ST. MARGARET'S SECONDARY SCHOOL

Preliminary Examinations 2020

CANDIDATE NAME

CLASS REGISTER NUMBER

ADDITIONAL MATHEMATICS **4047/01**

Paper 1 **20 August 2020**

Secondary 4 Express / 5 Normal (Academic) **2 hours**

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **20** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$, $\qquad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Solve the equation $25^{x+1} - 5(5^{x+1}) - 14 = 0$. [5]

2 **(i)** On the same axes, sketch the graphs of $y^2 = 3x$ and $y = \frac{-9}{\sqrt{x^5}}$ for $x > 0$. [2]

(ii) Calculate the *x*-coordinate of the point of intersection of the two graphs, giving your answer in exact form. [2]

3 Express $\frac{\sqrt{2}}{3+\sqrt{2}} - \frac{1}{1+\sqrt{2}}$ in the form $\frac{a+b\sqrt{2}}{7}$, where *a* and *b* are integers. [4]

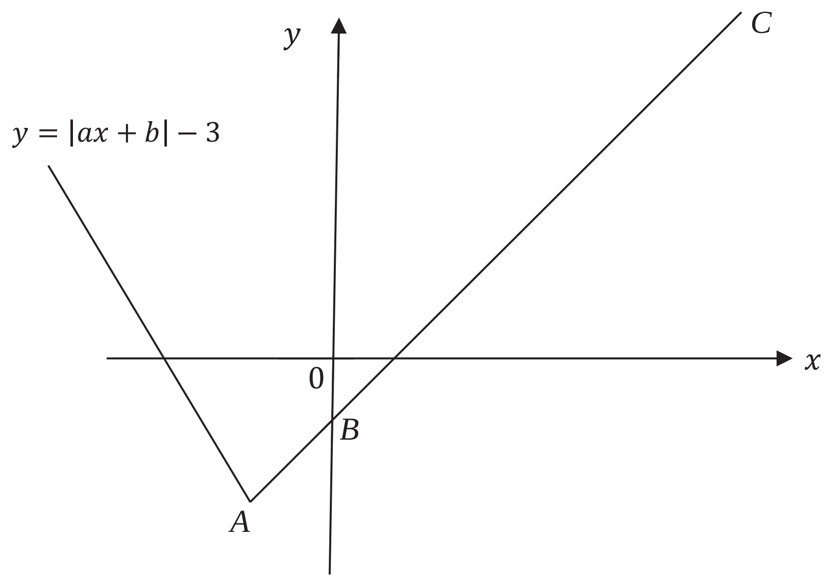
4 Given that $\log_2 a = h$ and $\log_2 b = k$, express $\log_2\sqrt{\frac{32a}{b^3}}$ in terms of h and k. [4]

5 (i) Express $\dfrac{7x^2+16x-3}{(2x-1)(1+x)^2}$ in partial fractions. [6]

(ii) Hence evaluate $\int \frac{7x^2+16x-3}{(2x-1)(1+x)^2}\,\mathrm{d}x$. [3]

6 A particle moves along the curve $y = \frac{x+1}{2x-1}$ in such a way that the x-coordinate of the particle is increasing at a constant rate of 0.03 units per second. Find the x-coordinates of the particle at the instants where the y-coordinate is decreasing at unit per second. [4]

7



The diagram shows the graph of $y = |ax + b| - 3$, where a and b are constants.
The graph crosses the y-axis at B where the point B is $(0, -1)$.

(i) Find the y-coordinate of A. [1]

(ii) Given that the x-coordinate of A is -2, find the value of a and of b. [2]

(iii) Given that $AB : BC = 1 : 4$, find the coordinates of C. [2]

8 **(i)** Show that $5\cos^2 x - 3\sin^2 x$ can be written as $a\cos 2x + 1$ where a is a positive integer. [3]

(ii) State the amplitude and period of $5\cos^2 x - 3\sin^2 x$. [2]

(iii) Sketch on the same axes, the graphs of $y = 5\cos^2 x - 3\sin^2 x$ and $y = |\tan x|$ for $0° \le x \le 180°$. [3]

(iv) Hence state the coordinates of the solutions of the equation $5\cos^2 x - 3\sin^2 x = |\tan x|$, $0° \le x \le 180°$. [2]

9 A curve is such that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{9}{(1-2x)^4}$, $x \neq \frac{1}{2}$ and $(1, 5)$ is a point on the curve.

(i) Find the equation of the curve. [4]

(ii) Find the range of values of x such that the gradient of the curve is decreasing. [3]

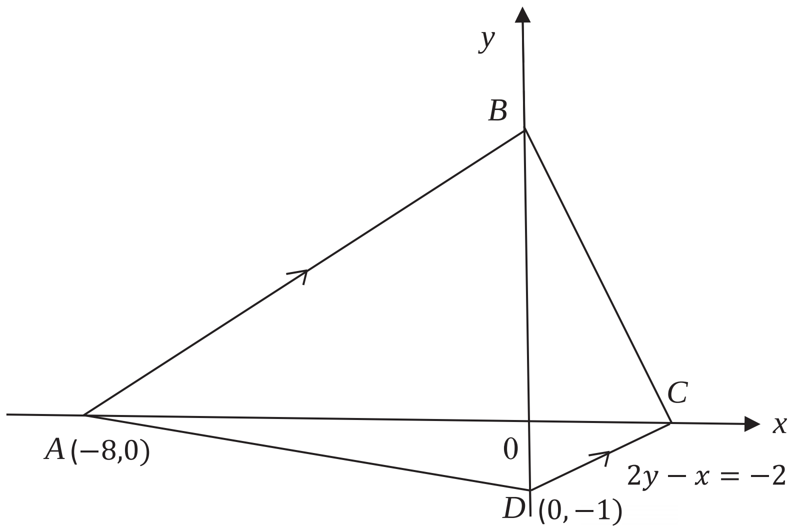
10 The mass m grams of a radioactive substance present at time t days after first being observed is given by the formula $m = 38e^{kt}$.
25 days after first being observed, the mass has dropped to 10.3 g.

(i) Find the initial mass of the radioactive substance. [1]

(ii) Find the number of days it takes after first being observed for the mass to become half its original value. [5]

(iii) Find the rate at which the mass is decreasing when $t = 40$ days. [2]

11 Solutions to this question by accurate drawing will not be accepted.



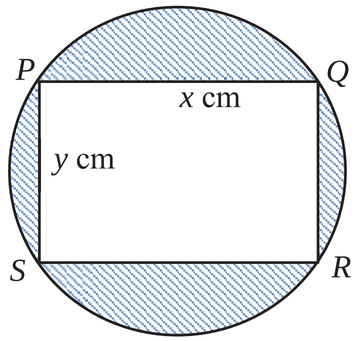
The diagram shows a trapezium *ABCD* such that *AB* is parallel to *DC* and *AB* is perpendicular to *BC*. The point *A* is $(-8, 0)$ and *D* is $(0, -1)$. The equation of the line *CD* is $2y - x = -2$.

(i) Find the coordinates of *B*. [3]

(ii) The perpendicular bisector of *AD* meets *AB* at point *E*.
Determine with explanation if it is possible to construct a circle with centre *E*, passing through the points *A* and *D*. [2]

(iii) Find the coordinates of E. [5]

12 The diagram shows a rectangle of length x cm and width y cm inscribed in a circle such that all the vertices of the rectangle $PQRS$ touch the circumference of the circle.



A brooch is designed by removing the inscribed rectangle $PQRS$ from the circle. The area of the rectangle is given to be 20 cm^2.

(i) Show that the radius of the circle is $\frac{\sqrt{x^4+400}}{2x}$. [3]

(ii) Ignoring the thickness of the brooch, show that the remaining area is given by

$$A = \frac{\pi x^2}{4} + \frac{100\pi}{x^2} - 20.$$

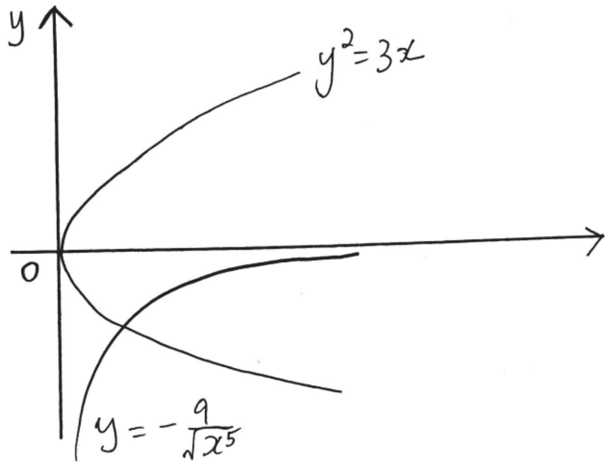
[2]

(iii) Given that x can vary, find the value of x for which A has a stationary value. [3]

(iv) Determine the nature of the stationary value of A. [2]

Answer

1. $x = 0.209$
2. (i)



(ii) $x = \sqrt{3}$

3. $\frac{5-4\sqrt{2}}{7}$
4. $\frac{1}{2}[5 + h - 3k]$
5. (i) $\frac{7x^2+16x-3}{(2x-1)(1+x)^2} = \frac{3}{2x-1} + \frac{2}{x+1} + \frac{4}{(x+1)^2}$

(ii) $\int \frac{7x^2+16x-3}{(2x-1)(1+x)^2}\,\mathrm{d}x = \int (\frac{3}{2x-1} + \frac{2}{x+1} + \frac{4}{(x+1)^2}\ dx$

$= \frac{3\ln(2x-1)}{2} + 2\ln(x+1) - \frac{4}{(x+1)} + c$

6. $x = 0.65$ or $x = 0.35$ [$x = \frac{13}{20}$ or $x = \frac{7}{20}$]
7. (i) -3

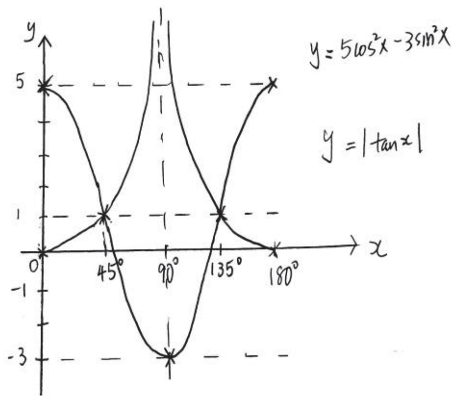
(ii) $a = 1\ or\ b = 2$

(iii) $C\ is\ (8, 7)$

8. (i) $4cos2x + 1$

(ii) Amplitude = 4, Period = 180°

8 (iii)



8 (iv) $(45°, 1)$ and $(135°, 1)$

9. (i) $y = \frac{3}{2(1-2x)^3} + \frac{13}{2}$

(ii) $x > \frac{1}{2}$

10. (i) Initial mass = 38 g.

(ii) $k = -0.052217$, $t = 13.3$ days

(iii) $\frac{dm}{dt} = -0.246$

11. (i) *B* is (0, 9)

(ii) Since *E* lies on the perpendicular bosector of *AD*, *AE* = *DE*.
If *E* is the centre, then *AE* = *DE* = Radius
Hence, yes it is possible.

(iii) *E* is $(-3\frac{2}{3}, 2\frac{1}{6})$

12. (iii) $x = 4.47$

(iv) *A* is a minimum

ST. MARGARET'S SECONDARY SCHOOL

Preliminary Examinations 2020

CANDIDATE NAME

CLASS | | | REGISTER NUMBER | |

ADDITIONAL MATHEMATICS 4047/02

Paper 2 **31 August 2020**

Secondary 4 Express / 5 Normal (Academic) **2 hours 30 minutes**

Additional Materials: Nil

READ THESE INSTRUCTIONS FIRST
Write your name and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of **20** printed pages and **0** blank page

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$, $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 The quadratic equation $2x^2 + 3x + 1 = 0$ has roots α and β.

(i) Show that $\alpha^2 + \beta^2 = \frac{5}{4}$. [3]

(ii) Find a quadratic equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$. [4]

2 The first three terms of the expansion of $(1 + ax)^n$ in ascending powers of x are $1 + 15x + 90x^2$.

(i) Find the values of a and n. [5]

.

(ii) Find the constant term in the expansion of $\left(2 - \frac{1}{3x}\right)^2(1 + ax)^n$. [3]

3 **(i)** Differentiate $3xe^{2x}$ with respect to x. [2]

(ii) Use your answer to part (i) to show that $\int_0^1 xe^{2x}\mathrm{d}x = \frac{e^2+1}{4}$. [4]

4 The table shows experimental values of two variables x and y.

x	0.5	1	1.5	2	2.5
y	3.6	6.5	7.95	7.2	3.25

It is known that x and y are related by an equation of the form $y = px^3 + qx$, where p and q are constants.

Plot $\frac{y}{x}$ against x^2 and draw a straight line graph. [3]

Use your graph to

(ii) estimate the value of p and of q, [3]

(iii) estimate the value of x when $y = 4x$. [2]

5 The equation of a curve is $y = 2(1 - 4x)^5 - 7$.

(i) Explain why the curve has only one stationary point and why this is a point of inflexion. [5]

(ii) Write down the coordinates of the stationary point. [1]

(iii) Explain why the normal to the curve at this point is parallel to the y-axis and write down the equation of the normal. [2]

6 **(i)** Prove the identity $(\sec\theta - \tan\theta)^2 = \frac{1-\sin\theta}{1+\sin\theta}$. [4]

(ii) Hence, solve the equation $2(\sec\theta - \tan\theta)^2 = 3$ for $-\pi \le \theta \le \frac{3}{2}\pi$. [4]

7 **(a)** By using long division, find the value of k for which $x^2 - 2x + k$ is a factor of $x^3 - 4x^2 + 8$. [4]

(b) The expressions $x^3 - ax + a^2$ and $ax^3 - 5x^2 - 32$ have the same remainder when divided by $x - 3$. Find the possible values of a. [4]

8 The velocity, v ms^{-1}, of a particle, travelling in a straight line, at time t s after leaving a fixed point O, is given by $v = 3 + kt - 2t^2$, where $t \geq 0$ and k is a constant. When $t = 0$, the particle is at O and its acceleration is 5 m/s^2.

(i) Find the value of k. [2]

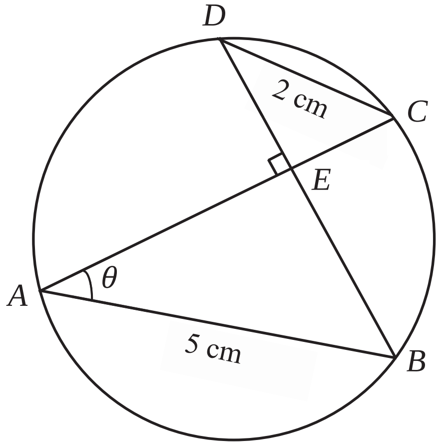
(ii) Find the displacement of the particle when it is instantaneously at rest. [5]

(iii) Explain why the distance the particle has travelled when it is again at O is twice the displacement found in **(ii)**. [1]

9 **(a)** Find the set of values of k for which the curve $y = x^2 + kx$ and the line $y = 5x - 1$ do not intersect. [4]

(b) Show that the equation $x^2 + 5x = 2ax + a$ has real and distinct roots for all real values of a. [4]

10 The diagram below shows the design of a medallion. The design consists of four chords. The chords AC and BD of the circle intersect at right angle at the point E. Angle $BAC = \theta$ and can vary. AB = 5 cm and CD = 2 cm.



(i) Express AC in the form $a\cos\theta + b\sin\theta$. [3]

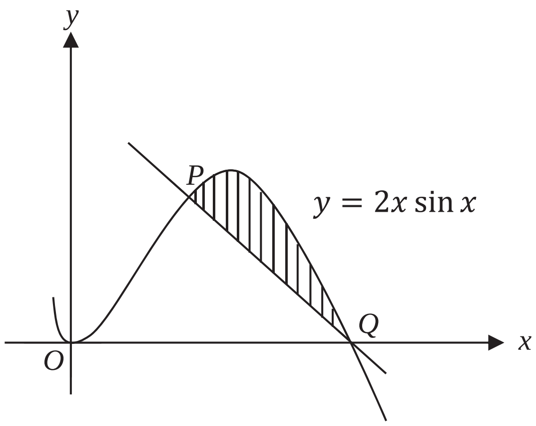
(ii) Express $a\cos\theta + b\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(iii) Find the maximum length of AC and the corresponding value of θ. [3]

(iv) The designer of the medallion specified that AE to EC must be in the ratio 3 : 2. Find the corresponding value of θ in this case. [2]

11 **(i)** Given that $\frac{\mathrm{d}}{\mathrm{d}x}(x\cos x) = \cos x - x\sin x$, find $\int x\sin x\,\mathrm{d}x$. [3]

The diagram below shows part of the graph of $y = 2x\sin x$ and a shaded region formed by the curve and the line PQ where $x = \frac{\pi}{2}$ at the point P. Q is the point where the curve intersects the x-axis.



(ii) Find the y-coordinate of P and the x-coordinate of Q. [3]

(iii) Hence, find the area of the shaded region. [4]

12 **(i)** Write down the equation of the circle with centre $A(-2, 3)$ and radius 5. [1]

(ii) Find the equation of the normal to the circle at the point $(1, -1)$. [3]

The circle intersects the x-axis at P and Q.

(iii) Find the length of PQ. [3]

A second circle, centre B, also passes through P and Q.

(iv) Find the smallest possible value of the radius of the second circle. [1]

(v) Find the equation of the second circle with this radius. [2]

Answer Keys

1 (ii) $x^2 - 5x + 4 = 0$

2 (i) $n = 5, a = 3$ (ii) –6

3 (i) $3e^{2x} + 6xe^{2x}$

4 (i) $p = -0.983\ (\pm 0.1)$, $q = 7.4 \pm 0.1$ (ii) $x^2 = 3.5\ (\pm 0.1), x = 1.87$

5 (ii) $\left(\frac{1}{4}, -7\right)$ (ii) $x = \frac{1}{4}$

6 (ii) $-2.94, -0.201, 3.34$

7 (i) –4 (ii) $a = 4$ or $a = 26$

8 (i) $k = 5$ (ii) 13.5 m

9 (a) $3 < k < 7$

10 (i) $AE = 5\cos\theta + 2\sin\theta$ (ii) $5.39\cos(\theta - 21.8°)$ or $\sqrt{29}\cos(\theta - 21.8°)$

(iii) Maximum length of AC = 5.39 cm, corresponding value of $\theta = 21.8°$

(iv) $\theta = 21.8°$

11 (i) $\int x \sin x \, dx = \sin x - x\cos x + c$

(ii) y-coordinate of P is π and x-coordinate of Q is π. (iii) 1.82 units^2

12 (i) $(x+2)^2 + (y-3)^2 = 25$ (ii) $y = -\frac{4}{3}x + \frac{1}{3}$ (iii) 8 units

(iv) 4 units (v) $(x+2)^2 + y^2 = 16$

ST. MARGARET'S SECONDARY SCHOOL

Preliminary Examinations 2020

Founded 1842

CANDIDATE NAME

CLASS

REGISTER NUMBER

ADDITIONAL MATHEMATICS **4047/01**

Paper 1 **20 August 2020**

Secondary 4 Express / 5 Normal (Academic) **2 hours**

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **20** printed pages.

1 Solve the equation $25^{x+1} - 5(5^{x+1}) - 14 = 0$. [5]

$5^{2x+2} - 5(5^x . 5) - 14 = 0$

$5^{2x} . 5^2 - 5^x . 5^2 - 14 = 0$

Let $u = 5^x$

$25u^2 - 25u - 14 = 0$

$(5u - 7)(5u + 2) = 0$

$u = \frac{7}{5}$ or $u = -\frac{2}{5}$

$5^x = \frac{7}{5}$ or $5^x = -\frac{2}{5}$ (NA)

$\lg 5^x = \lg(\frac{7}{5})$

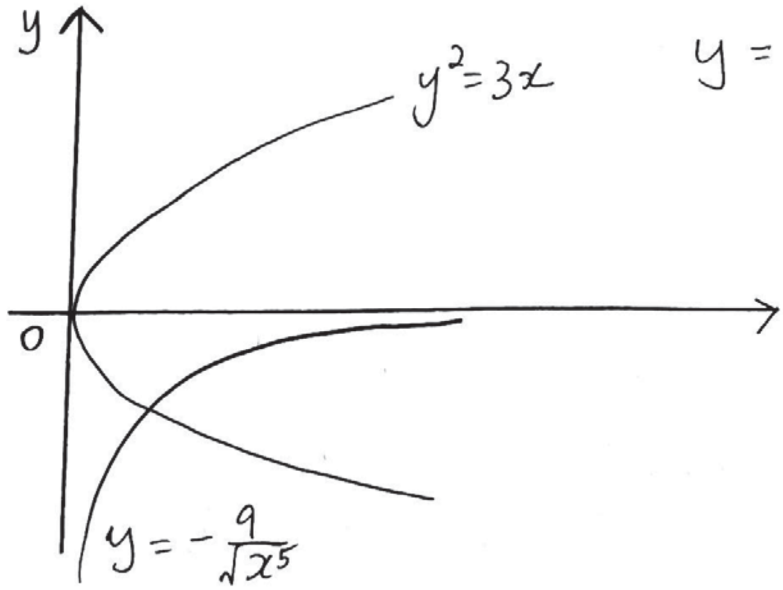
$x \lg 5 = \lg(\frac{7}{5})$

$x = \frac{\lg(\frac{7}{5})}{\lg 5}$

$x = 0.209$

2 **(i)** On the same axes, sketch the graphs of $y^2 = 3x$ and $y = \frac{-9}{\sqrt{x^5}}$ for $x > 0$. [2]

$y = -9x^{-\frac{5}{2}}$



(ii) Calculate the x-coordinate of the point of intersection of the two graphs, giving your answer in exact form. [2]

$$\left(-\frac{9}{\sqrt{x^5}}\right)^2 = 3x$$

$$\frac{81}{x^5} = 3x$$

$$x^6 = 27 = 3^3$$

$$x = 3^{\frac{3}{6}}$$

$$= 3^{\frac{1}{2}}$$

$$= \sqrt{3}$$ //

3 Express $\frac{\sqrt{2}}{3+\sqrt{2}} - \frac{1}{1+\sqrt{2}}$ in the form $\frac{a+b\sqrt{2}}{7}$, where a and b are integers. [4]

$$\frac{\sqrt{2}(1+\sqrt{2}) - (3+\sqrt{2})}{(3+\sqrt{2})(1+\sqrt{2})}$$

$$= \frac{\sqrt{2} + 2 - 3 - \sqrt{2}}{3 + 3\sqrt{2} + \sqrt{2} + 2}$$

$$= \frac{-1}{5 + 4\sqrt{2}} \times \frac{5 - 4\sqrt{2}}{5 - 4\sqrt{2}}$$

$$= \frac{-5 + 4\sqrt{2}}{25 - 32}$$

$$= \frac{-5 + 4\sqrt{2}}{-7}$$

$$= \frac{5 - 4\sqrt{2}}{7}$$

//

[Turn over

4 Given that $\log_2 a = h$ and $\log_2 b = k$, express $\log_2 \sqrt{\frac{32a}{b^3}}$ in terms of h and k. [4]

$$\log_2 \left(\frac{32a}{b^3}\right)^{\frac{1}{2}}$$

$$= \frac{1}{2}\left[\log_2 32a - \log_2 b^3\right]$$

$$= \frac{1}{2}\left[\log_2 32 + \log_2 a - 3\log_2 b\right]$$

$$= \frac{1}{2}\left[\log_2 2^5 + h - 3k\right]$$

$$= \frac{1}{2}\left[5 + h - 3k\right]$$

5 (i) Express $\frac{7x^2+16x-3}{(2x-1)(1+x)^2}$ in partial fractions. [6]

$$\frac{7x^2+16x-3}{(2x-1)(1+x)^2} = \frac{A}{2x-1} + \frac{B}{1+x} + \frac{C}{(1+x)^2}$$

$$7x^2+16x-3 = A(1+x)^2 + B(2x-1)(1+x) + C(2x-1)$$

$$\therefore \frac{7x^2+16x-3}{(2x-1)(1+x)^2} = \frac{3}{2x-1} + \frac{2}{x+1} + \frac{4}{(x+1)^2}$$

[Turn over

(ii) Hence evaluate $\int \frac{7x^2+16x-3}{(2x-1)(1+x)^2}\,dx$. [3]

$4(x+1)^{-2}$

$$\int \frac{7x^2+16x-3}{(2x-1)(1+x)^2}\,dx$$

$$= \int \frac{3}{2x-1} + \frac{2}{x+1} + \frac{4}{(x+1)^2}\,dx$$

$$= \frac{3\ln(2x-1)}{2} + 2\ln(x+1) + \frac{4(x+1)^{-1}}{-1} + C$$

$$= \frac{3\ln(2x-1)}{2} + 2\ln(x+1) - \frac{4}{(x+1)} + C$$

6 A particle moves along the curve $y = \frac{x+1}{2x-1}$ in such a way that the x-coordinate of the particle is increasing at a constant rate of 0.03 units per second. Find the x-coordinates of the particle at the instants where the y-coordinate is decreasing at 1 unit per second. [4]

given that $\frac{dx}{dt} = 0.03$ and $\frac{dy}{dt} = -1$ } find x

$$\frac{dy}{dx} = -\frac{3}{(2x-1)^2}$$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

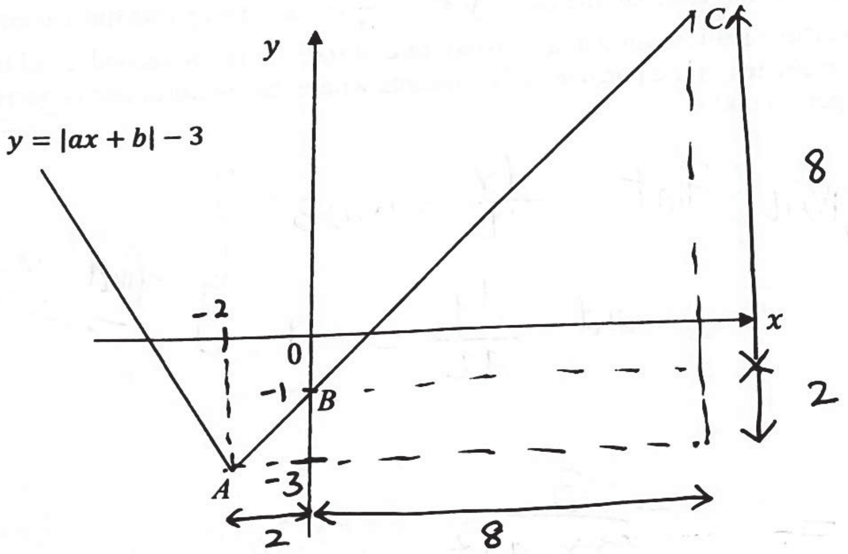
$$-1 = -\frac{3}{(2x-1)^2} \cdot (0.03)$$

$$(2x-1)^2 = 0.09$$

⋮

$x = 0.65$ $\left(\frac{13}{20}\right)$ or $x = 0.35$ $\left(\frac{7}{20}\right)$

7



The diagram shows the graph of $y = |ax + b| - 3$, where a and b are constants. The graph crosses the y-axis at B where the point B is $(0, -1)$.

(i) Find the y-coordinate of A. [1]

y-coord of A = −3

(ii) Given that the x-coordinate of A is -2, find the value of a and of b. [2]

$|b| - 3 = -1$
$|b| = 2$
$b = 2$

$-\frac{b}{a} = -2$
$\frac{b}{a} = 2$
$a = \frac{2}{2}$
$a = 1$

or $a = -1$, $b = -2$

$y = |x + 2| - 3$ or $y = |-x - 2| - 3$

(iii) Given that $AB : BC = 1 : 4$, find the coordinates of C. [2]

$x = 8$
$y = 8 - 1 = 7$

AB : BC
= 1 : 4
= 2 : 8

∴ C is (8, 7)

8 (i) Show that $5\cos^2 x - 3\sin^2 x$ can be written as $a\cos 2x + 1$ where a is a positive integer. [3]

$5\cos^2 x - 3(1-\cos^2 x)$

$= 8\cos^2 x - 3$

$= 8\left[\frac{\cos 2x + 1}{2}\right] - 3$

$= 4(\cos 2x + 1) - 3$

$= 4\cos 2x + 4 - 3$

$= 4\cos 2x + 1$

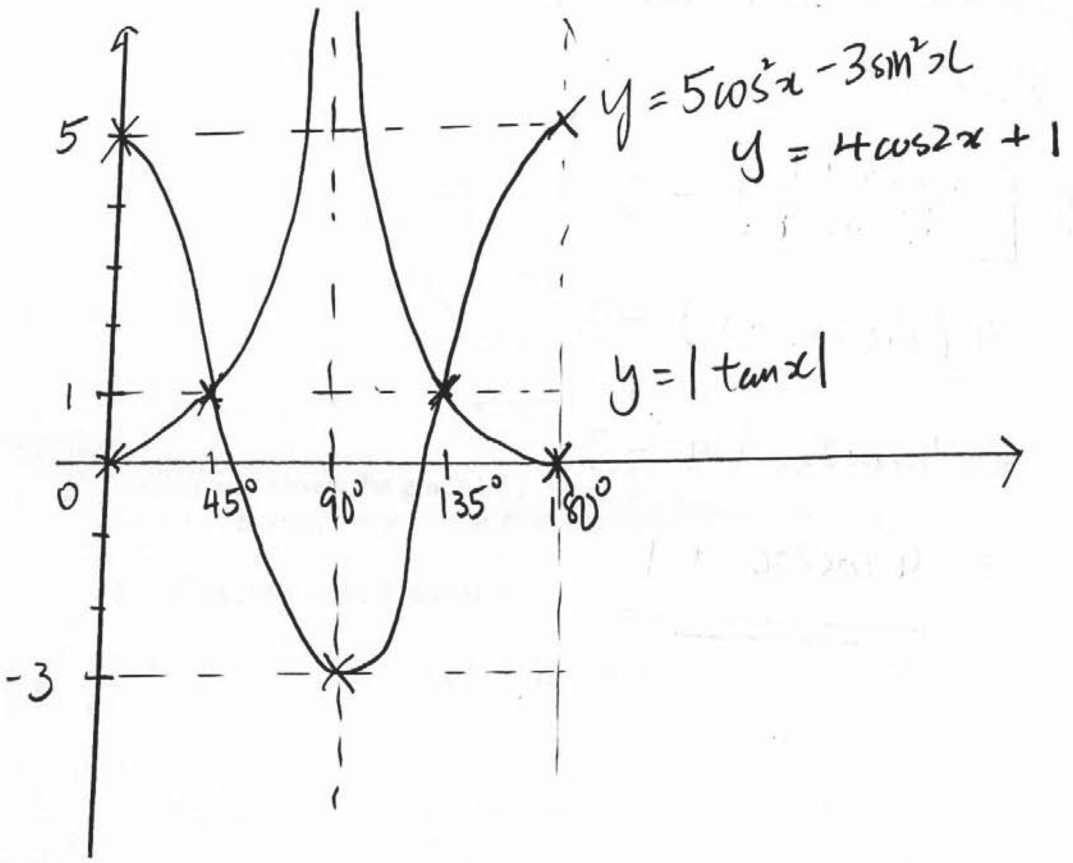
(ii) State the amplitude and period of $5\cos^2 x - 3\sin^2 x$. [2]

amp = 4

period = $180°$ or π radians.

[Turn over

(iii) Sketch on the same axes, the graphs of $y = 5\cos^2 x - 3\sin^2 x$ and $y = |\tan x|$ for $0° \leq x \leq 180°$. [3]



(iv) Hence state the coordinates of the solutions of the equation $5\cos^2 x - 3\sin^2 x = |\tan x|$, $0° \leq x \leq 180°$. [2]

(45°, 1) and (135°, 1)

9 A curve is such that $\frac{dy}{dx} = \frac{9}{(1-2x)^4}$, $x \neq \frac{1}{2}$ and $(1, 5)$ is a point on the curve.

(i) Find the equation of the curve. [4]

$$y = \int 9(1-2x)^{-4}\, dx$$

$$= \frac{9(1-2x)^{-3}}{-3(-2)} + C$$

sub $(1,5)$, $C = \frac{13}{2}$

curve is

$$y = \frac{3}{2(1-2x)^3} + \frac{13}{2}$$

(ii) Find the range of values of x such that the gradient of the curve is decreasing. [3]

$$\frac{d^2y}{dx^2} = \frac{72}{(1-2x)^5}$$

gradient is decreasing $\Rightarrow \frac{d^2y}{dx^2} < 0$

since $72 > 0$, $\Rightarrow (1-2x)^5 < 0$

$$1 - 2x < 0$$

$$x > \frac{1}{2}$$

10 The mass m grams of a radioactive substance present at time t days after first being observed is given by the formula $m = 38e^{kt}$.
25 days after first being observed, the mass has dropped to 10.3 g.

(i) Find the initial mass of the radioactive substance. [1]

$t = 0, \quad m = 38 \text{ g}$

(ii) Find the number of days it takes after first being observed for the mass to become half its original value. [5]

$38e^{k(25)} = 10.3$

$k = -0.0522217$

$38e^{-0.0522217t} = 19$

$t = 13.274$ days

$= \underline{13.3}$ days (3 sf)

(accept 14 days)

(iii) Find the rate at which the mass is decreasing when $t = 40$ days. [2]

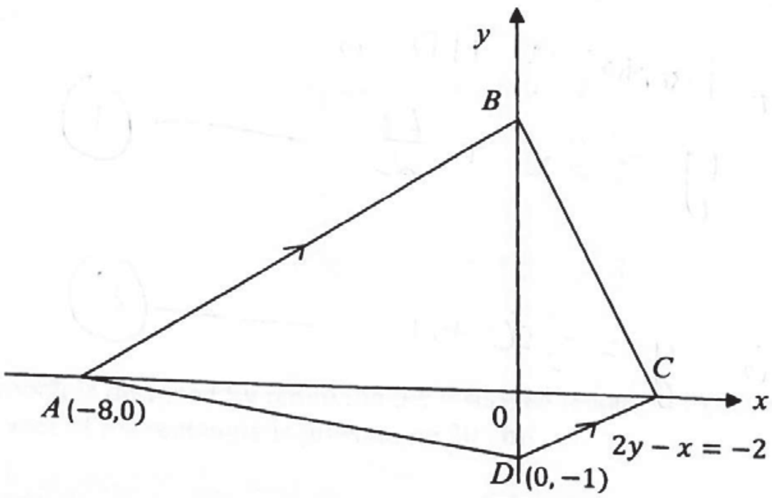
$$\frac{dm}{dt} = 38ke^{kt}$$

$$= 38(-0.052217)e^{-0.052217 \times 40}$$

$$\frac{dm}{dt} = -0.246 \quad (3sf)$$

mass is decreasing at a rate of 0.246 g/s.

11 Solutions to this question by accurate drawing will not be accepted.



The diagram shows a trapezium *ABCD* such that *AB* is parallel to *DC* and *AB* is perpendicular to *BC*. The point *A* is $(-8, 0)$ and *D* is $(0, -1)$. The equation of the line *CD* is $2y - x = -2$.

(i) Find the coordinates of *B*. [3]

Eqn of AB is $y = \frac{1}{2}x + 4$

B is $(0, 4)$

(ii) The perpendicular bisector of *AD* meets *AB* at point *E*.
Determine with explanation if it is possible to construct a circle with centre *E*, passing through the points *A* and *D*. [2]

Since E lies on the perpendicular bisector of AD, AE = DE

If E is the centre, then AE = DE
= Radius.

hence, Yes

(iii) Find the coordinates of *E*. [5]

Perpendicular bisector of AD is

$$y = 8x + \frac{63}{2} \quad \text{——— (1)}$$

line AB is $y = \frac{1}{2}x + 4$ ——— (2)

$\therefore$ E is $\left(-3\frac{2}{3}, 2\frac{1}{6}\right)$.

working:

grad of AD $= \frac{0-(-1)}{-8-0} = -\frac{1}{8}$

grad of ⊥ $= 8$

midpt of AD is $\left(-4, -\frac{1}{2}\right)$

Eqn of ⊥ bisector is

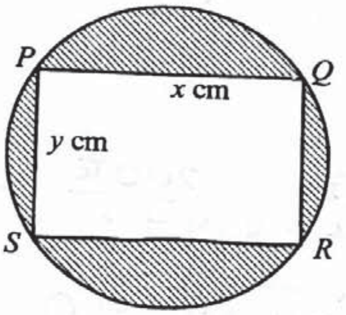
$y = 8x + c$

$-\frac{1}{2} = 8(-4) + c$

$c = \frac{63}{2}$

$\therefore \quad y = 8x + \frac{63}{2}$ //

12 The diagram shows a rectangle of length x cm and width y cm inscribed in a circle such that all the vertices of the rectangle $PQRS$ touch the circumference of the circle.



A brooch is designed by removing the inscribed rectangle $PQRS$ from the circle. The area of the rectangle is given to be 20 cm^2.

(i) Show that the radius of the circle is $\frac{\sqrt{x^4+400}}{2x}$. [3]

$$xy = 20 \Rightarrow y = \frac{20}{x}$$

$$SQ^2 = x^2 + \left(\frac{20}{x}\right)^2$$

$$= \frac{x^4+400}{x^2}$$

$$SQ = \sqrt{\frac{x^4+400}{x^2}} = \frac{\sqrt{x^4+400}}{x}$$

$$\therefore \text{radius} = \frac{\sqrt{x^4+400}}{2x} \text{ //}$$

(ii) Ignoring the thickness of the brooch, show that the remaining area is given by $A = \frac{\pi x^2}{4} + \frac{100\pi}{x^2} - 20$. [2]

$$A = \pi r^2 - 20$$

$$= \pi\left(\frac{\sqrt{x^4+400}}{2x}\right)^2 - 20$$

$$= \pi\left(\frac{x^4+400}{4x^2}\right) - 20$$

$$= \frac{\pi x^2}{4} + \frac{100\pi}{x^2} - 20$$

(iii) Given that x can vary, find the value of x for which A has a stationary value. [3]

$$\frac{dA}{dx} = \frac{\pi x}{2} - \frac{200\pi}{x^3}$$

$$\frac{dA}{dx} = 0, \quad \frac{\pi x}{2} - \frac{200\pi}{x^3} = 0$$

$$x^4 - 400 = 0$$

$$x^4 = 400$$

$$x = 400^{\frac{1}{4}}$$

$$x = 4.47$$

(iv) Determine the nature of the stationary value of A. [2]

$$\frac{d^2A}{dx^2} = \frac{\pi}{2} + \frac{600\pi}{x^4}$$

$$= 6.283 > 0$$

$\therefore$ Stat A is a minimum.