CEDAR GIRLS' SECONDARY SCHOOL
Preliminary Examination
Secondary Four

CANDIDATE NAME			
CLASS		INDEX NUMBER	
CENTRE/ INDEX NO	/		

ADDITIONAL MATHEMATICS

4049/01

Paper 1

13 September 2021

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

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Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
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Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 84.

For Examiner's Use
/ 84

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions.

1 P and Q are the points of intersection of the line $\frac{x}{a}+\frac{y}{b}=1$, $(a>0, b>0)$, with the x and y axes respectively. The distance PQ is 10 and the gradient of PQ is -2. Find the exact value of a and of b. [5]

2 The polynomial $P(x) = ax^3 + bx^2 - 19x + 4$, where a and b are constants, has a factor $x + 4$ and is such that $2P(1) = 5P(0)$.

(a) Find the value of a and of b. [4]

(b) Hence, factorise $P(x)$ completely. [2]

3 A function is defined by the equation $y = \frac{2x^2}{x-1}$, $x \neq 1$.

(a) Show that y is increasing for $x > 2$. [3]

(b) If y is increasing at the rate of 2 units per second, find the rate of change of x when $x = 3$. [2]

4 Consider the expansion $\left(\dfrac{a^2}{\sqrt{x}}-\dfrac{\sqrt{x}}{a}\right)^8$ where a is a positive constant.

Find, **in terms of *a***,

(a) the term independent of x. [2]

(b) the coefficient of x^2 in the expansion of $\left(\dfrac{3x^4-4x^2}{x^2}\right)\left(\dfrac{a^2}{\sqrt{x}}-\dfrac{\sqrt{x}}{a}\right)^8$. [4]

5 There are approximately 5 times as many animals of Species A as animals of Species B in a nature reserve.
The population of animals of Species A decreases at a rate of 5% per year and the population of animals of Species B increases at a rate of 10% per year.

It is given that N is the number of animals of Species B in the nature reserve.

(a) Write down, **in terms of** N,

(i) an expression, P_A, to model the population of animals of Species A after t years, [1]

(ii) write down an expression, P_B, to model the population of animals of Species B after t years. [1]

(b) Using your models, find the number of years that will elapse before the population of animals from Species A is equal to the population of animals from Species B. [3]

6 **(a)** Solve $\tan 3x = 0$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ giving your answers in terms of π. [2]

(b) Use your answers to part **(a)** to sketch the graph of $y = 4\tan 3x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ on the axes below. Show the coordinates of the points where the curve meets the axes. [3]

y

(c) On the same diagram given in part **(b)**, sketch the graph of $y = \sin x + 1$ for the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. Hence state the number of solutions of the equation $4\tan 3x - \sin x = 1$ in the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. [3]

7 **(a)** Show that $2\sin(A+45°)\cos(A+45°)=\cos 2A$. [3]

(b) Hence, solve the equation $\sin(A+45°)=\frac{1}{8}\sec(A+45°)$, for $0° \le A \le 360°$. [4]

8 A curve is such that $\frac{d^2y}{dx^2} = 5\cos 2x$. This curve has a gradient of $\frac{3}{4}$ at the point $\left(-\frac{\pi}{12}, \frac{5\pi}{4}\right)$.

Show that the equation of the curve is $y = -\frac{5}{4}\cos 2x + 2x + \frac{17\pi}{12} + \frac{5\sqrt{3}}{8}$. [6]

9 **(a)** Factorise $x^3 + y^3$. [1]

(b) Hence, prove the identity $\dfrac{\sin^3\theta + \cos^3\theta}{2\sin^2\theta - 1} = \dfrac{\sec\theta - \sin\theta}{\tan\theta - 1}$. [4]

Continuation of working space for Question (9b).

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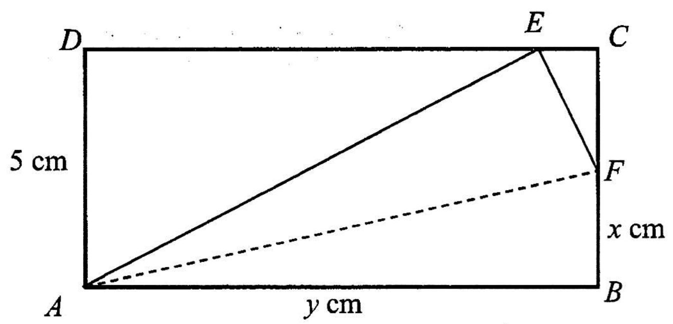


The diagram shows a rectangular piece of paper $ABCD$ such that $AD = 5$ cm, $BF = x$ cm and $AB = y$ cm. The paper is folded along AF such that B meets E on DC.

(a) Express EC and AE in terms of x. [4]

(b) Hence show that the area, A cm^2, of triangle AEF is given by

$$A = \frac{5x^2}{2\sqrt{10x - 25}}.$$ [2]

(c) Show that $x = \frac{10}{3}$ for which $\frac{dA}{dx} = 0$. [3]

(d) By considering the change in the sign of $\frac{dA}{dx}$, show that A is a minimum at $x = \frac{10}{3}$. Hence find the minimum value of A. [3]

11 The height, in metres, of a ball above the ground t seconds after it is thrown from a tower is given by the function $h(t) = -5t^2 + 10t + c$, where c is a constant.

(a) **(i)** Given that $h(0) = 6$, find the value of c. [1]

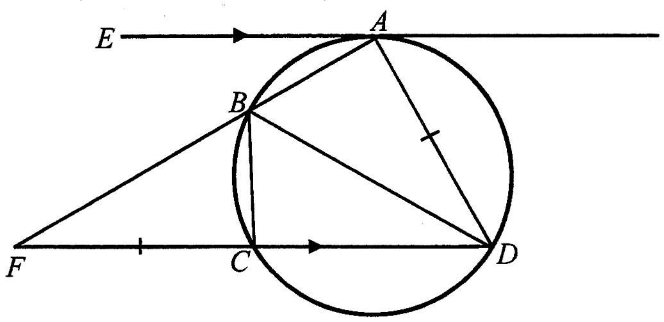
(ii) Interpret the meaning of the answer in **(a) (i)**. [1]

(b) **Without the use of differentiation** and using the value of c from **(a) (i)**, determine the maximum height reached by the ball and the corresponding value of t. [4]

Continuation of working space for Question (11b).

(c) Find the time it takes for the ball to drop to the ground. [2]

12



In the diagram above, EA is the tangent to the circle at A.
FBA and FCD are straight lines.
EA is parallel to FD and $FC = AD$.

(a) Show that $\angle BAD = \angle BCF$. [2]

(b) Hence, prove that triangle ABD is congruent to triangle CBF. [4]

13 A particle starts from rest at a fixed point O and moves in a straight line with its acceleration, a m/s^2, is given by $a = 9 - kt$, where t seconds is the time since leaving O, and k is a constant. When $t = 2$, its velocity is 15 m/s.

(a) Find the value of k. [3]

(b) The particle changes its direction of motion when $t = T$.
Find the value of T. [2]

(c) Show that the particle returns to O when $t = 18$. [3]

(d) Hence find the distance travelled by the particle between $t = 0$ and $t = 18$. [3]

END OF PAPER

ANS

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This document consists of **21** printed pages and **1** blank page.

[Turn over

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Answer **all** the questions.

1 P and Q are the points of intersection of the line $\frac{x}{a}+\frac{y}{b}=1$, $(a>0, b>0)$, with the x and y axes respectively. The distance PQ is 10 and the gradient of PQ is -2. Find the exact value of a and of b. [5]

Since P lies on x –axis, y = 0, $\frac{x}{a}+\frac{0}{b}=1$, $x=a$

$P(a,0)$

Since Q lies on y –axis, x = 0, $\frac{0}{a}+\frac{y}{b}=1$, $y=b$

$Q(0,b)$

Since gradient of PQ = -2, $\frac{b-0}{0-a}=-2$

$\therefore b=2a$.........................[1]

Since distance PQ = 10

$\sqrt{(a-0)^2+(0-b)^2}=10$

$\sqrt{(a-0)^2+(0-2a)^2}=10$

$5a^2=100$

$a=\sqrt{\frac{100}{5}}=\sqrt{20}=2\sqrt{5}$ $(a>0, b>0))$

$\therefore b=4\sqrt{5}$

2 The polynomial $P(x)=ax^3+bx^2-19x+4$, where a and b are constants, has a factor $x+4$ and is such that $2P(1)=5P(0)$.

(a) Find the value of a and of b. [4]

Since $x+4$ is a factor, $P(-4)=0$

$a(-4)^3+b(-4)^2-19(-4)+4=0$

$-4a+b--5$ ....(1)

Since $2P(1)=5P(0)$

$2(a+b-19+4)=20$

$a+b=25$ ....(2)

$\therefore a=6$ and $b=19$

(b) Hence, factorise $P(x)$ completely. [2]

By long division or factorization, $P(x)=(x+4)\left(6x^2-5x+1\right)$

$P(x)=(x+4)(3x-1)(2x-1)$

3 A function is defined by the equation $y = \frac{2x^2}{x-1}$, $x \neq 1$.

(a) Show that y is increasing for $x > 2$. [3]

$$\frac{dy}{dx} = \frac{(x-1)4x - 2x^2}{(x-1)^2} = \frac{2x(x-2)}{(x-1)^2}$$

Since $(x-1)^2 > 0$, and for $\frac{dy}{dx} > 0$, $2x(x-2) > 0 \Rightarrow x < 0$ or > 2

Hence function is increasing for $x > 2$.

(b) If y is increasing at the rate of 2 units per second, find the rate of change of x when $x = 3$. [2]

$$\frac{dx}{dt} = \frac{dx}{dy} \times \frac{dy}{dt}$$

$$= \frac{(x-1)^2}{2x(x-2)} \times 2$$

When $x = 3$, $\frac{dx}{dt} = \frac{4}{6} \times 2 = \frac{4}{3}$ units /s

4 Consider the expansion $\left(\frac{a^2}{\sqrt{x}}-\frac{\sqrt{x}}{a}\right)^8$ where a is a positive constant.

Find, **in terms of *a***,

(a) the term independent of x. [2]

General Term $=\binom{8}{r}\left(\frac{a^2}{\sqrt{x}}\right)^{8-r}\left(-\frac{\sqrt{x}}{a}\right)^r$

$=\binom{8}{r}a^{16-3r}x^{r-4}(-1)^r$

For independent term, $r-4=0\Rightarrow r=4$

Term independent of $x=\binom{8}{4}a^4(-1)^4=70\,a^4$

(b) the coefficient of x^2 in the expansion of $\left(\frac{3x^4-4x^2}{x^2}\right)\left(\frac{a^2}{\sqrt{x}}-\frac{\sqrt{x}}{a}\right)^8$. [4]

$\left(\frac{3x^4-4x^2}{x^2}\right)\left(\frac{a^2}{\sqrt{x}}-\frac{\sqrt{x}}{a}\right)^8=(3x^2-4)(\text{...Term in } x^2+\text{Independent Term+...})$

For term in x^2, $r-4=2\Rightarrow r=6$

Term in $x^2=\binom{8}{6}a^{16-18}x^2(-1)^6=\frac{28x^2}{a^2}$

$(3x^2-4)\left(...\ \frac{28x^2}{a^2}+70a^4+...\right)$

$=...+210a^4x^2-\frac{112}{a^2}x^2+...$

Coefficient of $x^2=210a^4-\frac{112}{a^2}$

5 There are approximately 5 times as many animals of Species A as animals of Species B in a nature reserve.
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It is given that N is the number of animals of Species B in the nature reserve.

(a) Write down, **in terms of N,**

(i) an expression, P_A, to model the population of animals of Species A after t years, [1]

$P_L = 5N(0.95)^t$

(ii) write down an expression, P_B, to model the population of animals of Species B after t years. [1]

$P_B = N(1.1)^t$

(b) Using your models, find the number of years that will elapse before the population of animals from Species A is equal to the population of animals from Species B. [3]

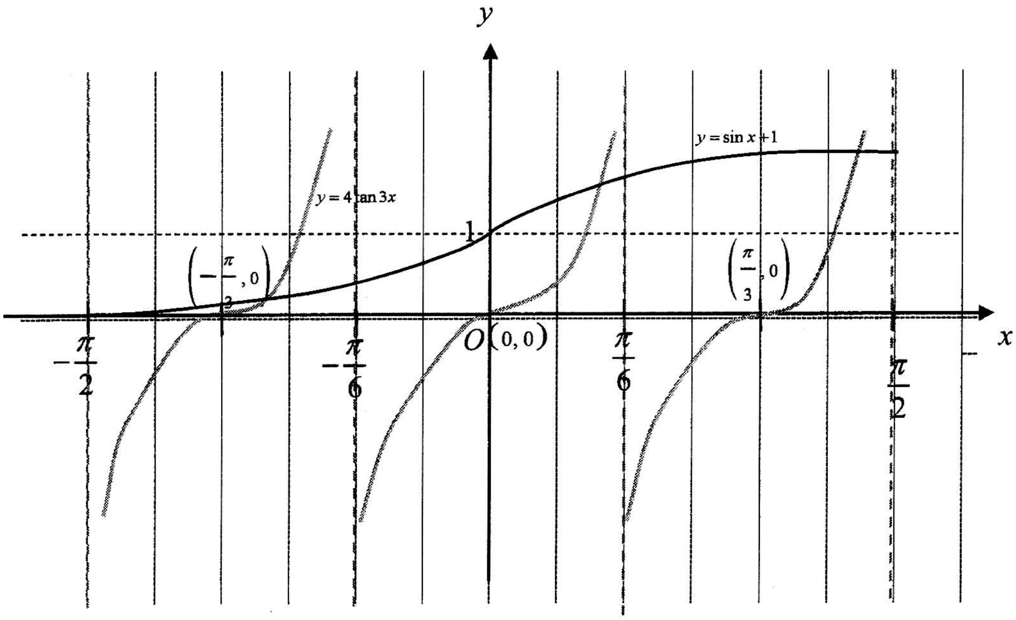
$$5N(0.95)^t = N(1.1)^t$$
$$\lg 5 + t\lg 0.95 = t\lg 1.1$$
$$t = \frac{\lg 5}{\lg 1.1 - \lg 0.95} = 10.978$$

No. of years =11.0

6 **(a)** Solve $\tan 3x = 0$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ giving your answers in terms of π. [2]

$\tan 3x = 0$
$3x = -\pi, 0, \pi$
$x = -\frac{\pi}{3}, 0, \frac{\pi}{3}$

(b) Use your answers to part **(a)** to sketch the graph of $y = 4\tan 3x$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ on the axes below. Show the coordinates of the points where the curve meets the axes. [3]



Period = $\frac{\pi}{3}$

(c) On the same diagram given in part **(b)**, sketch the graph of $y = \sin x + 1$ for the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. Hence state the number of solutions of the equation $4\tan 3x - \sin x = 1$ in the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. [3]

Correct shape of $y = \sin x + 1$

Correct points of (0,1), $(\frac{\pi}{2}, 0)$, and $(-\frac{\pi}{2}, 0)$

No. of Solutions = 3

7 **(a)** Show that $2\sin(A+45°)\cos(A+45°)=\cos 2A$. [3]

$LHS = 2\sin(A+45°)\cos(A+45°)$
$= \sin(2A+90°)$
$= \sin 2A\cos 90° + \cos 2A\sin 90°$
$= \sin 2A(0) + \cos 2A(1)$
$= \cos 2A$

Alternative :
LHS = $2(\sin A\cos 45° + \sin 45°\cos A)(\cos A\cos 45° - \sin A\sin 45°)$

$$=2\left(\sin A\left(\frac{\sqrt{2}}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)\cos A\right)\left(\cos A\left(\frac{\sqrt{2}}{2}\right)-\sin A\left(\frac{\sqrt{2}}{2}\right)\right)$$

$$=2\left[\left(\frac{\sqrt{2}}{2}\cos A\right)^2-\left(\frac{\sqrt{2}}{2}\sin A\right)^2\right]$$

$$=2\left(\frac{1}{2}\cos^2 A-\frac{1}{2}\sin^2 A\right)$$

$= \cos 2A$

(b) Hence, solve the equation $\sin(A+45°)=\frac{1}{8}\sec(A+45°)$, for $0° \le A \le 360°$. [4]

$$\sin(A+45°)=\frac{1}{8\cos(A+45°)}$$

$$2\sin(A+45°)\cos(A+45°)=\frac{1}{4}$$

$\cos 2A = \frac{1}{4}$, $0° \le 2A \le 720°$

Basic Angle = $\cos^{-1}\frac{1}{4} = 75.522°$

$2A$ can be in first or fourth quadrant.
$2A = 75.522°, 284.478°, 435.522°, 644.478°$
$A = 37.8°, 142.2°, 217.8°, 322.2°$

8 A curve is such that $\frac{d^2y}{dx^2} = 5\cos 2x$. This curve has a gradient of $\frac{3}{4}$ at the point $\left(-\frac{\pi}{12}, \frac{5\pi}{4}\right)$.

Show that the equation of the curve is $y = -\frac{5}{4}\cos 2x + 2x + \frac{17\pi}{12} + \frac{5\sqrt{3}}{8}$. [6]

When $x = -\frac{\pi}{12}$, $\frac{dy}{dx} = \frac{3}{4}$, $\frac{dy}{dx} = \frac{5\sin 2x}{2} + c$

$\frac{5}{2}\sin 2\left(-\frac{\pi}{12}\right) + c = \frac{3}{4}$

$c = \frac{3}{4} + \frac{5}{2}\left(\frac{1}{2}\right) = 2$

$y = \int \left[\frac{5}{2}\sin 2x + 2\right] dx$

$= -\frac{5}{4}\cos 2x + 2x + d$

When $x = -\frac{\pi}{12}$, $y = \frac{5\pi}{4}$

$\frac{5\pi}{4} = -\frac{5}{4}\cos 2\left(-\frac{\pi}{12}\right) + 2\left(-\frac{\pi}{12}\right) + d$ [

$d = \frac{17\pi}{12} + \frac{5\sqrt{3}}{8}$

Equation of curve: $y = -\frac{5}{4}\cos 2x + 2x + \frac{17\pi}{12} + \frac{5\sqrt{3}}{8}$. (proved)

n

9 **(a)** Factorise x^3+y^3. [1]

$$x^3+y^3=(x+y)\left(x^2-xy+y^2\right)$$ [B1]

(b) Hence, prove the identity $\dfrac{\sin^3\theta+\cos^3\theta}{2\sin^2\theta-1}=\dfrac{\sec\theta-\sin\theta}{\tan\theta-1}$. [4]

$$\frac{\sin^3\theta+\cos^3\theta}{2\sin^2\theta-1}=\frac{(\sin\theta+\cos\theta)\left(\sin^2\theta-\sin\theta\cos\theta+\cos^2\theta\right)}{2\sin^2\theta-\left(\sin^2\theta+\cos^2\theta\right)}$$

$$=\frac{(\sin\theta+\cos\theta)(1-\sin\theta\cos\theta)}{\sin^2\theta-\cos^2\theta}$$

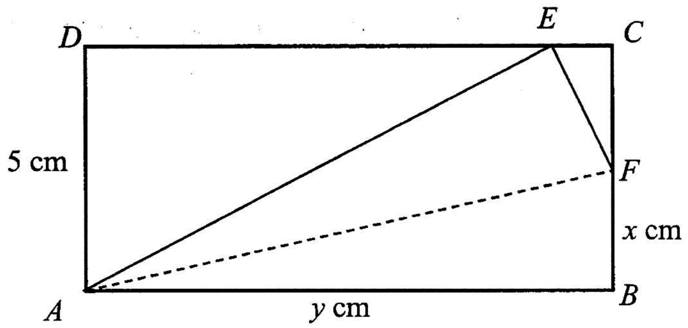
$$=\frac{(\sin\theta+\cos\theta)(1-\sin\theta\cos\theta)}{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}$$

$$=\frac{\dfrac{(1-\sin\theta\cos\theta)}{\cos\theta}}{\dfrac{(\sin\theta-\cos\theta)}{\cos\theta}}$$

$$=\frac{\sec\theta-\sin\theta}{\tan\theta-1}\text{ (proved)}$$

Continuation of working space for Question (9b).

10



The diagram shows a rectangular piece of paper $ABCD$ such that $AD = 5$ cm, $BF = x$ cm and $AB = y$ cm. The paper is folded along AF such that B meets E on DC.

(a) Express EC and AE in terms of x. [4]

$CF = (5-x)\text{cm}$

$EC = \sqrt{x^2 - (5-x)^2} = \sqrt{10x-25}$ cm

$AE = y$ and $DE = y - \sqrt{10x-25}$ cm

$5^2 + \left(y - \sqrt{10x-25}\right)^2 = y^2$

$25 + y^2 - 2y\sqrt{10x-25} + 10x - 25 = y^2$

$y = \dfrac{5x}{\sqrt{10x-25}}$

$AE = \dfrac{5x}{\sqrt{10x-25}}$ cm

(b) Hence show that the area, A cm^2, of triangle AEF is given by

$$A = \frac{5x^2}{2\sqrt{10x-25}}.$$ [2]

$A = \dfrac{1}{2}xy = \dfrac{1}{2}x \times \dfrac{5x}{\sqrt{10x-25}}$

$= \dfrac{5x^2}{2\sqrt{10x-25}}.$

(c) Show that $x=\frac{10}{3}$ for which $\frac{dA}{dx}=0$. [3]

$$\frac{dA}{dx}=\frac{5}{2}\left[\frac{\sqrt{10-2x}(2x)-x^2\left(\frac{1}{2}(10-2x)^{-\frac{1}{2}}(10)\right)}{10x-25}\right]$$

$$=\frac{5}{2}\left[\frac{(10x-25)^{-\frac{1}{2}}\left(2x(10x-25)-5x^2\right)}{10x-25}\right]$$

$$=\frac{5}{2}\left[\frac{15x^2-50x}{(10x-25)^{\frac{3}{2}}}\right]$$

For $\frac{dA}{dx}=0$, $\quad 15x^2-50x=0$

$5x(3x-10)=0$

$x=0$ (rej) or $x=\frac{10}{3}$

(d) By considering the change in the sign of $\frac{dA}{dx}$, show that A is a minimum at $x=\frac{10}{3}$. Hence find the minimum value of A. [3]

	$\frac{10}{3}^-$	$\frac{10}{3}$	$\frac{10}{3}^+$
$\frac{dA}{dx}$	Negative	Zero	Positive
Slope	\	—	/

Hence from the table, A is a minimum.

Minimum value of A in surd form $=\frac{5}{2}\left(\frac{\left(\frac{10}{3}\right)^2}{\sqrt{10\left(\frac{10}{3}\right)-25}}\right)=\frac{50\sqrt{3}}{9}=9.62$

11 The height, in metres, of a ball above the ground t seconds after it is thrown from a tower is given by the function $h(t) = -5t^2 + 10t + c$, where c is a constant.

(a) **(i)** Given that $h(0) = 6$, find the value of c. [1]

$-5(0)^2 + 10(0) + c = 6$
$c = 6$

(ii) Interpret the meaning of the answer in **(a) (i)**. [1]

When $t = 0$, height of ball = 6m. The ball was thrown from a tower which is 6 m tall.

(b) **Without the use of differentiation** and using the value of c from **(a) (i)**, determine the maximum height reached by the ball and the corresponding value of t. [4]

$h(t) = -5t^2 + 10t + 6$
$= -5\left(\left(t-1\right)^2 - 1\right) + 6$
$= -5\left(t-1\right)^2 + 11$

Maximum height = 11 m
Corresponding value of t = 1

Continuation of working space for Question (11b).

(c) Find the time it takes for the ball to drop to the ground. [2]

When it drops to the ground, $h(t) = 0$

$-5t^2 + 10t + 6 = 0$

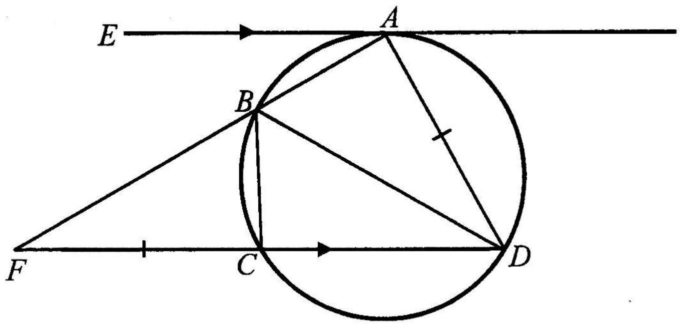
$-5(t-1)^2 + 11 = 0$

$(t-1)^2 = \frac{11}{5}$

$t = \sqrt{2.2} + 1 = 2.48$

Time = 2.48 s

12



In the diagram above, *EA* is the tangent to the circle at *A*.
FBA and *FCD* are straight lines.
EA is parallel to *FD* and $FC = AD$.

(a) Show that $\angle BAD = \angle BCF$. [2]

$\angle BAD + \angle BCD = 180°$ (Angles in the opposite segments)
$\angle BCF + \angle BCD = 180°$ (Adjacent angles on a straight line)
$\angle BCF + (180° - \angle BAD) = 180°$
$\angle BAD = \angle BCF$ (proved)

(b) Hence, prove that triangle *ABD* is congruent to triangle *CBF*. [4]

$\angle EAB = \angle AFC$ (alternate angles, $EA // FD$)
$\angle EAB = \angle ADB$ (Angle in alternate segment or Tangent Chord Theorem)

(1) $\angle ADB = \angle BFC$ (proved in above)
(2) $\angle BAD = \angle BCF$ (proved in part (a))
(3) $FC = AD$ (given)
$\Delta ABD \equiv \Delta CBF$ (ASA)

13 A particle starts from rest at a fixed point O and moves in a straight line with its acceleration, a m/s^2, is given by $a = 9 - kt$, where t seconds is the time since leaving O, and k is a constant. When $t = 2$, its velocity is 15 m/s.

(a) Find the value of k. [3]

$v = \int (9 - kt)\,\mathrm{d}t$

$v = 9t - \dfrac{kt^2}{2} + c$

When $t = 0,\ v = 0$

$c = 0$

When $t = 2,\ v = 15$

$18 - 2k = 15$

$\therefore k = \dfrac{3}{2}$

(b) The particle changes its direction of motion when $t = T$.
Find the value of T. [2]

When $v = 0$

$9T - \dfrac{3T^2}{4} = 0$

$T\left(9 - \dfrac{3}{4}T\right) = 0$

$T = 12$

(c) Show that the particle returns to O when $t = 18$. [3]

$$s = \int \left(9t - \frac{3t^2}{4}\right) dt$$
$$= \frac{9t^2}{2} - \frac{t^3}{4} + d$$

When $t = 0$, $s = 0, d = 0$

When $s = 0$,

$$\frac{9t^2}{2} - \frac{t^3}{4} = 0$$
$$t^2\left(\frac{9}{2} - \frac{t}{4}\right) = 0$$
$$t = 0 \text{ or } t = 18$$

Hence particle returns to O when $t = 18$.

(d) Hence find the distance travelled by the particle between $t = 0$ and $t = 18$. [3]

When $t = 12$, $s = \frac{9}{2}(12)^2 - \frac{12^3}{4} = 216\,\text{m}$.

Distance travelled between $t = 0$ and $t = 18 = 216 \times 2$
$= 432\,\text{m}$

END OF PAPER

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CEDAR GIRLS' SECONDARY SCHOOL
Preliminary Examination 2021
Secondary Four

CANDIDATE NAME	

CENTRE NUMBER		INDEX NUMBER	

ADDITIONAL MATHEMATICS

Paper 2

4049/02

14 September 2021

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your centre number, index number and name in the spaces at the top of this page.
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The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use
90

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$cosec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

Answer **all** the questions.

1 **(a)** Express $\dfrac{16x-14x^2-1}{(3x-1)(2x+1)^2}$ in partial fractions. [5]

(b) A right pyramid has a square base of side $\left(\sqrt{6}+\sqrt{3}\right)$ m and a height of h m. The volume of the pyramid is $\left(6+\sqrt{8}\right)$ m^3. Without using a calculator, show that h can be expressed as $a+b\sqrt{2}$, where a and b are integers. [3]

2 **(a)** Given that $p > \frac{1}{3}$, explain why the equation $3^{2x+1} = 6(3)^{x-1} - p$ has no real solutions. [4]

(b) Find the **exact** range of values of the constant a for which the line $y = 2x - \frac{a^2}{2}$ intersects the curve $y = x^2 - ax - 4$ at two distinct points. [5]

3 The equation of a curve is $y = \cos^3\left(\frac{x}{2} + \frac{\pi}{4}\right)$.

Find the value(s) of x for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ where the equation of the normal to the curve is $x = k$, where k is a constant. [4]

4 A glass of hot liquid is placed on a table to cool. The following table shows the measured values of the temperature of the liquid, T°C after n minutes.

n (minutes)	10	20	30	50
T (°C)	76.5	57.4	45.9	34.6

(i) On the grid on page 9, plot $\ln(T-28)$ against n and draw a straight line graph. [2]

use a graph paper instead

(ii) Use your graph to estimate the temperature of the liquid after 35 minutes. [2]

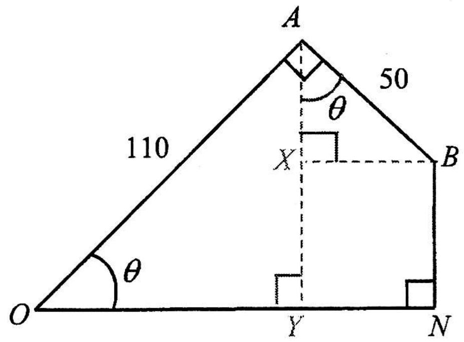
(iii) It is given that the formula connecting T and n is $T = 28 + ae^{-kn}$, where a and k are constants.

Use your graph to estimate

(a) the value of a and of k, [4]

(b) the time taken for the temperature to fall to half of its initial value. [2]

5



A farmer puts fences around the perimeter of his land $OABN$.

Angle OAB = angle $ONB = \frac{\pi}{2}$ and acute angle $AON = \theta$ radians.

$OA = 110$ m and $AB = 50$ m.

The length of fencing needed is L m.

(i) Show clearly that $L = p + 60\cos\theta + 160\sin\theta$, where p is a constant. [4]

(ii) Express L in the form $p + R\cos(\theta - \alpha)$ where $R > 0$ and α is acute. [3]

(iii) Given that the farmer uses exactly 315 m of fencing, find the value of θ. [3]

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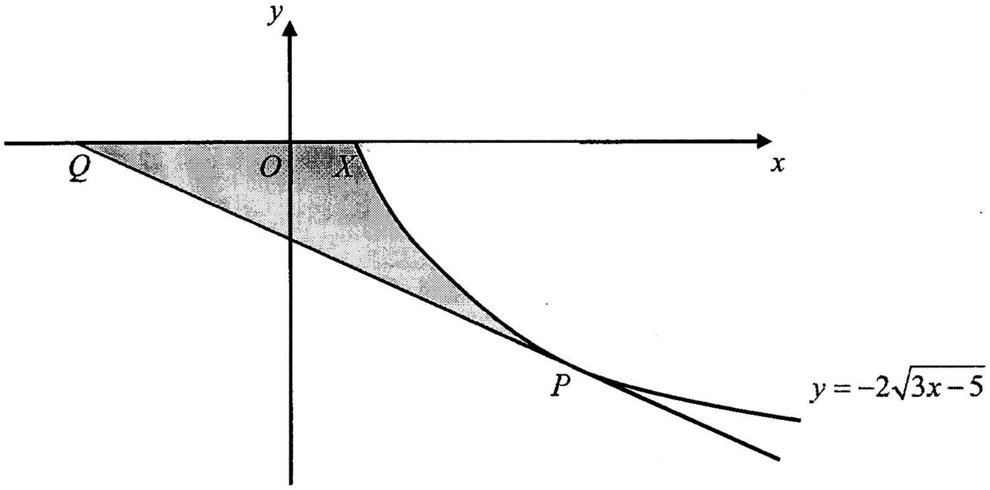


The diagram shows part of the curve $y=-2\sqrt{3x-5}$, $x \geq \frac{5}{3}$ passing through the point P where $x=10$. The curve meets the x-axis at the point X.
The tangent to the curve at P meets the x-axis at the point Q.

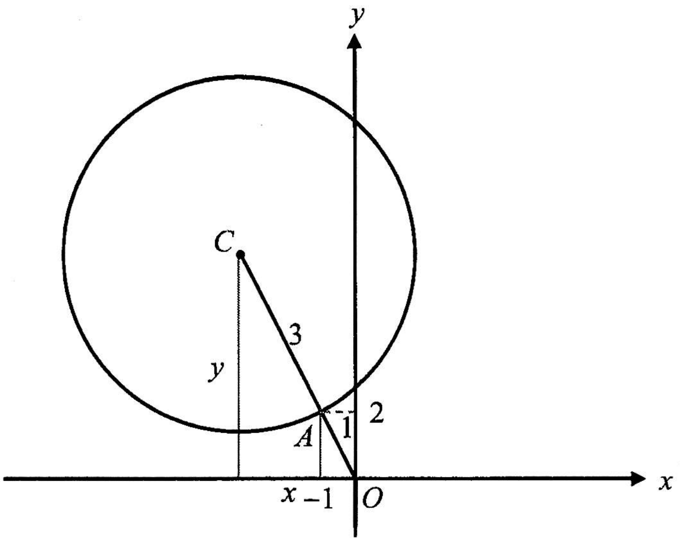
(i) Show that the x-coordinate of Q is $-\frac{20}{3}$. [4]

(ii) Find the area of the shaded region *PQX*. [5]

7 **(a)** Show that the solution of the equation $\dfrac{(\log_x y)^4}{2\log_y x} + \log_3 9 = -14$ can be written in the form $y = x^k$, where k is a constant. [4]

(b) Show that $\log_5(2x-7)-\log_5(x+4)=1$ has no real solutions. [3]

8



The diagram shows a circle, centre C passing through point A $(-1, 2)$.
A line from the origin O meets C such that $CA : AO$ is 3 : 1.

(i) Show that the centre of the circle is $(-4, 8)$ and hence find the equation of the circle. [3]

(ii) The circle is reflected along the y-axis.
State the equation of the reflected circle. [2]

(iii) Explain whether the point $(7,6)$ lies on the tangent to the circle at A. [3]

(iv) Find the equation of another tangent to the circle that has the same gradient as the tangent to the circle at A. [2]

9 **(i)** Find the value of the constant k for which $y = 2e^{-2x}x^2$ is a solution of the equation

$$4\frac{dy}{dx} + \frac{d^2y}{dx^2} + 4y = ke^{-2x}.$$ [6]

[3]

(ii) Find the exact coordinates of the stationary points of the curve $y = 2e^{-2x}x^2$.

(iii) Find the nature of the stationary points. [3]

10 **(i)** Express $\frac{9x^2}{3x-1}$ in the form $ax+b+\frac{c}{3x-1}$, where a, b and c are constants. [1]

(ii) Find $\int \frac{9x^2}{3x-1}\,\mathrm{d}x$. [3]

(iii) Given that $y=3x^2\ln(3x-1)$, find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(iv) Hence, evaluate $\int_1^2 x\ln(3x-1)\,dx$. [5]

End of Paper

ANS

CEDAR GIRLS' SECONDARY SCHOOL
Preliminary Examination 2021
Secondary Four

CANDIDATE NAME: Worked Solutions

CENTRE NUMBER: | | | | |

INDEX NUMBER: | | | | |

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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Answer **all** the questions.

1 **(a)** Express $\dfrac{16x-14x^2-1}{(3x-1)(2x+1)^2}$ in partial fractions. [5]

$$\frac{16x-14x^2-1}{(3x-1)(2x+1)^2}=\frac{A}{3x-1}+\frac{B}{2x+1}+\frac{C}{(2x+1)^2}$$

$$16x-14x^2-1=A(2x+1)^2+B(3x-1)(2x+1)+C(3x-1)$$

Let $x=-\frac{1}{2}$, Let $x=\frac{1}{3}$, Let $x=0$,

$-\frac{5}{2}C=-\frac{25}{2}$ $\quad$ $\frac{25}{9}A=\frac{25}{9}$ $\quad$ $B=-3$

$C=5$ $\quad$ $A=1$

$$\frac{16x-14x^2-1}{(3x-1)(2x+1)^2}=\frac{1}{3x-1}-\frac{3}{2x+1}+\frac{5}{(2x+1)^2}$$

(b) A right pyramid has a square base of side $\left(\sqrt{6}+\sqrt{3}\right)$ m and a height of h m. The volume of the pyramid is $\left(6+\sqrt{8}\right)$ m^3. Without using a calculator, show that h can be expressed as $a+b\sqrt{2}$, where a and b are integers. [3]

$$\frac{1}{3}\left(\sqrt{6}+\sqrt{3}\right)^2 \times h = 6+\sqrt{8}$$

$$h=\frac{18+6\sqrt{2}}{\left(\sqrt{6}+\sqrt{3}\right)^2}$$

$$=\frac{18+6\sqrt{2}}{9+6\sqrt{2}}$$

$$=\frac{18+6\sqrt{2}}{9+6\sqrt{2}}$$

$$=\frac{6+2\sqrt{2}}{3+2\sqrt{2}}$$

$$=\frac{6+2\sqrt{2}}{3+2\sqrt{2}}\times\frac{3-2\sqrt{2}}{3-2\sqrt{2}}$$

$$=10-6\sqrt{2}$$

2 **(a)** Given that $p > \frac{1}{3}$, explain why the equation $3^{2x+1} = 6(3)^{x-1} - p$ has no real solutions. [4]

$3^{2x+1} = 6(3)^{x-1} - p$

$3.3^{2x} = 6(3)^{x}3^{-1} - p$

$3.3^{2x} - 2(3)^{x} + p = 0$

Let $3^{x} = a$,
$3a^{2} - 2a + p = 0$

Discriminant $= (-2)^{2} - 4(3)(p)$
$= 4 - 12p$

Given $p > \frac{1}{3}$,
$-12p < -4$,
$4 - 12p < 0$

Since discriminant < 0, the equation has no real solutions.

(b) Find the **exact** range of values of the constant a for which the line $y = 2x - \frac{a^2}{2}$ intersects the curve $y = x^2 - ax - 4$ at two distinct points. [5]

$$x^2 - ax - 4 = 2x - \frac{a^2}{2}$$

$$x^2 + (-a-2)x + \frac{a^2}{2} - 4 = 0$$

$$\text{Discriminant} = (-a-2)^2 - 4(1)\left(\frac{a^2}{2} - 4\right)$$

$$= -a^2 + 4a + 20$$

Since line intersects the curve at 2 distinct points,

Discriminant > 0,

$-a^2 + 4a + 20 > 0$,

$a^2 - 4a - 20 < 0$

Let $a^2 - 4a - 20 = 0$

$$a = \frac{4 \pm \sqrt{16 - 4(1)(-20)}}{2}$$

$$= \frac{4 \pm \sqrt{96}}{2}$$

$$= \frac{4 \pm 4\sqrt{6}}{2} = 2 \pm 2\sqrt{6}$$

$$2 - 2\sqrt{6} < a < 2 + 2\sqrt{6}$$

3 The equation of a curve is $y = \cos^3\left(\frac{x}{2} + \frac{\pi}{4}\right)$.

Find the value(s) of x for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ where the equation of the normal to the curve is $x = k$, where k is a constant. [4]

$$y = \cos^3\left(\frac{x}{2} + \frac{\pi}{4}\right)$$

$$\frac{dy}{dx} = \frac{3}{2}\cos^2\left(\frac{x}{2} + \frac{\pi}{4}\right)\left(-\sin\left(\frac{x}{2} + \frac{\pi}{4}\right)\right)$$

$$\frac{3}{2}\cos^2\left(\frac{x}{2} + \frac{\pi}{4}\right)\left(-\sin\left(\frac{x}{2} + \frac{\pi}{4}\right)\right) = 0$$

$$\cos^2\left(\frac{x}{2} + \frac{\pi}{4}\right) = 0 \quad \text{or} \quad -\sin\left(\frac{x}{2} + \frac{\pi}{4}\right) = 0$$

$$\frac{x}{2} + \frac{\pi}{4} = \frac{\pi}{2}, \qquad \frac{x}{2} + \frac{\pi}{4} = 0$$

$$x = \frac{\pi}{2} \quad \text{or} \quad x = -\frac{\pi}{2}$$

4 A glass of hot liquid is placed on a table to cool. The following table shows the measured values of the temperature of the liquid, T°C after n minutes.

n (minutes)	10	20	30	50
T (°C)	76.5	57.4	45.9	34.6

(i) On the grid on page 9, plot $\ln(T-28)$ against n and draw a straight line graph. [2]

(ii) Use your graph to estimate the temperature of the liquid after 35 minutes. [2]

From the graph, $\ln(T-28)=2.64$

$T=42.0$°C

(iii) It is given that the formula connecting T and n is $T=28+ae^{-kn}$, where a and k are constants.

Use your graph to estimate

(a) the value of a and of k, [4]

$T=28+ae^{-kn}$

$\ln(T-28)=\ln(ae^{-kn})$

$\ln(T-28)=-kn+\ln a$

Gradient $=-k$

$y-$intercept $=\ln a$

$$\text{Gradient}=\frac{3.88-2.88}{10-30}$$

$$=-\frac{1}{20}$$

$k=\frac{1}{20}$ or 0.05

$y-$intercept $=4.38$

$\ln a=4.38$

$a=e^{4.38}=79.8$ (3 s.f)

(b) the time taken for the temperature to fall to half of its initial value. [2]

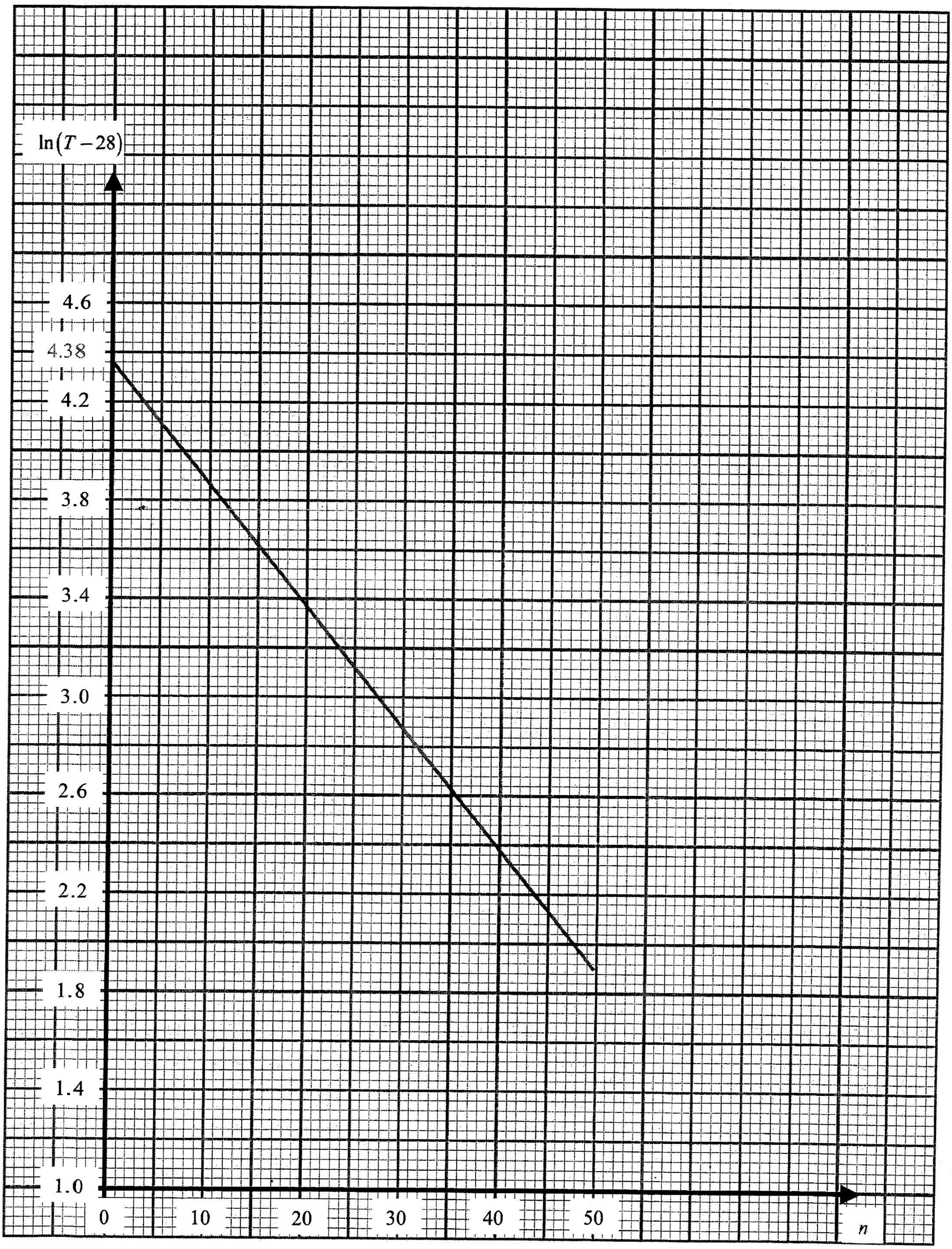
$\ln(T-28)=4.38$

Initial temperature $=107.8$°C

Half of its initial temperature $=53.9$°C

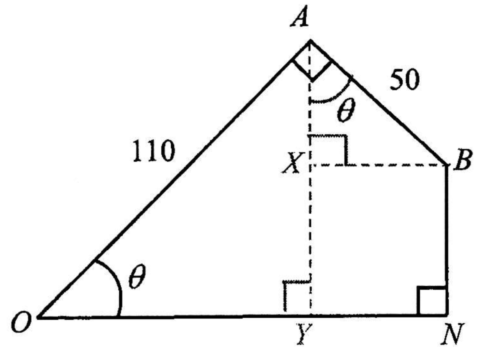
$\ln(53.9-28)=3.2542$

From graph, time taken = 23 minutes



ln(T − 28)
4.6
4.38
4.2
3.8
3.4
3.0
2.6
2.2
1.8
1.4
1.0
0
10
20
30
40
50
n

5



A farmer puts fences around the perimeter of his land *OABN*.

Angle OAB = angle $ONB = \frac{\pi}{2}$ and acute angle $AON = \theta$ radians.

$OA = 110$ m and $AB = 50$ m.

The length of fencing needed is L m.

(i) Show clearly that $L = p + 60\cos\theta + 160\sin\theta$, where p is a constant. [4]

$\cos\theta = \frac{OY}{110}$

$OY = 110\cos\theta$

$\sin\theta = \frac{AY}{110}$

$AY = 110\sin\theta$

$\angle BAX = \theta$

$\cos\theta = \frac{AX}{50}$

$AX = 50\cos\theta$

$\sin\theta = \frac{BX}{50}$

$BX = 50\sin\theta$

$= YN$

$BN = AY - AX$

$= 100\sin\theta - 50\cos\theta$

$L = 110 + 110\cos\theta + 50\sin\theta + 110\sin\theta - 50\cos\theta + 50$

$= 160 + 60\cos\theta + 160\sin\theta$ (shown)

(ii) Express L in the form $p+R\cos(\theta-\alpha)$ where $R>0$ and α is acute. [3]

$$L=160+60\cos\theta+160\sin\theta$$
$$=160+R\cos(\theta-\alpha)$$

$$R=\sqrt{60^2+160^2}$$
$$=\sqrt{29200} \quad \text{or} \quad 170.88\ (5\text{ s.f})$$

$$\tan\alpha=\frac{160}{60}$$
$$\alpha=1.2120\ (5\text{ s.f})$$

$$L=160+\sqrt{29200}\cos(\theta-1.21) \quad \text{or} \quad 160+171\cos(\theta-1.21)$$

(iii) Given that the farmer uses exactly 315 m of fencing, find the value of θ. [3]

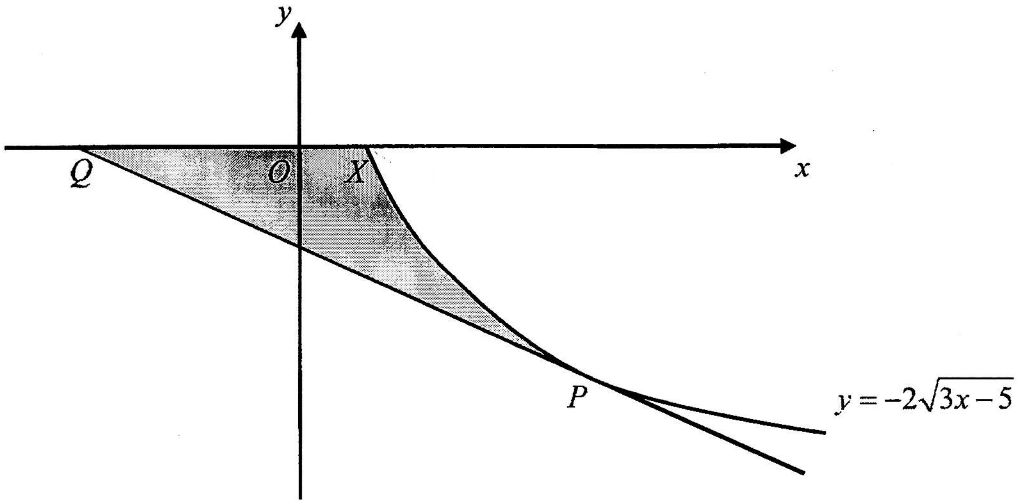
$$160+\sqrt{29200}\cos(\theta-1.2120)=315\ , \quad -1.2120<\theta-1.2120<0.35880$$

$$\cos(\theta-1.2120)=0.90707$$

$$\theta-1.2120=0.43453\ \text{(rej)}, \quad 2\pi-0.43453\ \text{(rej)}, \quad -0.43453$$

$$\theta=0.777(3\text{ s.f})$$

6



The diagram shows part of the curve $y=-2\sqrt{3x-5},\ x \geq \frac{5}{3}$ passing through the point P where $x=10$. The curve meets the x-axis at the point X.
The tangent to the curve at P meets the x-axis at the point Q.

(i) Show that the x-coordinate of Q is $-\frac{20}{3}$. [4]

$y=-2\sqrt{3x-5}$

$\frac{dy}{dx}=\frac{-3}{\sqrt{3x-5}}$

When $x=10$,

$\frac{dy}{dx}=-\frac{3}{\sqrt{30-5}}=-\frac{3}{5}$

and $y=-10$

Equation of tangent

$y+10=-\frac{3}{5}(x-10)$

$y=-\frac{3}{5}x-4$

When $y=0$, $\frac{3}{5}x=-4$

$x=-\frac{20}{3}$

(ii) Find the area of the shaded region *PQX*. [5]

When $y = 0$, $-2\sqrt{3x-5} = 0$

$$3x - 5 = 0$$

$$x = \frac{5}{3}$$

Area of shaded region *PQX*

$$= \frac{1}{2}\left(10 + \frac{20}{3}\right)(10) - \left|(-2)\int_{\frac{5}{3}}^{10} (3x-5)^{\frac{1}{2}}\ dx\right|$$

$$= \frac{250}{3} - \left|\left[\frac{(-2)(3x-5)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)(3)}\right]_{\frac{5}{3}}^{10}\right|$$

$$\frac{250}{3} - \left|\left[-\frac{4}{9}(3x-5)^{\frac{3}{2}}\right]_{\frac{5}{3}}^{10}\right|$$

$$\frac{250}{3} - \left|-\frac{4}{9}\left(25^{\frac{3}{2}} - 0\right)\right|$$

$$\frac{250}{3} - \frac{500}{9}$$

$$= \frac{250}{9} \text{ or } 27.8 \text{ (3 s.f) sq units}$$

7 **(a)** Show that the solution of the equation $\frac{(\log_x y)^4}{2\log_y x}+\log_3 9=-14$ can be written in the form $y=x^k$, where k is a constant. [4]

$$\frac{(\log_x y)^4}{2\log_y x}+\log_3 9=-14$$

$$\frac{(\log_x y)^4}{\frac{2\log_x x}{\log_x y}}+2=-14$$

$$\frac{(\log_x y)^5}{2}=-16$$

$$(\log_x y)^5=-32$$

$$\log_x y=-2$$

$$y=x^{-2}$$

(b) Show that $\log_5(2x-7)-\log_5(x+4)=1$ has no real solutions. [3]

$$\log_5\left(\frac{2x-7}{x+4}\right)=1$$

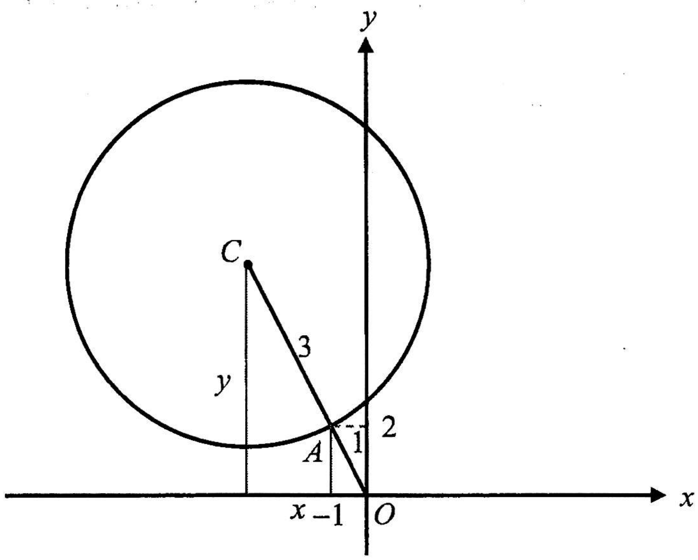
$$\frac{2x-7}{x+4}=5$$

$$2x-7=5x+20$$

$$x=-9$$

Sub $x=-9$ into $\log_5(2x-7)$ and $\log_5(x+4)$,
since $\log_5(-25)$ and $\log_5(-5)$ are undefined, the equation has no real solutions.

8



The diagram shows a circle, centre C passing through point $A\ (-1,2)$.
A line from the origin O meets C such that $CA : AO$ is 3 : 1.

(i) Show that the centre of the circle is $(-4,8)$ and hence find the equation of the circle. [3]

Let the vertical distance of the centre of circle from the x-axis be y and the horizontal distance of the centre of the circle from the y-axis be x.

Using similar triangles,

$\frac{x}{1}=\frac{4}{1}$, $\quad \frac{y}{2}=\frac{4}{1}$

$x=4 \quad y=8$

Thus, centre of circle is $(-4,8)$

Radius = $\sqrt{(-4+1)^2+(8-2)^2}$
$=\sqrt{45}$

Equation of circle
$(x+4)^2+(y-8)^2=45$

(ii) The circle is reflected along the y-axis.
State the equation of the reflected circle. [2]

Centre of reflected circle = $(4,8)$

Equation of circle
$(x-4)^2+(y-8)^2=45$

(iii) Explain whether the point $(7, 6)$ lies on the tangent to the circle at A. [3]

Gradient of $AC = \frac{8-2}{-4+1} = -2$

Gradient of tangent to circle at A $= \frac{1}{2}$ (tan $\perp$ rad)

Equation of tangent at A

$$y - 2 = \frac{1}{2}(x+1)$$
$$y = \frac{1}{2}x + \frac{5}{2}$$

When $x = 7, y = \frac{1}{2}(7) + \frac{5}{2} = 6$

Yes, the point (7, 6) lies on the tangent to the circle at A.

(iv) Find the equation of another tangent to the circle that has the same gradient as the tangent to the circle at A. [2]

The point on the circle whose tangent has the same gradient as the tangent at A would be the other end point of the diameter passing through A.

Let this end point be (p, q)

$$\frac{p-1}{2} = -4$$
$$p = -7$$

$$\frac{q+2}{2} = 8$$
$$q = 14$$

The end point is $(-7, 14)$

Equation of another tangent to the circle:

$$y - 14 = \frac{1}{2}(x+7)$$
$$y = \frac{1}{2}x + \frac{35}{2}$$

9 **(i)** Find the value of the constant k for which $y = 2e^{-2x}x^2$ is a solution of the equation

$$4\frac{dy}{dx} + \frac{d^2y}{dx^2} + 4y = ke^{-2x}.$$ [6]

$y = 2e^{-2x}x^2$

$\frac{dy}{dx} = 4xe^{-2x} - 4x^2e^{-2x}$

$= e^{-2x}(4x - 4x^2)$

$\frac{d^2y}{dx^2} = e^{-2x}(4 - 8x) - 2e^{-2x}(4x - 4x^2)$

$= e^{-2x}(8x^2 - 16x + 4)$

$4\frac{dy}{dx} + \frac{d^2y}{dx^2} + 4y$

$= 4e^{-2x}(4x - 4x^2) + e^{-2x}(8x^2 - 16x + 4) + 8e^{-2x}x^2$

$= e^{-2x}(16x - 16x^2 + 8x^2 - 16x + 4 + 8x^2)$

$= 4e^{-2x}$

Hence, $k = 4$

[3]

(ii) Find the exact coordinates of the stationary points of the curve $y = 2e^{-2x}x^2$.

For stationary points,

$$\frac{dy}{dx} = 0$$

$$e^{-2x}(4x - 4x^2) = 0$$

$$e^{-2x} = 0 \text{ (reject) or } 4x - 4x^2 = 0$$

$$x(1 - x) = 0$$

$$x = 0 \text{ or } x = 1$$

$$y = 0 \quad y = \frac{2}{e^2}$$

The stationary points are $(0, 0)$ and $\left(1, \frac{2}{e^2}\right)$

(iii) Find the nature of the stationary points. [3]

$$\frac{d^2y}{dx^2} = e^{-2x}(8x^2 - 16x + 4)$$

When $x = 0$,

$$\frac{d^2y}{dx^2} = e^0(4)$$

$$= 4 \ (> 0)$$

(0, 0) is a minimum point

When $x = 1$,

$$\frac{d^2y}{dx^2} = e^{-2}(8 - 16 + 4)$$

$$= \frac{-4}{e^2} \ (< 0)$$

$\left(1, \frac{2}{e^2}\right)$ is a maximum point

10 **(i)** Express $\dfrac{9x^2}{3x-1}$ in the form $ax+b+\dfrac{c}{3x-1}$, where a, b and c are constants. [1]

$$\frac{9x^2}{3x-1}=3x+1+\frac{1}{3x-1}$$

(ii) Find $\displaystyle\int \frac{9x^2}{3x-1}\,\mathrm{d}x$. [3]

$$\int \frac{9x^2}{3x-1}\,\mathrm{d}x$$
$$=\int 3x+1+\frac{1}{3x-1}\,\mathrm{d}x$$
$$=\frac{3}{2}x^2+x+\frac{1}{3}\ln(3x-1)+C$$

(iii) Given that $y=3x^2\ln(3x-1)$, find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [2]

$$\frac{\mathrm{d}y}{\mathrm{d}x}=3x^2\left(\frac{3}{3x-1}\right)+6x\ln(3x-1)$$
$$=\frac{9x^2}{3x-1}+6x\ln(3x-1)$$

(iv) Hence, evaluate $\int_1^2 x\ln(3x-1)\,dx$. [5]

$$\int_1^2 \frac{9x^2}{3x-1}+6x\ln(3x-1)\,dx=\left[3x^2\ln(3x-1)\right]_1^2$$

$$\left[\frac{3}{2}x^2+x+\frac{1}{3}\ln(3x-1)\right]_1^2+\int_1^2 6x\ln(3x-1)\,dx=12\ln 5-3\ln 2$$

$$\left(6+2+\frac{\ln 5}{3}\right)-\left(\frac{3}{2}+1+\frac{\ln 2}{3}\right)+6\int_1^2 x\ln(3x-1)\,dx=12\ln 5-3\ln 2$$

$$6\int_1^2 x\ln(3x-1)\,dx=17.234-5.8054$$

$$\int_1^2 x\ln(3x-1)\,dx=\frac{11.4286}{6}=1.90\text{ (3 s.f)}$$

End of Paper