SINGAPORE CHINESE GIRLS' SCHOOL
PRELIMINARY EXAMINATION 2021
SECONDARY FOUR
O-LEVEL PROGRAMME

CANDIDATE NAME										
CLASS	4					REGISTER NUMBER				
CENTRE NUMBER	S					INDEX NUMBER				

ADDITIONAL MATHEMATICS **4049/01**

Paper 1

Monday **30 August 2021** **2 hours 15 minutes**

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class, register number, centre number and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid / tape.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved electronic scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

FOR EXAMINERS' USE

Q1		**Q5**		**Q9**	
Q2		**Q6**		**Q10**	
Q3		**Q7**		**Q11**	
Q4		**Q8**			

/ 90

The Question Paper consists of **19** printed pages and **3** blank pages.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1. (a) Determine the range of values of m for which the equation $2m^2 + 2m = -x^2 - 2mx + 3$ has real roots. [3]

(b) Show that the curve $y = x^2 + (2k-1)x + 1 + 2k^2$ is always positive for all real values of k. [4]

2. **(a)** Find the value of p and of q for which $4x^2 + 4x - 3$ is a factor of

$$x^2(4x+p)-(qx+3).$$ [4]

(b) With these values of p and q, solve the equation $x^2(4x+p)=(qx+3)$. [3]

3. **(a)** Given that $\dfrac{5^{4-\frac{x}{2}}}{625(5^y)} = \dfrac{25^y}{\sqrt{125^x}}$, find the value of $\dfrac{x}{y}$. [3]

(b) Solve the equation $x\log_5 3 + \log_{25} 9 = \log_5(3^{2x-1} + 6)$. [5]

4. The equation of a curve is $y = \sqrt{4 + \sin^2 x}$ for $0 < x < \pi$.

(i) Given that $y\frac{\mathrm{d}y}{\mathrm{d}x} = k\sin 2x$, where k is a constant, find the value of k. [4]

(ii) Find the set of values of x for which y is an increasing function. [2]

5. (a) Express $\dfrac{8x-3}{(4x-1)^2}$ in partial fractions. [3]

(b) It is given that f(x) is such that $f'(x)=\dfrac{8x-3}{(4x-1)^2}$ and $f\left(\dfrac{1}{2}\right)=\dfrac{3}{4}$.

Find f(x). [6]

6. A stolen car, Q, travelling on a straight road, passes a roadblock, O, with a velocity of 5 m/s and travels at a constant acceleration of 1 m/s². A police car, P, parked at O sets off immediately to intercept the stolen car Q. P starts from rest and moves with an acceleration of a m/s², where $a = 1 + \frac{t}{5}$ and t seconds is the time since leaving O.

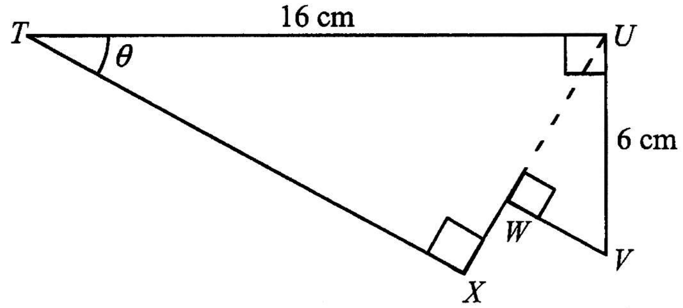
(i) Find the velocity of each car in terms of t. [3]

(ii) Explain clearly why the distance travelled by each car can be obtained by finding the displacement. [1]

(iii) Find the distance travelled by each car in terms of t. [3]

(iv) Hence find the distance from O at which P intercepts Q. [3]

7.



The diagram shows a pentagon *TUVWX* with three fixed points *T*, *U* and *V* such that *TU* = 16 cm, *UV* = 6 cm and angle *TUV* = 90°.
The lines *TX* and *VW* are perpendicular to the line *UX*. The angle θ can vary in such a way that the point *W* lies between the points *U* and *X*.

(i) Show that perimeter, *P* cm, of pentagon *TUVWX* is given by

$$P = 22 + 10\cos\theta + 22\sin\theta\,.$$

[3]

(ii) Express *P* in the form $22 + R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [4]

(iii) Explain why it is possible for the pentagon to have a perimeter of 45 cm. [1]

(iv) Find the values of the value of θ for which $P = 45$. [2]

8. A circle, C_1, passes through the points $A(0, 5)$ and $B(6, -7)$. Its centre lies on the line $2y + x + 3 = 0$.

(i) Find the equation of the circle, C_1. [6]

A second circle, C_2, is a reflection of the first circle along the x-axis.

(ii) Find the equation of the circle, C_2, in the form $x^2+y^2+gx+fy+c=0$ where g, f and c are integers. [1]

(iii) Write down the coordinates of the highest point of C_2. [1]

9. The table below shows experimental values of two variables x and y.

x	1.5	2.0	2.5	3.0
y	0.154	0.105	0.073	0.053

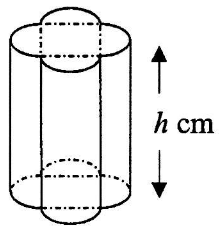
It is known that x and y are connected by an equation of the form $x = bx^3y + 2ay$ where a and b are constants.

Using suitable variables, this equation may be represented by a straight line graph with gradient b.

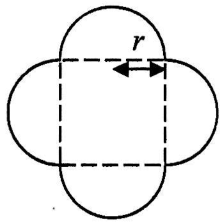
(i) On the grid on the next page, draw a straight line graph and use it to estimate the value of a and of b. [6]

(ii) Explain how the graph could be used to find the value of x when $\frac{x}{y} = 35$. [1]

10. The diagram shows a concrete slab with a vertical height of h cm. The slab has an uniform cross-section. The cross-section consists of 4 identical semicircles of radius of r cm attached to a square.



Concrete slab



Cross-section

(i) Given that the volume of the slab is 3200 cm^3, express h in terms of r. [2]

(ii) Show that the total surface area of the slab, A cm^2, is given by

$$A = 4(\pi + 2)r^2 + \frac{6400\pi}{r(\pi + 2)}.$$ [2]

Given that r can vary,

(iii) find the value of r for which A has a stationary value, [4]

(iv) determine whether this stationary value of A is a maximum or a minimum. [1]

11. On a particular day, the tide at a pier is modelled by the equation $y = -2\sin(bt) + 7$, where t is the number of hours past 06 00, y is the water level in metres and b is in radians per hour. The time between the successive low tides is 12 hours.

(i) Show that $b = \dfrac{\pi}{6}$ radians per hour. [1]

(ii) Find the time for the first high tide to occur after 06 00. [1]

(iii) Sketch the graph of $y = -2\sin(bt) + 7$ for $0 \leq t \leq 12$. [3]

The pier will be closed when the water level is less than 5.5 metres.

(iv) Find the duration, in hours, for which the pier remains closed in the first 12 hours. [4]

ANS

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Solution (Paper 1)

1. **(a)** Determine the range of values of m for which the equation $2m^2 + 2m = -x^2 - 2mx + 3$ has real roots. [3]

(b) Show that the curve $y = x^2 + (2k-1)x + 1 + 2k^2$ is always positive for all real values of k. [4]

1.	**(a)**	$2m^2 + 2m = -x^2 - 2mx + 3$ $x^2 + 2mx + 2m^2 + 2m - 3 = 0$ $(2m)^2 - 4(2m^2 + 2m - 3) \geq 0$ $-4m^2 - 8m + 12 \geq 0$ $m^2 + 2m - 3 \leq 0$ $(m+3)(m-1) \leq 0$ $-3 \leq m \leq 1$
	(b)	Discriminant $= (2k-1)^2 - 4(1+2k^2)$ $= -4k^2 - 4k - 3$ $= -4\left(k^2 + k + \frac{3}{4}\right)$ $= -4\left[\left(k + \frac{1}{2}\right)^2 + \frac{1}{2}\right]$ $= -4\left(k + \frac{1}{2}\right)^2 - 2$ Since $\left(k + \frac{1}{2}\right)^2 \geq 0$ for all real values of k, $-4\left(k + \frac{1}{2}\right)^2 - 2 \leq -2$ or $-4\left(k + \frac{1}{2}\right)^2 - 2 < 0$ $\therefore$ Since coefficient of $x^2 > 0$ and discriminant < 0, curve is always positive

2. **(a)** Find the value of p and of q for which $4x^2+4x-3$ is a factor of

$$x^2(4x+p)-(qx+3).$$ [4]

(b) With these values of p and q, solve the equation $x^2(4x+p)=(qx+3)$. [3]

---

2. **(a)**

$x^2(4x+p)-(qx+3)=\left(4x^2+4x-3\right)\left(x+1\right)$

Compare coefficient of x^2, $p=8$
Compare coefficient of x, $q=-1$

Alternative method

$4x^2+4x-3=(2x+3)(2x-1)$

Let $f(x)=x^2(4x+p)-(qx+3)$

$f\left(-\frac{3}{2}\right)=0$

$\left(-\frac{3}{2}\right)^2\left[4\left(-\frac{3}{2}\right)+p\right]-\left[q\left(-\frac{3}{2}\right)+3\right]=0$

$\frac{9}{4}(-6+p)+\frac{3}{2}q-3=0$

$3p+2q=22$ ......(1)

$f\left(\frac{1}{2}\right)=0$

$\left(\frac{1}{2}\right)^2\left[4\left(\frac{1}{2}\right)+p\right]-\left[q\left(\frac{1}{2}\right)+3\right]=0$

$\frac{1}{4}(2+p)-\frac{q}{2}-3=0$

$p-2q=10$ ......(2)

(1)+(2) $\quad 4p=32$

$p=8$

$q=-1$

(b)

$(4x^2+4x-3)(x+1)=0$

$(2x-1)(2x+3)(x+1)=0$

$x=\frac{1}{2},\ -\frac{3}{2}$ or -1

(7 marks)

3. **(a)** Given that $\dfrac{5^{4-\frac{x}{2}}}{625(5^y)} = \dfrac{25^y}{\sqrt{125^x}}$, find the value of $\dfrac{x}{y}$. [3]

(b) Solve the equation $x\log_5 3 + \log_{25} 9 = \log_5(3^{2x-1}+6)$. [5]

3.	**(a)**	$\dfrac{5^{4-\frac{x}{2}}}{625(5^y)} = \dfrac{25^y}{\sqrt{125^x}}$ $5^{4-\frac{x}{2}}.5^{\frac{3x}{2}} = 5^{2y}.5^{4+y}$ $4+x=3y+4$ $\dfrac{x}{y}=3$
	(b)	$x\log_5 3 + \log_{25} 9 = \log_5(3^{2x-1}+6)$ $\log_5 3^x + \dfrac{\log_5 9}{\log_5 25} = \log_5(3^{2x-1}+6)$ $\log_5 3^x + \dfrac{1}{2}\log_5 9 = \log_5(3^{2x-1}+6)$ $\log_5 3^{x+1} = \log_5(3^{2x-1}+6)$ $3^{x+1} = 3^{2x-1}+6$ $3^{2x} - 9(3^x) + 18 = 0$ $(3^x-6)(3^x-3)=0$ $x = \dfrac{\lg 6}{\lg 3}$ or 1 $= 1.63$ or 1
		(8 marks)

4. The equation of a curve is $y = \sqrt{4 + \sin^2 x}$ for $0 < x < \pi$.

(i) Given that $y\frac{dy}{dx} = k\sin 2x$, where k is a constant, find the value of k. [4]

(ii) Find the set of values of x for which y is an increasing function. [2]

4.	**(i)**	$\frac{dy}{dx} = \frac{1}{2}(4+\sin^2 x)^{-\frac{1}{2}}(2\sin x\cos x)$ $= \frac{1}{2\sqrt{4+\sin^2 x}} \times (\sin 2x)$ $= \frac{1}{2y}(\sin 2x)$ $k = \frac{1}{2}$
	(ii)	$\frac{dy}{dx} > 0 \Rightarrow \frac{\sin 2x}{2\sqrt{4+\sin^2 x}} > 0$ Since $\sqrt{4+\sin^2 x} > 0$ for all x (or $0 < x < \pi$), $\sin 2x > 0$ $0 < 2x < \pi$ $0 < x < \frac{\pi}{2}$ **(6 marks)**

5. **(a)** Express $\dfrac{8x-3}{(4x-1)^2}$ in partial fractions. [3]

(b) It is given that f(x) is such that $f'(x)=\dfrac{8x-3}{(4x-1)^2}$ and $f\left(\dfrac{1}{2}\right)=\dfrac{3}{4}$.

Find f(x). [6]

5.	**(a)**	Let $\dfrac{8x-3}{(4x-1)^2}=\dfrac{A}{4x-1}+\dfrac{B}{(4x-1)^2}$ $\dfrac{8x-3}{(4x-1)^2}=\dfrac{A(4x-1)+B}{(4x-1)^2}$ $8x-3=A(4x-1)+B$ Comparing coefficient of x, $4A=8$ $A=2$ Comparing constant term, $-A+B=-3$ $B=-1$ $\dfrac{8x-3}{(4x-1)^2}=\dfrac{2}{4x-1}-\dfrac{1}{(4x-1)^2}$ **Alternative Method** $\dfrac{8x-3}{(4x-1)^2}=\dfrac{8x-2-1}{(4x-1)^2}$ $=\dfrac{2(4x-1)-1}{(4x-1)^2}$ $=\dfrac{2}{4x-1}-\dfrac{1}{(4x-1)^2}$
	(b)	$f(x)=\int\dfrac{8x-3}{(4x-1)^2}\,dx$ $=\int\left[\dfrac{2}{4x-1}-\dfrac{1}{(4x-1)^2}\right]dx$ $=\dfrac{2\ln(4x-1)}{4}-\dfrac{(4x-1)^{-1}}{(-1)(4)}+C$ $=\dfrac{1}{2}\ln(4x-1)+\dfrac{1}{4(4x-1)}+C$ $f\left(\dfrac{1}{2}\right)=\dfrac{3}{4}$ $\dfrac{1}{2}\ln\left[4\left(\dfrac{1}{2}\right)-1\right]+\dfrac{1}{4\left[4\left(\dfrac{1}{2}\right)-1\right]}+C=\dfrac{3}{4}$ $C=\dfrac{1}{2}$ $f(x)=\dfrac{1}{2}\ln(4x-1)+\dfrac{1}{4(4x-1)}+\dfrac{1}{2}$ **(6 marks)**

6. A stolen car, Q, travelling on a straight road, passes a roadblock, O, with a velocity of 5 m/s and travels at a constant acceleration of 1 m/s^2. A police car, P, parked at O sets off immediately to intercept the stolen car Q. P starts from rest and moves with an acceleration of a m/s^2, where $a = 1 + \frac{t}{5}$ and t seconds is the time since leaving O.

(i) Find the velocity of each car in terms of t. [3]

(ii) Explain clearly why the distance travelled by each car can be obtained by finding the displacement. [1]

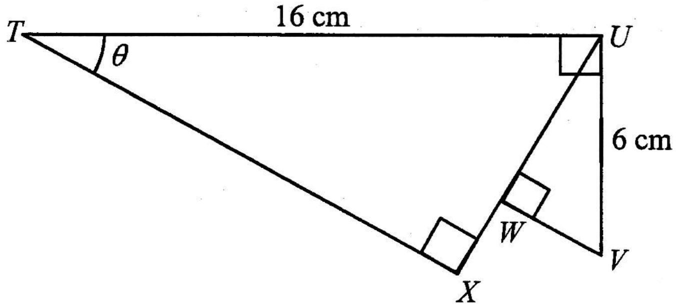
(iii) Find the distance travelled by each car in terms of t. [3]

(iv) Hence find the distance from O at which P intercepts Q. [3]

6.	**(i)**	$V_Q = 5 + t$ $V_P = \int \left(1 + \frac{t}{5}\right) dt$ $= t + \frac{t^2}{10} + C$ When $t = 0$, $V_P = 0$, $C = 0$ $\therefore V_P = t + \frac{t^2}{10}$
	(ii)	Since $t > 0$, $V_P = t + \frac{t^2}{10} > 0$ $V_Q = 5 + t > 0$ Since there is no change in direction of motion for both cars, the distance travelled takes the same value as the displacement of each car.
	(iii)	$S_Q = \int (5 + t) dt$ $= 5t + \frac{t^2}{2} + C$ When $t = 0$, $S_Q = 0$, $C = 0$ $\therefore S_Q = 5t + \frac{t^2}{2}$ $S_P = \int \left(t + \frac{t^2}{10}\right) dt$ $= \frac{t^2}{2} + \frac{t^3}{30} + C$ When $t = 0$, $S_P = 0$, $C = 0$ $\therefore S_P = \frac{t^2}{2} + \frac{t^3}{30}$

(iv)	$S_P = S_Q$ $\frac{t^2}{2}+\frac{t^3}{30}=5t+\frac{t^2}{2}$ $\frac{t^3}{30}=5t$ $t^3-150t=0$ $t(t^2-150)=0$ $t=0$ or $t=5\sqrt{6}$ $t>0,\ t=5\sqrt{6}$ or 12.2 Distance from O at which P intercept Q $=\frac{(5\sqrt{6})^2}{2}+\frac{(5\sqrt{6})^3}{30}$ $=136$ m **(10 marks)**

7.



The diagram shows a pentagon *TUVWX* with three fixed points *T*, *U* and *V* such that *TU* = 16 cm, *UV* = 6 cm and angle *TUV* = 90°.
The lines *TX* and *VW* are perpendicular to the line *UX*. The angle θ can vary in such a way that the point *W* lies between the points *U* and *X*.

(i) Show that perimeter, *P* cm, of pentagon *TUVWX* is given by
$$P = 22 + 10\cos\theta + 22\sin\theta .$$ [3]

(ii) Express *P* in the form 22 + *R* sin (θ + α), where *R* > 0 and 0° < α < 90°. [4]

(iii) Explain why it is possible for the pentagon to have a perimeter of 45 cm. [1]

(iv) Find the value of the values of θ for which *P* = 45. [2]

(i)	$TX = 16\cos\theta$ $WU = 16\sin\theta$ $WU = 6\cos\theta$ $WV = 6\sin\theta$ $P = 16 + 6 + 6\sin\theta + (16\sin\theta - 6\cos\theta) + 16\cos\theta$ $= 22 + 10\cos\theta + 22\sin\theta$	
(ii)	Let $22\sin\theta + 10\cos\theta = R\sin(\theta + \alpha)$ $R = \sqrt{22^2 + 10^2}$ $= \sqrt{584}$ or $2\sqrt{146}$ or 24.2 $\tan\alpha = \dfrac{10}{22}$ $\alpha = 24.443°$ $P = 22 + \sqrt{584}\sin(\theta + 24.4°)$ Or $22 + 2\sqrt{146}\sin(\theta + 24.4°)$ Or $22 + 24.2\sin(\theta + 24.4°)$	
(iii)	A perimeter of 45 cm is shorter than the maximum possible perimeter of 46.2. Hence it is possible for *P* to be 45 cm. When *P* = 45, $22 + \sqrt{584}\sin(\theta + 24.443°) = 45$ $\sqrt{584}\sin(\theta + 24.443°) = 23$ $\sin(\theta + 24.443°) = \dfrac{23}{\sqrt{584}}$ $\theta + 24.443° = 72.128°,\ 107.872°$ $\theta = 47.7°,\ 83.4°$	

8. A circle, C_1, passes through the points $A(0, 5)$ and $B(6, -7)$. Its centre lies on the line $2y+x+3=0$.

(i) Find the equation of the circle, C_1. [6]

A second circle, C_2, is a reflection of the first circle along the x-axis.

(ii) Find the equation of the circle, C_2, in the form $x^2+y^2+gx+fy+c=0$ where f, g and c are integers. [1]

(iii) Write down the coordinates of the highest point of C_2. [1]

(i)	Gradient of $AB = \dfrac{-7-(5)}{6-0}$ $= -2$ Gradient of perpendicular bisector of $AB = \dfrac{1}{2}$ midpoint $= \left(\dfrac{0+6}{2}, \dfrac{5-7}{2}\right)$ $= (3, -1)$ Equation of perpendicular bisector of AB: $y+1 = \dfrac{1}{2}(x-3)$ $y = \dfrac{x}{2} - \dfrac{5}{2}$ ----(1) $y = -\dfrac{x}{2} - \dfrac{3}{2}$ ----(2) $x = 1,\ y = -2$ $\therefore$ Centre of $C_1 = (1,\ -2)$ $r = \sqrt{1^2 + (-2-5)^2}$ $= 5\sqrt{2}$ or $\sqrt{50}$ or 6.08 units Equation of C_1, $(x-1)^2 + (y+2)^2 = 50$ or $x^2 + y^2 - 2x + 4y - 45 = 0$	
(ii)	Centre of image (1, 2) Equation of C_1, $x^2 + y^2 - 2x - 4y - 45 = 0$	
(iii)	$(1,\ 2+5\sqrt{2})$ or $(1,\ 2+\sqrt{50})$	

9. The table below shows experimental values of two variables x and y.

x	1.5	2.0	2.5	3.0
y	0.154	0.105	0.073	0.053

It is known that x and y are connected by an equation of the form $x = bx^3y + 2ay$ where a and b are constants.

Using suitable variables, this equation may be represented by a straight line graph with gradient b.

(i) Plot a straight line graph and use it to estimate the value of a and of b. [6]

(ii) Explain how the graph could be used to find the value of x when $\frac{x}{y} = 35$. [1]

(i)

$bx^3 = x - 2ay$

$\Rightarrow \frac{x}{y} = bx^3 + 2a$

x^3	3.38	8.00	15.60	27.00
$\frac{x}{y}$	9.74	19.00	34.20	56.60

Gradient$= \frac{40}{20} = 2$

$b = 2$ (accept 1.85 to 2.15)

Vertical intercept = 3

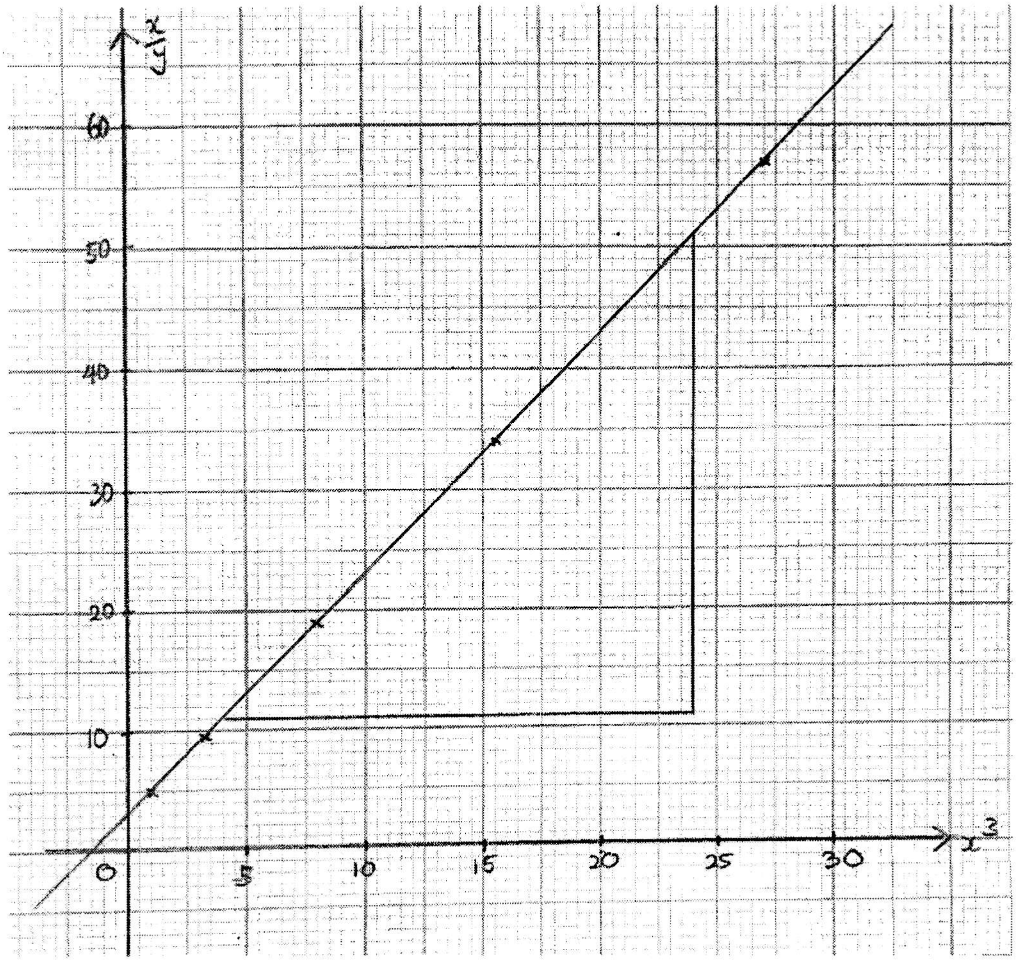
$2a = 3$

$a = 1.5$ (accept 1.25 to 1.75)

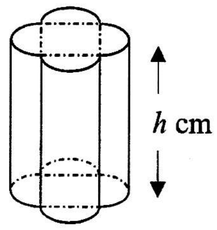
(ii)

When $\frac{x}{y} = 35$, read from the vertical axis at 35 and obtain the corresponding x^3 value from the straight line , which is 16.

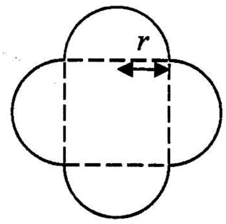
$x = \sqrt[3]{16} = 2.52$


y/x
60
50
40
30
20
10
0
5
10
15
20
25
30
x^3

10. The diagram shows a concrete slab with a vertical height of h cm. The slab has an uniform cross-section. The cross-section consists of 4 identical semicircles of radius of r cm attached to a square.



Concrete slab



Cross-section

(i) Given that the volume of the slab is 3200 cm^3, express h in terms of r. [2]

(ii) Show that the total surface area of the slab, A cm^2, is given by

$$A = 2(\pi + 2)r^2 + \frac{6400\pi}{r(\pi + 2)}.$$ [2]

Given that r can vary,

(iii) find the value of r for which A has a stationary value. [4]

(iv) determine whether this stationary value of A is a maximum or a minimum. [1]

(i)

Volume = 3200 cm^3

$2\pi r^2 h + 4r^2 h = 3200$

$r^2 h(\pi + 2) = 1600$

$$h = \frac{1600}{r^2(\pi + 2)}$$

$$A = 4(2\pi r^2) + 2(4r^2) + 2(2\pi r h)$$

$$= 4\pi r^2 + 8r^2 + 4\pi r\left[\frac{1600}{r^2(\pi + 2)}\right]$$

$$= 4(\pi + 2)r^2 + \frac{6400\pi}{r(\pi + 2)}$$

(ii)

$$\frac{dA}{dr} = 8(\pi + 2)r - \frac{6400\pi}{r^2(\pi + 2)}$$

When $\frac{dA}{dr} = 0$, $8(\pi + 2)r - \frac{6400\pi}{r^2(\pi + 2)} = 0$

$$8(\pi + 2)r = \frac{6400\pi}{r^2(\pi + 2)}$$

$$r^3 = \frac{800\pi}{(\pi + 2)^2}$$

$$r = \sqrt[3]{\frac{800\pi}{(\pi + 2)^2}}$$

$$r = 4.56$$

(iii)

$$\frac{d^2A}{dr^2} = 8(\pi + 2) + \frac{12800\pi}{r^3(\pi + 2)} > 0$$

A is a minimum value.

(10 marks)

11. On a particular day, the tide at a pier is modelled by the equation $y = -2\sin(bt) + 7$, where t is the number of hours past 06 00, y is the water level in metres and b is in radians per hour. The time between the successive low tides is 12 hours.

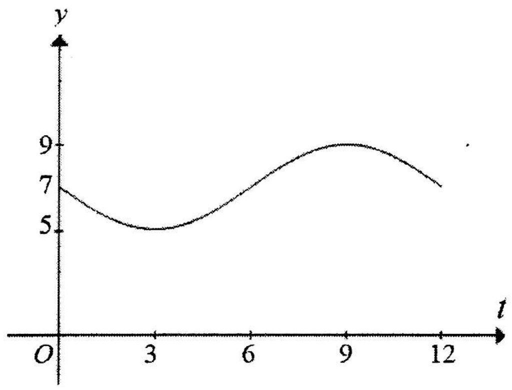
(i) Show that $b = \frac{\pi}{6}$ radians per hour. [1]

(ii) Find the time for the first high tide to occur after 06 00. [1]

(iii) Sketch the graph of $y = -2\sin(bt) + 7$ for $0 \le t \le 12$. [3]

The pier will be closed when the water level is less than 5.5 metres.

(iv) Find the duration, in hours, for which the pier remains closed in the first 12 hours. [4]

(i)	Period = 12 h $b = \frac{2\pi}{12} = \frac{\pi}{6}$
(ii)	15 00
(iii)	
(iv)	$-2\sin\left(\frac{\pi}{6}t\right)+7<5.5$ or $-2\sin\left(\frac{\pi}{6}t\right)+7=5.5$ $\sin\left(\frac{\pi}{6}t\right)<\frac{3}{4}$ Let $\sin\left(\frac{\pi}{6}t\right)=0.75$ Basic angle = 0.84806 $\frac{\pi}{6}t = 0.84806, 2.2935$ $t = 1.6196, 4.3802$ Duration = $4.3802 - 1.6196$ $= 2.76$ hours **(9 marks)**

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ADDITIONAL MATHEMATICS **4049/02**

Paper 2

Tuesday **31 August 2021** **2 hours 15 minutes**

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class, register number, centre number and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid / tape.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved electronic scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

FOR EXAMINERS' USE

Q1		**Q5**		**Q9**		**Q13**	
Q2		**Q6**		**Q10**			
Q3		**Q7**		**Q11**			/90
Q4		**Q8**		**Q12**			

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1. The value of a stock portfolio is given by the function $y = -\frac{1}{2}x^2 + 4x + 1$, where y is the value of the portfolio in thousands of dollars and x is the time in years from 2020.

(i) Write down the initial value of the stock portfolio. [1]

(ii) Express y in the form $a(x-h)^2 + k$. [2]

(iii) Determine the best time to sell off the stocks to make a maximum profit. Find its value at this time. [2]

2. **(i)** In January 2018, Mr Lee bought an antique for $2500. It was believed that the value of the antique would increase continuously with time such that it doubles after every 5 years. Explain why the value of the antique after n years is given by $2500(2^{0.2n})$. [2]

(ii) Find the year at which the value of the antique appreciates to $15 000. [3]

3. If p and q are the roots of the equation $x^2+2x-1=0$ and $p>q$, express $\frac{q}{p^2}$ in the form $a+b\sqrt{2}$, where a and b are integers. [4]

4. **(a)** State the principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$. [1]

(b) Solve, for $0 < x < 2\pi$, the equation $2\sin x = \tan x$. [4]

5. Given that $\sin A = \dfrac{8}{17}$ and $\tan B = -\sqrt{3}$, where A is an acute angle and $0 < B < \pi$, find the exact value of

(i) $\cos(\pi - B)$, [1]

(ii) $\cos A \sin B$, [2]

(iii) $\sin\left(\dfrac{\pi}{2} - B\right) + \tan\left(\dfrac{\pi}{2} - A\right)$. [2]

6. **(i)** Given that $\dfrac{\sin(A+B)}{\sin(A-B)} = \dfrac{1}{5}$, prove that $2\tan A + 3\tan B = 0$. [3]

(ii) Hence, solve the equation $5\sin(A+20°) = \sin(A-20°)$ for $-90° \le A \le 360°$. [3]

7. **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}(x\sin 2x - x^2) = \sin 2x - 4x\sin^2 x$. [3]

(b) Hence, find the value of each of the constants p and q for which

$$\int_0^{\frac{\pi}{3}} 36x\sin^2 x\,\mathrm{d}x = p + q\pi\sqrt{3} + \pi^2.$$ [7]

8. In the expansion of $\left(3x^3 - \dfrac{1}{2x}\right)^n$ in descending powers of x, where n is a positive integer, the seventh term of the expansion is a constant term.

(i) By considering the general term of the expansion of $\left(3x^3 - \dfrac{1}{2x}\right)^n$, show that n is 8. [3]

(ii) Using your answer in **(i)**, explain why there is a $\dfrac{1}{x^3}$ term in the expansion of

$(1-2x)\left(3x^3 - \dfrac{1}{2x}\right)^n$. [3]

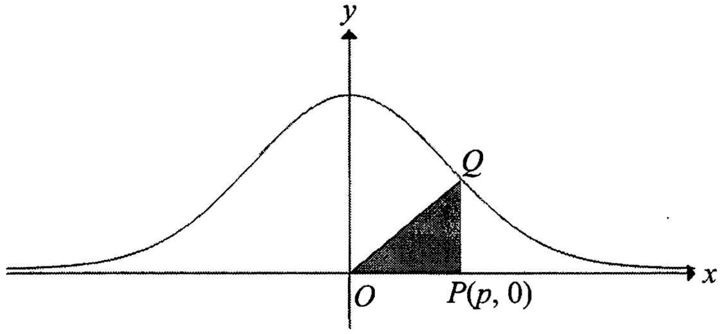
(iii) Hence, find the coefficient of $\dfrac{1}{x^3}$ in the expansion $(1-2x)\left(3x^3-\dfrac{1}{2x}\right)^n$. [2]

9. The function f is defined by $\mathrm{f}(x) = (x+2)\left(\frac{x}{3}-1\right)^3$.

(i) Find the coordinates of the stationary points of the curve. [5]

(ii) By considering the sign of $f'(x)$, determine the nature of the stationary points. Hence write down the range of values of x for which for which $f(x)$ is a decreasing function. [4]

10. The diagram shows part of the graph of $y = 4e^{-x^2}$. The point $P(p, 0)$, where $p > 0$, is on the x-axis and the point Q is on the curve.

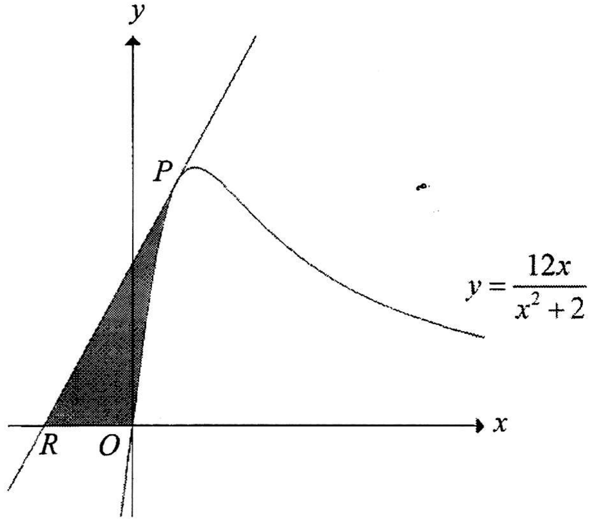


(i) Given that PQ is parallel to the y-axis, express the area, A, of the triangle OPQ in terms of p. [2]

(ii) If A is increasing at a rate of 0.8 units^2 per second, find the rate at which p is increasing at the instant when $p = 0.5$. [4]

11. **(a)** Find the derivative of $\ln(x^2 + 2)$. [2]

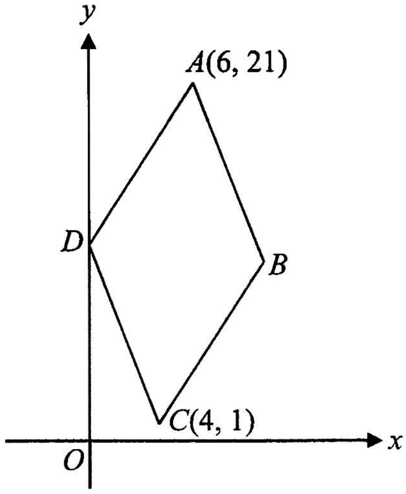
(b) The diagram shows part of the curve $y = \dfrac{12x}{x^2+2}$. The tangent at the point $P(1, 4)$ meets the x-axis at R.



(i) Show that the x-coordinate of R is -2. [7]

(ii) Find the area of the shaded region bounded by the tangent PR, the x-axis and the curve. [4]

12. Solutions to this question by accurate drawing will not be accepted.



The diagram shows a rhombus $ABCD$. The point D lies on the y-axis. A and C are the points (6, 21) and (4, 1) respectively.

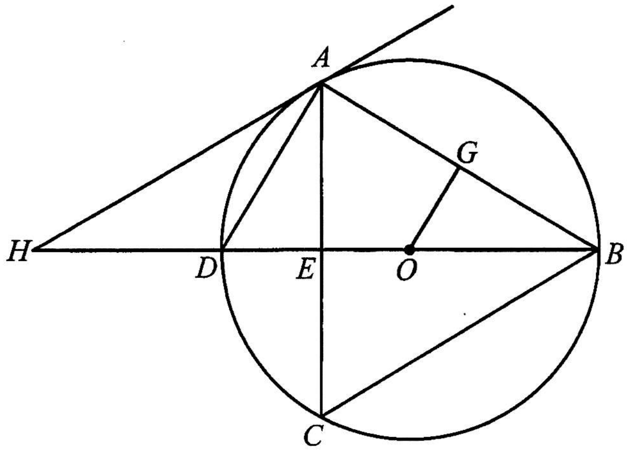
(i) Show that the coordinates of D are (0, 11.5). [4]

(ii) Find the area of the rhombus *ABCD*. [2]

A point *N* lies on *AC* produced such that the area of triangle *AND* is three times the area of triangle *ACD*.

(iii) Find the coordinates of *N*. [2]

13.



The diagram shows a circle, centre *O*, with points *A*, *B*, *C* and *D* lie on the circle.
HA is a tangent to the circle.
D and *G* are mid-points of *HB* and *AB* respectively.
AD is the angle bisector of angle *CAH*.

(a) Prove that *OG* is perpendicular to *AB*, [3]

(b) Prove that angle *ABD* = angle *CBD*, [3] ANS

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ADDITIONAL MATHEMATICS **4049/02**

Solution (Paper 2)

1. The value of a stock portfolio is given by the function $y=-\frac{1}{2}x^2+4x+1$, where y is the value of the portfolio in thousands of dollars and x is the time in years from 2020.

(i) Write down the initial value of the stock portfolio. [1]

(ii) Express y in the form $a(x-h)^2+k$. [2]

(iii) Determine the best time to sell off the stocks to make a maximum profit. Find its value at this time. [2]

1.	**(i)**	\$1000
	(ii)	$y=-\frac{1}{2}\left[x^2-8x-2\right]$ $=-\frac{1}{2}\left[(x-4)^2-4^2-2\right]$ $=-\frac{1}{2}\left[(x-4)^2-18\right]$ $=-\frac{1}{2}(x-4)^2+9$
	(iii)	4 years or in 2024 \$9000 **(5 marks)**

2. **(i)** In January 2018, Mr Lee bought an antique for $2500. It was believed that the value of the antique would increase continuously with time such that it doubles after every 5 years. Explain why the value of the antique after n years is given by $2500(2^{0.2n})$. [2]

(ii) Find the year at which the value of the antique appreciates to $15 000. [3]

2.	**(i)**	When $n = 0$, value $= 2500$ When $n = 5$, value $= 2500 \times 2^1$ When $n = 10$, value $= 2500 \times 2^2$ When $n = 15$, value $= 2500 \times 2^3$ $\therefore$ value $= 2500 \times 2^{\frac{n}{5}} = 2500 \times 2^{0.2n}$
	(ii)	$2500(2^{0.2n}) = 15\ 000$ $2^{0.2n} = 6$ $n = \dfrac{\lg 6}{0.2 \lg 2}$ $= 12.924$ Year = 2018 + 12 = 2030 **(5 marks)**

3. If p and q are roots of the equation $x^2+2x-1=0$ and $p>q$, express $\frac{q}{p^2}$ in the form $a+b\sqrt{2}$ where a and b are integers. [4]

3.	$x^2+2x-1=0$ $x=\frac{-2\pm\sqrt{4-4(-1)}}{2}$ $x=\frac{-2\pm 2\sqrt{2}}{2}$ $x=-1\pm\sqrt{2}$ $\frac{q}{p^2}=\frac{-1-\sqrt{2}}{(-1+\sqrt{2})(-1+\sqrt{2})}$ $=\frac{-1-\sqrt{2}}{3-2\sqrt{2}}\times\frac{3+2\sqrt{2}}{3+2\sqrt{2}}$ $=-7-5\sqrt{2}$ **(4 marks)**

4. (a) State the principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$. [1]

(b) Solve, for $0<x<2\pi$, the equation $2\sin x=\tan x$. [4]

4.	**(a)**	$-60°$ or $-\frac{\pi}{3}$
	(b)	$2\sin x=\tan x$ $2\sin x-\frac{\sin x}{\cos x}=0$ $\sin x\left(2-\frac{1}{\cos x}\right)=0$ $\sin x=0$ or $\cos x=\frac{1}{2}$ $x=\pi$ (or 3.14) or $x=\frac{\pi}{3}, \frac{5\pi}{3}$ (or 1.05, 5.24) **(4 marks)**

5. Given that $\sin A = \frac{8}{17}$ and $\tan B = -\sqrt{3}$, where A is an acute angle and $0 < B < \pi$, find the exact value of

(i) $\cos(\pi - B)$, [1]

(ii) $\cos A \sin B$, [2]

(iii) $\sin\left(\frac{\pi}{2} - B\right) + \tan\left(\frac{\pi}{2} - A\right)$. [2]

5. (i)	$\cos(\pi - B) = -\cos B = \frac{1}{2}$
(ii)	$\cos A \sin B$ $= \frac{15}{17} \times \frac{\sqrt{3}}{2}$ $= \frac{15\sqrt{3}}{34}$
(iii)	$\sin\left(\frac{\pi}{2} - B\right) + \tan\left(\frac{\pi}{2} - A\right)$ $= \cos B + \cot A$ $= -\frac{1}{2} + \frac{15}{8}$ $= \frac{11}{8}$ **(5 marks)**

6. **(i)** Given that $\frac{\sin(A+B)}{\sin(A-B)}=\frac{1}{5}$, prove that $2\tan A+3\tan B=0$. [3]

(ii) Hence, solve the equation $5\sin(A+20°)=\sin(A-20°)$ for $-90° \leq A \leq 360°$. [3]

6.	**(i)**	$\frac{\sin(A+B)}{\sin(A-B)}=\frac{1}{5}$ $\frac{\sin A\cos B+\cos A\sin B}{\sin A\cos B-\cos A\sin B}=\frac{1}{5}$ $5\sin A\cos B+5\cos A\sin B=\sin A\cos B-\cos A\sin B$ $4\sin A\cos B=-6\cos A\sin B$ $2\sin A\cos B=-3\cos A\sin B$ $2\tan A=-3\tan B$ $2\tan A+3\tan B=0$
	(ii)	$5\sin(A+20°)=\sin(A-20°)$ $\frac{\sin(A+20°)}{\sin(A-20°)}=\frac{1}{5}$ $2\tan A=-3\tan 20°$ $\tan A=\frac{-3}{2}\tan 20°$ Basic angle = 28.632° $A=-28.6°,\ 151.4°,\ 331.4°$ **(5 marks)**

7. **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}(x\sin 2x - x^2) = \sin 2x - 4x\sin^2 x$. [3]

(ii) Hence, find the value of each of the constants p and q for which

$$\int_0^{\frac{\pi}{3}} 36x\sin^2 x\mathrm{d}x = p + q\pi\sqrt{3} + \pi^2.$$ [7]

7.	**(i)**	$\frac{\mathrm{d}}{\mathrm{d}x}(x\sin 2x - x^2) = \sin 2x + 2x\cos 2x - 2x$ $= \sin 2x + 2x(1 - 2\sin^2 x) - 2x$ $= \sin 2x - 4x\sin^2 x$
	(ii)	$\int_0^{\frac{\pi}{3}} (\sin 2x - 4x\sin^2 x)\mathrm{d}x = \left[x\sin 2x - x^2\right]_0^{\frac{\pi}{3}}$ $\left[-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{3}} - \int_0^{\frac{\pi}{3}} 4x\sin^2 x\mathrm{d}x = \left[x\sin 2x - x^2\right]_0^{\frac{\pi}{3}}$ $\int_0^{\frac{\pi}{3}} 4x\sin^2 x\mathrm{d}x$ $= \left[-\frac{1}{2}\cos 2x - x\sin 2x + x^2\right]_0^{\frac{\pi}{3}}$ $= \left[-\frac{1}{2}\cos\frac{2\pi}{3} - \frac{\pi}{3}\sin\frac{2\pi}{3} + \left(\frac{\pi}{3}\right)^2\right] - \left(-\frac{1}{2}\cos 0\right)$ $= \left[-\frac{1}{2}\left(-\frac{1}{2}\right) - \frac{\pi}{3}\left(\frac{\sqrt{3}}{2}\right) + \frac{\pi^2}{9}\right] - \left(-\frac{1}{2}\right)$ $= \frac{3}{4} - \frac{\pi\sqrt{3}}{6} + \frac{\pi^2}{9}$ $\int_0^{\frac{\pi}{3}} 36x\sin^2 x\mathrm{d}x = \frac{27}{4} - \frac{3\pi\sqrt{3}}{2} + \pi^2$ $p = \frac{27}{4}$ $q = -\frac{3}{2}$ **(10 marks)**

8. In the expansion of $\left(3x^3 - \frac{1}{2x}\right)^n$ in descending powers of x, where n is a positive integer, the seventh term of the expansion is a constant term.

(i) By considering the general term of the expansion of $\left(3x^3 - \frac{1}{2x}\right)^n$, show that n is 8. [3]

(ii) Using your answer in **(i)**, explain why there is a $\frac{1}{x^3}$ term in the expansion of $(1-2x)\left(3x^3 - \frac{1}{2x}\right)^n$. [3]

(iii) Hence, find the coefficient of $\frac{1}{x^3}$ in the expansion $(1-2x)\left(3x^3 - \frac{1}{2x}\right)^n$. [2]

8.	**(i)**	$T_7 = \binom{n}{6}(3x^3)^{n-6}\left(-\frac{1}{2x}\right)^6$ $3n - 24 = 0$ $n = 8$
	(ii)	$T_{r+1} = \binom{8}{r}3^{8-r}\left(-\frac{1}{2}\right)^r x^{24-4r}$ For $\frac{1}{x^3}$ to exist in $(1-2x)\left(3x^3 - \frac{1}{2x}\right)^n$, $24 - 4r = -3$ or $24 - 4r = -4$ $r = 6\frac{3}{4}$ $r = 7$ Since one of the r values is a positive integer, $\frac{1}{x^3}$ exists.
	(iii)	Coefficient of $\frac{1}{x^3}$ $= -2\binom{8}{7}3\left(-\frac{1}{2}\right)^7$ $= \frac{3}{8}$ **(8 marks)**

9. The function f is defined by $f(x)=(x+2)\left(\frac{x}{3}-1\right)^3$.

(i) Find the coordinates of the stationary points of the curve. [5]

(ii) By considering the sign of $f'(x)$, determine the nature of the stationary points. Hence write down the range of values of x for which for which f(x) is a decreasing function. [4]

9. (i)

$$f'(x) = \left(\frac{x}{3}-1\right)^3 + 3(x+2)\left(\frac{x}{3}-1\right)^2\left(\frac{1}{3}\right)$$

$$= \left(\frac{x}{3}-1\right)^3 + (x+2)\left(\frac{x}{3}-1\right)^2$$

Or $\frac{1}{3}\left(\frac{x}{3}-1\right)^2(4x+3)$

When $f'(x)=0$, $\left(\frac{x}{3}-1\right)^2=0$, $4x+3=0$

$x=3$ $\quad x=-\frac{3}{4}$

$y=0$ $\quad y=-\frac{625}{256}$

The two points are (3, 0) and $\left(-\frac{3}{4}, -\frac{625}{256}\right)$

(ii)

x	$\left(-\frac{3}{4}\right)^-$	$\left(-\frac{3}{4}\right)$	$\left(-\frac{3}{4}\right)^+$	3^-	3	3^+
$f'(x)$	–	0	+	+	0	+
slope	\	–	/	/	–	/

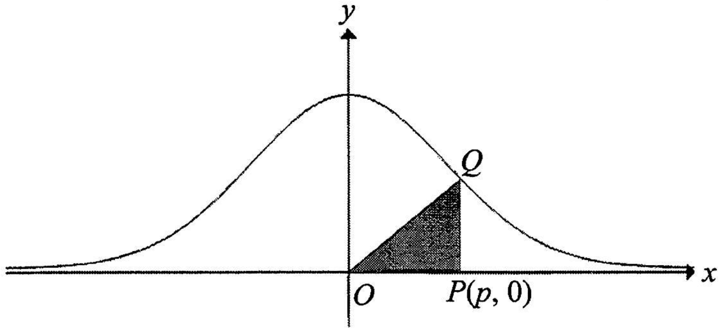
$\left(-\frac{3}{4}, -\frac{625}{256}\right)$ is a minimum point

(3, 0) is a point of inflexion.

$x<-\frac{3}{4}$

(9 marks)

10. The diagram shows part of the graph of $y = 4e^{-x^2}$. The point $P(p, 0)$ where $p > 0$ is on the x-axis and the point Q is on the curve.



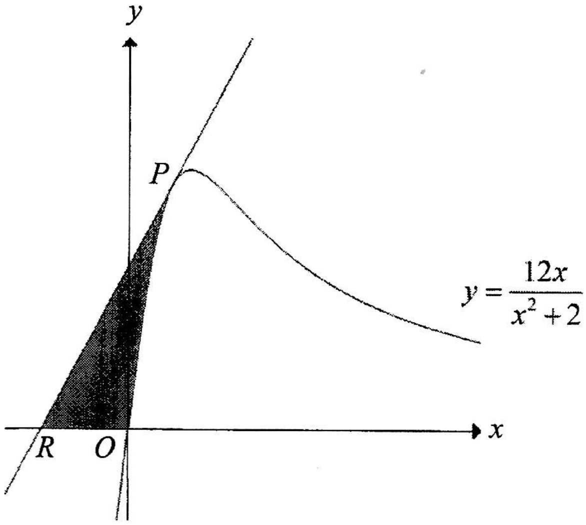
(i) Given that PQ is parallel to the y-axis, express the area, A, of the triangle OPQ in terms of p. [2]

(ii) If A is increasing at a rate of 0.8 unit^2 per second, find the rate at which p is increasing at the instant when $p = 0.5$. [4]

10.	(i)	At Q, $x = p,\ y = 4e^{-p^2}$ $A = \frac{1}{2}p(4e^{-p^2})$ $= 2pe^{-p^2}$
	(ii)	$\frac{dA}{dp} = 2e^{-p^2} + 2p(e^{-p^2})(-2p)$ $= 2e^{-p^2}(1-2p^2)$ When $p = 0.5$, $\frac{dA}{dt} = \frac{dA}{dp} \times \frac{dp}{dt}$ $0.8 = 2e^{-0.5^2}(1-2\times 0.5^2)\times\frac{dp}{dt}$ $\frac{dp}{dt} = \frac{0.8}{2e^{-0.5^2}(1-2\times 0.5^2)}$ $= 1.03$ units per second **(6 marks)**

11. **(a)** Find the derivative of $\ln(x^2+2)$. [2]

(b) The diagram shows part of the curve $y=\dfrac{12x}{x^2+2}$. The tangent at the point $P(1, 4)$ meets the x-axis at R.



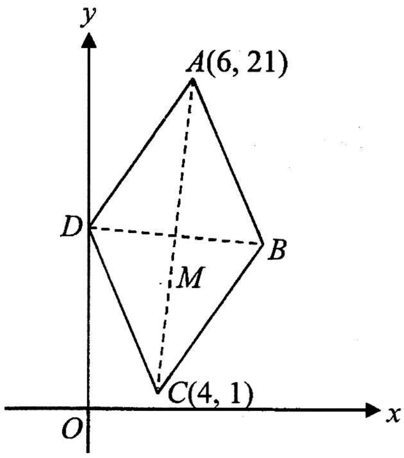
(i) Show that the x-coordinate of R is -2. [7]

(ii) Find the area of the shaded region bounded by the tangent PR, the x-axis and the curve. [4]

11. (a)	$\dfrac{d}{dx}\left(\ln(x^2+2)\right) = \dfrac{2x}{x^2+2}$
(b)(i)	$\dfrac{dy}{dx} = \dfrac{12(x^2+2)-12x(2x)}{(x^2+2)^2}$ $= \dfrac{24-12x^2}{(x^2+2)^2}$ At $P(1, 4)$, $\dfrac{dy}{dx} = \dfrac{24-12}{3^2} = \dfrac{4}{3}$ Equation of tangent, $y-4=\dfrac{4}{3}(x-1)$ $y=\dfrac{4x}{3}+\dfrac{8}{3}$ At R, $y=0$, $\dfrac{4x}{3}+\dfrac{8}{3}=0$ $x=-2$

(b)(ii)	Shaded Area $=\frac{1}{2}\times 3\times 4-\int_0^1 \frac{12x}{x^2+2}\mathrm{d}x$ $=6-6\left[\ln(x^2+2)\right]_0^1$ $=6-6(\ln 3-\ln 2)$ $=3.57\text{ units}^2$ **(12 marks)**

12. Solutions to this question by accurate drawing will not be accepted.



The diagram shows a rhombus $ABCD$. The point D lies on the y-axis and M is the mid-point of AC. A and C are the points (4, 1) and (6, 21) respectively. Find

(i) Show that the coordinates of D are (0, 11.5). [4]

(ii) Find the area of the rhombus $ABCD$. [3]

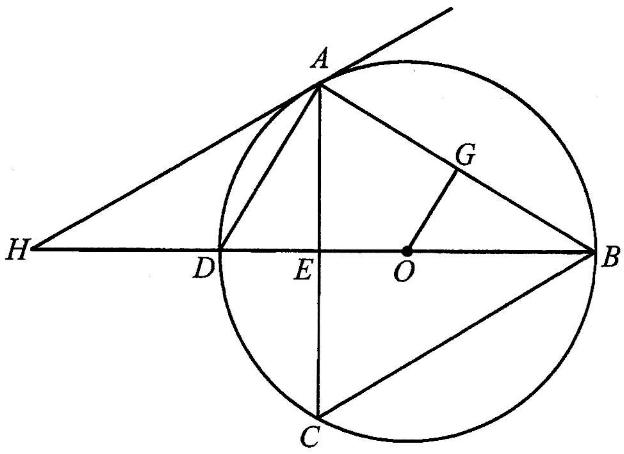
A point N lies on AC produced such that area of triangle AND is three times the area of triangle ADC.

(iii) Find the coordinates of N. [2]

12.	**(i)**	Midpoint, $M(5,11)$ Gradient of AC $=\dfrac{21-1}{6-4}$ $=10$ Equation of BD, $y-11=-\dfrac{1}{10}(x-5)$ $y=-\dfrac{1}{10}x+\dfrac{23}{2}$ $D(0, 11.5)$
	(ii)	Area of rhombus $=2\times\dfrac{1}{2}\begin{vmatrix}0 & 4 & 6 & 0\\ 11.5 & 1 & 21 & 11.5\end{vmatrix}$ $=2\times\dfrac{1}{2}(84+69-46-6)$ $=101\text{ unit}^2$ **Alternative Method** $\left(\dfrac{x+0}{2},\dfrac{y+11.5}{2}\right)=(5,\ 11)$ $\Rightarrow x=10,\ y=10.5$ $B(10,10.5)$

	Area of rhombus $=2\times\frac{1}{2}\begin{vmatrix}0 & 4 & 10 & 6 & 0\\11.5 & 1 & 10.5 & 21 & 11.5\end{vmatrix}$ $=\frac{1}{2}(42+210+69-46-10-63)$ $=101\text{ unit}^2$
(iii)	$\frac{\text{Area of }\Delta ADC}{\text{Area of }\Delta ADN}=\frac{1}{3}$ $\frac{\frac{1}{2}\times AN\times DM}{\frac{1}{2}\times AC\times DM}=\frac{1}{3}$ $\frac{AC}{AN}=\frac{1}{3}$ $x=6-3(2)=0$ $y=21-3(20)=-39$ Coordinates of $N=(0,-39)$

13.



The diagram shows a circle, centre *O*, with points *A*, *B*, *C* and *D* lie on the circle.
HA is a tangent to the circle.
G are mid-points of *AB* respectively.
AD is the angle bisector of angle *CAH*.

(a) Prove that *OG* is perpendicular to *AB*, [3]
(b) Prove that angle *ABD* = angle *CBD*, [3]

13.	**(a)**	In $\triangle ADB$, *G* is the mid-point of *AB* (given) *O* is the mid-point of *DB* (given) $\therefore GO // AD$ (Mid-point Theorem) $\angle DAB = 90^\circ$ (angle in semi-circle) $\angle GOB = 90^\circ$ (coresponding angle) Hence, *OG* is perpendicular to *AB*.
	(b)	$\angle DAH = \angle CAD$ (*AD* bisects $\angle CAH$) $\angle ABD = \angle DAH$ (alternate segment theorem) $\angle CBD = \angle CAD$ (angles in the same segment) $\angle CBD = \angle DAH = \angle ABD$ Hence, $\angle ABD = \angle CBD$