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| Name: | Class: | Class Register Number: |
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中正中學

CHUNG CHENG HIGH SCHOOL (MAIN)

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**PRELIMINARY EXAMINATION 2021
SECONDARY 4**

MATHEMATICS

4048/01

Paper 1

Wednesday 15 September 2021

2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on **all** the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 80.

| For Examiner's Use | |
|--------------------|------|
| Total | / 80 |

This document consists of **19** printed pages and 1 blank page.

[Turn over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} a b \sin C$$

$$\text{Arc length} = r \theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2 b c \cos A$$

Statistics

$$\text{Mean} = \frac{\sum f x}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum f x^2}{\sum f} - \left(\frac{\sum f x}{\sum f} \right)^2}$$

Answer all the questions.

- 1 State which of the following number(s) is/are irrational.

$$0.\dot{5}, \frac{\pi}{2}, 2\sqrt{2}, 3\sqrt{3} \times \sqrt{3}$$

Answer [1]

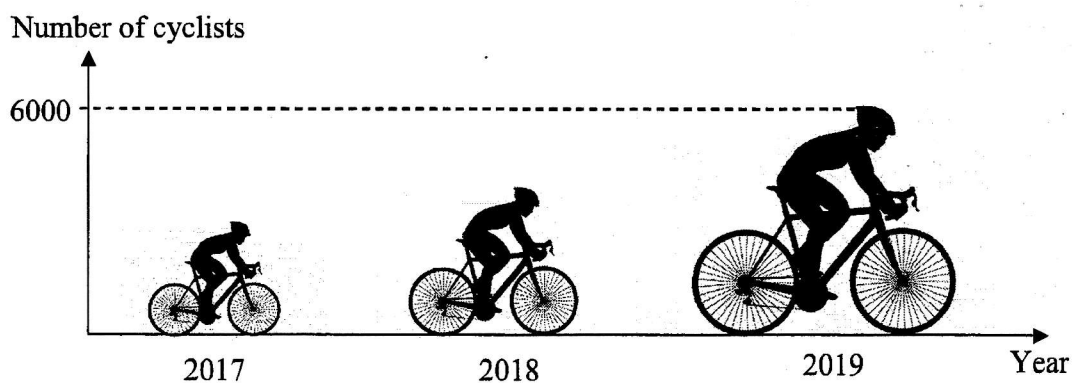
- 2 x is an integer greater than 1.

Write the following in order of size, starting with the largest.

$$x^0, x^{-\frac{3}{2}}, x^{\frac{3}{4}}, x^{\frac{3}{2}}$$

Answer [1]

- 3 The graph shows the number of cyclists in an annual cycling marathon for the years 2017 to 2019.



- (a) State one aspect of the graph that may be misleading.

.....
 [1]

- (b) Explain how this may lead to a misinterpretation of the graph.

.....
 [1]

4 (a) Simplify $\left(\frac{2^{a-1}\sqrt{2}}{2^a}\right)^2$.

Answer [2]

(b) If $27m^{3x} = 1$ and $m > 0$, find $m^{2x} + m^{-x}$.

Answer [2]

5 n is a positive integer.

Show that, for all n , $(5n+3)^2 - (5n-3)^2$ is a multiple of 4.

.....

 [2]

- 6 Gladys wrote down four numbers.
The mean of these numbers is 13, the median is 12 and the mode is 10.
Find the four numbers.

Answer [2]

- 7 (a) Mrs Boey deposited \$20 000 in a bank that pays 0.8% per annum compound interest compounded half-yearly. Find the total amount of money she will receive at the end of 5 years, leaving your answer to the nearest cent.

Answer [2]

(b)

Singapore Budget 2018: GST to be raised from 7% to 9%
some time between 2021 and 2025

Shane and Glen came across an article in the newspaper with the headline shown above and made the following comments.

Shane: Oh no, the GST will increase by 2% soon!

Glen: No! I disagree. It did not increase by 2%, in fact it is **MORE** than 2%!

Whose statement is correct? Support your answer with mathematical calculations.

Answer [2]

- 8 (a) Find the range of values of x which satisfy the inequalities

$$\frac{17-8x}{3} < \frac{2(3x-1)}{5} \leq 4.$$

Answer [3]

- (b) Hence, state the smallest prime number that satisfies the inequalities.

Answer [1]

- 9 (a) Factorise completely $9b^2 - 6ab + a^2 - x^2$.

Answer [2]

- (b) Simplify $\frac{5}{2x^2-7x-4} - \frac{8}{4-x}$.

Answer [3]

10 Roy made a model of his yacht with a scale of 1: 40.

- (a) The actual length of the yacht is 12.5 m, find the length of the model in cm.

Answercm [1]

- (b) Roy wants to spray paint his yacht. He can select either of the following options to paint his yacht.

Option A: A lump sum payment of \$1 000

Option B: Payment for cost of paint at \$5.50 per m^2

[Cost of manpower is included in both options]

If the total surface area of the model to be painted is 937.5 cm^2 , find the surface area of his yacht. Which one of the two options should Roy select to paint his yacht?

[Show your workings clearly]

.....

.....

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.....

.....

.....

.....

.....

.....[3]

11 $\xi = \{x : x \text{ is an integer and } 1 \leq x \leq 10\}$

$A = \{x : x \text{ is a factor of } 20\}$

$B = \{x : x \text{ is a perfect square}\}$

Find

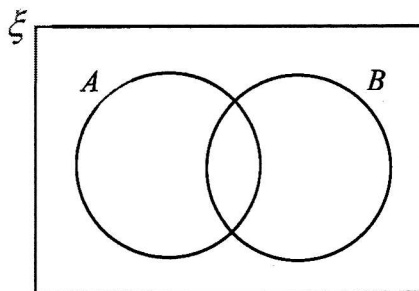
(a) $n(B)$,

Answer [1]

(b) $A' \cap B'$.

Answer [1]

(c) On the Venn Diagram, shade the region which represents $(A \cup B)' \cup (A \cap B)$.



[1]

- 12 When written as the product of their prime factors,

$$p \text{ is } 2^2 \times 3 \times 5,$$

$$q \text{ is } 2^6 \times 3^3,$$

$$r \text{ is } 2^2 \times 3^2 \times 11.$$

Find

- (a) the value of the cube root of q ,

Answer [1]

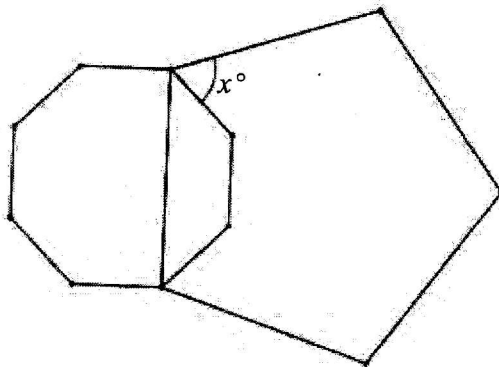
- (b) the LCM of p , q and r , giving your answer as the product of its prime factors,

Answer [1]

- (c) the greatest number that will divide p , q and r exactly.

Answer [1]

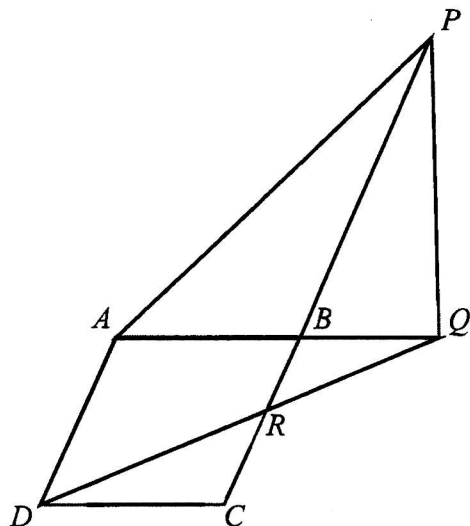
- 13 The diagram shows a regular pentagon and a regular octagon. Calculate the value of x .



Answer $x =$ [3]

- 14 The diagram shows a rhombus $ABCD$.

ABQ , PBC and DRQ are straight lines and $AQ = BP$.



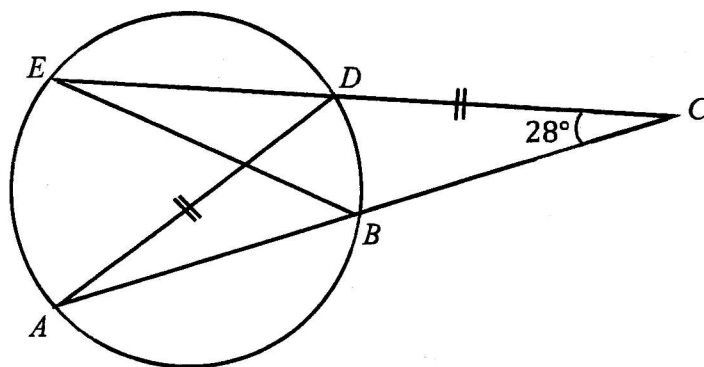
- (a) State the triangle that is similar to triangle BRQ .

Answer Triangle [1]

- (b) Prove that triangle DQA is congruent to triangle APB .

.....

[2]



[3]

- 16 The masses of 20 bags, in kg, are measured.
The results are shown on the stem-and-leaf diagram.

| Mass of 20 bags | |
|-----------------|---------------|
| Stem | Leaf |
| 0 | 9 |
| 1 | 2 |
| 2 | 0 6 6 6 |
| 3 | 0 3 3 4 4 8 8 |
| 4 | 1 2 9 9 9 |
| 5 | 0 1 |

Key: 0|9 means 0.9 kg

- (a) Find the mean mass of the bags.

Answerkg [1]

- (b) Find the standard deviation of the masses of the bags.

Answerkg [1]

- (c) It was found later that the weighing machine has an error.
Each bag was actually 0.08 kg heavier.
Explain how this will affect the mean and standard deviation.

.....

.....

.....

.....

.....[2]

17 y is inversely proportional to the square of x , $x > 0$.

(a) If x is increased by 25%, find the percentage change in y .

Answer% [2]

(b) Given that $y = 16$ when $x = \frac{1}{2}$, find

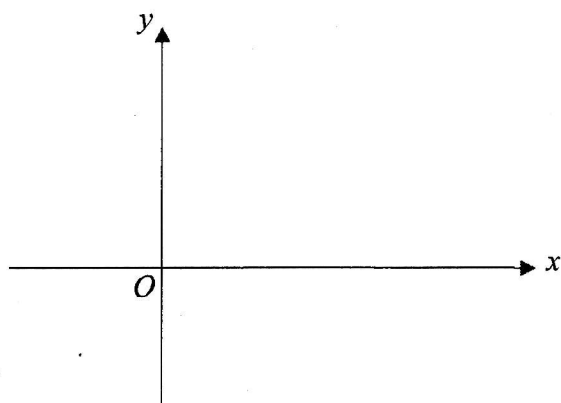
(i) the equation connecting y and x ,

Answer [2]

(ii) the value of x when $y = 100$.

Answer $x = \dots\dots\dots$ [2]

(c) Sketch the graph which represents the relation between y and x .



[1]

18 In the following sequence,

$$(1 \times 2) - 2 = 0$$

$$(2 \times 3) - 4 = 2$$

$$(3 \times 4) - 6 = 6$$

$$(4 \times 5) - 8 = 12$$

.....

.....

$$(a \times 12) - b = c$$

.....

.....

$$(d \times e) - f = g$$

(a) Find the values of a , b and c .

Answer $a = \dots\dots\dots$, $b = \dots\dots\dots$, $c = \dots\dots\dots$ [2]

(b) Express g in terms of d .

Answer $\dots\dots\dots$ [2]

(c) Explain why 279 cannot be the result of an equation in this sequence.

.....

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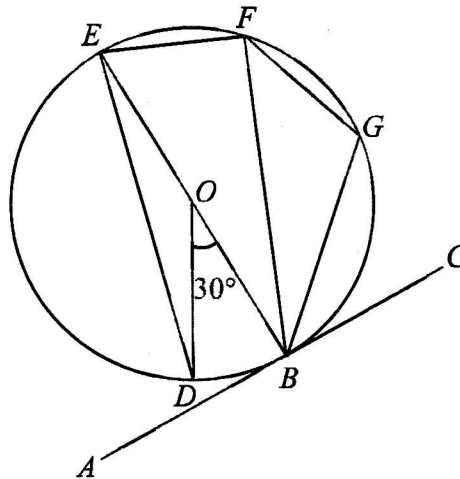
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.....[1]

- 19 The diagram shows a circle $BDEFG$, centre O and diameter BE . The line AC is a tangent to the circle at B and angle $BOD = 30^\circ$. The ratio of angle EBF to angle FBG to angle GBC is $1 : 1 : 2$.



- (a) State a fact about the arcs EF and FG .

.....
[1]

- (b) Showing all reasons clearly, find

- (i) angle DEB ,

Answer [1]

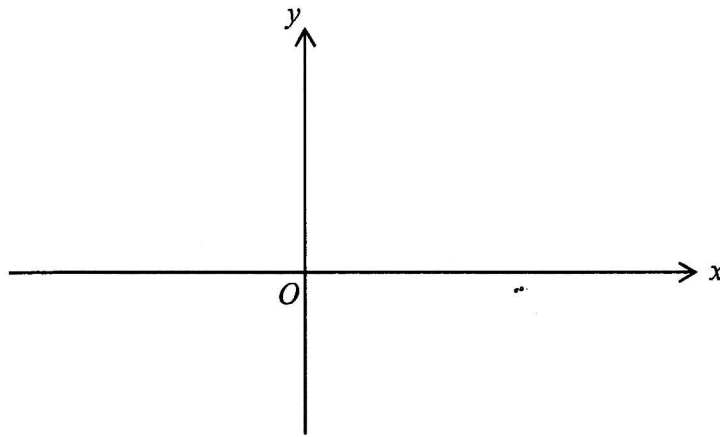
- (ii) angle EFG .

Answer [2]

- 20 (a) Express $-x^2 + 4x + 12$ in the form $-(x + p)(x - q)$.

Answer [1]

- (b) Sketch the graph of $y = -x^2 + 4x + 12$ on the axes below.
Indicate clearly the values where the graph crosses both axes. [2]



- (c) Find the coordinates of the turning point.

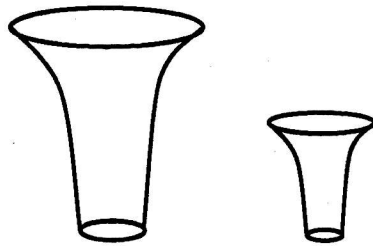
Answer (.....,) [1]

- (d) Without solving the equation algebraically, explain why $-x^2 + 4x + 12 = 18$ has no solution.

.....

 [1]

- 21 The diagram shows two geometrically similar containers. The cost of painting the base area of the smaller container is $\frac{25}{64}$ of the cost of painting the base of the larger container.



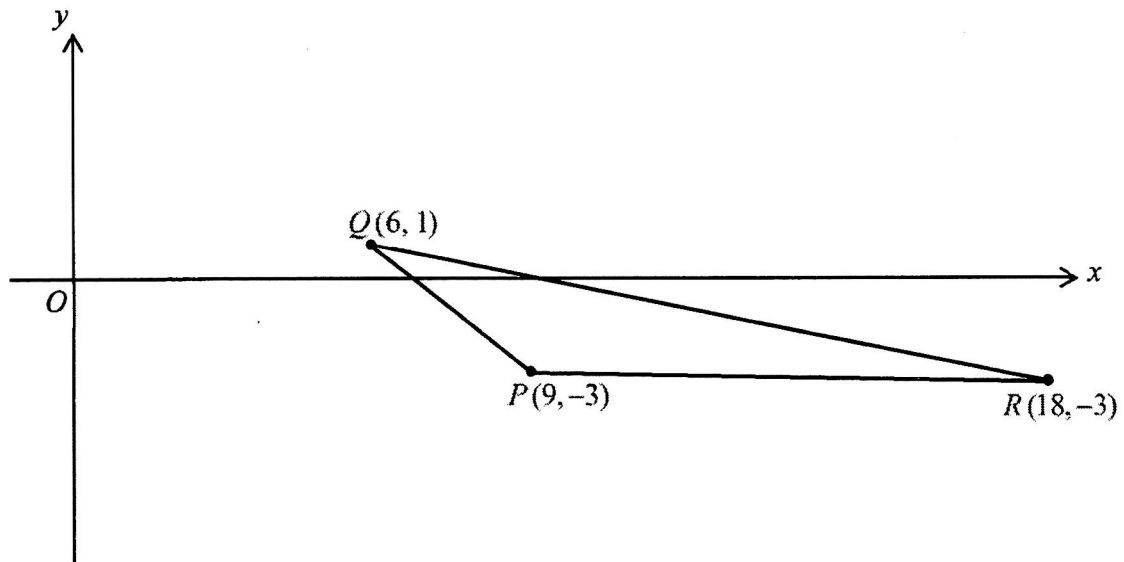
- (a) The top of the larger container has a circumference of 24 cm.
Find the circumference of the top of the smaller container.

Answer cm [2]

- (b) The capacity of the smaller container is 0.45 litres.
Find the capacity of the larger container, giving your answer to 2 decimal places.

Answer litres [1]

22



In the diagram, the points P , Q and R have coordinates $(9, -3)$, $(6, 1)$ and $(18, -3)$ respectively.

(a) Find the length of PQ .

Answerunits [1]

(b) Find the value of $\cos \angle QPR$, giving your answer as a fraction in its simplest form.

Answer [1]

(c) Find the area of triangle PQR .

Answerunits² [1]

- (d) Find the equation of the line PQ .

Answer [2]

- (e) The equation of a line passing through the point R is $4x + 21y = 9$.

Find the coordinates of the point of intersection of this line and the line PQ .

Answer [3]

~ End of Paper ~

ANS

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Total

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Answer **all** the questions.

- 1 State which of the following number(s) is/are irrational.

$$0.\dot{5}, \frac{\pi}{2}, 2\sqrt{2}, 3\sqrt{3} \times \sqrt{3}$$

$$\frac{\pi}{2}, 2\sqrt{2}$$

$0.\dot{5}$ can be expressed as $\frac{5}{9}$, so $0.\dot{5}$ is a rational number

Answer [1]

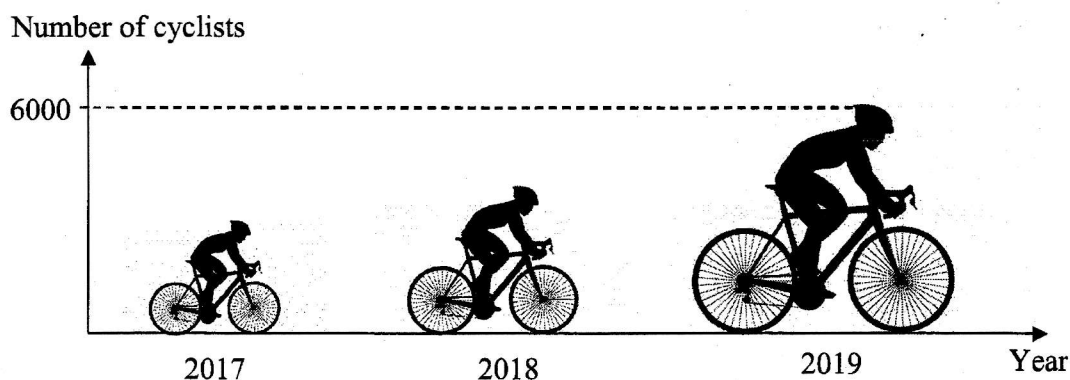
- 2 x is an integer greater than 1.
Write the following in order of size, starting with the largest.

$$x^0, x^{-\frac{3}{2}}, x^{\frac{3}{4}}, x^{\frac{3}{2}}$$

$$x^{\frac{3}{2}}, x^{\frac{3}{4}}, x^0, x^{-\frac{3}{2}}$$

Answer [1]

- 3 The graph shows the number of cyclists in an annual cycling marathon for the years 2017 to 2019.



- (a) State one aspect of the graph that may be misleading.

The vertical axis is not labelled at the start / No "0" on the y-axis. **OR**

The size of the picture not only increases with height, but also in the width.

- (b) Explain how this may lead to a misinterpretation of the graph.

It is difficult to estimate the number of cyclists for 2017 and 2018. **OR**

Unclear as to whether the height or the area is used to determine the number of cyclists

- 4 (a) Simplify $\left(\frac{2^{a-1}\sqrt{2}}{2^a}\right)^2$.

$$\begin{aligned}\left(\frac{2^{a-1}\sqrt{2}}{2^a}\right)^2 &= \left(\frac{2^a \cdot 2^{-1} \cdot \sqrt{2}}{2^a}\right)^2 \\ &= \left(\frac{\sqrt{2}}{2}\right)^2 \\ &= \frac{2}{4} \\ &= \frac{1}{2}\end{aligned}$$

2^{-1} is considered as not simplified, need to simplify to $\frac{1}{2}$

Answer [2]

- (b) If $27m^{3x} = 1$ and $m > 0$, find $m^{2x} + m^{-x}$.

$$27m^{3x} = 1$$

$$m^{3x} = \frac{1}{27}$$

$$m^x = \frac{1}{3}$$

$$\begin{aligned}m^{2x} + m^{-x} &= (m^x)^2 + (m^x)^{-1} \\ &= \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^{-1} \\ &= 3\frac{1}{9}\end{aligned}$$

Some obtained $m^{3x} = 3^{-3}$ and from this step, some went to equate $m = 3$ and $x = -1$ which is not always true as $m = \frac{1}{3}, x = 1$ or $m = 9, x = -\frac{1}{2}$.
As there are 2 unknown variables in this equation so we need two equations to be solved simultaneously in order to solve for the value of m and x .

Answer [2]

- 5 n is a positive integer.

Show that, for all n , $(5n+3)^2 - (5n-3)^2$ is a multiple of 4. [2]

Answer

$$\begin{aligned}(5n+3)^2 - (5n-3)^2 &= (5n+3+5n-3)(5n+3-(5n-3)) \\ &= (10n)(6) \\ &= 60n \\ &= 4(15n)\end{aligned}$$

Since n is a positive integer and $60n = 4(15n)$, $(5n+3)^2 - (5n-3)^2$ is a multiple of 4.

- 6 Gladys wrote down four numbers.
The mean of these numbers is 13, the median is 12 and the mode is 10.
Find the four numbers.

Let the four numbers be a , b , c and d in ascending order.
Since the mode is smaller than the median, a and b will be 10.
Since median is 12,

$$\frac{10+c}{2} = 12$$

$$c = 24 - 10$$

$$= 14$$

Since mean is 13,
 $10 + 10 + 14 + d = 13 \times 4$
 $d = 18$

The four numbers are 10, 10, 14 and 18.

Answer [2]

- 7 (a) Mrs Boey deposited \$20 000 in a bank that pays 0.8% per annum compound interest compounded half-yearly. Find the total amount of money she will receive at the end of 5 years, leaving your answer to the nearest cent.

$$\text{Total amount} = \$20\,000 \left(1 + \frac{0.8}{2(100)} \right)^{5 \times 2}$$

$$= \$20\,814.55468$$

$$= \$20\,814.55 \text{ (nearest cent)}$$

Answer [2]

(b)

Singapore Budget 2018: GST to be raised from 7% to 9%
some time between 2021 and 2025

Shane and Glen came across an article in the newspaper with the headline shown above and made the following comments.

Shane: Oh no, the GST will increase by 2% soon!

Glen: No! I disagree. It did not increase by 2%, in fact it is **MORE** than 2%!

Whose statement is correct? Support your answer with mathematical calculations.

$$\frac{9-7}{7} \times 100\% = 28.6\% \text{ (3sf)}$$

Percentage point is defined as the **difference** between two percentages. So the increase from 7% to 9% is two percentage points, and not two percent.

Glen's statement is correct as the GST will increase by 28.6%, which is more than 2%.

Answer [2]

- 8 (a) Find the range of values of x which satisfy the inequalities

$$\frac{17-8x}{3} < \frac{2(3x-1)}{5} \leq 4.$$

$$\frac{2(3x-1)}{5} \leq 4$$

$$3x-1 \leq 10$$

$$3x \leq 11$$

$$x \leq 3\frac{2}{3}$$

and

$$\frac{17-8x}{3} < \frac{2(3x-1)}{5}$$

$$5(17-8x) < 6(3x-1)$$

$$85-40x < 18x-6$$

$$-58x < -91$$

$$x > \frac{91}{58}$$

$$x > 1\frac{33}{58}$$

$$\therefore 1\frac{33}{58} < x \leq 3\frac{2}{3}$$

Answer [3]

- (b) Hence, state the smallest prime number that satisfies the inequalities.

Smallest prime number is 2.

Answer [1]

- 9 (a) Factorise completely $9b^2 - 6ab + a^2 - x^2$.

$$\begin{aligned} 9b^2 - 6ab + a^2 - x^2 &= (3b-a)^2 - x^2 \\ &= (3b-a+x)(3b-a-x) \end{aligned}$$

Or

$$\begin{aligned} 9b^2 - 6ab + a^2 - x^2 &= (a-3b)^2 - x^2 \\ &= (a-3b+x)(a-3b-x) \end{aligned}$$

Answer [2]

- (b) Simplify $\frac{5}{2x^2-7x-4} - \frac{8}{4-x}$.

$$\begin{aligned} \frac{5}{2x^2-7x-4} - \frac{8}{4-x} &= \frac{5}{(2x+1)(x-4)} + \frac{8}{x-4} \\ &= \frac{5+8(2x+1)}{(2x+1)(x-4)} \\ &= \frac{5+16x+8}{(2x+1)(x-4)} \\ &= \frac{16x+13}{(2x+1)(x-4)} \end{aligned}$$

Answer [3]

10 Roy made a model of his yacht with a scale of 1: 40.

(a) The actual length of the yacht is 12.5 m, find the length of the model in cm.

1: 40

1 cm: 0.4 m

$$\begin{aligned}\text{Length of model} &= \frac{12.5}{0.4} \\ &= 31.25 \text{ cm}\end{aligned}$$

The value 31.25 is exact, so it should not be expressed as 31.3 (correct to 3 s.f.)

Answercm [1]

(b) Roy wants to spray paint his yacht. He can select either of the following options to paint his yacht.

Option A: A lump sum payment of \$1 000

Option B: Payment for cost of paint at \$5.50 per m^2

[Cost of manpower is included in both options]

If the total surface area of the model to be painted is 937.5 cm^2 , find the surface area of his yacht. Which one of the two options should Roy select to paint his yacht?

[Show your workings clearly]

1 cm: 0.4 m

$1 \text{ cm}^2 : (0.4)^2 \text{ m}^2$

$1 \text{ cm}^2 : 0.16 \text{ m}^2$

$$\begin{aligned}\text{Cost of paint} &= 937.5 \times 0.16 \times \$5.50 \\ &= \$825\end{aligned}$$

Roy should select Option B as it is cheaper.

Answer

.....[3]

11 $\xi = \{x : x \text{ is an integer and } 1 \leq x \leq 10\}$

$A = \{x : x \text{ is a factor of } 20\}$

$B = \{x : x \text{ is a perfect square}\}$

Find

(a) $n(B)$,

$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 2, 4, 5, 10\}$

$B = \{1, 4, 9\}$

** 1 is a perfect square

$n(B) = 3$

Answer [1]

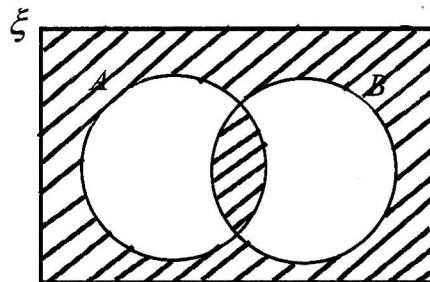
(b) $A' \cap B'$.

$A' \cap B' = \{3, 6, 7, 8\}$

It is a set, so curly brackets must be included.

Answer [1]

(c) On the Venn Diagram, shade the region which represents $(A \cup B)' \cup (A \cap B)$.



[1]

- 12 When written as the product of their prime factors,

$$p \text{ is } 2^2 \times 3 \times 5,$$

$$q \text{ is } 2^6 \times 3^3,$$

$$r \text{ is } 2^2 \times 3^2 \times 11.$$

Find

- (a) the value of the cube root of q ,

$$\begin{aligned} \text{Cube root of } q &= 2^2 \times 3 \\ &= 12 \end{aligned}$$

Answer [1]

- (b) the LCM of p , q and r , giving your answer as the product of its prime factors,

$$\text{LCM of } p, q \text{ and } r = 2^6 \times 3^3 \times 5 \times 11$$

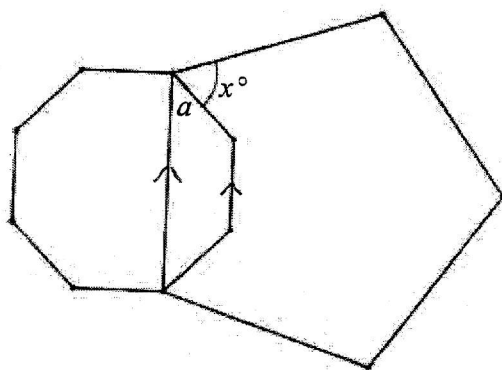
Answer [1]

- (c) the greatest number that will divide p , q and r exactly.

$$\text{The greatest number} = \text{HCF} = 2^2 \times 3 = 12$$

Answer [1]

- 13 The diagram shows a regular pentagon and a regular octagon. Calculate the value of x .



$$\begin{aligned} \text{Each interior angle of pentagon} &= \frac{(5-2)}{5} \times 180^\circ \\ &= 108^\circ \end{aligned}$$

$$\begin{aligned} \text{Each interior angle of octagon} &= \frac{(8-2)}{8} \times 180^\circ \\ &= 135^\circ \end{aligned}$$

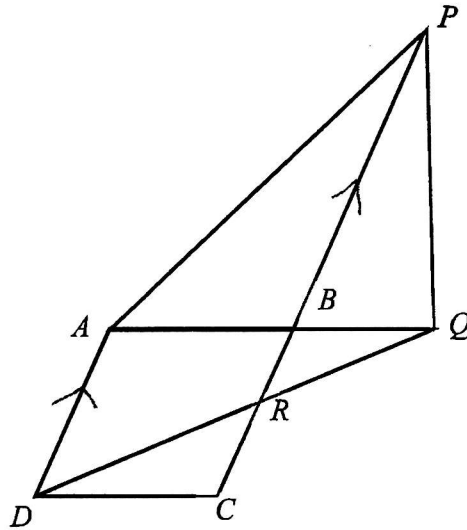
$$\begin{aligned} a &= \frac{1}{2} (360^\circ - 2 \times 135^\circ) \quad (\angle \text{ sum of isosceles trapezium}) \\ &= 45^\circ \end{aligned}$$

$$\begin{aligned} x^\circ &= 108^\circ - 45^\circ \\ &= 63^\circ \end{aligned}$$

Answer $x =$ [3]

- 14 The diagram shows a rhombus $ABCD$.

ABQ , PBC and DRQ are straight lines and $AQ = BP$.



- (a) State the triangle that is similar to triangle BRQ .

Triangle ADQ or Triangle CRD

Answer Triangle [1]

- (b) Prove that triangle DQA is congruent to triangle APB .

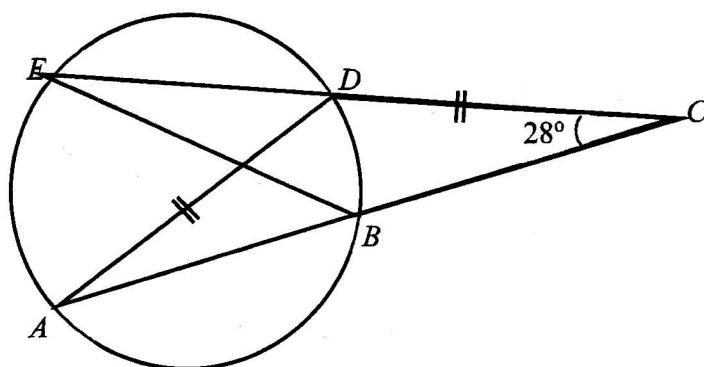
$AD = AB$ (given $ABCD$ is a rhombus)

angle $DAQ =$ angle ABP (alternate angles, AD parallel to BC and BP)

$AQ = BP$ (given)

Therefore triangle DQA is congruent to triangle APB . (SAS)

[2]



A, B, D and E are points on the circle such that $AD = CD$ and angle $BCD = 28^\circ$.
Explain with geometrical reasons, why the length of BC equals to BE .

Angle $DAC = 28^\circ$ (base angles of isosceles triangle ACD)

Angle $DEB = \text{Angle } DAC$ (angles in same segment)

$$= 28^\circ$$

$$= \text{Angle } BCD$$

So triangle BCE is an isosceles triangle.

Therefore the length of BC equals to BE .

Many students are not able to spell the word **isosceles** correctly.

[3]

- 16 The masses of 20 bags, in kg, are measured.
The results are shown on the stem-and-leaf diagram.

| Mass of 20 bags | |
|-----------------|---------------|
| Stem | Leaf |
| 0 | 9 |
| 1 | 2 |
| 2 | 0 6 6 6 |
| 3 | 0 3 3 4 4 8 8 |
| 4 | 1 2 9 9 9 |
| 5 | 0 1 |

Key: 0|9 means 0.9 kg

- (a) Find the mean mass of the bags.

$$\begin{aligned}\text{Mean mass} &= \frac{69}{20} \text{ kg} \\ &= 3.45 \text{ kg}\end{aligned}$$

A few forgot to check the key and did not realise the decimal point, hence giving the answer as 34.5 kg which is incorrect.

Answerkg [1]

- (b) Find the standard deviation of the masses of the bags.

$$\begin{aligned}\text{Standard deviation} &= \sqrt{\frac{266.8}{30} - 3.45^2} \text{ kg} \\ &= 1.20 \text{ kg (3 s.f.)}\end{aligned}$$

Answerkg [1]

- (c) It was found later that the weighing machine has an error.
Each bag was actually 0.08 kg heavier.
Explain how this will affect the mean and standard deviation.

Answer

The mean will increase by 0.08 kg.
The standard deviation will remain the same.

[2]

- 17 y is inversely proportional to the square of x , $x > 0$.

- (a) If x is increased by 25%, find the percentage change in y .

$$y = \frac{k}{x^2} \text{ where } k \text{ is a constant}$$

$$\text{New } y = \frac{k}{\left(\frac{125}{100}x\right)^2}$$

$$= \frac{16}{25} \left(\frac{k}{x^2}\right)$$

$$= \frac{16}{25}y$$

$$\text{Percentage change} = \frac{\text{new} - \text{original}}{\text{original}} \times 100\%$$

$$\text{Percentage change in } y = \frac{\frac{16}{25}y - y}{y} \times 100\%$$

$$= -36\%$$

Quite a few missed the negative sign as it is a decrease in y .

Answer% [2]

- (b) Given that $y = 16$ when $x = \frac{1}{2}$, find

- (i) the equation connecting y and x ,

$$k = yx^2$$

$$= 16 \left(\frac{1}{2}\right)^2$$

$$= 4$$

$$\therefore y = \frac{4}{x^2}$$

Answer [2]

- (ii) the value of x when $y = 100$.

$$\text{When } y = 100,$$

$$x^2 = \frac{4}{100}$$

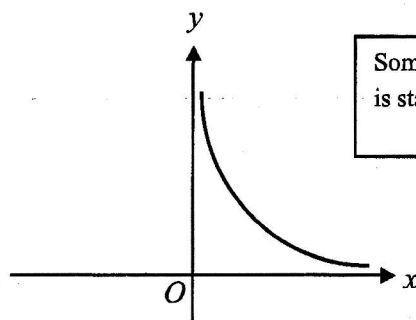
$$= \frac{1}{25}$$

$$x = \frac{1}{5}$$

Some forgot to reject the negative value as it is given at the beginning of the question that $x > 0$.

Answer $x =$ [2]

- (c) Sketch the graph which represents the relation between y and x . [1]



Some still include the negative part even though it is stated at the beginning of the question that $x > 0$.

- 18 In the following sequence,

$$(1 \times 2) - 2 = 0$$

$$(2 \times 3) - 4 = 2$$

$$(3 \times 4) - 6 = 6$$

$$(4 \times 5) - 8 = 12$$

.....

.....

$$(a \times 12) - b = c$$

.....

.....

$$(d \times e) - f = g$$

- (a) Find the values of a , b and c .

$$a = 11$$

$$b = 2 \times 11$$

$$c = 11 \times 12 - 22$$

$$= 22$$

$$= 110$$

Answer $a = \dots\dots\dots$, $b = \dots\dots\dots$, $c = \dots\dots\dots$ [2]

- (b) Express g in terms of d .

$$e = d + 1$$

$$f = 2d$$

$$g = d(d + 1) - 2d$$

$$g = d^2 + d - 2d$$

$$g = d^2 - d$$

$$g = d(d - 1)$$

Some left out the " $g =$ " part in their answer blank which is part of the required answer.

Answer $\dots\dots\dots$ [2]

- (c) Explain why 279 cannot be the result of an equation in this sequence.

Answer $\dots\dots\dots$ [1]

g is the product of an odd and an even number. Therefore g must be even. Since 279 is odd, 279 cannot be the result of an equation.

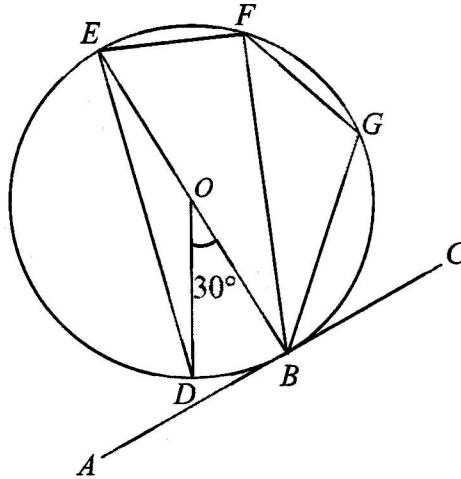
Alternatively,

$$279 = 9 \times 31$$

From (b), g is the product of 2 consecutive integers which is an even number.

279 is odd, therefore 279 cannot be the result of an equation in this sequence..

- 19 The diagram shows a circle $BDEFG$, centre O and diameter BE . The line AC is a tangent to the circle at B and angle $BOD = 30^\circ$. The ratio of angle EBF to angle FBG to angle GBC is $1 : 1 : 2$.



- (a) State a fact about the arcs EF and FG .

Answer Arc $EF = \text{Arc } FG$ [1]

- (b) Showing all reasons clearly, find

- (i) angle DEB ,

angle $DEB = 15^\circ$ (angle at centre = $2 \times$ angle at circumference)

Answer [1]

- (ii) angle EFG .

angle $EBC = 90^\circ$ (tangent perpendicular to radius)

$$\begin{aligned} \text{angle } EBG &= \frac{2}{4} \times 90^\circ \\ &= 45^\circ \end{aligned}$$

$$\begin{aligned} \text{angle } EFG &= 180^\circ - 45^\circ \text{ (angles in opposite segments)} \\ &= 135^\circ \end{aligned}$$

Some are still not using the standardised geometrical reasons/statements for stating the reasons.

Students must use only the given standardised geometrical properties/reasons.

Some are using very long-winded methods even though it is only 2 marks.

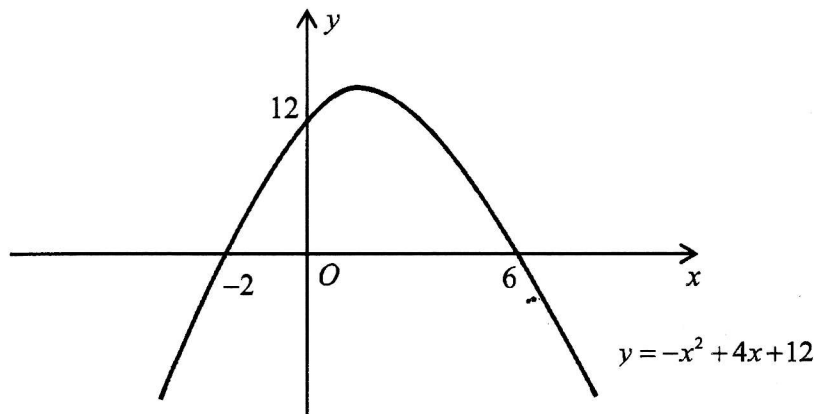
Answer [2]

- 20 (a) Express $-x^2 + 4x + 12$ in the form $-(x + p)(x - q)$.

$$\begin{aligned} -x^2 + 4x + 12 &= -(x^2 - 4x - 12) \\ &= -(x + 2)(x - 6) \end{aligned}$$

Answer [1]

- (b) Sketch the graph of $y = -x^2 + 4x + 12$ on the axes below.
Indicate clearly the values where the graph crosses both axes. [2]



- (c) Find the coordinates of the turning point.

Turning point is (2, 16).

Answer (.....,) [1]

- (d) Without solving the equation algebraically, explain why $-x^2 + 4x + 12 = 18$ has no solution.

Since the maximum value of the curve y is 16, there is no point of intersection between the curve and the horizontal straight line $y = 18$.

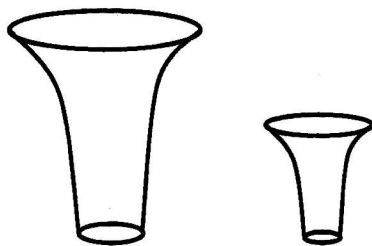
Or

The line $y = 18$ is always above the curve, so there is no point of intersection between the curve and the straight line $y = 18$.

Answer [1]

Some are still using algebraic method to solve despite the instruction saying otherwise.

- 21 The diagram shows two **geometrically similar** containers. The cost of painting the base area of the smaller container is $\frac{25}{64}$ of the cost of painting the base of the larger container.



- (a) The top of the larger container has a circumference of 24 cm.
Find the circumference of the top of the smaller container.

$$\left(\frac{l_s}{l_l}\right) = \sqrt{\frac{25}{64}} = \frac{5}{8}$$

$$\begin{aligned}\text{Circumference of the top of smaller container} &= \frac{5}{8} \times 24 \\ &= 15 \text{ cm}\end{aligned}$$

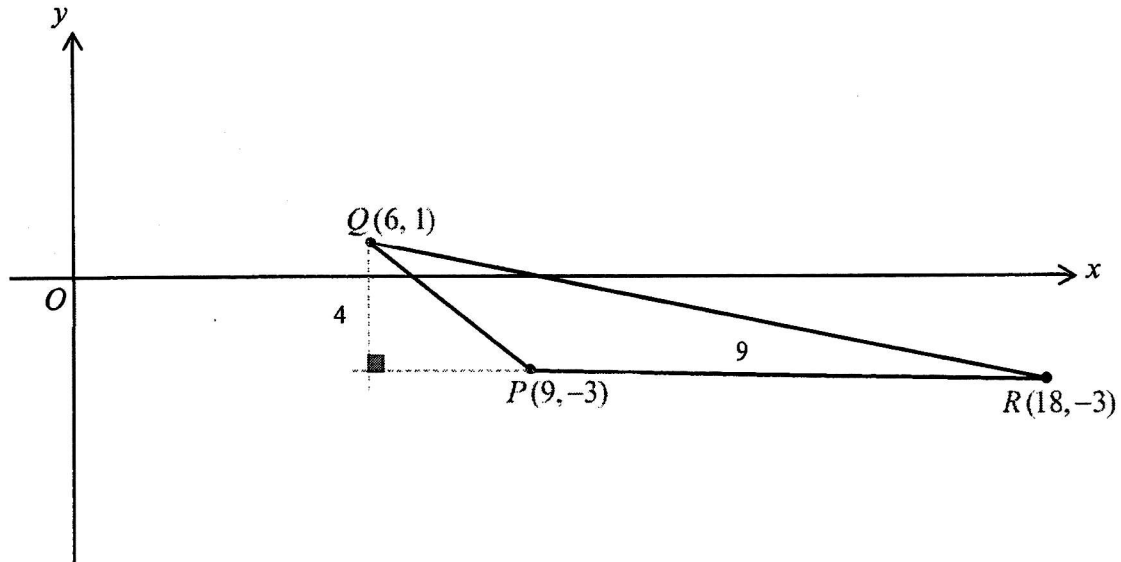
Answer cm [2]

- (b) The capacity of the smaller container is 0.45 litres.
Find the capacity of the larger container, giving your answer to 2 decimal places.

$$\begin{aligned}\text{Capacity of the larger container} &= \left(\frac{8}{5}\right)^3 \times 0.45 \text{ litres} \\ &= 1.8432 \text{ litres} \\ &= 1.84 \text{ litres (2 decimal places)}\end{aligned}$$

Answer litres [1]

22



In the diagram, the points P , Q and R have coordinates $(9, -3)$, $(6, 1)$ and $(18, -3)$ respectively.

(a) Find the length of PQ .

$$\begin{aligned}
 PQ &= \sqrt{(9-6)^2 + (-3-1)^2} \\
 &= \sqrt{3^2 + (-4)^2} \\
 &= 5 \text{ units}
 \end{aligned}$$

Answerunits [1]

(b) Find the value of $\cos \angle QPR$, giving your answer as a fraction in its simplest form.

$$\cos \angle QPR = -\frac{3}{5}$$

Some could not identify the right - angled triangle and some forget that $\cos A = -\cos (180^\circ - A)$

Answer [1]

(c) Find the area of triangle PQR .

$$\begin{aligned}
 \text{Area of triangle } PQR &= \frac{1}{2} \times 9 \times 4 \text{ units}^2 \\
 &= 18 \text{ units}^2
 \end{aligned}$$

Some are using other more time-consuming methods such as "shoelace", $\frac{1}{2}ab \sin C$ even though the question is only 1 mark.

Answer [1]

- (d) Find the equation of the line PQ .

$$\begin{aligned}\text{Gradient of } PQ &= \frac{1 - (-3)}{6 - 9} \\ &= -\frac{4}{3}\end{aligned}$$

$$\text{Equation of } PQ \text{ is } y - 1 = -\frac{4}{3}(x - 6)$$

$$y - 1 = -\frac{4}{3}x + 8$$

$$y = -\frac{4}{3}x + 9 \quad \text{or } 4x + 3y = 27$$

Answer [2]

- (e) The equation of the line passing through the point R is $4x + 21y = 9$.

Find the coordinates of the point of intersection of this line and the line PQ .

$$4x + 3y = 27 \quad \text{..... (1)}$$

$$4x + 21y = 9 \quad \text{..... (2)}$$

$$(2) - (1):$$

$$18y = -18$$

$$y = -1$$

Sub $y = -1$ into (1):

$$4x + 3(-1) = 27$$

$$4x = 30$$

$$x = 7\frac{1}{2}$$

\therefore The point of intersection is $\left(7\frac{1}{2}, -1\right)$.

Students must remember to put the brackets for the coordinates on their answer blank.

Answer [3]

~ End of Paper ~

| | | |
|-------|--------|------------------------|
| Name: | Class: | Class Register Number: |
|-------|--------|------------------------|



中正中學

CHUNG CHENG HIGH SCHOOL (MAIN)

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**PRELIMINARY EXAMINATION 2021
SECONDARY 4**

MATHEMATICS

4048/02

Paper 2

Wednesday 1 September 2021

2 hours 30 minutes

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer. Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 100.

| For Examiner's Use | |
|--------------------|----------------|
| Question Number | Marks Obtained |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| Total Marks | |

This document consists of **21** printed pages and 1 blank page.

[Turn Over

Mathematical Formulae*Compound interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} a b \sin C$$

$$\text{Arc length} = r \theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2 b c \cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f} \right)^2}$$

1 (a) Simplify $\left(\frac{a^6 b^3}{8}\right)^{\frac{1}{3}} \div \frac{3b^{-1}}{a^2}$.

Answer [3]

(b) Solve $\frac{2}{x-2} + \frac{3}{x+2} = 0$.

Answer $x =$ [2]

(c) Make q the subject in the equation $pq + 9 = p^2 + 3q$ and simplify your answer.

Answer [3]

- (d) (i) Simplify $(4x+3y)^2 + (3x-4y)^2$.

Answer [2]

- (ii) It is given that $x^2 + y^2 = 1$. Using your answer in part (i), explain why the maximum value of $(4x+3y)^2$ is 25.

Answer

[3]

- 2 During nuclear fusion, it is observed that when two hydrogen atoms are fused together to form one helium atom, there is a loss in mass, M .

The atomic masses of one hydrogen atom and one helium atom are given in the table below. The masses are given in atomic mass unit (u).

| Atom | Hydrogen | Helium |
|-----------------|----------|--------|
| Atomic Mass (u) | 2.0141 | 4.0026 |

- (a) Find M (in u), leaving your answer in standard form.

Answer u [2]

- (b) Express M as a percentage of the atomic mass of one helium atom.

Answer % [2]

- (c) Given that $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$, find M in kilograms.

Answer kg [1]

- 3 Town P and Town Q are 40 km apart. A car is travelling from Town P to Town Q at an average speed of x km/h. During its return journey, the car later travelled back to Town P from Town Q at an average speed of $(x+8)$ km/h and the time taken for the return journey was 12 minutes less.

- (i) Write down an equation to represent this information and show that it simplifies to $x^2 + 8x - 1600 = 0$.

Answer

[4]

- (ii) Solve the equation $x^2 + 8x - 1600 = 0$, giving your solutions correct to two decimal places.

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [3]

- (iii) Given that the car left Town P at 09 00, find the time when the car reached Town Q . Give your answer correct to the nearest minute.

Answer [2]

- 4 The variables x and y are connected by the equation $y = \frac{3}{2}x + \frac{5}{x} - 7$.

Some corresponding values of x and y are given in the table below.

| | | | | | | | | | |
|-----|------|------|-------|-----|-------|------|-----|------|------|
| x | 0.5 | 1 | 1.5 | 2 | 3 | 4 | 5 | 6 | 7 |
| y | 3.75 | -0.5 | -1.42 | p | -0.83 | 0.25 | 1.5 | 2.83 | 4.21 |

- (a) Find the value of p . [1]

Answer $p = \dots\dots\dots$

- (b) On the grid opposite, draw the graph of $y = \frac{3}{2}x + \frac{5}{x} - 7$ for $0.5 \leq x \leq 7$.

Use the scale of 2 cm to represent 1 unit on both axes. [3]

- (c) By drawing a tangent, find the gradient of the curve at $x = 2.5$.

Answer $\dots\dots\dots$ [2]

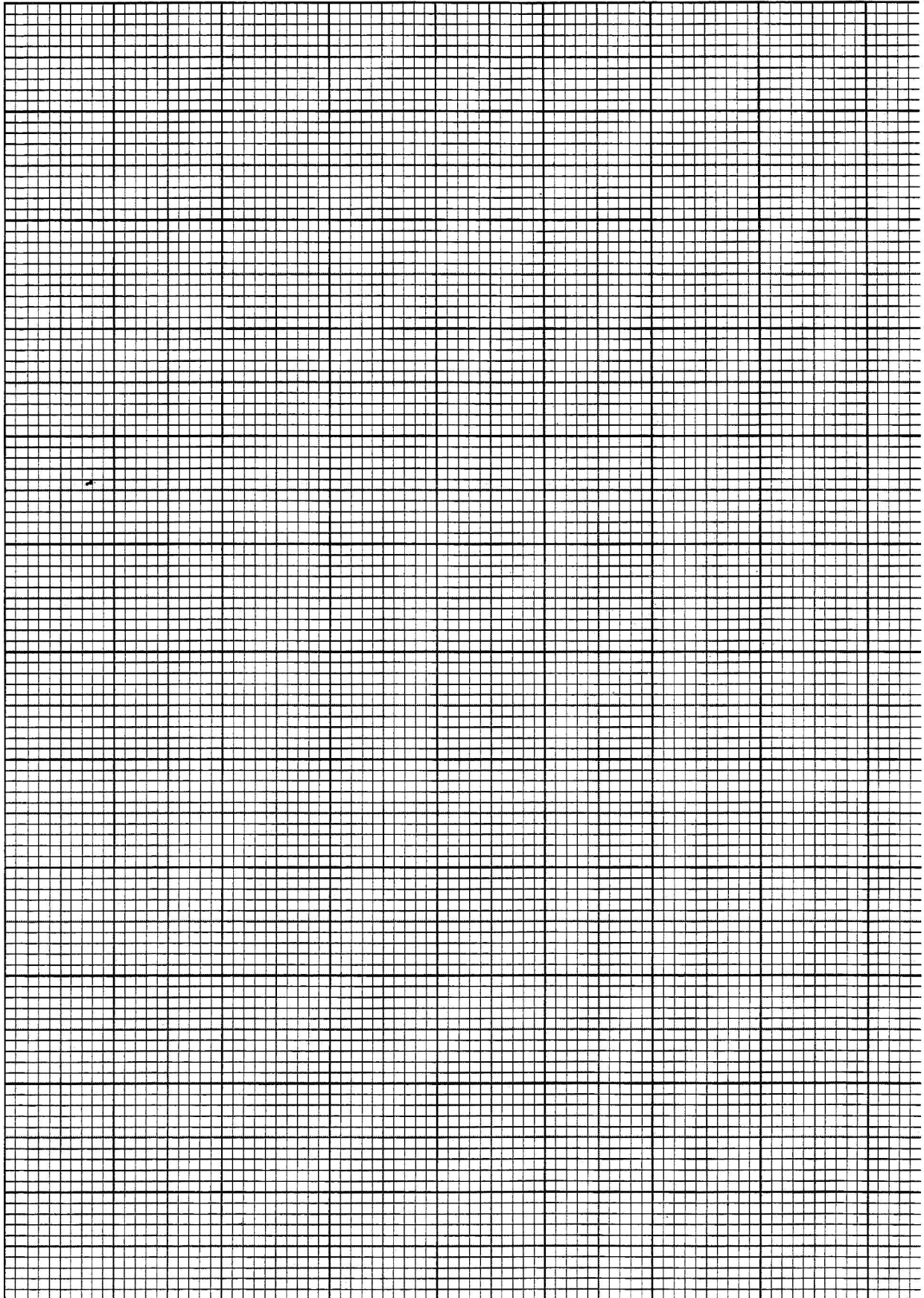
- (d) (i) On the grid in part (b), draw the line $2y + x = 2$ for $0 \leq x \leq 7$.

- (ii) Write down the x -coordinates of the points where the line intersects the curve. [2]

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [2]

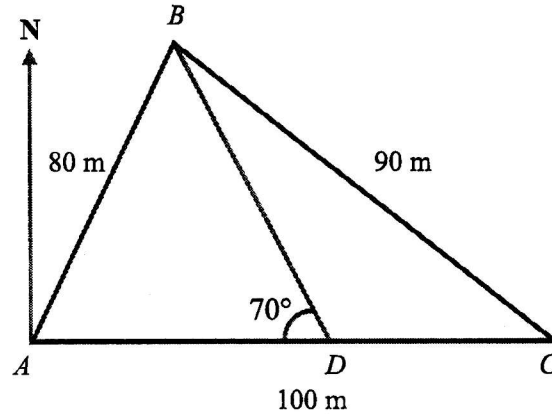
- (iii) Show that the points of intersection of the line and the curve give the solutions to the equation $2x^2 - 8x + 5 = 0$.

[2]



- 5 The diagram below shows three points, A , B and C on flat ground.

$AB = 80$ m, $AC = 100$ m and $BC = 90$ m. C is due east of A .



- (a) (i) Show that angle $ACB = 49.5^\circ$, correct to one decimal place.

[3]

- (ii) Hence, calculate the bearing of C from B .

Answer $^\circ$ [2]

- (iii) Point D lies along AC such that angle $BDA = 70^\circ$. Find the distance CD .

Answer m [3]

- (iv) Find the ratio of area of triangle ABD to area of triangle BDC .

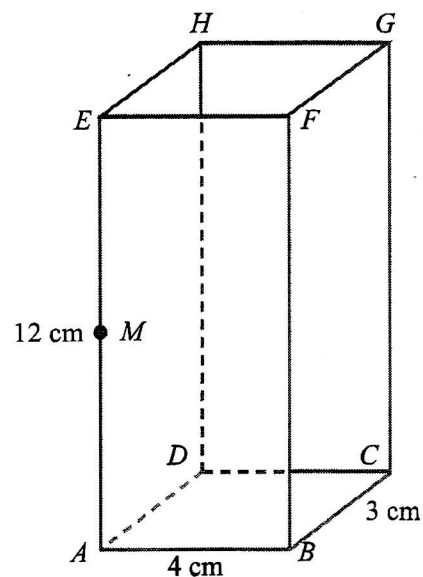
Give your answer correct to one decimal place.

Answer [2]

- (b) The diagram below shows a cuboid with rectangular base $ABCD$.

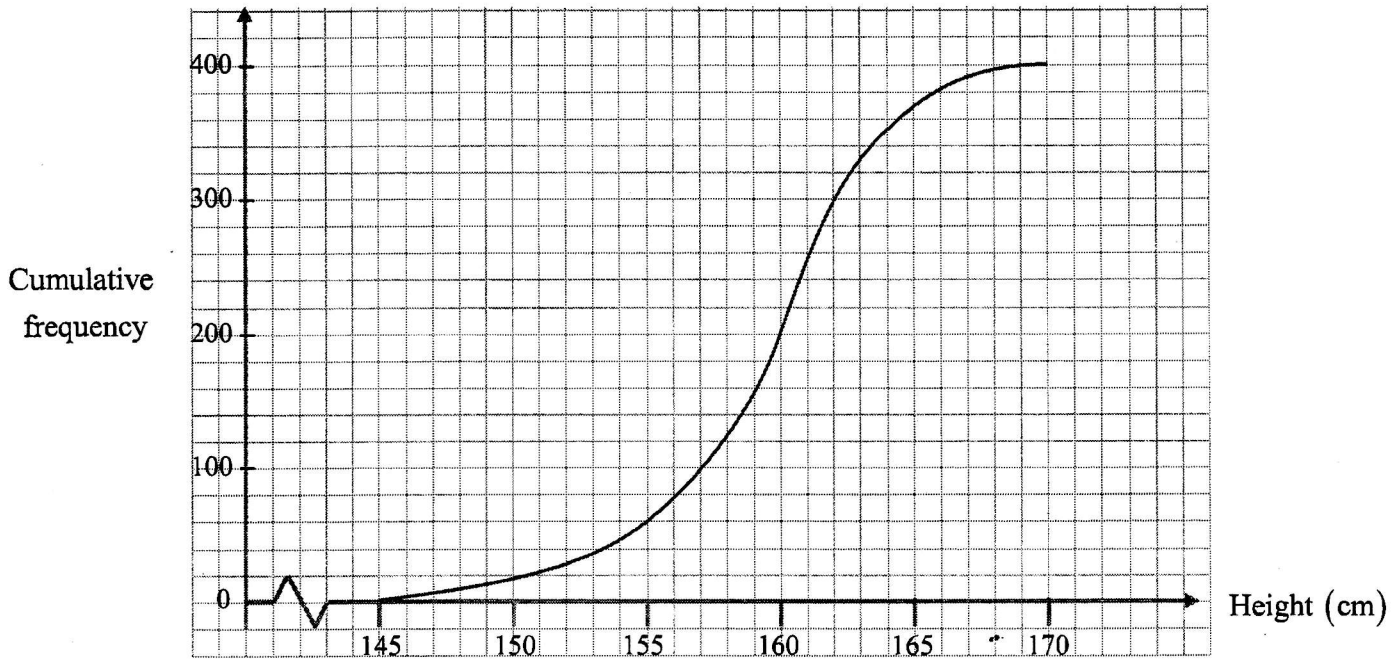
$AB = 4$ cm, $BC = 3$ cm and $AE = 12$ cm. M is the midpoint of AE .

Find the angle of elevation of G from M .



Answer [3]

- 6 (a) The heights of 400 students in School A are recorded. The cumulative frequency curve below shows the distribution of their heights.



- (i) State the median.

Answer cm [1]

- (ii) Find the interquartile range of the distribution.

Answer cm [2]

- (iii) In School B, the median and interquartile range of the heights of another 400 students are 165 cm and 8 cm respectively. Make two comments comparing the heights of students in School A and School B.

Answer

.....

.....

.....

.....

.....

[2]

- (b) John flipped a coin five times and obtained five successive heads. Is it conclusive that the coin is biased? Explain your answer clearly.

Answer

.....

.....

.....

.....

[2]

- (c) A two-digit number is formed by drawing a random card from Bag A followed by another random card from Bag B. Bag A contains three cards labelled '1', '2' and '3'. Bag B contains three cards labelled '4', '5' and '7'. For example, drawing a '1' and "5" from Bag A and Bag B respectively will form the number '15'.

- (i) Complete the possibility diagram below.

| Bag B \ Bag A | 1 | 2 | 3 |
|---------------|-----------|-----------|---|
| 4 | 14 | | |
| 5 | | 25 | |
| 7 | | | |

[2]

Find, as a fraction in its simplest form, the probability that the two-digit number formed is

- (ii) odd,

Answer [1]

- (iii) a perfect square,

Answer [1]

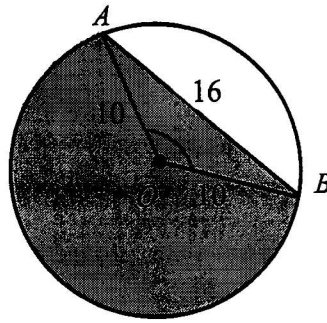
- (iv) a prime number,

Answer [1]

- (v) a factor of 135.

Answer [2]

- 7 The diagram shows a circle, centre O , radius 10 cm. AB is a chord of length 16 cm.



- (a) Show that angle $AOB = 1.855$ radians, correct to four significant figures.

Answer

[2]

Hence, find the

- (b) perimeter of the shaded segment,

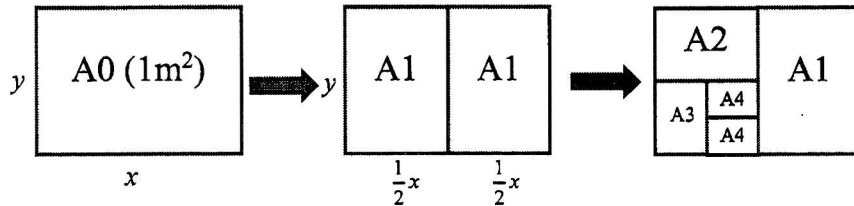
Answer cm [3]

(c) area of the shaded segment.

Answer cm^2 [2]

- 8 A piece of A0-sized paper has a total surface area of 1 m^2 . The length and breadth of the A0-sized paper are $x \text{ m}$ and $y \text{ m}$ respectively.

An A1 paper is an A0 paper folded in half, length-wise. An A2 paper is an A1 paper folded in half, so on and so forth.



- (i) Write down the relationship between x and y .

Answer [1]

- (ii) Given that all A-sized papers are geometrically similar, show that $\frac{x}{y} = \sqrt{2}$.

Answer

[2]

- (iii) Hence, using parts (i) and (ii), find the values of x and y .

Answer $x = \dots\dots\dots, y = \dots\dots\dots$ [4]

- (iv) Write down the dimensions of an A4-sized paper in millimetres.

Answer mm \times mm [2]

- (v) Given that N is an integer, express the number of pieces of A_N -sized paper that can be cut out from an A_0 -sized paper in terms of N .

Answer [1]

- 9 Robert has 3 options to get to school every weekday morning.

Option A: He takes the bus.

Option B: He takes the MRT.

Option C: He takes a taxi.

The matrices **A**, **B** and **C** represent his journey to school if he were to choose option **A**, **B** or **C** respectively.

$$\begin{matrix} & \begin{matrix} \text{Bus} & \text{MRT} & \text{Taxi} \end{matrix} \\ \begin{matrix} \text{A} = \end{matrix} & \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{matrix}, \begin{matrix} & \begin{matrix} \text{Bus} & \text{MRT} & \text{Taxi} \end{matrix} \\ \begin{matrix} \text{B} = \end{matrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{matrix}, \begin{matrix} & \begin{matrix} \text{Bus} & \text{MRT} & \text{Taxi} \end{matrix} \\ \begin{matrix} \text{C} = \end{matrix} & \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{matrix}.$$

- (a) In a particular week, Robert took the bus 3 times. He took the MRT and taxi in each of the other two days. The above information can be represented by a matrix **S** where $\mathbf{S} = 3\mathbf{A} + \mathbf{B} + \mathbf{C}$. Find **S**.

Answer [2]

- (b) The table below shows the travelling time taken, in minutes, for each of the 3 options that Robert can take to get to school.

| Option | Bus | MRT | Taxi |
|-------------------------|-----|-----|------|
| Time taken (in minutes) | 30 | 20 | 15 |

- (i) Write down a column matrix **N** to represent the information in the table.

Answer **N** = [1]

- (ii) Evaluate the matrix SN .

Answer $SN = \dots\dots\dots$ [2]

- (iii) State what the elements in matrix SN represent.

Answer

.....

[1]

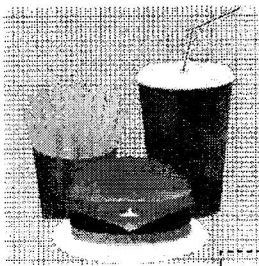
- (c) Write down a matrix J such that product of J and the answer in part (b)(ii) will give the average travelling time taken by Robert in a particular week.

Answer [1]

- (d) Hence, find the average travelling time taken by Robert per day in a particular week.

Answer minutes [2]

- 10 The diagram below shows the menu from a fast food restaurant.



Combo A

- Burger
- Medium Fries
- Medium Drink

\$5.90 only!

***Upsize your
drink and fries
to Large-size
for only \$1.10***

| Ala carte | Medium (\$) | Large (\$) |
|-----------------|-------------|------------|
| Burger (1 size) | 3.20 | |
| Fries | 2.20 | 2.90 |
| Drink | 2.80 | 3.60 |

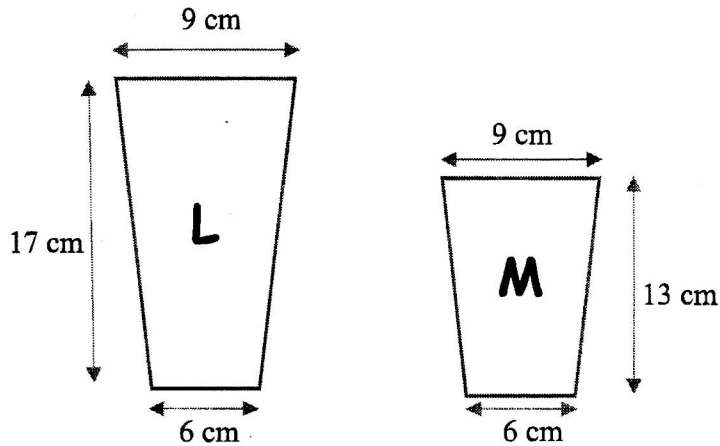
- (a) Albert wants to buy a burger for both himself and his brother Bart. His brother wants at least a medium-sized fries. Albert wants a large drink while Bart wants a medium drink.

By considering three options, suggest how Albert should place his order.
Justify any decisions that you make and show your calculations clearly.

Answer

[4]

- (b) The diagram below shows the cross-sectional area of the large (L) and medium (M) drink cups sold by the fast food restaurant in part (a). The Large and Medium cups have a height of 17 cm and 13 cm respectively. The top and bottom **diameters** of both cups are 9 cm and 6 cm respectively



The volume of a cup is given by $V = \frac{1}{3}\pi h(R^2 + Rr + r^2)$ where h is the height,

R and r are the radii of the top and bottom of the cup respectively. Using the information from part (a), determine whether the medium cup or large cup gives better value for money.

Answer

[4]

ANS

| | | |
|-------|--------|------------------------|
| Name: | Class: | Class Register Number: |
|-------|--------|------------------------|



中正中學

CHUNG CHENG HIGH SCHOOL (MAIN)

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PRELIMINARY EXAMINATION 2021 SECONDARY 4

MATHEMATICS

4048/02

Paper 2

Wednesday 1 September 2021

2 hours 30 minutes

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use paper clips, glue or correction fluid.

Answer all questions.

If working is needed for any question it must be shown with the answer. Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 100.

For Examiner's Use

| Question Number | Marks Obtained |
|-----------------|----------------|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| Total Marks | |

This document consists of 21 printed pages and 1 blank page.

[Turn Over

Mathematical Formulae*Compound interest*

$$\text{Total amount} = P \left(1 + \frac{r}{100} \right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4 \pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3} \pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3} \pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} a b \sin C$$

$$\text{Arc length} = r \theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2 b c \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2}$$

1 (a) Simplify $\left(\frac{a^6b^3}{8}\right)^{\frac{1}{3}} \div \frac{3b^{-1}}{a^2}$.

$$\begin{aligned}\left(\frac{a^6b^3}{8}\right)^{\frac{1}{3}} \div \frac{3b^{-1}}{a^2} &= \frac{a^2b}{2} \times \frac{a^2}{3b^{-1}} \\ &= \frac{a^2b}{2} \times \frac{a^2b}{3} \quad [\text{M2} - \text{Any 2 Laws of Indices}] \\ &= \frac{a^4b^2}{6} \quad [\text{A1}]\end{aligned}$$

Answer [3]

(b) Solve $\frac{2}{x-2} + \frac{3}{x+2} = 0$.

$$\begin{aligned}\frac{2(x+2)+3(x-2)}{(x-2)(x+2)} &= 0 \quad [\text{M1} - \text{Single Fraction}] \\ 2x+4+3x-6 &= 0 \\ 5x-2 &= 0 \\ x &= \frac{2}{5} \text{ or } 0.4 \quad [\text{A1}]\end{aligned}$$

Answer $x =$ [2]

(c) Make q the subject in the equation $pq+9=p^2+3q$ and simplify your answer.

$$\begin{aligned}pq+9 &= p^2+3q \\ pq-3q &= p^2-9 \quad [\text{M1} - \text{Grouping } q \text{ terms}] \\ q(p-3) &= (p-3)(p+3) \quad [\text{M1} - \text{Difference of squares}] \\ q &= \frac{(p-3)(p+3)}{p-3} \\ q &= p+3 \quad [\text{A1}]\end{aligned}$$

Answer [3]

- (d) (i) Simplify $(4x+3y)^2 + (3x-4y)^2$.

$$\begin{aligned}
 & (4x+3y)^2 + (3x-4y)^2 \\
 &= 16x^2 + 24xy + 9y^2 + 9x^2 - 24xy + 16y^2 \quad [\text{M1} - \text{Expansion}] \\
 &= 25x^2 + 25y^2 \\
 &= 25(x^2 + y^2) \quad \left. \vphantom{\begin{aligned} & (4x+3y)^2 + (3x-4y)^2 \\ &= 16x^2 + 24xy + 9y^2 + 9x^2 - 24xy + 16y^2 \\ &= 25x^2 + 25y^2 \end{aligned}} \right\} [\text{A1}]
 \end{aligned}$$

Answer [2]

- (ii) It is given that $x^2 + y^2 = 1$. Using your answer in part (i), explain why the maximum value of $(4x+3y)^2$ is 25.

$$\begin{aligned}
 & (4x+3y)^2 + (3x-4y)^2 = 25(x^2 + y^2) \\
 & \quad = 25(1) \quad [\sqrt{\text{M1}} - \text{Substitute for } x^2 + y^2 = 1] \\
 & \quad = 25 \\
 & (4x+3y)^2 = 25 - (3x-4y)^2 \quad [\text{B1}] \\
 & \text{Since } (3x-4y)^2 \geq 0, -(3x-4y)^2 \leq 0 \quad [\text{A1} - \text{Recognising that } (3x-4y)^2 \geq 0] \\
 & 25 - (3x-4y)^2 \leq 25 \\
 & (4x+3y)^2 \leq 25 \\
 & \text{Max value of } (4x+3y)^2 = 25 \text{ (shown)} \quad [\text{AG}]
 \end{aligned}$$

[3]

- 2 During nuclear fusion, it is observed that when two hydrogen atoms are fused together to form one helium atom, there is a loss in mass, M .

The atomic masses of one hydrogen atom and one helium atom are given in the table below. The masses are given in atomic mass unit (u).

| Atom | Hydrogen | Helium |
|-----------------|----------|--------|
| Atomic Mass (u) | 2.0141 | 4.0026 |

- (a) Find M (in u), leaving your answer in standard form.

$$\begin{aligned}
 M_D &= H + H - \text{He} \\
 &= 2.0141 + 2.0141 - 4.0026 \quad [\text{B1} - \text{Substituting correct values}] \\
 &= 0.0256 \\
 &= 2.56 \times 10^{-2} \quad [\text{B1} - \text{Correct Answer}]
 \end{aligned}$$

Answer u [2]

- (b) Express M as a percentage of the atomic mass of one helium atom.

$$\begin{aligned}
 \frac{M}{\text{He}} \times 100\% &= \frac{2.56 \times 10^{-2}}{4.0026} \times 100\% \quad [\text{M1} - \text{Substitution of values}] \\
 &= 0.639585 \\
 &= 0.640 \quad (3 \text{ sig. fig.}) \quad [\text{A1}]
 \end{aligned}$$

Answer % [2]

- (c) Given that $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$, find M in kilograms.

$$\begin{aligned}
 \text{mass} &= (1.66 \times 10^{-27}) M \\
 &= (1.66 \times 10^{-27}) (0.0256) \\
 &= 0.042496 \times 10^{-27} \\
 &= 4.2496 \times 10^{-29} \\
 &= 4.25 \times 10^{-29} \text{ kg} \quad (3 \text{ sig. fig.}) \quad [\text{B1 for either}]
 \end{aligned}$$

Answer kg [1]

- 3 Town P and Town Q are 40 km apart. A car is travelling from Town P to Town Q at an average speed of x km/h. During its return journey, the car later travelled back to Town P from Town Q at an average speed of $(x+8)$ km/h and the time taken for the return journey was 12 minutes less.

- (i) Write down an equation to represent this information and show that it simplifies to $x^2 + 8x - 1600 = 0$.

Answer

$$\begin{aligned} \frac{40}{x} - \frac{40}{x+8} &= \frac{1}{5} && [\text{M1} - \text{Correct expression for time (either)}] \\ &&& [\text{B1} - \text{Forming equation with } \frac{1}{5}] \\ \frac{40(x+8) - 40x}{x(x+8)} &= \frac{1}{5} && [\text{M1} - \text{Single fraction}] \\ \frac{320}{x^2 + 8x} &= \frac{1}{5} && [\text{M1} - \text{Simplifying}] \\ x^2 + 8x &= 1600 \\ x^2 + 8x - 1600 &= 0 \text{ (shown)} && [\text{AG}] \end{aligned}$$

[4]

- (ii) Solve the equation $x^2 + 8x - 1600 = 0$, giving your solutions correct to two decimal places.

$$\begin{aligned} x^2 + 8x - 1600 &= 0 \\ x &= \frac{-8 \pm \sqrt{8^2 - 4(1)((-1600))}}{2(1)} && [\text{M1} - \text{Formula or completing the square}] \\ &= \frac{-8 \pm \sqrt{6464}}{2} \\ x &= 36.199 \text{ or } x = -44.199 \\ x &= 36.20 \text{ or } x = -44.20 \text{ (2 dec. places)} && [\text{A1 each}] \end{aligned}$$

Note: -1 mark if left in 3 sig. fig.

Answer $x = \dots\dots\dots$ or $\dots\dots\dots$ [3]

- (iii) Given that the car left Town *P* at 0900, find the time when the car reached Town *Q*. Give your answer correct to the nearest minute.

Since $x > 0$, $x = 36.199$

$$\begin{aligned} \text{Time taken} &= \frac{40}{36.199} && [\text{M1} - \text{Time formula}] \\ &= 1.10498 \text{ hours} \\ \text{Time} &= 1006 && [\text{A1}] \end{aligned}$$

Answer [2]

- 4 The variables x and y are connected by the equation $y = \frac{3}{2}x + \frac{5}{x} - 7$.

Some corresponding values of x and y are given in the table below.

| | | | | | | | | | |
|-----|------|------|-------|-----|-------|------|-----|------|------|
| x | 0.5 | 1 | 1.5 | 2 | 3 | 4 | 5 | 6 | 7 |
| y | 3.75 | -0.5 | -1.42 | p | -0.83 | 0.25 | 1.5 | 2.83 | 4.21 |

- (a) Find the value of p .

-1.5 [B1]

[1]

Answer $p = \dots\dots\dots$

- (b) On the grid opposite, draw the graph of $y = \frac{3}{2}x + \frac{5}{x} - 7$ for $0.5 \leq x \leq 7$.

Use the scale of 2 cm to represent 1 unit on both axes.

[3]

- (c) By drawing a tangent, find the gradient of the curve at $x = 2.5$.

From graph,

Drawing tangent [M1]

Gradient = 0.7 [0.6-0.8] [A1]

Answer [2]

- (d) (i) On the grid in part (b), draw the line $2y + x = 2$ for $0 \leq x \leq 7$.

$$2y + x = 2$$

$$2y = -x + 2 \quad [\text{M1} - \text{Make } y \text{ the subject}]$$

$$y = -\frac{1}{2}x + 1 \quad [\text{G1} - \text{Draw } y] [\text{G2} - \text{If draw graph only}]$$

[2]

- (ii) Write down the x -coordinates of the points where the line intersects the curve.

$$x = 0.75 [0.7-0.8], x = 3.25 [3.2-3.3] \quad [\text{A1}]$$

Answer $x = \dots\dots\dots$ or [2]

- (iii) Show that the points of intersection of the line and the curve give the solutions to the equation $2x^2 - 8x + 5 = 0$.

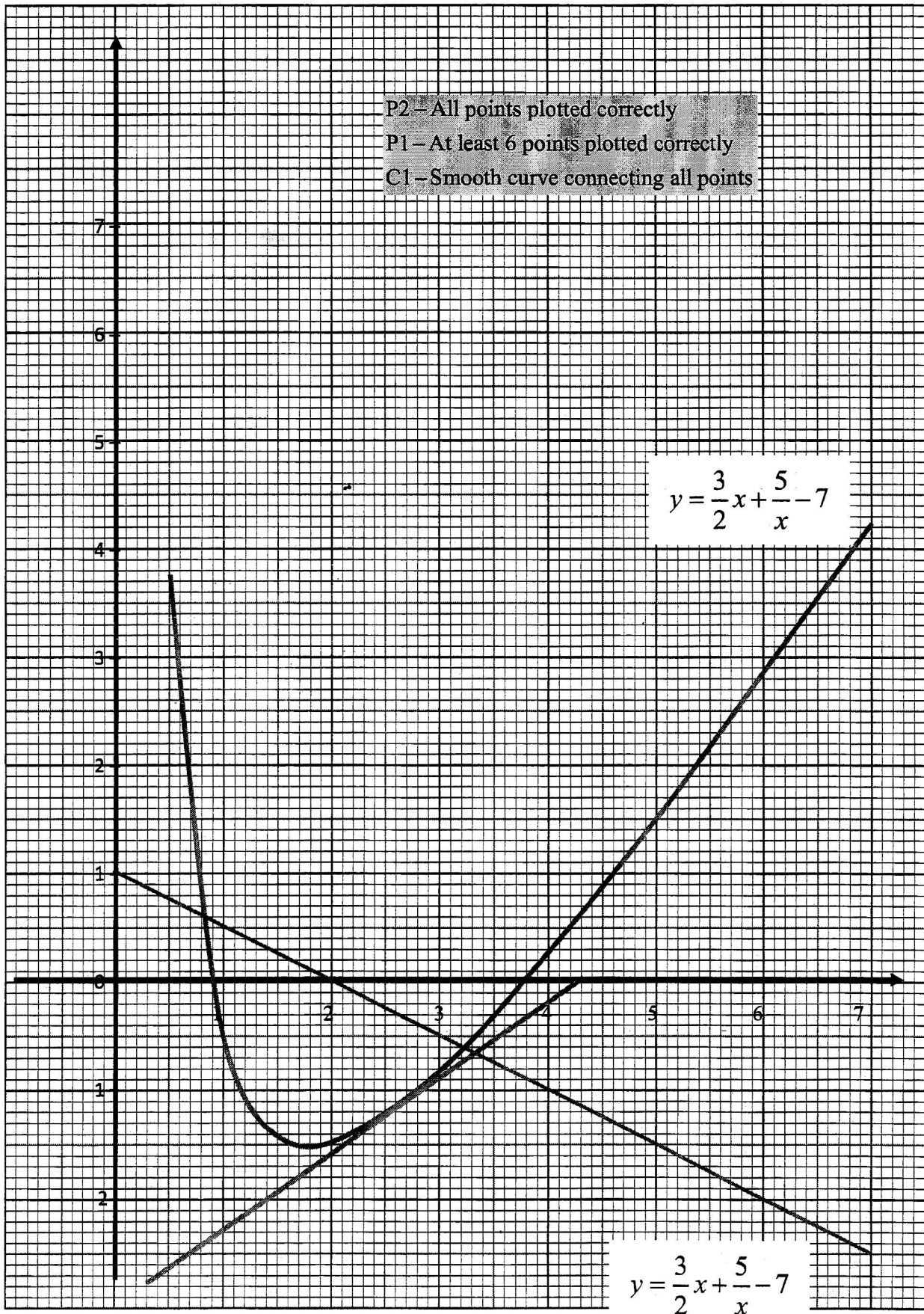
$$\frac{3}{2}x + \frac{5}{x} - 7 = -\frac{1}{2}x + 1 \quad [\text{M1} - \text{Equating}]$$

$$3x^2 + 10 - 14x = -x^2 + 2x$$

$$4x^2 - 16x + 10 = 0 \quad [\text{A1}]$$

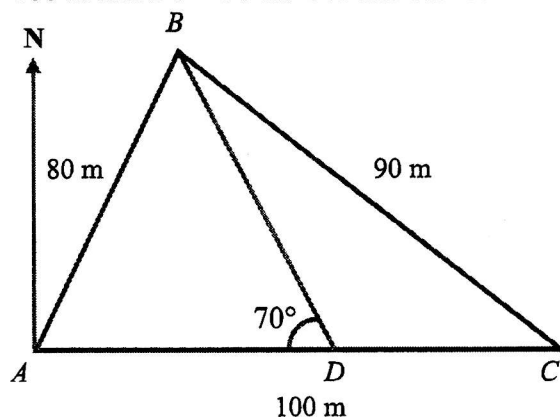
$$2x^2 - 8x + 5 = 0 \quad [\text{AG}]$$

Answer [2]



- 5 The diagram below shows three points, A , B and C on flat ground.

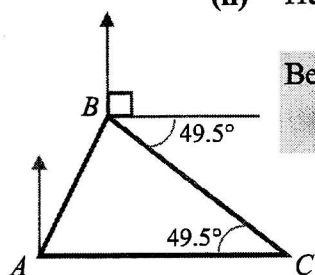
$AB = 80$ m, $AC = 100$ m and $BC = 90$ m. C is due east of A .



- (a) (i) Show that angle $ACB = 49.5^\circ$, correct to one decimal place.

$$\begin{aligned}\cos \angle ACB &= \frac{90^2 + 100^2 - 80^2}{2(90)(100)} && [\text{M1} - \text{Cosine Rule or other method}] \\ \angle ACB &= \cos^{-1} \left(\frac{90^2 + 100^2 - 80^2}{2(90)(100)} \right) && [\text{M1} - \text{Cos } C \text{ the subject}] \\ &= 49.458^\circ && [\text{A1}] \\ &= 49.5^\circ \text{ (shown)} && [3]\end{aligned}$$

- (ii) Hence, calculate the bearing of C from B .



$$\begin{aligned}\text{Bearing of } C \text{ from } B &= 90^\circ + 49.458^\circ && [\text{M1} - 90^\circ \text{ due east}] \\ &= 139.5^\circ \text{ (1 dec. place)} && [\text{A1}]\end{aligned}$$

Answer ° [2]

- (iii) Point D lies along AC such that angle $BDA = 70^\circ$. Find the distance CD .

$$\begin{aligned}\angle BDC &= 180^\circ - 70^\circ \text{ (Adjacent } \angle\text{s on a straight line)} \\ &= 110^\circ \\ \angle CBD &= 180^\circ - 110^\circ - 49.458^\circ \\ &= 20.542^\circ && [\text{M1} - \text{Finding } \angle CBD] \\ \frac{90}{\sin 110^\circ} &= \frac{CD}{\sin 20.542^\circ} && [\text{M1} - \text{Sine Rule}] \\ CD &= \frac{90 \sin 20.542^\circ}{\sin 110^\circ} \\ &= 33.606 \\ &= 33.6 \text{ m (3 sig. fig.)} && [\text{A1}]\end{aligned}$$

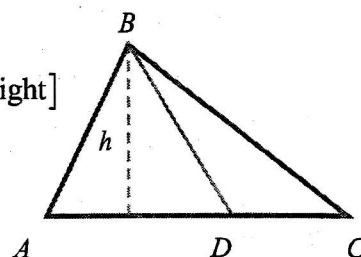
Answer m [3]

- (iv) Find the ratio of area of triangle ABD to area of triangle BDC .

Give your answer correct to one decimal place.

Since they share a common height,

$$\begin{aligned} \frac{\text{Area of } \triangle ABD}{\text{Area of } \triangle BDC} &= \frac{\frac{1}{2}(AD)(h)}{\frac{1}{2}(DC)(h)} \quad [\text{M1} - \text{Compare ratio with common height}] \\ &= \frac{AD}{DC} \\ &= \frac{100 - 33.606}{33.606} \\ &= 1.975 \\ &= 2.0 \text{ (1 dec. place)} \quad [\text{A1} - \text{Accept } 2.0:1.0] \end{aligned}$$



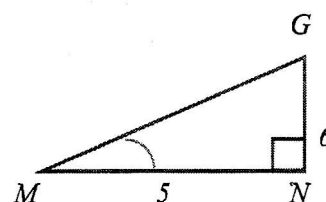
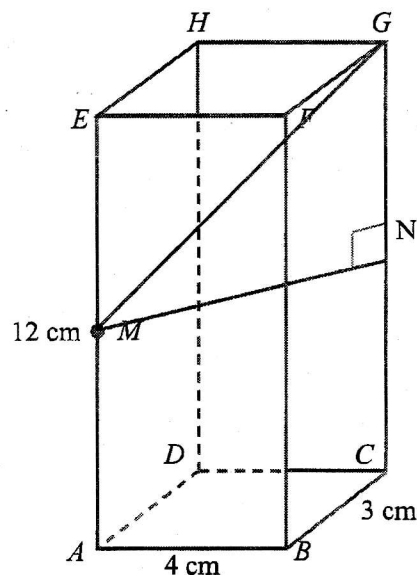
Answer [2]

- (b) The diagram below shows a cuboid with rectangular base $ABCD$.

$AB = 4$ cm, $BC = 3$ cm and $AE = 12$ cm. M is the midpoint of AE .

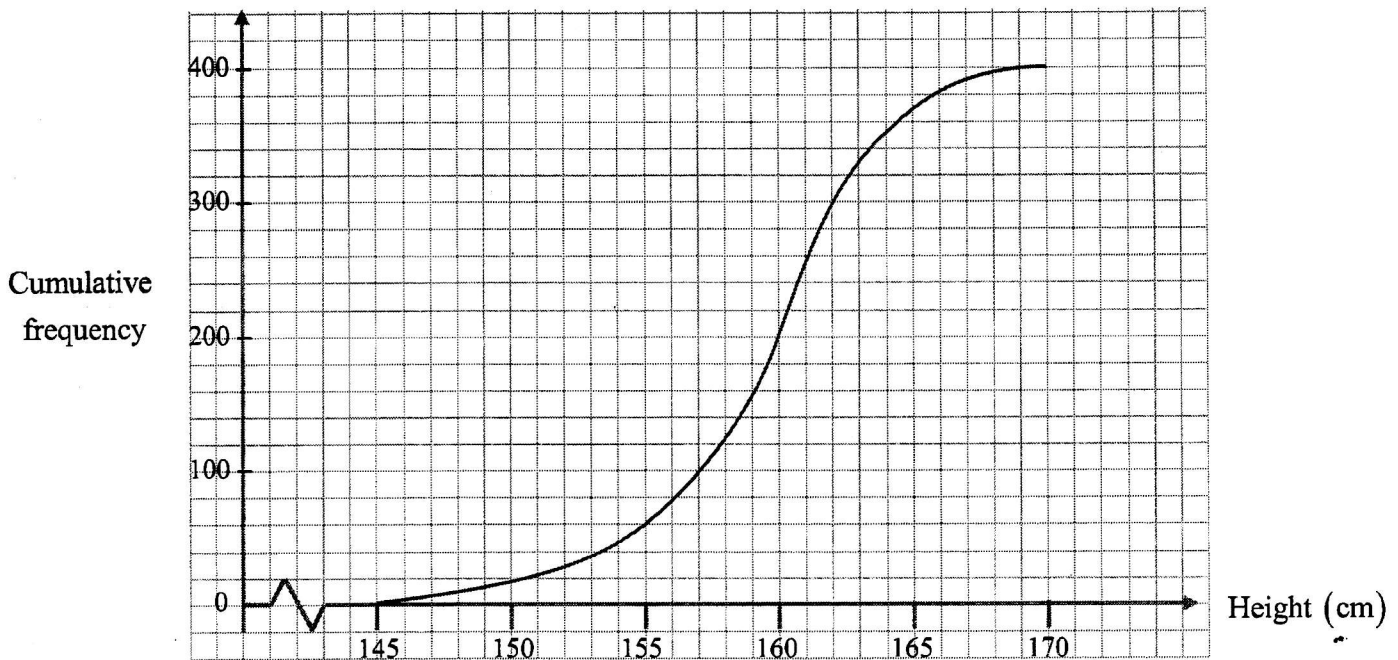
Find the angle of elevation of G from M .

$$\begin{aligned} \text{Let } N \text{ be midpoint of } GC. \\ AC &= \sqrt{4^2 + 3^2} \\ &= 5 \text{ cm} \quad [\text{M1} - \text{Finding } AC] \\ GN &= \frac{12}{2} = 6 \text{ cm} \\ \tan \angle GMN &= \frac{GN}{MN} \\ &= \frac{6}{5} \quad [\text{M1} - \text{Using tangent}] \\ \angle GMN &= \tan^{-1}\left(\frac{6}{5}\right) \\ &= 50.194^\circ \\ &= 50.2^\circ \text{ (1 dec. place)} \quad [\text{A1}] \end{aligned}$$



Answer [3]

- 6 (a) The heights of 400 students in School A are recorded. The cumulative frequency curve below shows the distribution of their heights.



- (i) State the median.

160 cm [B1]

Answer cm [1]

- (ii) Find the interquartile range of the distribution.

$Q_1 = 157, Q_3 = 162$
 $IQR = 162 - 157$ [B1]
 $= 5$ cm [B1]

Answer cm [2]

- (iii) In School B, the median and interquartile range of the heights of another 400 students are 165 cm and 8 cm respectively. Make two comments comparing the heights of students in School A and School B.

Answer

The students in School B are generally taller as they have a higher median of
 165 cm as compared to School A (160 cm).

The height of students in School B are generally more widespread as they have a
 higher interquartile range of 8 cm as compared to School A (5 cm). [B1 each]

[2]

- (b) John flipped a coin five times and obtained five successive heads. Is it conclusive that the coin is biased? Explain your answer clearly.

Answer

No, given that John only flipped the coin 5 times, there is a 1 in 32 chance that of obtaining 5 successive heads which is still possible. The sample size is **too small** for any logical conclusion that the coin is biased.

[2]

- (c) A two-digit number is formed by drawing a random card from Bag A followed by another random card from Bag B. Bag A contains three cards labelled '1', '2' and '3'. Bag B contains three cards labelled '4', '5' and '7'. For example, drawing a '1' and '5' from Bag A and Bag B respectively will form the number '15'.

- (i) Complete the possibility diagram below.

Any 1 row correct [B1]

All rows correct [B2]

| Bag B \ Bag A | 1 | 2 | 3 |
|---------------|----|----|----|
| 4 | 14 | 24 | 34 |
| 5 | 15 | 25 | 35 |
| 7 | 17 | 27 | 37 |

[2]

Find, as a fraction in its simplest form, the probability that the two-digit number formed is

- (ii) odd,

$$\frac{2}{3} \text{ [B1]}$$

- (iii) a perfect square,

Answer [1]

$$P(\text{Number is a perfect square}) = P(\text{Number is 25})$$

$$= \frac{1}{9} \text{ [B1]}$$

[1]

- (iv) a prime number,

$$P(\text{Number is prime}) = P(\text{Number is 17, 37})$$

$$= \frac{2}{9} \text{ [B1]}$$

[1]

- (v) a factor of 135.

$$135 = 3 \times 3 \times 3 \times 5 \text{ [M1 - Prime factorisation of 135]}$$

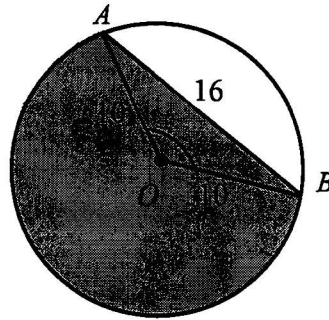
Factors of 135 are: 1, 3, 5, 9, 15, 27, 45, 135

$$P(\text{Factor of 135}) = P(15, 27)$$

$$= \frac{2}{9} \text{ [A1]}$$

Answer [2]

- 7 The diagram shows a circle, centre O , radius 10 cm. AB is a chord of length 16 cm.



- (a) Show that angle $AOB = 1.855$ radians, correct to four significant figures.

Answer

$$\begin{aligned}\sin \frac{\angle AOB}{2} &= \frac{8}{10} \\ \angle AOB &= 2 \sin^{-1} \frac{8}{10} && \text{[M1 – Any method]} \\ &= 1.8545 && \text{[A1]} \\ &= 1.855 \text{ rad (shown)} && \text{[AG]}\end{aligned}$$

[2]

Hence, find the

- (b) perimeter of the shaded segment,

$$\begin{aligned}\text{Perimeter} &= (2\pi - 1.8545)(10) + 16 && \text{[M1, M1]} \\ &= 60.28 \\ &= 60.3 \text{ cm (3 sig. fig.)} && \text{[A1]}\end{aligned}$$

Answer cm [3]

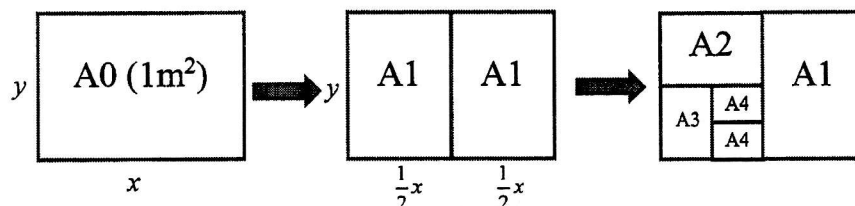
(c) area of the shaded segment.

$$\begin{aligned}
 \text{Area of shaded segment} &= \frac{1}{2}(10)^2(2\pi - 1.8545) + \frac{1}{2}(10)^2 \sin 1.8545 \\
 &\quad [\text{M1} - \text{Area of sector or Area of triangle}] \\
 &= 269.43 \\
 &= 269 \text{ cm}^2 \text{ (3 sig. fig.)} \quad [\text{A1}]
 \end{aligned}$$

Answer cm² [2]

- 8 A piece of A0-sized paper has a total surface area of 1 m^2 . The length and breadth of the A0-sized paper are $x \text{ m}$ and $y \text{ m}$ respectively.

An A1 paper is an A0 paper folded in half, length-wise. An A2 paper is an A1 paper folded in half, so on and so forth.



- (i) Write down the relationship between x and y .

$$xy = 1 \quad [\text{B1 oe}]$$

Answer [1]

- (ii) Given that all A-sized papers are geometrically similar, show that $\frac{x}{y} = \sqrt{2}$.

Since the papers are geometrically similar, ratio of corresponding sides are equal.

$$\begin{aligned} \frac{\text{length}_{A0}}{\text{breadth}_{A0}} &= \frac{\text{length}_{A1}}{\text{breadth}_{A1}} \\ \frac{x}{y} &= \frac{y}{\left(\frac{1}{2}x\right)} \quad [\text{M1 or similar}] \\ \frac{x^2}{y^2} &= 2 \quad [\text{A1}] \\ \frac{x}{y} &= \sqrt{2} \quad (\text{Shown}) \quad [\text{AG}] \end{aligned}$$

[2]

- (iii) Hence, using parts (i) and (ii), find the values of x and y .

$$\begin{aligned} xy &= 1 \\ y &= \frac{1}{x} \quad \text{--- (1)} \quad [\text{M1 - Making } x \text{ or } y \text{ the subject}] \\ \frac{x}{y} &= \sqrt{2} \quad \text{--- (2)} \\ \text{Substitute (1) into (2):} \\ \frac{x}{\frac{1}{x}} &= \sqrt{2} \quad [\text{M1 - Substitution}] \\ x^2 &= \sqrt{2} \\ x &= \sqrt{\sqrt{2}} \quad [\text{M1 - Solving}] \\ &= 1.1892.. \\ &= 1.19 \\ y &= 0.841 \quad (3 \text{ sig. fig}) \quad [\text{A1 - Both answers correct}] \end{aligned}$$

[4]

- (iv) Write down the dimensions of an A4-sized paper in millimetres.

$$\begin{aligned}
 x &= \frac{1.1892 \times 1000}{4} \\
 &= 297 \text{ (3 sig fig) [M1 – Converting to mm]} \\
 y &= \frac{0.84089 \times 1000}{4} \\
 &= 210 \text{ (3 sig fig) [A1 – Both correct] [B2 – Both correct]}
 \end{aligned}$$

Answer mm × mm [2]

- (v) Given that N is an integer, express the number of pieces of A_N -sized paper that can be cut out from an A_0 -sized paper in terms of N .

$$\text{Number of pieces} = 2^N \quad [\text{B1}]$$

Answer [1]

- 9 Robert has 3 options to get to school every weekday morning.

Option A: He takes the bus.

Option B: He takes the MRT.

Option C: He takes a taxi.

The matrices **A**, **B** and **C** represent his journey to school if he were to choose option **A**, **B** or **C** respectively.

$$\begin{array}{ccc} \text{Bus} & \text{MRT} & \text{Taxi} \\ \mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & \begin{array}{ccc} \text{Bus} & \text{MRT} & \text{Taxi} \\ \mathbf{B} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & \begin{array}{ccc} \text{Bus} & \text{MRT} & \text{Taxi} \\ \mathbf{C} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \end{array} \end{array}$$

- (a) In a particular week, Robert took the bus 3 times. He took the MRT and taxi in each of the other two days. The above information can be represented by a matrix **S** where $\mathbf{S} = 3\mathbf{A} + \mathbf{B} + \mathbf{C}$. Find **S**.

$$\begin{aligned} \mathbf{S} &= 3 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} & \text{[M1]} \\ &= \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} & \text{[A1]} \end{aligned}$$

Answer [2]

- (b) The table below shows the travelling time taken, in minutes, for each of the 3 options that Robert can take to get to school.

| Option | Bus | MRT | Taxi |
|-------------------------|-----|-----|------|
| Time taken (in minutes) | 30 | 20 | 15 |

- (i) Write down a column matrix **N** to represent the information in the table.

$$\mathbf{N} = \begin{pmatrix} 30 \\ 20 \\ 15 \end{pmatrix} \quad \text{[B1]}$$

Answer $\mathbf{N} = \dots\dots\dots$ [1]

- (ii) Evaluate the matrix SN .

$$SN = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 30 \\ 20 \\ 15 \end{pmatrix} = \begin{pmatrix} 3(30) \\ 1(20) \\ 1(15) \end{pmatrix} \quad [M1]$$

$$= \begin{pmatrix} 90 \\ 20 \\ 15 \end{pmatrix} \quad [A1]$$

Answer $SN = \dots\dots\dots$ [2]

- (iii) State what the elements in matrix SN represent.

Answer

The elements represent the total time taken in minutes, in that particular

 week by Robert to go to school by via option A , B and C respectively.

[1]

- (c) Write down a matrix J such that product of J and the answer in part (b)(ii) will give the average travelling time taken by Robert in a particular week.

$$J = \begin{pmatrix} 0.2 & 0.2 & 0.2 \end{pmatrix} \quad [B1 - \text{Accept fractions}]$$

Answer [1]

- (d) Hence, find the average travelling time taken by Robert per day in a particular week.

$$J(SN) = \begin{pmatrix} 0.2 & 0.2 & 0.2 \end{pmatrix} \begin{pmatrix} 90 \\ 20 \\ 15 \end{pmatrix} \quad [M1 - \text{FT but order must be correct}]$$

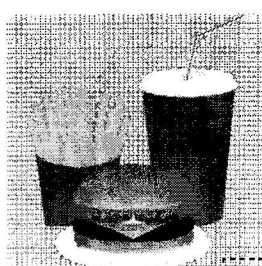
$$= (18 + 4 + 3)$$

$$= (25) \quad [B1 - \text{with brackets}]$$

Average time taken is 25 minutes

Answer minutes [2]

- 10 The diagram below shows the menu from a fast food restaurant.



Combo A

- Burger
- Medium Fries
- Medium Drink

\$5.90 only!

**Upsize your
drink and fries
to Large-size
for only \$1.10**

| Ala carte | Medium (\$) | Large (\$) |
|-----------------|-------------|------------|
| Burger (1 size) | 3.20 | |
| Fries | 2.20 | 2.90 |
| Drink | 2.80 | 3.60 |

- (a) Albert wants to buy a burger for both himself and his brother Bart. His brother wants at least a medium-sized fries. Albert wants a large drink while Bart wants a medium drink.

By considering **three options**, suggest how Albert should place his order.
Justify any decisions that you make and show your calculations clearly.

Answer

Choose 3 out of the 4 possible options

Option A: (Combo A + Upsize) + (Burger + Medium Drink)

Total cost = \$ 5.90 + 1.10 + 3.20 + 2.80

= \$13.00 [B1]

Option B: Combo A + Combo A + Upsize

Total cost = \$ 5.90 + 5.90 + 1.10

= \$12.90 [B1]

Option C: 2 × Burger + M.Fries + M.Drink + L.Drink

Total cost = \$2(3.20) + 2.20 + 2.80 + 3.60

= \$15.00 [B1]

Option D: ComboA + Burger + L.Drink

Total cost = \$ 5.90 + 3.20 + 3.60

= \$12.70 [B1]

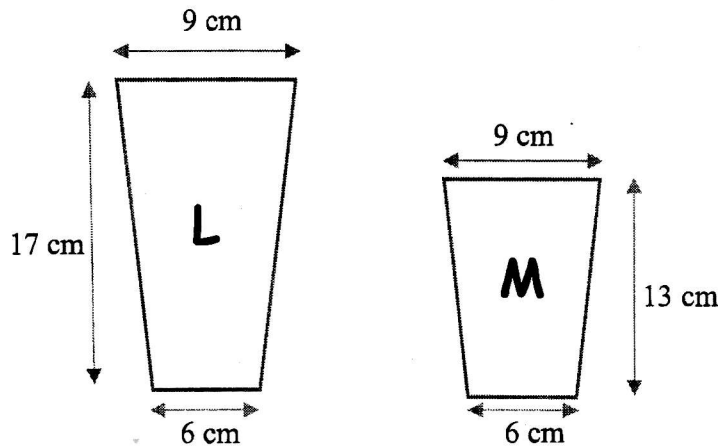
He should choose Option D to minimise his spending

[B1 – Correct Option with logical and reasonable explanation]

[4]

OR He should choose Option C as for \$0.20 more, he can get an extra packet of medium fries so it is the most value – for – money option.

- (b) The diagram below shows the cross-sectional area of the large (L) and medium (M) drink cups sold by the fast food restaurant in part (a). The Large and Medium cups have a height of 17 cm and 13 cm respectively. The top and bottom **diameters** of both cups are 9 cm and 6 cm respectively



The volume of a cup is given by $V = \frac{1}{3}\pi h(R^2 + Rr + r^2)$ where h is the height,

R and r are the radii of the top and bottom of the cup respectively. Using the information from part (a), determine whether the medium cup or large cup gives better value for money.

Answer

$$V_L = \frac{1}{3}\pi(17)\left[\left(\frac{9}{2}\right)^2 + \frac{9}{2}\left(\frac{6}{2}\right) + \left(\frac{6}{2}\right)^2\right]$$

$$= 242.25\pi \text{ cm}^3 \text{ or } 761 \text{ cm}^3 (3sf) \quad [\text{B1}]$$

$$V_M = \frac{1}{3}\pi(13)\left[\left(\frac{9}{2}\right)^2 + \frac{9}{2}\left(\frac{6}{2}\right) + \left(\frac{6}{2}\right)^2\right]$$

$$= 185.25\pi \text{ cm}^3 \text{ or } 582 \text{ cm}^3 (3sf) \quad [\text{B1}]$$

For L cup:

$$\text{Volume per \$1} = \frac{357\pi}{3.60}$$

$$= 3.11541... \text{ cm}^3$$

[M1 – Find cost/unit volume]

For M cup:

$$\text{Volume per \$} = \frac{273\pi}{2.80}$$

$$= 306.305 \text{ cm}^3$$

\therefore Large cup gives more value for money [A1]

OR

$$\frac{\text{Price of L}}{\text{Price of M}} = \frac{P_L}{P_M} = \frac{3.6}{2.8} = \frac{9}{7}$$

$$\frac{V_L}{V_M} = \frac{17}{13}$$

By comparing ratio:

$$\frac{P_L}{P_M} = \frac{9}{7} \times \frac{13}{13} = \frac{117}{91} \quad [\text{M1 – Comparing ratio}]$$

$$\frac{V_L}{V_M} = \frac{17}{13} \times \frac{7}{7} = \frac{119}{91}$$

$$\text{Since } \frac{V_L}{V_M} > \frac{P_L}{P_M},$$

buying a Large drink gives more value for money.

[A1 – Conclusion]