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ANDERSON SECONDARY SCHOOL
Preliminary Examination 2021
Secondary Four Express

CANDIDATE NAME:

CLASS: / INDEX NUMBER:

ADDITIONAL MATHEMATICS **4049/01**

Paper 1

23 August 2021

2 hours 15 minutes

1100 – 1315h

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid/tape.

Answer **all** the questions.
Omission of essential working will result in loss of marks.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Setter: Mr Low Kok Ming

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 A curve is such that $\frac{d^2y}{dx^2} = 2(1-2x)$. The equation of the normal to the curve at the point $(-1, 7)$ is $9y = x + 64$. Find the equation of the curve. [6]

2 **(i)** Show that $p+q$ is a factor of the expression $p^3-3p^2q+4q^3$. [2]

(ii) Factorise $p^3-3p^2q+4q^3$ completely. [2]

(iii) Hence solve the equation $(m+2)^3-3(m+2)^2(m-3)+4(m-3)^3=0$. [2]

3 **(a)** Solve $7\cos\left(\frac{\theta}{2}\right) = 10\sin\theta$, for $0 \leq \theta \leq 2\pi$. [5]

(b) Prove that $\frac{\sec x \tan x + \sec^2 x}{(\tan x + \sec x)^2 + 1} = \frac{1}{2}$. [3]

4 **(i)** Write down and simplify, in descending powers of x, the first three terms in the expansion of $\left(x^5 + \frac{2}{x^6}\right)^n$, where $n > 0$. [2]

(ii) When the third term of the expansion is divided by the second term, $\frac{8}{x^{11}}$ is obtained. Show that $n = 9$. [2]

(iii) Using the value of n found in **(ii)**, without expanding $\left(x^5+\frac{2}{x^6}\right)^n$, show that there is no constant term in the expansion. [3]

5 **(a)** Solve the equation $3\left(9^k\right)+2\left(4^k\right)=5\left(6^k\right)$. [5]

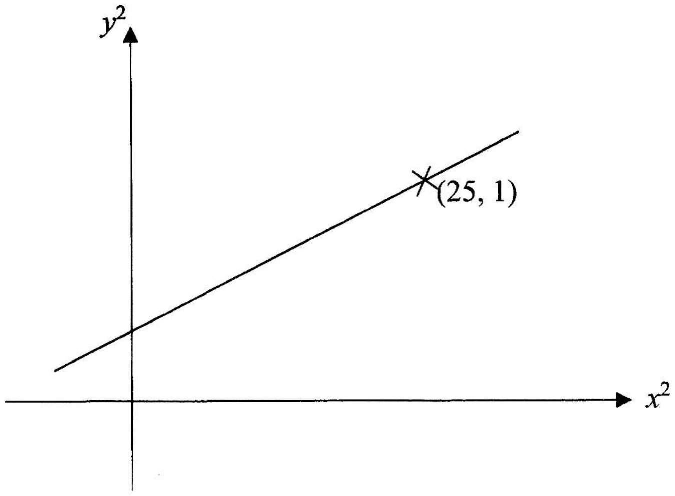
(b) Show that $\dfrac{\sqrt{\left(a-b^2\right)^3\left(a+b^2\right)}}{\left(\sqrt{a}+b\right)\sqrt{a^2-b^4}}$ can be expressed in the form $m\sqrt{a}+nb$ where m and n are constants to be determined. [4]

6 **(i)** Express $\dfrac{4x^3+9x^2-17x-5}{(2x-3)(x^2+1)}$ in partial fractions. [6]

(ii) Differentiate $\ln\left(x^2+1\right)$ with respect to x. [1]

(iii) Hence find $\int \frac{4x^3+9x^2-17x-5}{(2x-3)\left(x^2+1\right)}\,dx$. [3]

7 **(a)**



Two variables x and y are related by the equation $\frac{x^2}{p^2} = 1 + \frac{3y^2}{q^2}$, where p and q are constants. When the graph of y^2 against x^2 is drawn, a straight line is obtained. Given that the line passes through the point (25, 1) and has a gradient $\frac{1}{15}$, find

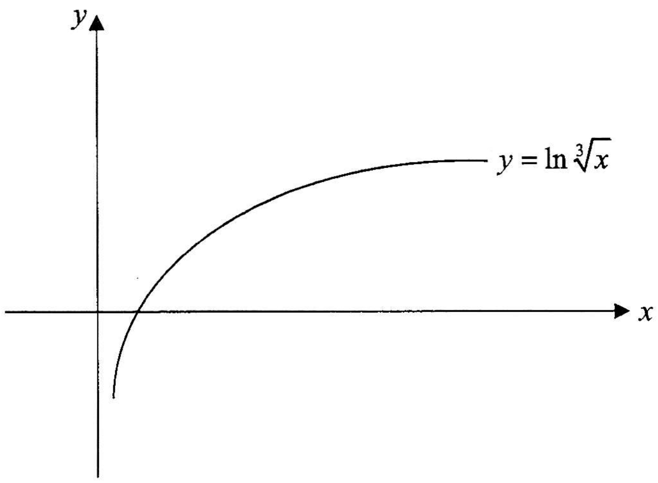
(i) the exact values of p and q. [4]

(ii) the values of x when $y = \sqrt{\frac{2}{5}}$. [2]

(b) Determine, showing your working, if it is possible for any line that passes through the point (2, 1) to be a tangent to the circle with equation $x^2 + y^2 - 6x + 8y - 11 = 0$. [5]

8 **(a)** A curve has the equation $y = \dfrac{4x+15}{x-9}$, where $x \neq 9$. Find the gradient function of the curve and determine, with explanation, whether y is an increasing or decreasing function. [4]

(b) The diagram shows part of the curve $y = \ln \sqrt[3]{x}$.



Calculate the area of the region bounded by the curve, the line $x = 3$ and the x-axis. [5]

9 **(a)** Write down the principal value, in radians as a multiple of π, of $\cos^{-1}\left(-\sin\frac{\pi}{3}\right)$. [1]

(b) x, y and z are three angles of a triangle. Given that x and y are acute angles such that $\sin x = \frac{8}{17}$ and $\sin y = \frac{3}{5}$, find the exact value of $\tan z$ without the use of a calculator. [4]

10 A factory is tasked to design an open cylindrical container with a surface area of 243π cm^2. The radius and height of the cylinder is r cm and h cm respectively.

(i) Show that the volume, V cm^3, of the cylinder is $V = \frac{\pi}{2}\left(243r - r^3\right)$. [4]

(ii) Find, in terms of π, the maximum volume of the cylinder. [6]

11 A particle moves in a straight line so that, t seconds after passing a fixed point O, its velocity, v m/s, is given by $v = 2t^2 - 16t + 30$.

(i) Find an expression, in terms of t, for the displacement of the particle. [2]

(ii) Sketch the velocity-time graph and hence find the range of times when the particle is travelling towards O. [3]

(iii) Calculate the total distance travelled by the particle in the first 7 seconds. [4]

End of Paper

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Answers:

1. $y = -\frac{2}{3}x^3 + x^2 - 5x + \frac{1}{3}$

2(ii) $(p+q)(p-2q)^2$ (iii) $m = \frac{1}{2}$ or 8

3(a) $\theta = \pi$, 0.715, 5.57

4(i) $x^{5n} + 2nx^{5n-11} + 2n(n-1)x^{5n-22} + \ldots$

5(a) $k = -1$ or 0 (b) $\sqrt{a} - b$

6(i) $2 + \frac{1}{(2x-3)} + \frac{7x}{(x^2+1)}$ (ii) $\frac{2x}{x^2+1}$

(iii) $2x + \frac{1}{2}\ln(2x-3) + \frac{7}{2}\ln(x^2+1) + C$

7(a)(i) $p = \pm\sqrt{10}$, $q = \pm\sqrt{2}$ (ii) $x = \pm 4$ (b) not possible

8(a) $\frac{dy}{dx} = -\frac{51}{(x-9)^2}$; decreasing function (b) 0.432 units^2

9(a) $\frac{5\pi}{6}$ (b) $-\frac{77}{36}$

10(ii) 729π cm^2

11(i) $s = \frac{2t^3}{3} - 8t^2 + 30t$ (ii) $3 < t < 5$ (iii) 52 m

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ANDERSON SECONDARY SCHOOL
Preliminary Examination 2021
Secondary Four Express

CANDIDATE NAME:

CLASS: /

INDEX NUMBER:

ADDITIONAL MATHEMATICS

4049/02

Paper 2

26 August 2021

2 hours 15 minutes

0800 – 1015h

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This document consists of **17** printed pages.

Setter: Ms Lee Siew Lin

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1 If $y = (1+x)e^{3x}$, find the value of the constant k for which $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + ky = 0$. [6]

2 A hot stone with a temperature of 220°C was dropped into a reservoir of water. As the stone cools, its temperature, T°C, t minutes after it enters the water is given by $T = P + 190e^{-kt}$, where P and k are constants.

(i) Show that $P = 30$. [2]

(ii) It took 4 seconds for the temperature of the stone to reach 120°C. Find the value of k. [3]

(iii) Sketch the graph of T against t. [2]

3 **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\tan^3 x\right) = 3\sec^4 x - 3\sec^2 x$. [3]

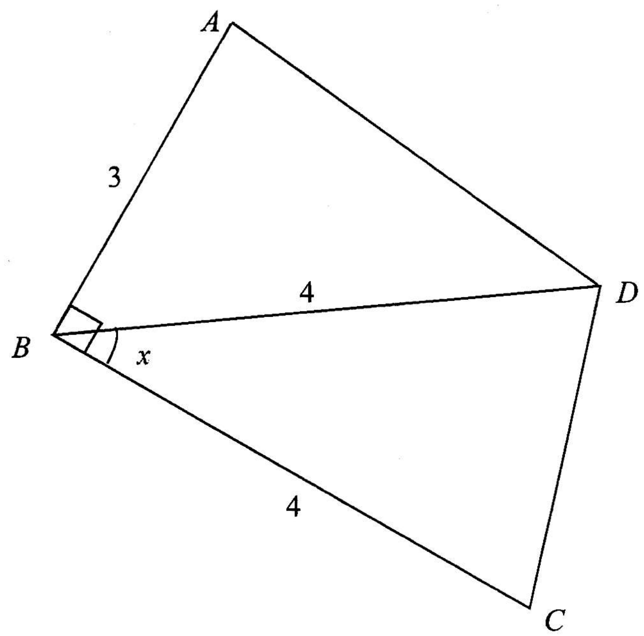
(ii) Hence evaluate $\int_0^{\frac{\pi}{4}} \sec^4 x - 2\sec^2 x \, \mathrm{d}x$. [5]

4 A curve has the equation $y = \ln\sqrt{1+2x^3}$.

(i) Show that the curve has only one stationary point and determine the nature of this stationary point. [6]

(ii) A particle moves along the curve in such a way that the y-coordinate of the particle is decreasing at a constant rate of 0.1 units per second. Find the rate of change of the x-coordinate at the instant when $x = 2$. [3]

5 The diagram below shows a quadrilateral $ABCD$ with $AB = 3$ cm, $BC = BD = 4$ cm and $\angle ABC = 90°$. The acute angle DBC is x.



(i) Show that the area, A cm^2, of the quadrilateral is given by $A = 6\cos x + 8\sin x$. [3]

(ii) Express A in the form $R\cos(x - \alpha)$, where $R > 0$ and α is acute. [4]

(iii) Hence state the maximum area of the quadrilateral. [1]

(iv) Find x for which the area of *ABCD* is 7 cm^2. [3]

6 (i) Show that $\dfrac{\sin^2 x - 2\sin x}{1-\cos 2x} = \dfrac{1}{2}(1 - 2\operatorname{cosec} x)$. [3]

(ii) Hence find the equation of the tangent to the curve $y = \dfrac{\sin^2 x - 2\sin x}{1-\cos 2x}$ at the point where $x = \dfrac{\pi}{4}$. [5]

7 The equation of a curve is $2y = 3x^2 - px - 2q - 1$, where p and q are constants. The line $y = p - q - 2x$ is a tangent to the curve at the point A.

(i) Show that the possible values of p are -2 and -14. [4]

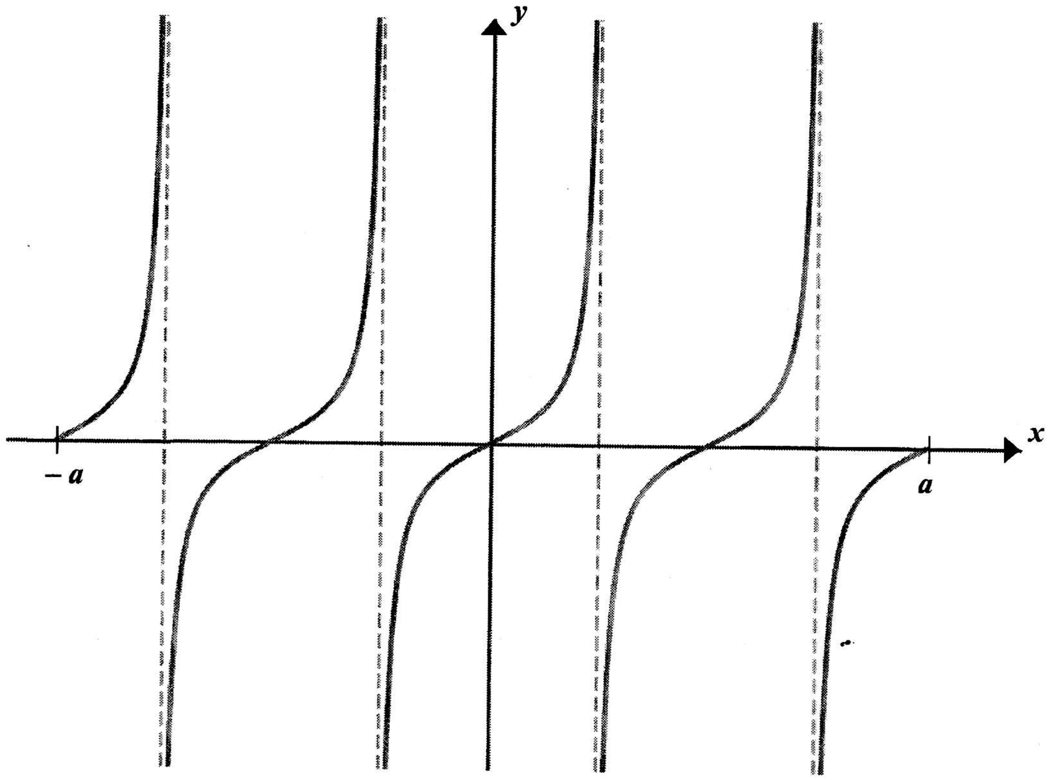
(ii) It is given that the curve passes through the point $(0, -5.5)$.
For the case where $p = -2$, find the coordinates of A. [4]

8 **(a)** Solve the equation $\log_4 x^2 - 3\log_x 4 = 1$. [6]

(b) **(i)** Solve the equation $2\log_2(1-x)-\log_2 x-\log_2 2x-3=0$. [5]

(ii) Hence solve $2\log_2(-y)-\log_2(y+1)-\log_2(2y+2)-3=0$. [2]

9 **(a)** The figure shows the graph of $y = \tan 2x$ for $-a \le x \le a$, where a is in radians.



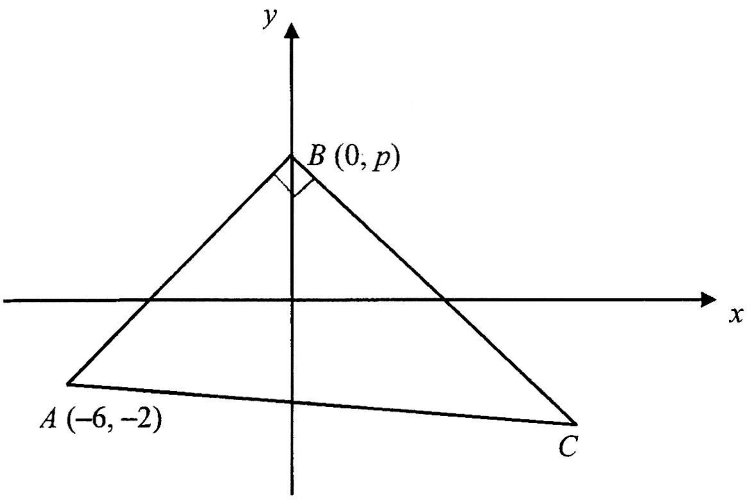
(i) State the value of a in terms of π. [1]

(ii) State the amplitude and the period of $y = -4\cos 3x$. [2]

(iii) Sketch, on the same axes above, the graph of $y = -4\cos 3x$ for $-a \le x \le a$. [4]

(b) Eric claims that $x^2 + mx - 2m + \frac{17}{4} > 2$ for all real values of x if $-5 < m < 0$.
Justify whether Eric's claim is true. [5]

10 The diagram shows a triangle with vertices A $(-6, -2)$, B $(0, p)$ and C.
BC has a gradient of $-\frac{3}{2}$. AB is perpendicular to BC and $3AB = 2BC$.



(i) Show that $p = 2$. [3]

(ii) Explain why AC is not parallel to the x-axis. [3]

(iii) Find the area of the triangle ABC. [2]

End of Paper

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ANDERSON SECONDARY SCHOOL
Sec 4 Express Additional Mathematics
PRELIM EXAM 2021 PAPER 2

Answer Key

Qn	Answer
1	$k=9$
2	(ii) $k=11.2$
3	(ii) $-\frac{2}{3}$
4	(ii) $-\frac{17}{120}$ units/s
5	(ii) $A=10\cos(x-53.1°)$
	(iii) 10 cm^2
	(iv) $x=7.6°$
6	(ii) $y-\left(\frac{1}{2}-\sqrt{2}\right)=\sqrt{2}\left(x-\frac{\pi}{4}\right)$
7	(ii) $(-1,-5)$
8	(a) $x=8$ or $x=\frac{1}{4}$
	(b) (i) $x=\frac{1}{5}$
	(b) (ii) $y=-\frac{4}{5}$
9	(a) (i) $a=\pi$
	(a) (ii) Amplitude = 4, Period = $\frac{2\pi}{3}$
9	(b) Eric's claim is true.
10	(iii) 39 $units^2$