Name: _____ (     )          Class: _____

# ADDITIONAL MATHEMATICS                                   4049/01

Paper 1                                                   **23 August 2021**
                                                          **2 hours 15 minutes**

Candidates answer on the Question Paper.

No Additional Materials are required.

## READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
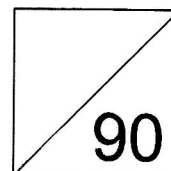You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [  ] at the end of each question or part question.

The total number of marks for this paper is **90**.

## FOR EXAMINER'S USE

| Q1 |   | Q4 |   | Q7 |   | Q10 |   |
|----|---|----|---|----|---|-----|---|
| Q2 |   | Q5 |   | Q8 |   | Q11 |   |
| Q3 |   | Q6 |   | Q9 |   | Q12 |   |

90

This document consists of **15** printed pages and **1** blank page.

圣尼各拉女校
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**[Turn over**

*Mathematical Formulae*

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial expansion*

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

*Formulae for $\triangle ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

**1** The line $2x = 1 - 3y$ intersects the curve $3y^2 - x^2 = 2$ at points $A$ and $B$.
The $x$-coordinate of $B$ is greater than the $x$-coordinate of $A$.

**(a)** Find the coordinates of $A$. [4]

Point $C$ lies on $AB$ such that $AC : CB = 2 : 1$.

**(b)** Find the $x$-coordinate of $C$. [1]

**2**  **(a)**  The function f is defined by $f(x) = \dfrac{2x+5}{1-3x}$, $x \neq \dfrac{1}{3}$.

Determin whether f is an increasing or decreasing function. [3]

**(b)**  Find $\displaystyle\int \left( \sec^2 x - \dfrac{\pi}{2} \sin(3x-5) \right) dx$. [2]

**3**   The equation of a curve is $y = 2\cos 2x + c,$ where $c$ is a constant.
The graph passes through the point $(30°, 5)$.

**(a)**   Find the value of $c$.                                                                    [2]

**(b)**   State the amplitude and period of $y$.                                      [2]

**(c)**   Sketch the graph of $y = 2\cos 2x + c$ for $0° \le x \le 360°$.          [3]

4   A curve is such that $\dfrac{d^2y}{dx^2} = \dfrac{3}{(2x-1)^2}$ and the point $(1, 7)$ lies on the curve. The gradient of the tangent to the curve at $x = 2$ is $\dfrac{3}{2}$. Find the equation of the curve.   [5]

**5** **(a)** Write down and simplify the first 3 terms in the expansion of $(1+3x)^6$, in ascending powers of $x$.

Hence find the value of $a$ and of $b$ in the expansion of $(1+ax+bx^2)(1+3x)^6$ if the coefficients of $x$ and $x^2$ are 23 and 223 respectively. [4]

**(b)** Find the term independent of $x$ in the expansion of $\left(x-\dfrac{1}{2x^2}\right)^{15}$. [3]

6   The equation of a polynomial is $f(x) = 9x^3 - 6x^2 - 11x + 4$.

   **(i)**   Factorise completely the polynomial $f(x)$.   [3]

   **(ii)**   Sketch the graph of $f(x) = 9x^3 - 6x^2 - 11x + 4$.   [2]

   **(iii)**   Find the range of values of $x$ for which $f(x) \geq 0$.   [2]

**7**    **(i)**    Prove the identity $\dfrac{1}{\sin x + 1} - \dfrac{1}{\sin x - 1} = 2\sec^2 x$.      [3]

     **(ii)**    Hence, solve the equation $\dfrac{1}{2(\sin x + 1)} - \dfrac{1}{2(\sin x - 1)} = 3 + \tan x$ for $0° \le x \le 360°$.    [5]

**8** Michael bought a brand new Ducati motorcycle. As the motorcycle is a limited edition model, many enthusiasts predict that its value, $P$, will appreciate so that after $t$ months, its value can be modelled by $P = 65000e^{kt}$. The motorcycle is expected to have a value of \$72000 after 2 years.

**(i)** How much did Michael pay for the motorcycle? [1]

**(ii)** Calculate the value of the motorcycle after 5 years. [4]

**(iii)** After how many years will the motorcycle have a value of \$90000? [3]

**9** **(a)** The equation of a curve is $y = x^2 + (2a-1)x + a^2$, where $a$ is a constant.

    **(i)** Find the range of values of $a$ for which the curve has no real roots. [3]

    **(ii)** In the case where $a = 2$, find the range of values of $x$ for which the curve lies above the line $y = 8$. [3]

**(b)** Explain why the greatest value of $-2x^2 + 4x - 5$ is $-3$ for all values of $x$. [3]

**10**   **(a)**   It is given that $\tan A = c$, where $c$ is a constant and $180° \le A \le 270°$.
Express each of the following, in terms of $c$,

        **(i)**    $\cot A$,                            [1]

        **(ii)**    $\sin 2A$.                          [2]

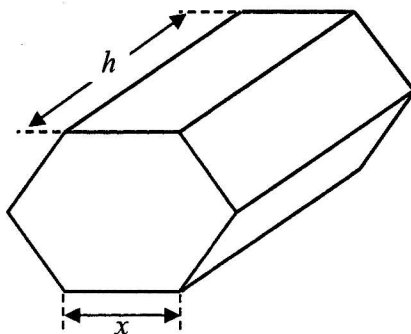   **(b)**   Given that $\sin x = 2\sin(30° - x)$, find the exact value of $\tan x$.    [5]

**11** A particle $P$, moves in a straight line so that its velocity, $v$ m/s from a fixed point $O$ is given by $v = 2t^2 - 9t + 7$, where $t$ is the time in seconds after passing $O$.

**(a)** Find the deceleration of $P$ when $t = 2$. [3]

**(b)** Find the values of $t$ when $P$ is instantaneously at rest. [3]

**(c)** Find the distance $P$ travelled in the fourth second. [4]

**12**



The diagram shows part of a solid metal allen key that has length $h$ cm and uniform regular hexagonal cross-section of side $x$ cm.

**(i)** Find an expression, in terms of $x$, for the cross-sectional area of the allen key. [2]

**(ii)** The volume of the allen key is 4 cm$^3$. Show that the surface area of the allen key, $A$ cm$^2$, is given by $A = 3\sqrt{3}x^2 + \dfrac{16}{\sqrt{3}x}$. [3]

Given that $x$ can vary, find

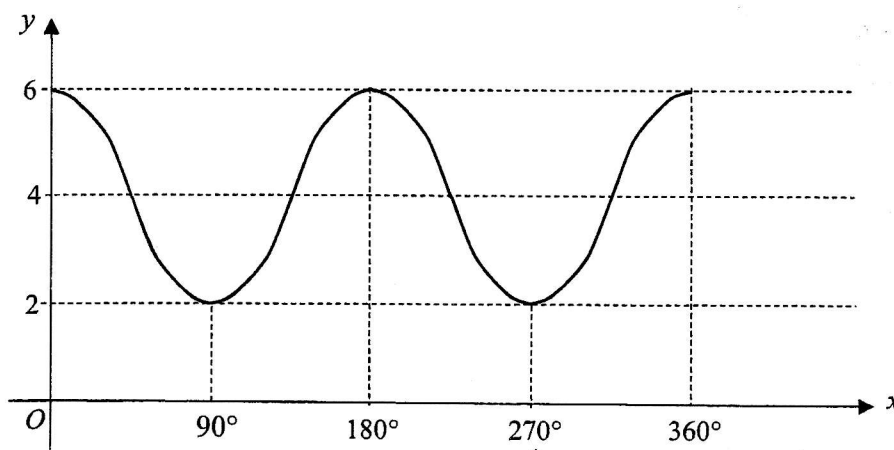**(iii)**    the value of $x$ for which $A$ has a stationary value,          [3]

**(iv)**    the stationary value of $A$ and determine whether this value is maximum or minimum.    [3]

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**16**

**Answers:**

1    (a)    $A(-1, 1)$       (b)    $x_C = 3$

2    (a)    increasing function       (b)    $\tan x + \dfrac{\pi}{6}\cos(3x-5) + c$

3    (a)    $c = 4$       (b)    amplitude = 2, period = 180°

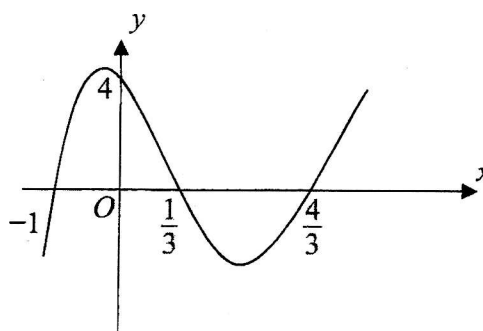     (c)



4    $y = -\dfrac{3}{4}\ln(2x-1) + 2x + 5$

5    (a)    $1 + 18x + 135x^2 + \ldots$ ; $a = 5, b = -2$

     (b)    $-93\dfrac{27}{32}$

6    (i)    $f(x) = (x+1)(3x-4)(3x-1)$    (ii)

     (iii)    $-1 \le x \le \dfrac{1}{3}$, $x \ge \dfrac{4}{3}$



7    (ii)    $x = 63.4°, 135°, 243.4°, 315°$

8    (i)    $65000       (ii)    $83935.38       (iii)    6.36 years

9    (a)(i)    $a > \dfrac{1}{4}$       (ii)    $x < -4$ or $x > 1$

10    (a)(i)    $\dfrac{1}{c}$       (ii)    $\dfrac{2c}{c^2+1}$       (b)    $= \dfrac{1}{2}\left(\sqrt{3}-1\right)$

11    (a)    $1 \text{ m/s}^2$       (b)    $t = 3.5, 1$       (c)    1.25 m

12    (i)    $\dfrac{3\sqrt{3}}{2}x^2$       (iii)    $x = 0.961$       (iv)    $14.4 \text{ cm}^2$, min

Name: _____ (   )          Class: _____

# ADDITIONAL MATHEMATICS          **4049/02**

Paper 2                                        **24 August 2021**
                                               **2 hours 15 minutes**

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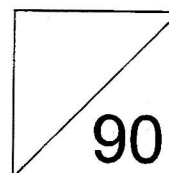You are reminded of the need for clear presentation in your answers.

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/90

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where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

## 2. TRIGONOMETRY

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*Formulae for $\triangle ABC$*

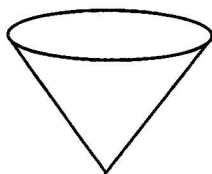$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

**1**   Show that $5^n + 5^{n+1} + 5^{n+2}$ is odd for all positive integer values of $n$.   [3]

2    Express $\dfrac{2x^3+10x-9}{(2-x)(1+2x^2)}$ in partial fractions.    [5]

**3**    A viscous liquid is poured into an empty inverted right circular cone.



The volume of the viscous liquid in the cone is increasing at a steady rate of $2\pi$ cm³/s. Given that the dimensions of the cone is such that the height of the cone is always 3 times the radius, find

**(i)**    the radius of the viscous liquid 4 seconds after the pouring has commenced,    [3]

**(ii)**    the rate of the increase of the radius at that instant.    [4]

4  Given that $y = x^2\sqrt{2x-1}$, where $x > \dfrac{1}{2}$, express $\dfrac{dy}{dx}$ in the form $\dfrac{x(Ax+B)}{\sqrt{2x-1}}$, where $A$ and $B$ are integers.

Hence evaluate $\displaystyle\int_1^4 \dfrac{5x^2-2x-2}{\sqrt{2x-1}}\, dx$.　　　　　　　　　　　　　[7]

**5** Solve the equation

    **(i)**     $5^{2x+1} = 39 - 2(5^x)$,                                               [4]

    **(ii)**    $\log_4 \sqrt{x-5} - 1 = \log_{16}(1-2x)$.                             [5]

**6** **(a)** The equation of a curve is $y = 3x^2 - x - 16$.

Find the set of values of $x$ for which $y + 2 < 0$. Represent this set on a number line. [3]

**(b)** A curve has equation $y = px^2 - mx + 12m$ and a line has equation $y = -3mx$, where $m$ and $p$ are positive constants.

**(i)** Find, in terms of $m$ and $p$, the condition for which the curve and the line intersect. [4]

**(ii)** In the case when the line is a tangent to the curve at point $A$,

    **(a)** state the value of $\dfrac{m}{p}$,              [2]

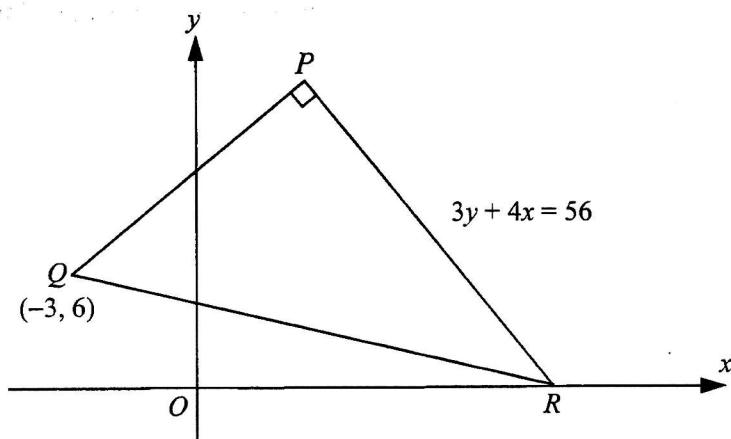    **(b)** find the $x$-coordinate of point $A$.           [2]

7    The expression $5\cos\theta - \sin\theta$ is defined for $0 \le \theta \le \pi$.

    **(i)**    Using $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$, solve the equation $5\cos\theta - \sin\theta = 2$.   [4]

    **(ii)**    Using your answer in part (i), solve $5\cos\left(\dfrac{\theta}{2}\right) - \sin\left(\dfrac{\theta}{2}\right) = 2$.     [1]

**(iii)** State the largest and smallest values of $\left(10\cos\theta - 2\sin\theta\right)^2$ and find the corresponding values of $\theta$. [4]

8



The diagram shows a triangle $PQR$ in which the point $Q$ is $(-3, 6)$, the point $R$ lies on the $x$-axis and angle $QPR$ is $90°$. The equation of $PR$ is $3y + 4x = 56$.

**(a)**  Find the coordinates of $P$.  [5]

**(b)** A circle is drawn passing through the points $P$, $Q$ and $R$.
$x = a$ is a tangent to the circle.
    **(i)** Find the equation of the circle. [4]

    **(ii)** Find the exact value(s) of $a$. [2]

**9** Given that $y = e^{-x}(\sin 2x - 2\cos 2x)$,

**(i)** Find and simplify an expression for $\dfrac{dy}{dx}$. [2]

Hence find

**(ii)** the value of $x$ between $\dfrac{\pi}{2}$ and $\pi$ for which $y$ is stationary, [3]

**(iii)**  the $y$-coordinate of this stationary point and determine its nature.  **[3]**

10 Recorded values of the volume, $v$ ml, of diffuser perfume in a jar, $t$ hours after observation began are shown in the table below.

| $t$ (hours) | 25 | 50 | 75 | 100 |
|---|---|---|---|---|
| $v$ (millilitres) | 250 | 112 | 36 | 12 |

It is known that $v$ and $t$ are related by the equation $v = v_0 10^{-kt}$, where $v_0$ and $k$ are constants.

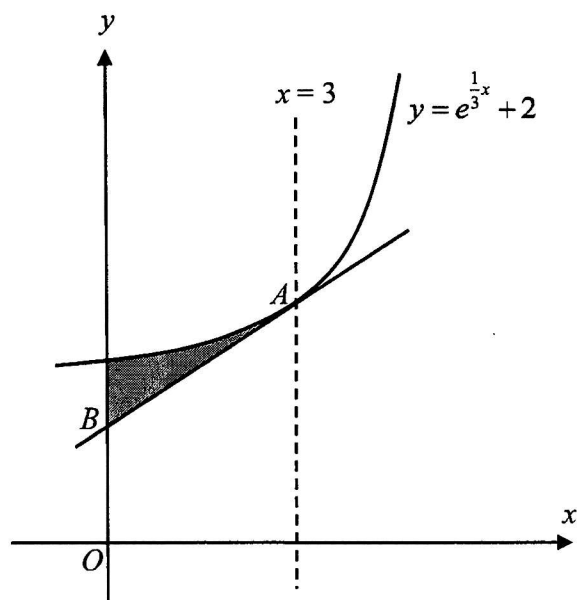**(a)** Plot $\lg v$ against $t$ and draw a straight line graph. [3]
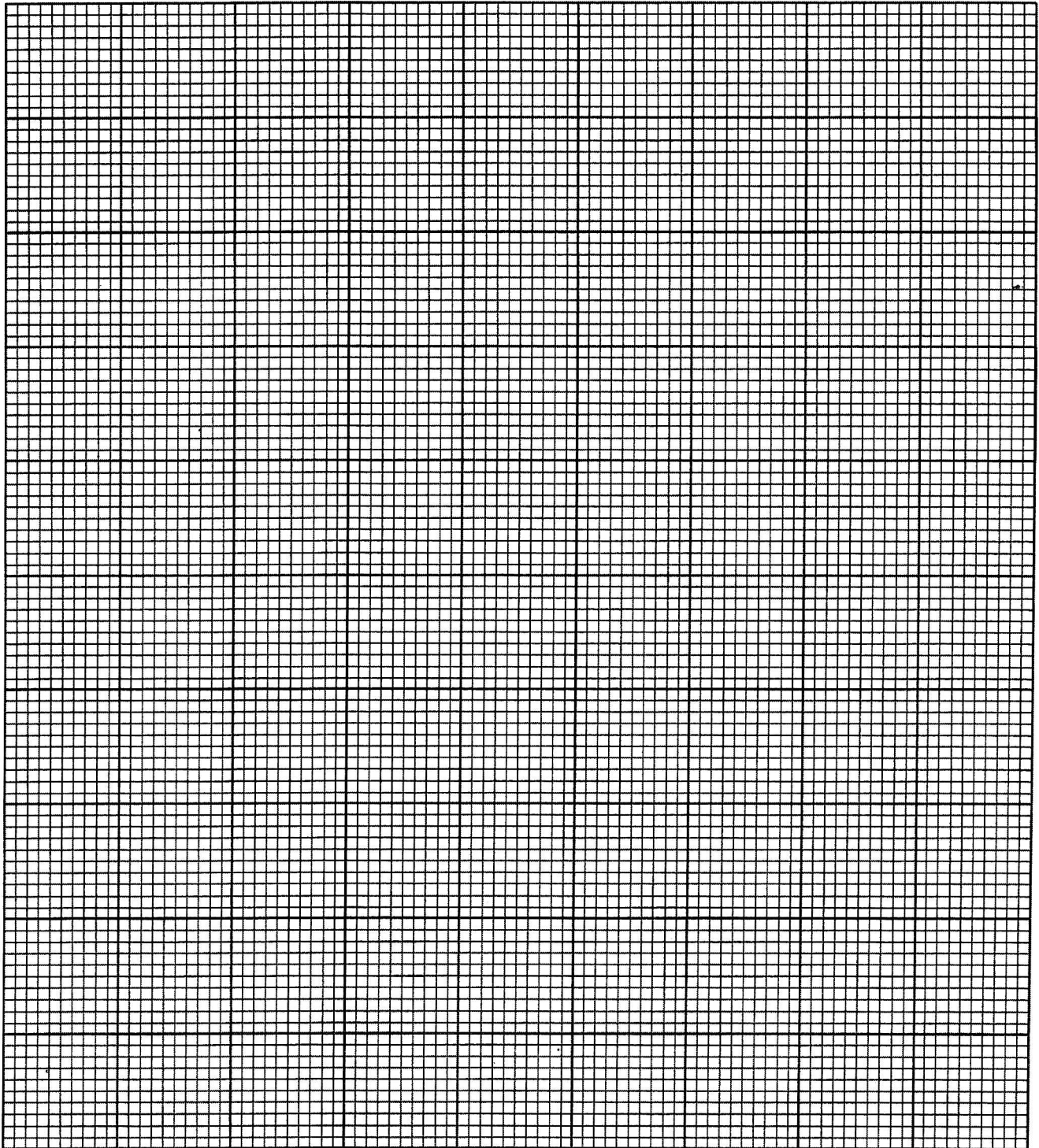
**(b)** Use your graph to estimate the volume of perfume, to the nearest ml, at the start of the observation. [3]

**(c)** Use your graph to find the value of $k$. [1]

**(d)** Use your graph to find the time taken for the perfume to be 10 percent of its orginal volume. [2]

**11**



The diagram shows part of the curve $y = e^{\frac{1}{3}x} + 2$. The tangent to the curve at $A$ intersects the $y$-axis at $B$.

**(a)** Find the exact area of the shaded region bounded by the tangent $AB$, the curve and the $y$-axis. [8]

**(b)** Find the equation of the normal to the curve at $x = 0$. [3]

**Answers:**

2     $\dfrac{2x^3+10x-9}{(2-x)(1+2x^2)} = -1 + \dfrac{3}{2-x} + \dfrac{2x-5}{1+2x^2}$

3     (i)     2 cm          (ii)     $\dfrac{1}{6}$ cm/s

4     $\dfrac{dy}{dx} = \dfrac{x(5x-2)}{\sqrt{2x-1}}$ ;     $14\sqrt{7}+1$

5     (i)     0.594          (ii)     no solution

6     (a)     $-2 < x < 2\dfrac{1}{3}$

      (b)(i)     $m \geq 12p$   or   $m - 12p \geq 0$   or   $4m^2 - 48pm \geq 0$

      (ii)     $\dfrac{m}{p} = 12$

      (iii)     $x = -12$

7     (i)     $\theta = 0.970$     (ii)     $\theta = 1.94$

      (iii)     Smallest value = 0,     $\theta = 1.37$

              Largest value = 104,     $\theta = 2.94$

8     (a)     $P(5, 12)$     (ii)     $\left(x - \dfrac{11}{2}\right)^2 + (y-3)^2 = \dfrac{325}{4}$     (iii)     $a = \dfrac{11 \pm 5\sqrt{13}}{2}$

9     (i)     $\dfrac{dy}{dx} = e^{-x}(3\sin 2x + 4\cos 2x)$

      (ii)     $x = 2.68$     (iii)     $-0.137$ ; minimum point

10    (b)     $v_0 = 891$ ml     (c)     $k = 0.0168$     (d)     52.5 hours

11    (a)     $\dfrac{3}{2}e - 3$ units$^2$ (b)     $y = -3x + 3$