Anglican High. P1

1 Solve the equation $x+1+\sqrt{5-x}=0$. [3]

2 (a) (i) Given the equation $13\left(3^{2x}\right)=5^{3x}$, show that $\left(\frac{125}{9}\right)^{x}=13$. [1]

(ii) Hence find the value of x. [2]

2 (b) Solve the equation $\log_x(2x-1)=\log_{x^2}(4-5x)$. [4]

3 **(i)** State the third term in the expansion of $(2+x)^n$ in terms of a and n. [2]

(ii) Write down the first three terms in the expansion of $(1-3x)^5$. [2]

(iii) In the expansion of $(1-3x)^5(2+ax)^6$, the coefficient of x is -384. Find the value of a. [4]

4 Differentiate the following with respect to x.

(i) $3\tan^2\left(\dfrac{x}{2}\right)$ [2]

(ii) $e^{-x}x^2$ [3]

5 Given that $y=\frac{e^{2x}}{\sqrt{1-4x}}$, show that $\frac{dy}{dx}=\frac{4e^{2x}(1-2x)}{(1-4x)\sqrt{1-4x}}$. [3]

6 (i) The equation of a curve is given by $y=x+\sin^2 x$ for $0\le x\le\pi$.

Write down the expression for $\frac{dy}{dx}$. [1]

(ii) Find stationary point of the curve. [3]

7 Find the range of values of x such that the graph $y = \ln\left(\dfrac{x-2}{x-3}\right)^2$ is decreasing. [4]

8 **(a)** The expression $f(x)=ax^3+(a-3b)x^2+3bx+c$ is exactly divisible by x^2+3x. When $f(x)$ is divided by $x+2$, the remainder is 10. Find the values of *a*, *b* and *c*. Hence factorise $f(x)$ completely. [6]

(b) Given that $3x^3-2x^2+x-4=A(x-1)+B(x-1)(x+1)+Cx(x^2-1)+D$ for all values of *x*, find the value of *A*, *B*, *C* and *D*. [4]

9 The function f is defined, for $x \geq 0°$, by $f(x) = 3\cos\left(\frac{2x}{3}\right) - 1$.

(i) State the maximum and minimum values of $f(x)$. [2]

(ii) State the amplitude of $f(x)$. [1]

(iii) State the period of $f(x)$. [1]

(iv) Sketch the graph of $y = 3\cos\left(\frac{2x}{3}\right) - 1$ for $0° \leq x \leq 270°$. [2]

10 **(i)** Express the equations $y = 2x^2 - 3x + 9$ and $y = -2x^2 - 3x + \frac{27}{4}$ in the form of

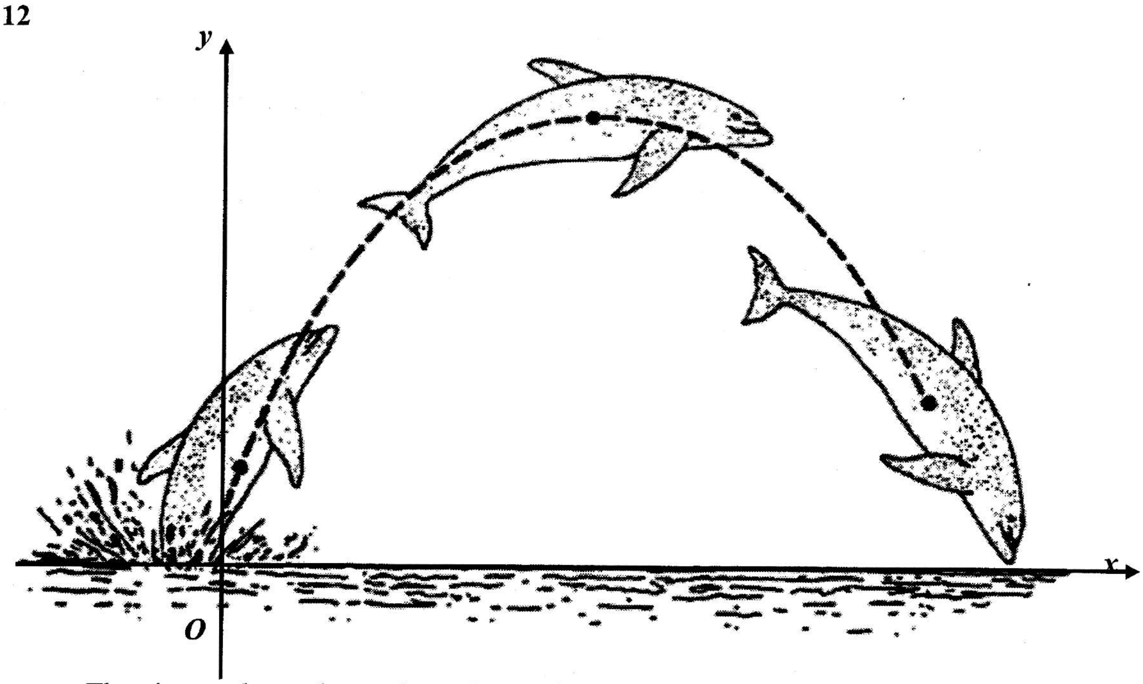
$$y = a(x+b)^2 + c.$$ [4]

(ii) Hence explain whether the curves will intersect or not. [3]

11 **(a)** Find the range of values of k for which the line $y = kx + 3$ lies entirely below the curve $xy + 20 = 5y$ for all real values of x. [5]

(b) The quadratic equation $mx^2 + 5nx + 9m = 0$ has equal roots. Given that m and n are positive, find $\frac{m}{n}$. Determine also the range of values of $\frac{m}{n}$ for which the quadratic equation has real roots. [5]

12



The picture above shows the path of a dolphin jumping out of the water to see clearly and to watch the surface of the oceans. The path is modelled by a quadratic function, $y = ax^2 + bx + c$ as shown in the diagram. The dimensions of the path is given in the table below.

How high can the dolphin jump out of the water?	450 cm
How far can the dolphin jump out of the water?	600 cm

(i) Find the coordinates of the highest point in the path. [1]

(ii) Find the equation, in the form $y = ax^2 + bx + c$, of the path of the dolphin, where a, b and c are real numbers. [4]

13(i)	Factorise $x^4 - 1$ completely.

(ii)	Given that $\dfrac{x^4}{x^4 - 1} = a + \dfrac{b}{x^4 - 1}$, find the values of a and of b.

(iii)	Express $\dfrac{x^4}{x^4 - 1}$ in partial fractions.

14 John deposits \$ P into a bank which pays a compound interest r % annually. The table shows the total amount \$ A in John's bank account over a period of 5 years.

n	1	2	3	4	5
A	8960	10035	11239	12588	14099

It is known that n and A are related by the equation $A = P\left(1+\frac{r}{100}\right)^n$.

(i) Express this equation in a form suitable for drawing a straight line graph. [2]

(ii) Draw a straight line graph for the given data. [2]

(iii) Use your graph to estimate the value of

(a) P, correct to the nearest thousand, and [2]

(b) r, correct to 2 significant figures. [2]

P1 ANS

1 Solve the equation $x+1+\sqrt{5-x}=0$. [3]

$x+1+\sqrt{5-x}=0$

$x+1=-\sqrt{5-x}$

$(x+1)^2=5-x$

$x^2+2x+1-5+x=0$

$x^2+3x-4=0$

$(x+4)(x-1)=0$

$x=-4$ or $x=1$ (rejected)

2 (a) (i) Given the equation $13\left(3^{2x}\right)=5^{3x}$, show that $\left(\frac{125}{9}\right)^x=13$. [1]

$13\left(3^{2x}\right)=5^{3x}$

$13=\frac{\left(5^3\right)^x}{\left(3^2\right)^x}$

$\left(\frac{125}{9}\right)^x=13$

(ii) Hence find the value of x. [2]

$\left(\frac{125}{9}\right)^x=13$

$\lg\left(\frac{125}{9}\right)^x=\lg 13$

$x=\lg 13\div\lg\left(\frac{125}{9}\right)$

$x\approx 0.975$ (3 s.f.)

2 (b) Solve the equation $\log_x(2x-1)=\log_{x^2}(4-5x)$. [4]

	$\log_x(2x-1)=\log_{x^2}(4-5x)$ $\log_x(2x-1)=\dfrac{\log_x(4-5x)}{\log_x x^2}$ $\log_x(2x-1)=\dfrac{\log_x(4-5x)}{2}$ $2\log_x(2x-1)-\log_x(4-5x)=0$ $\log_x\dfrac{(2x-1)^2}{4-5x}=0$ $\dfrac{(2x-1)^2}{4-5x}=1$ $4x^2-4x+1=4-5x$ $4x^2+x-3=0$ $(4x-3)(x+1)=0$ $x=\dfrac{3}{4}$ or $x=-1$ (N.A.)

3 **(i)** State the third term in the expansion of $(2+x)^n$ in terms of a and n. [2]

(ii) Write down the first three terms in the expansion of $(1-3x)^5$. [2]

(iii) In the expansion of $(1-3x)^5(2+ax)^6$, the coefficient of x is -384. Find the value of a. [4]

3(i)	$T_3=\binom{n}{2}2^{n-2}(x)^2$ $=\dfrac{n(n-1)}{2}\dfrac{2^n}{2^2}x^2$ $=n(n-1)2^{n-3}x^2$ *or* $\dfrac{n(n-1)2^{n-2}x^2}{2}$
(ii)	$(1-3x)^5$ $=1+\binom{5}{1}(-3x)+\binom{5}{2}(-3x)^2+\ldots$ $=1-15x+90x^2+\ldots$

(iii)	$(1-3x)^5(2+ax)^6$ $=(1-15x+90x^2+...)\left[2^6+\binom{6}{1}2^5(ax)+...\right]$ $=(1-15x+90x^2+...)(64+192ax+...)$ Coefficient of $x=(1)(192a)+(-15)(64)=-384$ $192a-960=-384$ $192a=576$ $a=3$

4 Differentiate the following with respect to x.

(i) $3\tan^2\left(\frac{x}{2}\right)$ [2]

(ii) $e^{-x}x^2$ [3]

4(i)	$\frac{d}{dx}\left[3\tan^2\left(\frac{x}{2}\right)\right]=3\left(2\tan\frac{x}{2}\right)\left(\sec^2\frac{x}{2}\right)\left(\frac{1}{2}\right)$ $=3\tan\frac{x}{2}\sec^2\frac{x}{2}$
(ii)	$\frac{d}{dx}\left[e^{-x}x^2\right]=e^{-x}(2x)+(-e^{-x})x^2$ $=e^{-x}(2x-x^2)$ OR $xe^{-x}(2-x)$

5 Given that $y = \dfrac{e^{2x}}{\sqrt{1-4x}}$, show that $\dfrac{dy}{dx} = \dfrac{4e^{2x}(1-2x)}{(1-4x)\sqrt{1-4x}}$. [3]

5	$\dfrac{dy}{dx} = \dfrac{e^{2x}}{\sqrt{1-4x}}$ $= \dfrac{\sqrt{1-4x}\dfrac{d}{dx}e^{2x} - e^{2x}\dfrac{d}{dx}\sqrt{1-4x}}{\left(\sqrt{1-4x}\right)^2}$ $= \dfrac{2\sqrt{1-4x}\,e^{2x} - e^{2x}\left(\dfrac{1}{2}\right)(1-4x)^{-\frac{1}{2}}(-4)}{1-4x}$ $= \dfrac{e^{2x}\left(2\sqrt{1-4x} + 2(1-4x)^{-\frac{1}{2}}\right)}{1-4x}$ $= \dfrac{2e^{2x}(1-4x+1)}{(1-4x)\sqrt{1-4x}}$ $= \dfrac{4e^{2x}(1-2x)}{(1-4x)\sqrt{1-4x}}$ (shown)

6 **(i)** The equation of a curve is given by $y = x + \sin^2 x$ for $0 \le x \le \pi$.

Write down the expression for $\dfrac{dy}{dx}$. [1]

(ii) Find stationary point of the curve. [3]

(iii) Determine the nature of the stationary point. [2]

6	(i) $y = x + \sin^2 x$ $\frac{dy}{dx} = 1 + 2\sin x \cos x \ (or = 1 + \sin 2x)$ (ii) At stationary point, $\frac{dy}{dx} = 0$ $1 + \sin 2x = 0$ $\sin 2x = -1$ $2x = \frac{3\pi}{2} or\ 2\pi + \frac{3\pi}{2} rej, \sin ce\ \ 0 \le 2x \le 2\pi$ $x = \frac{3\pi}{4}\ or\ 2.36$ $y = \frac{3\pi}{4} + \sin^2\left(\frac{3\pi}{4}\right)$ $= \frac{3\pi}{4} + \frac{1}{2}\ or\ \frac{3\pi+2}{4}\ or\ 2.86$ Therefore the stationary point is $(2.36, 2.86)$ $or \left(\frac{3\pi}{4}, \frac{3\pi+2}{4}\right) or \left(\frac{3\pi}{4}, 0.5 + \frac{3\pi}{4}\right)$
	(iii)

x	$\frac{3\pi}{4}^-$	$\frac{3\pi}{4}$	$\frac{3\pi}{4}^+$
$\frac{dy}{dx}$	+ve	0	+ve

Therefore the stationary point is a point of inflexion

7 Find the range of values of x such that the graph $y = \ln\left(\frac{x-2}{x-3}\right)^2$ is decreasing. [4]

7	

$$y = \ln\left(\frac{x-2}{x-3}\right)^2$$

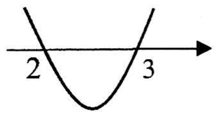
$$= 2\left(\ln(x-2) - \ln(x-3)\right)$$

$$\frac{dy}{dx} = 2\left(\frac{1}{x-2} - \frac{1}{x-3}\right)$$

$$= 2\left(\frac{x-3-x+2}{(x-2)(x-3)}\right)$$

$$= \frac{-2}{(x-2)(x-3)}$$

$$(x-2)(x-3) > 0$$



$x < 2$ or $x > 3$

8 **(a)** The expression $f(x) = ax^3 + (a-3b)x^2 + 3bx + c$ is exactly divisible by $x^2 + 3x$. When $f(x)$ is divided by $x+2$, the remainder is 10. Find the values of *a*, *b* and *c*. Hence factorise $f(x)$ completely. [6]

(b) Given that $3x^3 - 2x^2 + x - 4 = A(x-1) + B(x-1)(x+1) + Cx(x^2-1) + D$ for all values of *x*, find the value of *A*, *B*, *C* and *D*. [4]

8(a)

$f(0) = c = 0 \quad \Rightarrow c = 0$

$f(-3) = 0$

$-27a + 9(a-3b) - 9b = 0$

$-18a - 36b = 0$

$a = -2b$

	$f(-2)=10$ $-8a+4(a-3b)-6b=10$ $-4a-18b=10$ $8b-18b=10$ $b=-1$ $a=2$ $f(x)=2x^3+5x^2-3x$ $=x(2x^2+5x-3)$ $=x(x+3)(2x-1)$
(b)	$3x^3-2x^2+x-4=A(x-1)+B(x-1)(x+1)+Cx(x^2-1)+D$ When $x=1$, $D=-2$ When $x=-1$, $A=4$ When $x=0$, $B=-2$ When $x=2$, $C=3$

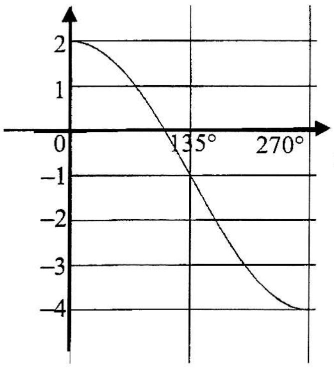
9 The function f is defined, for $x \geq 0°$, by $f(x) = 3\cos\left(\frac{2x}{3}\right) - 1$.

(i) State the maximum and minimum values of $f(x)$. [2]

(ii) State the amplitude of $f(x)$. [1]

(iii) State the period of $f(x)$. [1]

(iv) Sketch the graph of $y = 3\cos\left(\frac{2x}{3}\right) - 1$ for $0° \leq x \leq 270°$. [2]

9(i)	Max value of $f(x) = 2$ Min value of $f(x) = -4$
(ii)	Amplitude = 3
(iii)	Period = $360° \div \frac{2}{3} = 540°$
(iv)	

10 **(i)** Express the equations $y = 2x^2 - 3x + 9$ and $y = -2x^2 - 3x + \frac{27}{4}$ in the form of

$$y = a(x+b)^2 + c.$$ [4]

(ii) Hence explain whether the curves will intersect or not. [3]

10(i)

$$y = 2x^2 - 3x + 9$$

$$y = 2\left(x^2 - \frac{3}{2}x\right) + 9$$

$$y = 2\left[\left(x - \frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right] + 9$$

$$y = 2\left(x - \frac{3}{4}\right)^2 - \frac{9}{8} + 9$$

$$y = 2\left(x - \frac{3}{4}\right)^2 + \frac{63}{8}$$

$$y = -2x^2 - 3x + \frac{27}{4}$$

$$y = -2\left(x^2 + \frac{3}{2}x\right) + \frac{27}{4}$$

$$y = -2\left[\left(x + \frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right] + \frac{27}{4}$$

$$y = 2\left(x + \frac{3}{4}\right)^2 + \frac{9}{8} + \frac{27}{4}$$

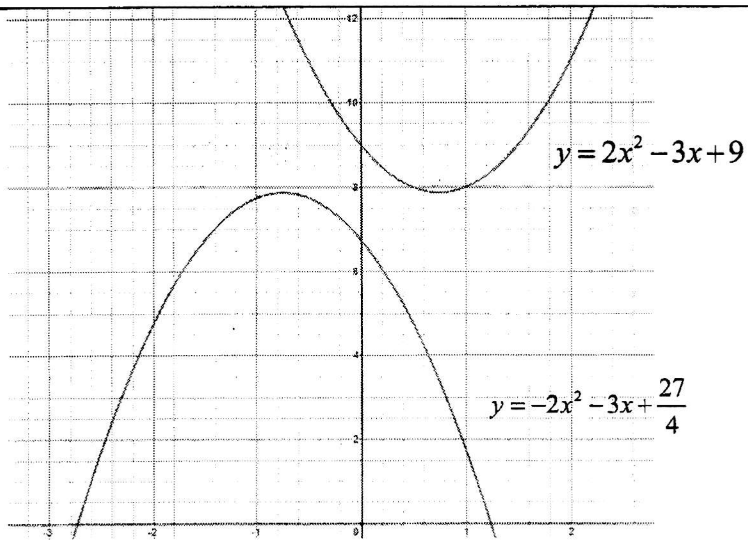
$$y = -2\left(x + \frac{3}{4}\right)^2 + \frac{63}{8}$$

(ii)

The minimum point of $y = 2x^2 - 3x + 9$ is $\left(\frac{3}{4}, \frac{63}{8}\right)$.

The maximum point of $y = -2x^2 - 3x + \frac{27}{4}$ is $\left(-\frac{3}{4}, \frac{63}{8}\right)$.

Both of the y coordinates are the same but their x coordinates are different.



From the graphs, the two graphs will not intersect.

11 **(a)** Find the range of values of k for which the line $y = kx + 3$ lies entirely below the curve $xy + 20 = 5y$ for all real values of x. [5]

(b) The quadratic equation $mx^2 + 5nx + 9m = 0$ has equal roots. Given that m and n are positive, find $\frac{m}{n}$. Determine also the range of values of $\frac{m}{n}$ for which the quadratic equation has real roots. [5]

(a)

$y = kx + 3$ -------- (1)

$xy + 20 = 5y$ ------- (2)

Substitute (1) into (2)

$x(kx + 3) + 20 = 5(kx + 3)$

$kx^2 + 3x + 20 = 5kx + 15$

$kx^2 + 3x - 5kx + 5 = 0$

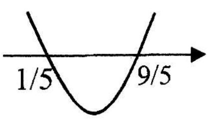
Since line and curve does not intersect,

$(3 - 5k)^2 - 4(k)(5) < 0$

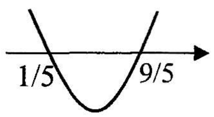
$9 - 30k + 25k^2 - 20k < 0$

$25k^2 - 50k + 9 < 0$

$(5k - 1)(5k - 9) < 0$

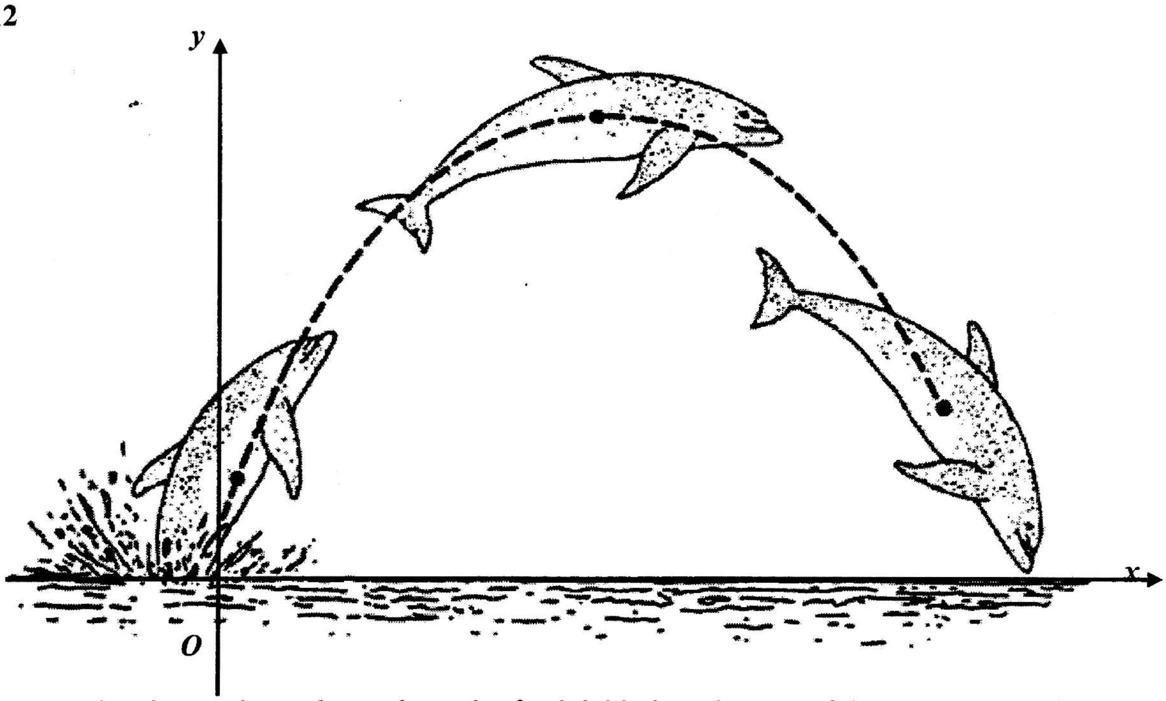


$\frac{1}{5} < k < \frac{9}{5}$ *or* $0.2 < k < 1.8$

	Alternatively, $y = kx + 3$ $xy + 20 = 5y$ $20 = 5y - xy$ $y(5-x) = 20$ $y = \dfrac{20}{5-x}$ $kx + 3 < \dfrac{20}{5-x}$ $(kx+3)(5-x) < 20$ $5kx - kx^2 + 15 - 3x < 20$ $-kx^2 + (5k-3)x - 5 < 0$ $kx^2 + (3-5k)x + 5 > 0$ for all real values x. $b^2 - 4ac < 0$ $(3-5k)^2 - 4k(5) < 0$ $9 - 30k + 25k^2 - 20k < 0$ $25k^2 - 50k + 9 < 0$ $(5k-1)(5k-9) < 0$  $\dfrac{1}{5} < k < \dfrac{9}{5}$ *or* $0.2 < k < 1.8$
11(b)	For equal roots, $b^2 - 4ac = 0$ $(5n)^2 - 4m(9m) = 0$ $25n^2 = 36m^2$ $\dfrac{m^2}{n^2} = \dfrac{25}{36}$ $\dfrac{m}{n} = \dfrac{5}{6}$ or $\dfrac{m}{n} = -\dfrac{5}{6}$(rejected)

	For real roots, $b^2 - 4ac \geq 0$ $(5n)^2 - 4m(9m) \geq 0$ $25n^2 - 36m^2 \geq 0$ $36m^2 - 25n^2 \leq 0$ $(6m - 5n)(6m + 5n) \leq 0$ $-5n \leq 6m \leq 5n$ $-\frac{5}{6} \leq \frac{m}{n} \leq \frac{5}{6}$ Since $\frac{m}{n} \geq 0$, $0 \leq \frac{m}{n} \leq \frac{5}{6}$.

12



The picture above shows the path of a dolphin jumping out of the water to see clearly and to watch the surface of the oceans. The path is modelled by a quadratic function, $y = ax^2 + bx + c$ as shown in the diagram. The dimensions of the path is given in the table below.

How high can the dolphin jump out of the water?	450 cm
How far can the dolphin jump out of the water?	600 cm

(i) Find the coordinates of the highest point in the path. [1]

(ii) Find the equation, in the form $y = ax^2 + bx + c$, of the path of the dolphin, where a, b and c are real numbers. [4]

12(i)	$x=\dfrac{600}{2}$ $=300$ Coordinates of the turning point $=(300,\ 450)$
(ii)	$y=a(x-300)^2+450$ When $x=0,\ y=0$ $0=a(-300)^2+450$ $a=-\dfrac{1}{200}$ or -0.005 Equation of the path is $y=-\dfrac{1}{200}(x-300)^2+450$ $=-\dfrac{1}{200}(x^2-600x+90000)+450$ $=-\dfrac{1}{200}x^2+3x-450+450$ $=-\dfrac{1}{200}x^2+3x.$
12(ii)	Alternatively, When $x=0,\ y=0,\ y=ax^2+bx+c$ becomes $0=a(0)^2+b(0)+c$ $c=0$ $y=ax^2+bx$ When $x=300,\ y=450,$ $450=a(300)^2+b(300)$ $b=\dfrac{3}{2}-300a$ ------------ (1) When $y=0,\ x=600,$ $0=a(600)^2+b(600)$ $b=-600a$ ------------ (2) Equating (1) and (2), $-600a=\dfrac{3}{2}-300a$ $a=-\dfrac{1}{200}$ From (2), $b=-600a$ $b=3$ Equation of the path is $y=-\dfrac{1}{200}x^2+3x.$

13(i)	Factorise x^4-1 completely.
	$x^4-1 = (x^2-1)(x^2+1)$ $= (x-1)(x+1)(x^2+1)$
(ii)	Given that $\dfrac{x^4}{x^4-1} = a + \dfrac{b}{x^4-1}$, find the values of a and of b.
	$\dfrac{x^4}{x^4-1} = \dfrac{(x^4-1)+1}{x^4-1}$ $= 1 + \dfrac{1}{x^4-1}$ $a=1,\ b=1$
(iii)	Express $\dfrac{x^4}{x^4-1}$ in partial fractions.
	$\dfrac{x^4}{x^4-1} = 1 + \dfrac{1}{x^4-1}$ $\dfrac{1}{x^4-1} = \dfrac{1}{(x-1)(x+1)(x^2+1)}$ $= \dfrac{A}{x-1} + \dfrac{B}{x+1} + \dfrac{Cx+D}{x^2+1}$ $1 = A(x+1)(x^2+1) + B(x-1)(x^2+1) + (Cx+D)(x-1)(x+1)$ When $x=1$, $4A=1$ $A=\dfrac{1}{4}$ When $x=-1$ $-4B=1$ $B=-\dfrac{1}{4}$ When $x=0$ $A-B-D=1$ $D=\dfrac{1}{4}-\left(-\dfrac{1}{4}\right)-1=-\dfrac{1}{2}$ By comparing coefficient of x^3

	$0 = A + B + C$ $C = -\frac{1}{4} - \left(-\frac{1}{4}\right) = 0$ $\frac{x^4}{x^4 - 1} = 1 + \frac{1}{4(x-1)} - \frac{1}{4(x+1)} - \frac{1}{2(x^2+1)}$

14 John deposits \$ P into a bank which pays a compound interest r % annually. The table shows the total amount \$ A in John's bank account over a period of 5 years.

n	1	2	3	4	5
A	8960	10035	11239	12588	14099

It is known that n and A are related by the equation $A = P\left(1 + \frac{r}{100}\right)^n$.

(i) Express this equation in a form suitable for drawing a straight line graph. [2]

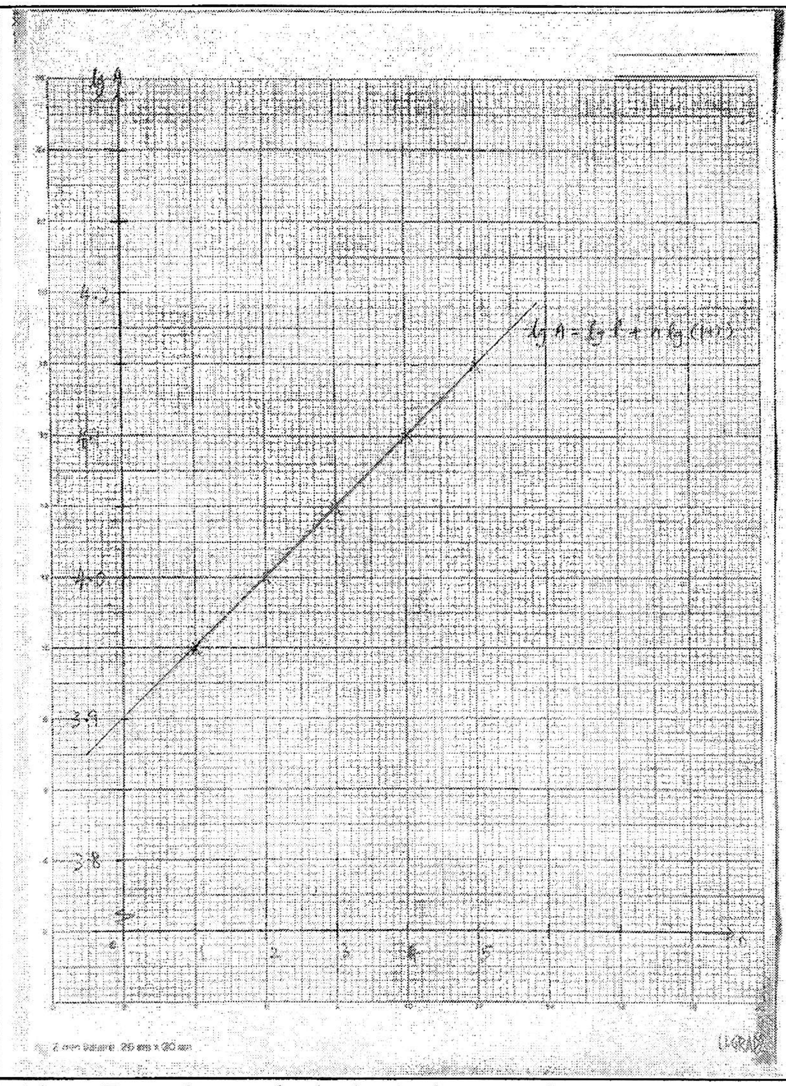
(ii) Draw a straight line graph for the given data. [2]

(iii) Use your graph to estimate the value of
(a) P, correct to the nearest thousand, and [2]
(b) r, correct to 2 significant figures. [2]

14(i)	$A = P\left(1 + \frac{r}{100}\right)^n$ $\lg A = \lg P\left(1 + \frac{r}{100}\right)^n$ $\lg A = \lg P + \lg\left(1 + \frac{r}{100}\right)^n$ $\lg A = n \lg\left(1 + \frac{r}{100}\right) + \lg P$

(ii)

n	1	2	3	4	5
A	8960	10035	11239	12588	14099
$\lg A$	3.95	4.00	4.05	4.10	4.15



(iii) (a) From the graph, the $\lg A$ – intercept is 3.9

Therefore, $\lg P = 3.9$

$$P = 10^{3.9}$$

$$P = 7943.28$$

$$P \approx 8000 \text{ (nearest thousand)}$$

(b) Gradient of graph is $\lg\left(1+\frac{r}{100}\right) = \frac{4.15-9.95}{5-1}$

$$\lg\left(1+\frac{r}{100}\right) = 0.05$$

$$1+\frac{r}{100} = 10^{0.05}$$

$$\frac{r}{100} = 1.122-1$$

$$r \approx 12 \text{ (2 s.f.)}$$

NAME: ______________________________ (   ) CLASS: 4 (   )

ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2021

S4

ADDITIONAL MATHEMATICS | **4049 / 02**
Paper 2 | **31st August 2021 Tuesday**
Candidates answer on the Question Paper. | **2 hours 15 minutes**
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the space at the top of this page.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters and glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiners' Use

Question	Marks	Question	Marks	Table of Penalties	
1		7			
2		8		Clarity/Logic	
3		9		Accuracy/Precision	
4		10		Units	
5		11		Total:	
6		12			
Parent's Name & Signature: Date:				90	

This document consists of **21** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \Delta = \frac{1}{2}ab \sin C$$

1 Angles A and B are both obtuse and $\tan(A+B) = \frac{3}{4}$.

Without the use of calculators,

(i) state the range of angle $(A+B)$, [1]

(ii) find the value of $\sin(A+B)$, [2]

(iii) given that $\tan B = -\frac{12}{5}$, find the value of $\cos A$. [3]

2 A slice of bread is toasted in an oven to a temperature of 154 °C. It is subsequently left to cool in such a way that its temperature, T °C, after removal from the oven, is

given by $T = 26 + Ae^{-kt}$, where A and k are constants, and t is the time after removal from oven in minutes.

(i) Find the value of A. [1]

(ii) What does the value "26" represent? [1]

When $t = 2$, the temperature of the bread is 116 °C.

(iii) Find the value of k. [2]

The toasted bread is recommended to be consumed at a temperature of 45 °C.

(iv) Determine, with clear working, the waiting time, in minutes, after removal from the oven before consuming the toasted bread. [2]

3 Given that the function $f(x) = 1 + 2\sin^2 3x + 4\cos^2 3x$,

(i) Express $f(x)$ in the form $a + b\cos 6x$. [2]

(ii) Hence, find the exact value of $f(x) = 3.5$ for $0 \leq x \leq \frac{\pi}{2}$. [3]

4 **(i)** Find the derivative of the functions $y = \ln x$ and $y = \ln 2x$ respectively. [1]

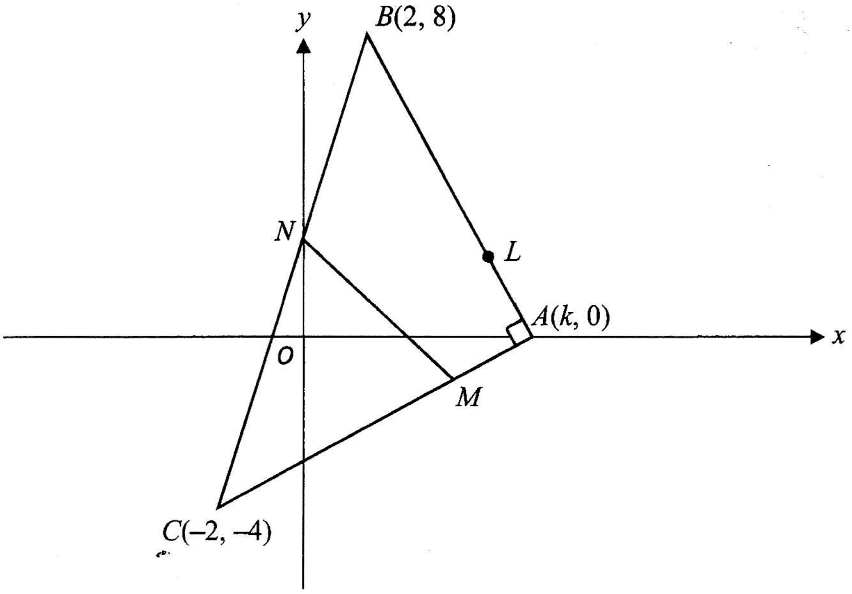
(ii) Explain why, in **(i)**, having the same derivative does not mean that the two functions are the same. [1]

5 **(i)** Using $R\cos(\theta - \alpha)$, where $R > 0$ and $0° \leq \alpha \leq 90°$, solve the equation

$$5\cos\theta + 8\sin\theta = 6 \text{ for } 0° \leq \theta \leq 180°.$$ [4]

(ii) State the largest value of $20-(5\cos x+8\sin x)^2$ and find the corresponding value of x, such that $0° \le x \le 180°$ [2]

6 **Solutions to this question by accurate drawing will NOT be accepted.**



The figure shows a right-angled triangle ABC, where points A, B and C are $(k,0)$, $(2,8)$ and $(-2,-4)$ respectively. BC cuts the y-axis at N. M is a point on AC.

(i) Given that $k > 0$, find the value of k. [3]

(ii) Find the coordinates of N and show that N is the midpoint of BC. [3]

(iii) Given that the area of quadrilateral *ABNM* is 25 units^2, find the coordinates of *M*. [6]

7 **(i)** Show that $\frac{d}{dx}\ln(e^{x}\cos x)=1-\tan x$. [3]

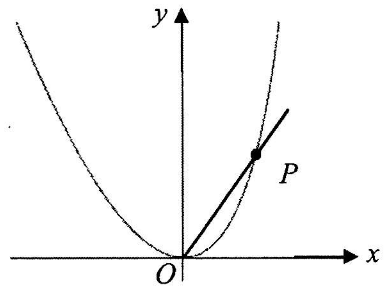
(ii) Using part (i), find $\int \tan x \, dx$. [2]

(iii) Hence show that $\int_0^{\frac{\pi}{3}} \tan x \, dx = \ln 2$. [2]

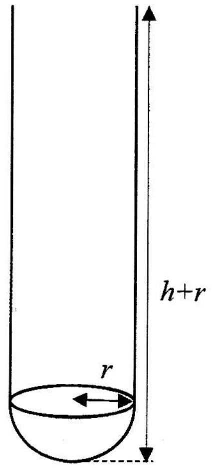
8 Point P lies on the curve $y = \dfrac{kx^2}{\sqrt{2-x}}$, where k is a constant. The x-coordinate of P is -2.

(i) Given that the gradient of the normal at P is $\frac{1}{7}$, show that $k = 4$. [4]

(i) The point P moves along the curve. Given that x is increasing at a rate of 0.9 units per second, find the rate of change of $(OP)^2$ at the point $x = 0.5$. [4]



9 The diagram shows the glass test tube of radius r cm and height $(h+r)$ cm with a hemisphere of radius r cm as the base. The surface area of the test tube is 12π.



(i) Show that $h = \dfrac{6-r^2}{r}$. [2]

(ii) Show that the volume, V cm^3, of the test tube is $V = 6\pi r - \dfrac{1}{3}\pi r^3$. [2]

(iii) Calculate the stationary value of V and determine if the volume is a maximum or minimum. [5]

10 The equation of a circle, C_1, is $x^2+y^2-4x+8y+11=0$.

(i) Find the coordinates of the centre of C_1 and its radius. [3]

(ii) Find the equation of the circle, C_2, which is a reflection of C_1 in the line $y=1$. [2]

(iii) Given that the line $y-x+3=0$ intersects the circle, C_1, at two points, R and S, find the equation of the perpendicular bisector of the line RS. [4]

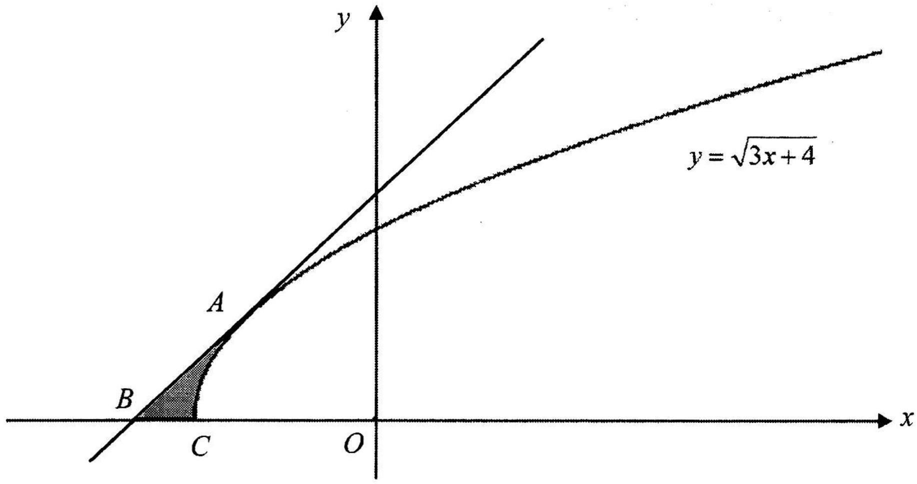
11 A particle moves in a straight line, so that, t seconds after leaving a fixed point O its velocity, v is given by $v = \frac{36}{(t-3)^2} - 9$. Find

(i) the initial acceleration of the particle. [2]

(ii) the values of t when the particle is at instantaneous rest. [2]

(iii) the total distance travelled by the particle in the first 2 seconds. [6]

12



The diagram shows part of the curve $y = \sqrt{3x+4}$. The line *AB* is tangent to the curve at *A*. The points *B* and *C* lie on the *x*-axis, and *C* is a point on the curve. The gradient of the

tangent of the curve at *A* is $\frac{3}{2}$. Find

(i) the coordinates of *A* and equation of the line *AB*. [5]

(ii) the coordinates of *B* and of *C*. [2]

(iii) the area of the shaded region *ABC*. [3]

End of Paper

ANS

NAME: ______________________________ (   )   CLASS: 4 (   )

ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2021

S4

ADDITIONAL MATHEMATICS

Paper 2
Candidates answer on the Question Paper.
No Additional Materials are required.

4049 / 02
31st August 2021 Tuesday
2 hours 15 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the space at the top of this page.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters and glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiners' Use

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1 Angles A and B are both obtuse and $\tan(A+B)=\frac{3}{4}$.

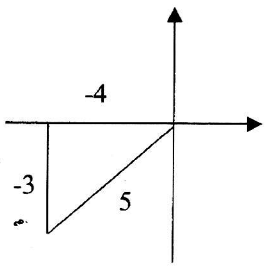
Without the use of calculators,

(i) state the range of angle $(A+B)$, [1]

(i)	$180° < (A+B) < 270°$

(ii) find the value of $\sin(A+B)$, [2]

(ii)



$$\sin(A+B)=-\frac{3}{5}$$

(iii) given that $\tan B=-\frac{12}{5}$, find the value of $\cos A$. [3]

(iii)

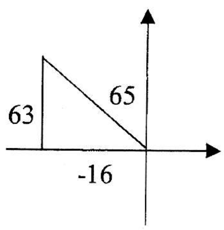
$$\tan(A+B)=\frac{3}{4}$$

$$\frac{\tan A+\tan B}{1-\tan A\tan B}=\frac{3}{4}$$

$$\frac{\tan A-\frac{12}{5}}{1+\frac{12}{5}\tan A}=\frac{3}{4}$$

$$-\frac{16}{5}\tan A=\frac{63}{5}$$

$$\tan A=-\frac{63}{16}$$



$$\cos A=-\frac{16}{65}$$

2 A slice of bread is toasted in an oven to a temperature of 154 °C. It is subsequently left to cool in such a way that its temperature, T °C, after removal from the oven, is

given by $T = 26 + Ae^{-kt}$, where A and k are constants, and t is the time after removal from oven in minutes.

(i) Find the value of A. [1]

(i)	At $t=0$, $T=154$. $154 = 26 + Ae^0$ $A = 128$	B1

(ii) What does the value "26" represent? [1]

(ii)	"26" represents the room temperature.	B1

When $t = 2$, the temperature of the bread is 116 °C.

(iii) Find the value of k. [2]

(iii)	At $t=2$, $T=116$. $116 = 26 + 128e^{-2k}$ $e^{-2k} = \frac{116-26}{128}$ $-2k = \ln\left(\frac{90}{128}\right)$ $k \approx 0.17611$ $k \approx 0.176$ (3 s.f.)	

The toasted bread is recommended to be consumed at a temperature of 45 °C.

(iv) Determine, with clear working, the waiting time, in minutes, after removal from the oven before consuming the toasted bread. [2]

(iv)	When $T = 45$, $45 = 26 + 128e^{-0.17611t}$ $e^{-0.17611t} = \frac{45-26}{128}$ $-0.17611t = \ln\left(\frac{19}{128}\right)$ $t \approx 10.8$ (1 d.p.) Waiting time is 10.8 min after removal from oven.	

3 Given that the function $f(x) = 1 + 2\sin^2 3x + 4\cos^2 3x$,

(i) Express $f(x)$ in the form $a + b\cos 6x$. [2]

(i)	$f(x) = 1 + 2\sin^2 3x + 4\cos^2 3x$ $= 1 + 2(1 - \cos^2 3x) + 4\cos^2 3x$ $= 3 + 2\cos^2 3x$ $= 3 + \cos 6x + 1$ $= 4 + \cos 6x$	

(ii) Hence, find the exact value of $f(x) = 3.5$ for $0 \le x \le \frac{\pi}{2}$. [3]

(ii)	$f(x) = 3.5$ $4 + \cos 6x = 3.5$ $\cos 6x = -\frac{1}{2}$ basic angle $= \cos^{-1}\left(\frac{1}{2}\right)$ $= \frac{\pi}{3}$ $0 \le x \le \frac{\pi}{2}$ $0 \le 6x \le 3\pi$ $6x = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}$ $x = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}$	

4 **(i)** Find the derivative of the functions $y = \ln x$ and $y = \ln 2x$ respectively. [1]

(i)	$\frac{dy}{dx} = \frac{d}{dx}\ln x$ $= \frac{1}{x}$ $\frac{dy}{dx} = \frac{d}{dx}\ln 2x$ $= \frac{1}{2x} \times 2$ $= \frac{1}{x}$	 B1 B1

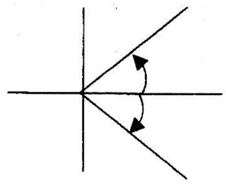
(ii) Explain why, in **(i)**, having the same derivative does not mean that the two functions are the same. [1]

(ii)	Since $y = \ln x$ $y = \log_e x$ $x = e^y$ And $y = \ln 2x$ $y = \log_e 2x$ *or* $y = \ln 2 + \ln x$ $2x = e^y$ $\ln x = y - \ln 2$ $x = \frac{1}{2}e^y$ $x = e^{y - \ln 2}$ $x = \frac{e^y}{e^{\ln 2}} = \frac{e^y}{2}$ Hence, they are different functions.	

5 **(i)** Using $R\cos(\theta - \alpha)$, where $R > 0$ and $0° \le \alpha \le 90°$, solve the equation

$$5\cos\theta + 8\sin\theta = 6 \text{ for } 0° \le \theta \le 180°.$$ [4]

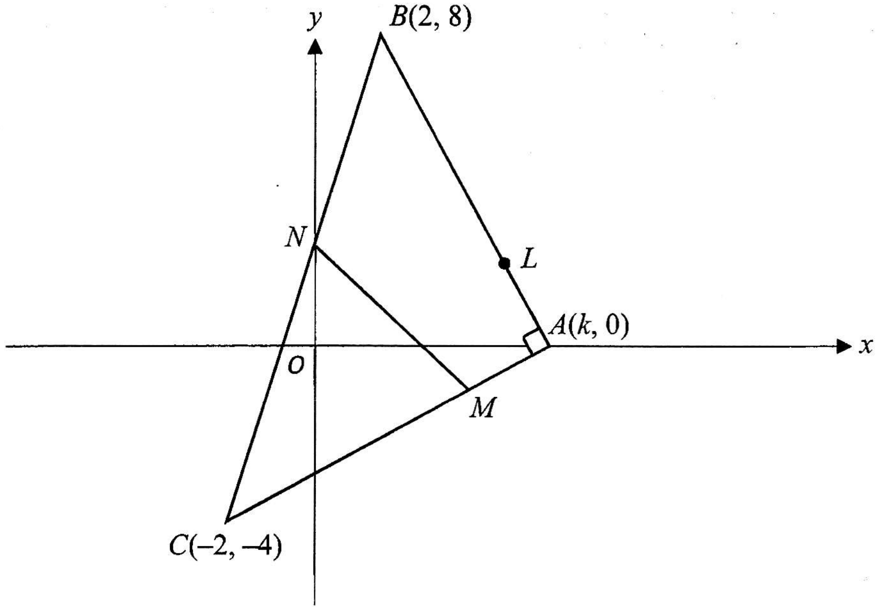
(i)	$5\cos\theta + 8\sin\theta = 6$ $R\cos(\theta - \alpha) = 6$ $R = \sqrt{5^2 + 8^2}$ $\tan\alpha = \frac{8}{5}$ $= \sqrt{89}$ $\alpha = 57.9946°$	

<table>
<tr><td></td><td>$\sqrt{89}\cos(\theta-57.9946°)=6$
$\cos(\theta-57.9946°)=\dfrac{6}{\sqrt{89}}$
basic angle $=\cos^{-1}\left(\dfrac{6}{\sqrt{89}}\right)$
$=50.5059°$
$\theta-57.9946°=-50.5059°,\ 50.5059°$
$\theta=7.4887°,108.5005°$
$\theta=7.5°,108.5°$
</td><td></td></tr>
</table>

(ii) State the largest value of $20-(5\cos x+8\sin x)^2$ and find the corresponding value of x, such that $0°\le x\le 180°$ [2]

<table>
<tr><td>(ii)</td><td>$20-(5\cos x+8\sin x)^2=20-\left(\sqrt{89}\cos(x-57.9946°)\right)^2$
$=20-89\cos^2(x-57.9946°)$
Largest value = 20
$\cos(x-57.9946°)=0$
$x-57.9946°=90°$
$x=147.9946°$
$=148.0°$</td><td></td></tr>
</table>

6 **Solutions to this question by accurate drawing will NOT be accepted.**



The figure shows a right-angled triangle *ABC*, where points *A*, *B* and *C* are $(k,0)$, $(2,8)$ and $(-2,-4)$ respectively. *BC* cuts the y-axis at *N*. *M* is a point on *AC*.

(i) Given that $k>0$, find the value of k. [3]

(i)	Gradient of AB × gradient of $AC = -1$ $\left(\frac{0-8}{k-2}\right)\times\left(\frac{0-(-4)}{k-(-2)}\right)=-1$ $(-8)(4)=-(k-2)(k+2)$ $-32=-k^2+4$ $k^2=36$ $k=\pm 6$ Since $k>0$, $k=6$	

(ii) Find the coordinates of N and show that N is the midpoint of BC. [3]

(ii)	Let the coordinates of N be $(0,n)$. Gradient of BN = gradient of BC $\left(\frac{8-n}{2-0}\right)=\left(\frac{8-(-4)}{2-(-2)}\right)$ $\left(\frac{8-n}{2}\right)=3$ $8-n=6$ $n=2$ Coordinates of N is $(0,2)$. Midpoint of BC is $\left(\frac{2-2}{2},\frac{8-4}{2}\right)=(0,2)$. Hence, N is the midpoint of BC.	

(iii) Given that the area of quadrilateral *ABNM* is 25 units^2, find the coordinates of *M*. [6]

(iii)	Gradient of $AC = \frac{0-(-4)}{6-(-2)} = \frac{1}{2}$ Equation of AC is $y - 0 = \frac{1}{2}(x-6)$ $y = \frac{1}{2}x - 3$ Let the coordinates of M be $\left(a, \frac{1}{2}a - 3\right)$ Area of quadrilateral $ABNM$ $= 25$ units^2 $\frac{1}{2}\begin{vmatrix} 6 & 2 & 0 & a & 6 \\ 0 & 8 & 2 & \frac{1}{2}a-3 & 0 \end{vmatrix} = 25$ $\frac{1}{2}\left[(48+4) - \left(2a + 6\left(\frac{1}{2}a - 3\right)\right)\right] = 25$ $52 - 2a - 3a + 18 = 50$ $5a = 20$ $a = 4$ Coordinates of M is $(4, -1)$	

7 (i) Show that $\frac{d}{dx}\ln(e^x \cos x) = 1 - \tan x$. [3]

(i)	$\frac{d}{dx}\ln(e^x \cos x) = \frac{d}{dx}\left(\ln e^x + \ln \cos x\right)$ $= \frac{d}{dx}x + \frac{1}{\cos x}\frac{d}{dx}\cos x$ $= 1 + \frac{-\sin x}{\cos x}$ $= 1 - \tan x$ (shown) OR $\frac{d}{dx}\ln(e^x \cos x) = \frac{1}{e^x \cos x}\left[\left(e^x(-\sin x)\right) + \cos x(e^x)\right]$ $= \frac{e^x\left[(-\sin x) + \cos x\right]}{e^x \cos x}$ $= \frac{(-\sin x) + \cos x}{\cos x}$ $= 1 - \tan x$ (*shown*)	

(ii) Using part (i), find $\int \tan x \, dx$. [2]

(ii)	$\frac{d}{dx}\ln(e^x \cos x) = 1 - \tan x$ $\tan x = 1 - \frac{d}{dx}\ln(e^x \cos x)$ $\int \tan x \, dx = \int 1 \, dx - \int \frac{d}{dx}\ln(e^x \cos x) \, dx$ $= x - \ln(e^x \cos x) + c$	

(iii) Hence show that $\int_0^{\frac{\pi}{3}} \tan x \, dx = \ln 2.$ [2]

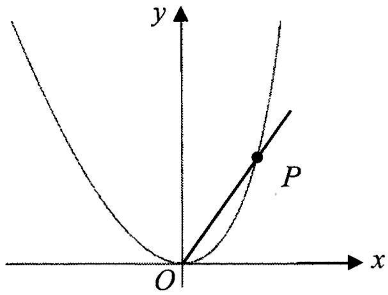
(iii)		
	$\int_0^{\frac{\pi}{3}} \tan x \, dx = \left[x - \ln(e^x \cos x) \right]_0^{\frac{\pi}{3}}$ $= \left[\frac{\pi}{3} - \ln\left(e^{\frac{\pi}{3}} \cos\frac{\pi}{3} \right) \right] - \left[0 - \ln\left(e^0 \cos 0 \right) \right]$ $= \left[\frac{\pi}{3} - \ln\left(e^{\frac{\pi}{3}} \right) - \ln\left(\cos\frac{\pi}{3} \right) \right] - 0$ $= \left[\frac{\pi}{3} - \frac{\pi}{3} - \ln\left(\frac{1}{2} \right) \right]$ $= -\ln 2^{-1}$ $= \ln 2$ (shown)	

8 Point P lies on the curve $y = \frac{kx^2}{\sqrt{2-x}}$, where k is a constant. The x-coordinate of P is -2.

(i) Given that the gradient of the normal at P is $\frac{1}{7}$, show that $k = 4$. [4]

(i)	$\frac{dy}{dx}=\frac{2kx\sqrt{2-x}-kx^2\left(\frac{1}{2}\right)(2-x)^{-\frac{1}{2}}(-1)}{2-x}$ $=\frac{2kx(2-x)+\frac{k}{2}x^2}{(2-x)^{\frac{3}{2}}}$ At $x = -2$, $\frac{dy}{dx}=-7$ $\frac{2k(-2)(2+2)+\frac{k}{2}(-2)^2}{(2+2)^{\frac{3}{2}}}=-7$ $\frac{-16k+2k}{8}=-7$ $-14k=-56$ $k=4\ (shown)$	

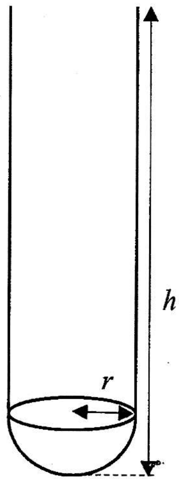
(i) The point P moves along the curve. Given that x is increasing at a rate of 0.9 units per second, find the rate of change of $(OP)^2$ at the point $x = 0.5$. [4]



(ii)	$(OP)^2 = x^2 + y^2$ $= x^2 + \left(\frac{4x^2}{\sqrt{2-x}}\right)^2$ $= x^2 + \frac{16x^4}{2-x}$ $\frac{d(OP)^2}{dx} = 2x + \frac{64x^3(2-x)-16x^4(-1)}{(2-x)^2}$ $= 2x + \frac{128x^3 - 48x^4}{(2-x)^2}$ At $x = 0.5$, $\frac{d(OP)^2}{dx} = 2(0.5) + \frac{128(0.5)^3 - 48(0.5)^4}{(2-0.5)^2}$ $= \frac{61}{9}$ By chain rule, $\frac{d(OP)^2}{dt} = \frac{d(OP)^2}{dx} \times \frac{dx}{dt}$ $= \frac{61}{9} \times 0.9$ $= 6.1 \quad or \quad 6\frac{1}{10} \ units/s$ The rate of change of $(OP)^2 = 6.1$ units/s	

9 The diagram shows the glass test tube of radius r cm and height $(h+r)$ cm with a hemisphere of radius r cm as the base. The surface area of the test tube is 12π.

(i) Show that $h=\dfrac{6-r^2}{r}$. [2]



(i)	$12\pi = 2\pi rh + 2\pi r^2$ $6 = rh + r^2$ $h=\dfrac{6-r^2}{r}$	

(ii) Show that the volume, V cm^3, of the test tube is $V = 6\pi r - \dfrac{1}{3}\pi r^3$. [2]

(ii)	$V=\pi r^2 h + \dfrac{2}{3}\pi r^3$ $V=\pi r^2\left(\dfrac{6-r^2}{r}\right)+\dfrac{2}{3}\pi r^3$ $V=6\pi r-\pi r^3+\dfrac{2}{3}\pi r^3$ $V=6\pi r-\dfrac{1}{3}\pi r^3$	

(iii) Calculate the stationary value of V and determine if the volume is a maximum or minimum. [5]

(iii)

$$\frac{dV}{dr} = 6\pi - \pi r^2$$

For stationary value of V, $\frac{dV}{dr} = 0$

$$6\pi - \pi r^2 = 0$$

$$\pi r^2 = 6\pi$$

$$r = \sqrt{6}, \quad \text{reject } r = -\sqrt{6}$$

$$= 2.4495$$

Stationary volume of $V = 6\pi(2.4495) - \frac{1}{3}\pi(2.4495)^3$

$$= 30.781$$

$$= 30.8\,\text{cm}^3$$

$$\frac{d^2V}{dr^2} = -2\pi r$$

$$= -2\sqrt{6}\,\pi < 0$$

Alternatively,

r	2.4495-	2.4495	2.4495+
$\frac{dV}{dr}$	+ve	0	-ve

Therefore volume is maximum.

10 The equation of a circle, C_1, is $x^2+y^2-4x+8y+11=0$.

(i) Find the coordinates of the centre of C_1 and its radius. [3]

(i)	$x^2+y^2-4x+8y+11=0$ $(x-2)^2-4+(y+4)^2-16+11=0$ $(x-2)^2+(y+4)^2=3^2$ Centre $\equiv(2,-4)$ Radius $=3$ units	

(ii) Find the equation of the circle, C_2, which is a reflection of C_1 in the line $y=1$. [2]

(ii)	New centre $=(2,6)$ Equation of new circle is $(x-2)^2+(y-6)^2=9$ or $x^2+y^2-4x-12y+31=0$	

(iii) Given that the line $y-x+3=0$ intersects the circle, C_1, at two points, R and S, find the equation of the perpendicular bisector of the line RS. [4]

(iii)	$x^2+y^2-4x+8y+11=0$ ---------------(1) Sub $y-x+3=0$ into (1) $x^2+(x-3)^2-4x+8(x-3)+11=0$ $x^2-x-2=0$ $(x+1)(x-2)=0$ $x=2$ or $x=-1$ when $x=2$, $y=-1$ when $x=-1$, $y=-4$ Gradient of $RS=\dfrac{-1+4}{2+1}=1$ Midpoint of $RS=\left(\dfrac{2-1}{2},\dfrac{-1+4}{2}\right)=\left(\dfrac{1}{2},-\dfrac{5}{2}\right)$ Gradient of normal of $RS=-1$ Equation of the perpendicular bisector of the line RS: $y+\dfrac{5}{2}=-\left(x-\dfrac{1}{2}\right)$ $y=-x-2$	

11 A particle moves in a straight line, so that, t seconds after leaving a fixed point O its velocity, v is given by $v = \frac{36}{(t-3)^2} - 9$. Find

(i) the initial acceleration of the particle. [2]

(i)	$v = \frac{36}{(t-3)^2} - 9$ $= 36(t-3)^{-2} - 9$ $\frac{dv}{dt} = 36(-2)(t-3)^{-3}$ $= \frac{-72}{(t-3)^3}$ Initial acceleration is when $t = 0$ Initial acceleration $= \frac{-72}{(0-3)^3}$ $= \frac{8}{3}$ *or* $2\frac{2}{3}$ ms^{-2}	

(ii) the values of t when the particle is at instantaneous rest. [2]

(ii)	At instantaneous rest, $v = 0$, $\frac{36}{(t-3)^2} - 9 = 0$ $\frac{36}{(t-3)^2} = 9$ $(t-3)^2 = \frac{36}{9}$ $(t-3) = \pm\sqrt{4}$ $t = 3+2$ *or* $t = 3-2$ $t = 1$ *or* 5	

(iii) the total distance travelled by the particle in the first 2 seconds. [6]

(iii)

$$s=\int\left(\frac{36}{(t-3)^2}-9\right)dt$$

$$=\int 36(t-3)^{-2}dx-\int 9\,dt$$

$$=\frac{36(t-3)^{-2+1}}{(-2+1)}-9t+c$$

$$=-36(t-3)^{-1}-9t+c$$

$$s=\frac{-36}{(t-3)}-9t+c$$

At the fixed point, $t=0, s=0$

$$0=\frac{-36}{-3}-9(0)+c$$

$$c=-12$$

Therefore, $s=\dfrac{-36}{(t-3)}-9t-12$

Displacement, s from $t=0$ to

$t=1$ (first instantaneous rest)

$$s=\frac{-36}{(1-3)}-9(1)-12$$

$$=18-9-12$$

$$=-3$$

Displacement, s when $t=2$

$$s=\frac{-36}{(2-3)}-9(2)-12$$

$$=36-30$$

$$=6$$

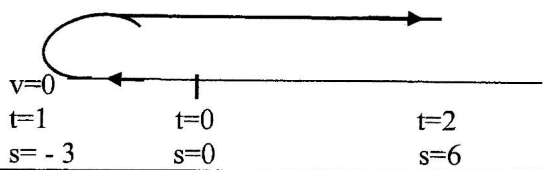
Therefore total distance travelled

$=3$ (on the left of fixed point O when $t=1$)

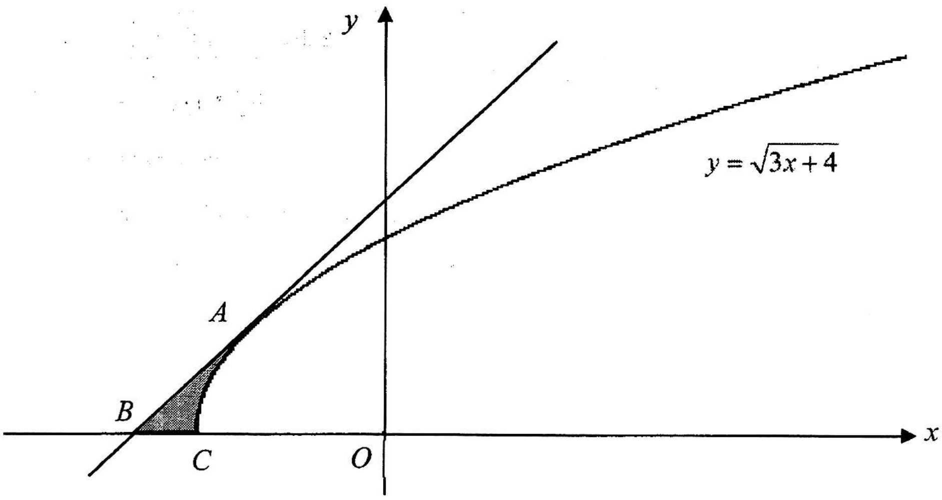
$+3$ (back to fixed point from the left)

$+6$ (from fixed point when $t=2$

$=12\,\text{m}$



12



The diagram shows part of the curve $y=\sqrt{3x+4}$. The line AB is tangent to the curve at A. The points B and C lie on the x-axis, and C is a point on the curve. The gradient of the

tangent of the curve at A is $\frac{3}{2}$. Find

(i) the coordinates of A and equation of the line AB. [5]

(i)	$\frac{dy}{dx}=\frac{1}{2}(3x+4)^{\frac{1}{2}-1}(3)$ $=\frac{3}{2}(3x+4)^{-\frac{1}{2}}$ $=\frac{3}{2\sqrt{3x+4}}$ Grad at A, $\frac{dy}{dx}=\frac{3}{2}$ $\frac{3}{2}=\frac{3}{2\sqrt{3x+4}}$ $x=-1$ When $x=-1, y=\sqrt{3(-1)+4}$ $=1$ Coordinates of A are $(-1,1)$ Equation of AB is $y-1=\frac{3}{2}(x-(-1))$ Equation of AB is $y=\frac{3}{2}x+\frac{5}{2}$	

(ii) the coordinates of B and of C. [2]

(ii)	y-coordinate of B is 0, x-coordinate of B is $0=\frac{3}{2}x+\frac{5}{2}$ $x=-\frac{5}{3}$ Coordinates of B are $\left(-\frac{5}{3},0\right)$ y-coordinate of C is 0, x-coordinate of C is $0=\sqrt{3x+4}$ $x=-\frac{4}{3}$ Coordinates of C are $\left(-\frac{4}{3},0\right)$	

(iii) the area of the shaded region ABC. [3]

(iii)	Area of ABC = Area under line - Area under curve $=\int_{-\frac{5}{3}}^{-1}\left(\frac{3}{2}x+\frac{5}{2}\right)dx-\int_{-\frac{4}{3}}^{-1}\sqrt{3x+4}\,dx$ $=\left[\frac{3x^2}{4}+\frac{5}{2}x\right]_{-\frac{5}{3}}^{-1}-\left[\frac{(3x+4)^{\frac{3}{2}}}{\frac{3}{2}\times 3}\right]_{-\frac{4}{3}}^{-1}$ $=\left[\frac{3}{4}-\frac{5}{2}-\left(\frac{\frac{25}{3}}{4}-\frac{25}{6}\right)\right]-\frac{2}{9}\left[(3(-1)+4)^{\frac{3}{2}}-\left(3\times-\frac{4}{3}+4\right)^{\frac{3}{2}}\right]$ $=\frac{1}{3}-\frac{2}{9}$ $=\frac{1}{9}$ units^2	

	Alternatively, Area of ABC = Area of triangle - Area under curve $=\frac{1}{2}\left(\frac{5}{3}-1\right)\times 1-\int_{-\frac{4}{3}}^{-1}\sqrt{3x+4}\,dx$ $=\frac{1}{3}-\frac{2}{9}\left[(3(-1)+4)^{\frac{3}{2}}-\left(3\times -\frac{4}{3}+4\right)^{\frac{3}{2}}\right]$ $=\frac{1}{9}$ units^2	