Write down the value of P.

Answer $P = 129$ [1]

(iii) Write down the number of small right-angled triangles in Diagram 24.

Answer 576 [1]

(iv) Express q in terms of n.

Answer $q = \dfrac{3}{2}n^2 + \dfrac{3}{2}n$ [1]

[Turn Over

(b) A missile launcher is programmed to fire missiles at enemy aircraft until the aircraft is hit. Each time a missile is fired, the probability that it hits the target is 0.85 .

(i) Find the probability that

(a) the first two missiles miss the target,

P(the first two missiles miss the target) = 0.15^2

$$= 0.0225 \text{ or } \frac{9}{400}$$

Answer $0.0225 \text{ or } \dfrac{9}{400}$ [1]

(b) the first missile misses the target and the second missile hits the target,

P(the first missile misses the target and the second missile hits the target)
= 0.15(0.85)
$= 0.1275 \text{ or } \dfrac{51}{400}$

Answer $0.1275 \text{ or } \dfrac{51}{400}$ [1]

(c) the target is hit in at most three attempts by the missile launcher.

P(hit in at most 3 attempts)
$= 0.85 + 0.15(0.85) + (0,15)^2(0.85)$
$= 0.996625 \text{ or } \dfrac{7973}{8000}$

Or,

P(hit in at most 3 attempts) $= 1 - (0.15)^3$
$$= 0.996625 \text{ or } \frac{7973}{8000}$$

Answer $0.996625 \text{ or } \dfrac{7973}{8000}$ [2]

**(ii)** Find the probability, in terms of n, that the target is still not hit after n missiles are fired.

Answer 0.15^n [1]

(iii) Write down the probability, in terms of n, that the target is hit after n missiles are fired.

Answer $0.85(0.15)^{n-1}$ [1]

9 **(a)** *ABCDEF* is a wedge with a horizontal rectangular base *ABCD*.
Rectangle *AFED* is vertical.
AF = *DE* = 10 cm and *AB* = 24 cm.
The angle of elevation of *E* from *B* is 12°.
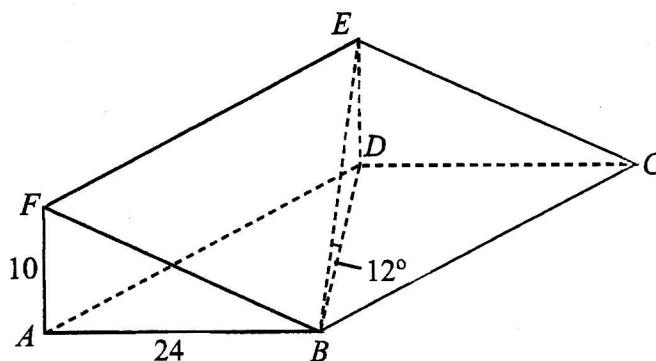
(i) Calculate *BD*.

$$\tan 12° = \frac{10}{BD}$$

$$BD = 47.046 \text{ cm}$$

$$= 47.0 \text{ cm}$$

Answer   47.0 cm   [2]

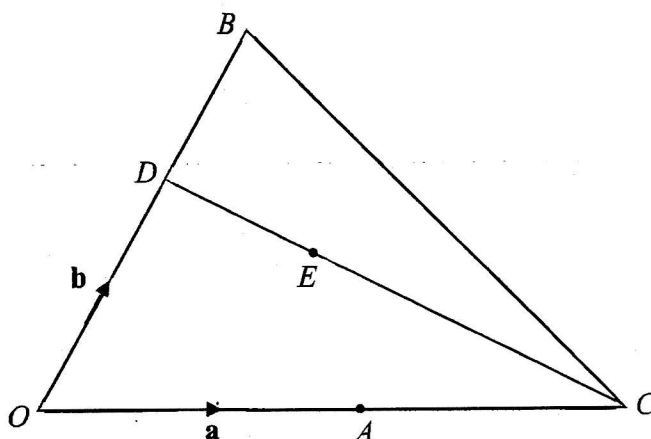**(ii)** Calculate the volume of the wedge.

$$BC = \sqrt{47.046^2 - 24^2}$$

$$= 40.464 \text{ cm}$$

$$\text{Volume} = \frac{1}{2}(10)(24)(40.464)$$

$$= 4860 \text{ cm}^3$$

Answer   4860 cm³   [3]

(b)

OBC is a triangle.
A is a point on *OC* such that *OA* : *AC* = 5 : 4.
D is a point *OB* such that 2*OD* = 3*DB*.

[Turn Over

$\overrightarrow{OA}$ = **a** and $\overrightarrow{OB}$ = **b**.

(i) Express $\overrightarrow{OC}$ in terms of **a** and/or **b**, as simply as possible.

Answer .... [1]

(ii) Express $\overrightarrow{AB}$ in terms of **a** and/or **b**, as simply as possible.

Answer .... [1]

(iii) Express $\overrightarrow{CD}$ in terms of **a** and/or **b**, as simply as possible.

Answer .... [1]

It is given that $\overrightarrow{CE} = \frac{2}{3}\overrightarrow{CD}$.

(iv) Express $\overrightarrow{AE}$ in terms of **a** and/or **b**, as simply as possible.

Answer .... [1]

(v) Hence, write down two facts about *A*, *E* and *B*.

Answer ....

....

....

....

.... [2]

(vi) Find the ratio   area of triangle *ODE* : area of triangle *CEA*.

Answer .... : .... [1]

10 On 30 December 2020, a nurse working at the National Centre for Infectious Diseases (NCID) became the first person in Singapore to receive a COVID-19 vaccination.

As of 14 June 2021, a total of 1 990 994 persons have completed the full vaccination regimen.

(a) Given that Singapore has a population of approximately 5 900 000 in June 2021, calculate the percentage of the population who have completed the full vaccination regimen in Singapore as of 14 June 2021. Give your answer to a reasonable degree of accuracy.

$$\text{Percentage} = \frac{1990994}{5900000} \times 100$$
$$= 34\% \text{ (2sf)}$$

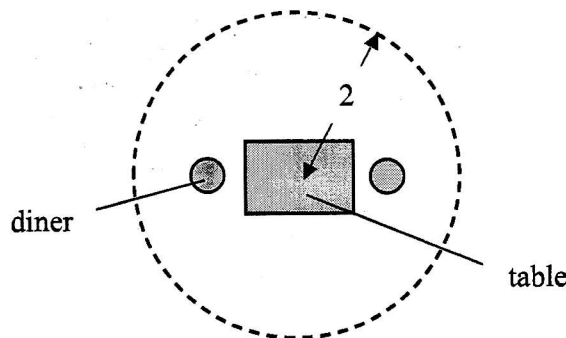*Answer* 34%   [2]

(b) Peter owns a restaurant.

During the COVID-19 Phase 2 (Heightened Alert), dining-in at his restaurant was restricted to group sizes of up to 2 persons starting 21 June 2021. This meant that a maximum number of 2 diners could be seated at the same table at any time.

In consideration of the Safety Management Measures which require a minimal distance between the diners, he decided to arrange the tables in his restaurant such that each table is placed in a circular area of radius 2 m starting 21 June 2021. The circular areas of adjacent tables should not overlap.

The restaurant has a rectangular seating area of 26 m by 9 m.

Write down the greatest number of diners that could be seated in the restaurant at any time on 21 June 2021.

Answer 24   [1]

(c) The restaurant temporarily suspended operations in mid-May 2021 and resumed operations on 21 June 2021 with the following operating hours.

Operating Hours
11.00am to 10.30pm

NO Service Charge or Goods and Services Tax (GST)!
Food delivery service is unavailable.

Open for business every day of the week!

The costs involved in operating his restaurant in a typical week starting21 June 2021 are given below.

Costs involved in operating restaurant in a typical week	
Salary for each cook (3 cooks are required)	$ 775
Salary for each staff member (excluding cooks) (6 staff members, excluding cooks, are required)	$ 450
Cost of ingredients for food	$ 6 000
Rental of restaurant	$ 3 000
General utilities	$ 2 000
Miscellaneous costs	$ 800

Based on his experience, Peter estimated that the restaurant would seat an average of 60 diners per day of operation and each diner would typically spend a median amount of $ 28. He also estimated that the restaurant would collect an average of $ 400 per day of operation from takeaway orders.

Peter knew that he would make a loss by operating his restaurant for 3 weeks starting from 21 June 2021 and his friend has agreed to give him a loan to help him tide over this period.

Suggest a suitable amount Peter should loan from his friend to cover his loss over these 3 weeks.
Justify any decisions you make and show your calculations clearly.

Total operation costs in a week
= 3(775) + 6(450) + 6 000 + 3 000 + 2 000 + 800
= $ 16 825

Estimated average revenue per day = 60(28) + 400
= $ 2 080

Estimated average revenue per week = 2080(7)
= $14 560

Estimated loss over the 3 weeks = 3(16825 – 14560)
= $ 6 795

Peter should take a loan of $ 7 000 from his friend to cover his loss over the 3 weeks.
This will also give him a buffer of approximately
$ 205 to cover any unexpected costs which may arise during this period.

Or,

Total operation costs in a week
= 3(3775) + 6(450) + 6000 + 3000 + 2000 + 800
= $ 16 825

Total operation costs in 3 weeks = 3(16825)
= $ 50 475

Estimated average revenue per day = 60(28) + 400
= $ 2 080

Estimated average revenue in 3 weeks = 2 080 (21)
= $ 43 680

Estimated loss over the 3 weeks = 50 475 – 43 680
= $ 6 795

Peter should take a loan of $ 7 000 from his friend to cover his loss over the 3 weeks.
This will also give him a buffer of approximately
$ 205 to cover any unexpected costs which may arise during this period. [7]