**Name:** **Register Number:** **Class:**

| For Marker's Use |
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| 90 |

南僑中學

# NAN CHIAU HIGH SCHOOL

## PRELIMINARY EXAMINATION 2021
## SECONDARY FOUR EXPRESS

Parents' signature:

**ADDITIONAL MATHEMATICS** **4049/01**
**Paper 1** **23 August 2021, Monday**

Candidates answer on the Question Paper. **2 hours 15 minutes**

No Additional Materials are required.

**READ THESE INSTRUCTIONS FIRST**

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

## *Mathematical Formulae*

### 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial expansion*

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \cdots + \binom{n}{r} a^{n-r}b^r + \cdots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

### 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

*Formulae for $\Delta ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The loudness, $L$ dB, of a sound is given by $L = 10\, lg\left(\frac{I}{I_0}\right)$, where $I$ is the sound intensity to be measured and $I_0$ is the minimum sound intensity that can be heard by a human.

**(a)** Given that the loudness of a thunder is 120 dB, find the ratio of the sound intensity of thunder to the minimum sound intensity that can be heard by a human. [2]

**(b)** Given that the minimum sound intensity that can be heard by a human occurs when $L$ = 90 dB and $I = 10^{-3}$, find the loudness of a plane taking-off if its intensity is 100. [3]

**2** The coordinates of three points $A$, $B$ and $C$ are $(-3, 1)$, $(6, 3)$ and $(1, 8)$ respectively. Given that $ABDC$ is a parallelogram, find

**(a)** the coordinates of $D$, [3]

**(b)** the area of the parallelogram $ABDC$. [2]

3 **(a)** Show that $4\sin^2 x - 2\cos^2 x$ can be written as $a + b\cos 2x$, where $a$ and $b$ are integers. [2]

**(b)** Hence sketch the graph of $y = 4\sin^2 x - 2\cos^2 x$ for $-90° \le x \le 270°$. [3]

4 **(a)** It is given that $(3x + k)$ is a factor of the polynomial

$81x^4 - 6x^3 - 9k^2x^2 - 11x - \frac{35}{9}$, show that $2k^3 + 33k - 35 = 0$. [2]

**(b)** Show that the equation $2k^3 + 33k - 35 = 0$ has only one real root for all real values of $k$. [4]

**5** Integrate each of the following with respect to $x$.

**(a)** $\frac{1}{3}\tan^2\left(\frac{2}{5}x\right)$ [2]

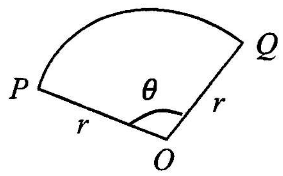
**(b)** $\frac{2-3x}{5-3x}$ [3]

6 **(a)** By squaring $\sin^2 x + \cos^2 x$, or otherwise, show that

$$\sin^4 x + \cos^4 x = 1 - \frac{1}{2}\sin^2 2x.$$ [3]

**(b)** Hence, find the values of $x$ for which $\sin^4 x + \cos^4 x = \frac{3}{5}$ for $\frac{3\pi}{2} \leq x \leq 2\pi$. [3]

7 A piece of wire with a fixed length, $L$ cm, is bent to form the shape a sector of a circle, $OPQ$, centre $O$ and radius $r$ cm as shown in the diagram. The angle at the centre is $\theta$ radians.




**(a)** Show that the area, $A$ cm$^2$, of the sector is given by $A = \frac{1}{2}rL - r^2$. [2]

**(b)(i)** Given that $r$ can vary and $A$ has a stationary value, find $r$ in term of $L$ and its corresponding value of $\theta$. [3]

**(b)(ii)** Determine the nature of this stationary value. [1]

**8** **(a)** The variables $x$ and $y$ increase in such a way that, when $x$ is a particular value, the rate of increase of $y$ with respect to time is half the rate of increase of $x$ with respect to time. Given that $y = \frac{2\sqrt{6x+7}}{3}$, find this particular value of $x$. [3]

**(b)** The drug concentration, $C(t)$, in a person's bloodstream $t$ hours after digesting a certain amount a drug, can be modelled by the function $C(t) = 0.845t - \frac{3t^3}{200}$.

Find the time interval when the drug concentration in the bloodstream is increasing. [4]

9 The height, $h$ metres, of a stone from the ground after it has been thrown can be modelled by the equation $h = -ax^2 + x + \frac{2}{5}$, where $x$ is the horizontal distance travelled by the stone in metres and $a$ is a constant.

(a) Given that the maximum height attained by the stone is 2.4 metres, find the value of $a$ and the corresponding horizontal distance, in metres, travelled by the stone. [5]

(b) Hence, solve the equation $-ax^2 + x + \frac{2}{5} = 0$ and explain the significant of the answer(s). [2]

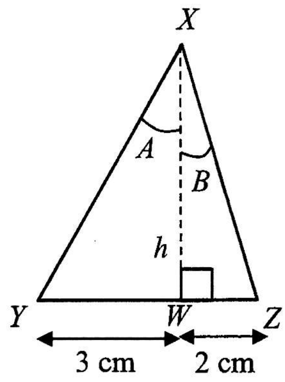
**10** **(a)** Without the use of a calculator, express the principal value of $cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ in radians as a multiple of $\pi$. [1]

**(b)** Without using a calculator, express $\cos\frac{7\pi}{12}$ in the form $\frac{\sqrt{a}-\sqrt{b}}{c}$, where $a$, $b$ and $c$ are integers. [2]

**(c)** Angles $A$ and $B$ both lie between 0° and 360°. Given that $\sin A$ and $\cos B$ are both negative, explain whether you agree or disagree that $360° < A + B < 540°$. [2]

**(d)** The diagram shows a triangle $XYZ$ of height $h$ cm, $YW = 3$ cm, $WZ = 2$ cm and angle $XWZ = 90°$. Angles $A$ and $B$ are such that $A + B = 45°$, find the value of $h$.

[4]



**11** **(a)** Given that $\int e^{-2x}\,\mathrm{f}(x)\,\mathrm{d}x = e^{-2x}\sin 4x + c$, find the value of $\mathrm{f}\left(\frac{\pi}{4}\right)$. [4]

**(b)** It is given that f($x$) is such that $f''(x) = 4\sin\left(3x + \frac{\pi}{2}\right) + e^{2x}$. Given also that $f'(0) = \frac{2}{3}$ and $f(0) = \frac{65}{36}$, find the expression for f($x$). [5]

**12** The velocity, $v$ m/s, of a particle moving in a straight line, $t$ seconds after passing through a fixed point $O$, is given by $v = \frac{27}{2(3t+1)^2} - \frac{3t+1}{2}$.

**(a)** Find the initial acceleration of the particle. [2]

**(b)** Determine, with appropriate working, whether the velocity of the particle is increasing or decreasing. [2]

**(c)** Find the average speed of the particle during the first 6 seconds. [6]

13 **(a)(i)** The ninth term in the expansion of $\left(px - \frac{q}{x}\right)^n$, where $p$ and $q$ are constants, is independent of $x$. Find the value of $n$. [3]

**(a)(ii)** Show that the ninth term is a positive number. [1]

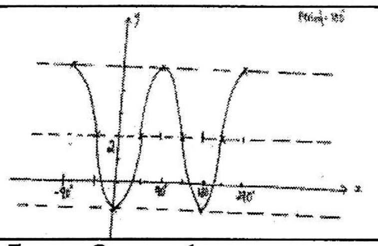
**(b)(i)** Write down, and simplify, the first three terms in the expansion $\left(2 - \frac{x}{4}\right)^n$, where $n$ is a positive integer greater than 2, in ascending power of $x$. [2]

**(b)(ii)** The first two non-zero terms in the expansion, in ascending power of $x$, of $(2+x)\left(2-\frac{x}{4}\right)^n$ are $a+bx^2$, where $a$ and $b$ are constants. Find the values of $n$, $a$ and $b$. [4]

End of paper

P1

**Answer Keys for 2021 NCHS Prelim Exam Add Math Paper 1 (4049/01)**

| | | | |
|---|---|---|---|
| 1a | $l : l_0 = 10^{12} : 1$ | 12a | $a = -82.5\ m/s^2$ |
| 1b | 140 dB | 12b | Since $\frac{dv}{dt} < 0, \therefore v\ is\ decreasing$ |
| 2a | $D$ (10, 10) | 12c | 5.07 m/s (3sf) |
| 2b | 55 units$^2$ | 13ai | 16 |
| 3a | $1 - 3\cos 2x$ | 13bi | $2^n - n2^{n-3}x + n(n-1)2^{n-7}x^2 + \cdots$. |
| 3b |  | 13bii | $n = 4$<br>$a = 32$<br>$b = -5$ |
| 5a | $\frac{5}{6}\tan\left(\frac{2}{5}x\right) - \frac{1}{3}x + c$ | | |
| 5b | $x + \ln(5 - 3x) + c$ | | |
| 6b | $x = 5.27, 5.73$ | | |
| 7bi | $r = \frac{L}{4}; \theta = 2$ | | |
| 7bii | $A$ is a maximum value | | |
| 8a | 1.5 | | |
| 8b | $0 \leq t < 4\frac{1}{3}$ | | |
| 9a | $a = \frac{1}{8}; x = 4$ | | |
| 9b | $x = 8.38\ \ or\ \ x = -0.382$<br><br>Since distance > 0, $x = -0.382$ is rejected. 8.38m represents the **horizontal distance travelled by the stone when it hits the ground.** | | |
| 10a | $\frac{5\pi}{6}$ | | |
| 10b | $\frac{\sqrt{2} - \sqrt{6}}{4}$ | | |
| 10c | Disagree as<br>$270° < A + B < 630°$ and not $360° < A + B < 540°$. | | |
| 10d | 6 | | |
| 11a | - 4 | | |
| 11b | $f(x) = -\frac{4}{9}\sin\left(3x + \frac{\pi}{2}\right) + \frac{e^{2x}}{4} + \frac{1}{6}x + 2$ | | |

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# NAN CHIAU HIGH SCHOOL

## PRELIMINARY EXAMINATION 2021
## SECONDARY FOUR EXPRESS

**ADDITIONAL MATHEMATICS** **4049/02**

**Paper 2** **25 August 2021, Wednesday**

**2 hour 15 minutes**

Candidates answer on the Question Paper.

No Additional Materials are required.

**READ THESE INSTRUCTIONS FIRST**

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# *Mathematical Formulae*

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$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \cdots + \binom{n}{r} a^{n-r}b^r + \cdots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\cdots(n-r+1)}{r!}$

## 2. TRIGONOMETRY

*Identities*

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$$\sin 2A = 2 \sin A \cos A$$
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*Formulae for $\Delta ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

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1 **(a)** Given the solution set of $3x^2 < hx + k$ is $-2\frac{1}{3} < x < 2\frac{1}{2}$, where $h$ and $k$ are constants. Find the value of $h$ and of $k$. [3]

**(b)** Find the range of values of $m$ for which the curve $y = x^2 + (m+2)x + 1 - m$ does not intersect the line $y + x = 1$. [3]

**2** The equation of a curve is $y = e^{3x-5x^2+ln2x}$, where $x > 0$.

**(a)** Obtain an expression for $\frac{dy}{dx}$. [2]

**(b)** Find the coordinates of the stationary point of the curve and leave your answer in exact form. [3]

**(c)** Determine the nature of the stationary point of the curve. [2]

**3** Express $\frac{8x^3-5x+3}{x(4x^2-9)}$ in partial fractions. Hence find $\int \frac{8x^3-5x+3}{2x(4x^2-9)}\,dx$. [8]

4 Solve the following equations.

**(a)** $\log_3\sqrt{3x^2 - 3} = 2 + \dfrac{1}{\log_{(3x-3)}9}$. [4]

**(b)** $2^{4x+3} + 14(4^x) = 15$. [4]

**5** **(a)** Show that $\frac{1+\sec 2x}{\tan 2x} = \cot x$. [4]

**(b)** Given that $y = \ln[\sin(nx)]$ where $n$ is a constant, find an expression for $\frac{dy}{dx}$. [2]

**(c)** Using the results from parts **(a)** and **(b)**, find $\int \frac{1+\sec 8x}{2\tan 8x} - 1 \, dx$. [3]

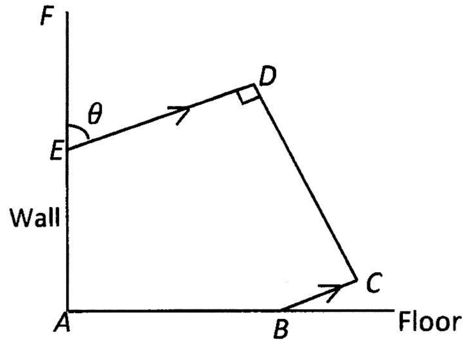
**6** A circle with centre $C$ touches the $y$-axis and intersects $x$-axis at $x = -2$ and $x = -8$. Point $C$ lies above the $x$-axis.

**(a)** Find the equation of the circle. [4]

**(b)** If the point $(-3, k)$ lies inside the circle, find the range of values of $k$. [3]

**(c)** Find the length of the chord such that the coordinates of the mid-point of the chord is $(-3, 5)$. [3]

**7** The side view of a sculpture $ABCDE$ lying on the horizontal floor $AB$ and leaning against a vertical wall $AEF$ is shown below. Given $DE = 17$ m, $CD = 20$ m, $BC = 2$ m, angle $EDC = \frac{\pi}{2}$ radians, angle $DEF = \theta$ and $ED$ is parallel to $BC$.



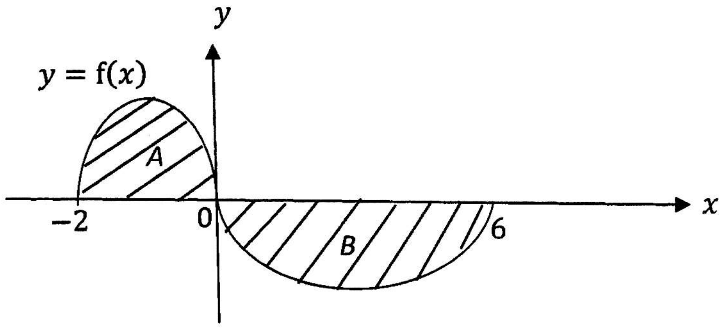

**(a)** Show that the length $AB$ is $15\sin\theta + 20\cos\theta$. [2]

**(b)** Express $AB$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [3]

**(c)** Find the values of $\theta$ for which $AB = 23$ m. [3]

**(d)** Write down the maximum value of $AB$ and find the corresponding value of $\theta$. [3]

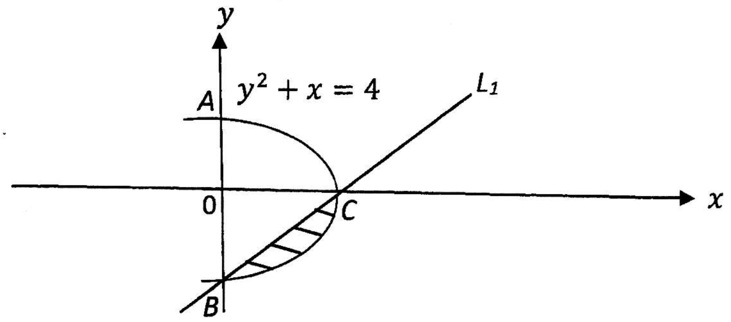
**8** **(a)** The diagram shows part of the curve $y = \mathrm{f}(x)$. Shaded area $A$ and $B$ are bounded by the curve and $x$-axis. Given $\int_{-2}^{0} \mathrm{f}(x)\mathrm{d}x = h$ and $\int_{0}^{6} \mathrm{f}(x)\mathrm{d}x = q$, where $h$ and $q$ are constants, find an expression in terms of $h$ and $q$ for each of the following. Hence interpret how each expression is related to the area $A$ and $B$.




**(i)** $\int_{-2}^{6} 5\,\mathrm{f}(x)\mathrm{d}x$ [3]

**(ii)** $\int_{6}^{0} \mathrm{f}(x)\,\mathrm{d}x - \int_{0}^{-2} \mathrm{f}(x)\,\mathrm{d}x$ [3]

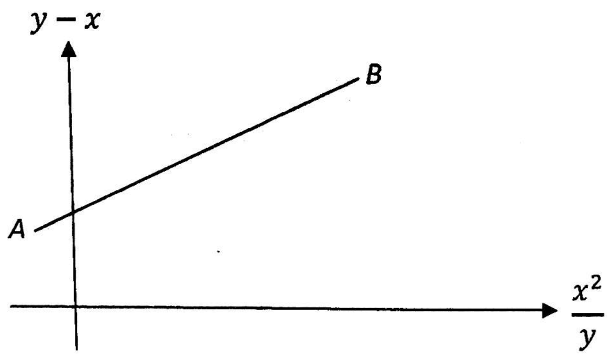
**(b)** The diagram shows the curve $y^2 + x = 4$ intersecting the line $L_1$ at point $B$ and $C$. Both points $A$ and $B$ lie on the $y$-axis while point $C$ lies on the $x$-axis. Find the area of the shaded region bounded by the line and curve. [6]



**9** **(a)** Express $h$ and $k$ in terms of $x$ given that $(6^{x-3})(24^{3x+1})(27^{3-2x}) = (a^{h})(b^{k})$, where $a$ and $b$ are prime numbers and $a < b$. [4]

**(b)** Hence find the value of $x$ given further that $(6^{x-3})(24^{3x+1})(27^{3-2x}) = (ab)^{q}$, where $q$ is a rational number. [2]

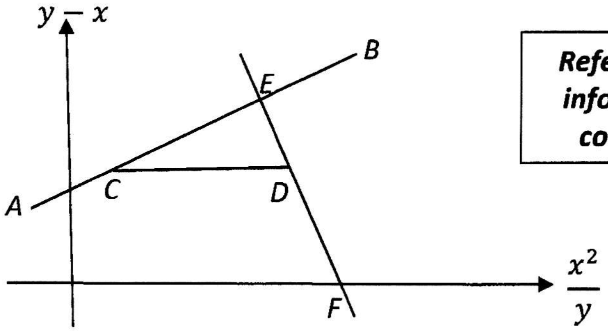
**10 (a)**




The diagram shows part of a straight line graph $AB$ drawn to represent the equation $y^2 = hx^2 + y(k + x)$, where $h$ and $k$ are constants. Given that the line passes through $\left(\frac{1}{2}, 3\right)$ and $(2, 5)$, find the value of $h$ and of $k$. [4]

**(b)** If the diagram from part **(a)** is being drawn to scale on a graph paper, find the equation of a suitable straight line which should be added to the graph in order to find the value of $x$ for which $h + 2 + \frac{ky}{x^2} = \frac{5y}{x^2}$. [2]

**(c)**




***Refer to part (a) information for continuation***

$EF$ is a normal to the line $AB$ at $E(6, p)$, intersects the $x$-axis at $F$ where $p$ is a constant. A horizontal line meets $AB$ at $C$ and $EF$ at $D$. $ACEB$ and $EDF$ are straight lines.

**(i)** Find the value of acute angle *ECD*. [2]

**(ii)** Find the value of $p$. [2]

**(iii)** Find the coordinates of $F$. [3]

***End of Paper***

NHAN CHIAM



P2

## ***Answers***

1a) $h = \frac{1}{2}, \quad k = 17.5$

1b) $-9 < m < -1$

2a) $\frac{dy}{dx} = 2xe^{3x-5x^2}\left(3 - 10x + \frac{1}{x}\right)$ OR $2e^{3x-5x^2}(-10x^2 + 3x + 1)$

2b) $\left(\frac{1}{2}, e^{\frac{1}{4}}\right)$

2c) $\left(\frac{1}{2}, e^{\frac{1}{4}}\right)$ is a maximum stationary point.

3) $2 - \frac{1}{3x} + \frac{5}{2(2x-3)} - \frac{11}{6(2x+3)}$, $\quad x - \frac{1}{6}\ln x + \frac{5}{8}\ln(2x-3) - \frac{11}{24}\ln(2x+3) + c$

4a) $x = 80$

4b) $x = -0.208$

5b) $\frac{dy}{dx} = n\cot(nx)$

5c) $\frac{1}{8}\ln\sin(4x) - x + C$

6a) $radius = 5, \quad (x+5)^2 + (y-4)^2 = 25$

6b) $-0.583 < k < 8.58$

6c) Length of cord = $4\sqrt{5} = 8.94$

7b) $AB = 25\cos(\theta - 0.644)$

7c) $\theta = 1.05$ or $0.241$

7d) $max\ AB = 25, \ \theta = 0.644$

8ai) $5(h+q)$, 5 times of (area A – area B)

8aii) $-q + h$, Area A + area B

8b) $\frac{4}{3}$

9a) $h = 10x$ and $k = 7 - 2x$

9b) $x = \frac{7}{12}$

10a) $h = \frac{4}{3}$ and $k = \frac{7}{3}$

10b) $y - x = -2\frac{x^2}{y} + 5$

10ci) $\angle ECD = 53.1$ or $0.927$ radian

10cii) $p = 10\frac{1}{3}$

10ciii) $F\left(19\frac{7}{9}, 0\right)$