ADDITIONAL MATHEMATICS 4047/1

PAPER 1
SECONDARY 4
2020 PRELIMINARY EXAMINATION

14 September 2020 (Monday) **2 hours**

CANDIDATE NAME	

CLASS		INDEX NUMBER		

READ THESE INSTRUCTIONS FIRST
Do not turn over the page until you are told to do so.
Write your name, class and index number in the spaces above.
Write in dark blue or black pen in the space provided for each question.

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INFORMATION FOR CANDIDATES
Answer **all** the questions in the space provided.

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Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

For Examiner's Use		
Q1	4	
Q2	4	
Q3	5	
Q4	4	
Q5	7	
Q6	7	
Q7	7	
Q8	5	
Q9	10	
Q10	8	
Q11	8	
Q12	11	
Total		/80

This document consists of **20** printed pages including the Cover Sheet.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n *is a positive integer and* $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for Δ *ABC*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} ab \sin C$$

Answer **all** questions.

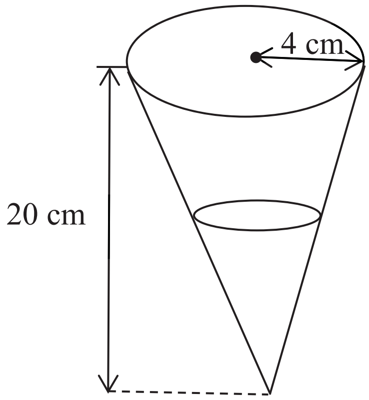
1 Given that $\tan\alpha = p$, where $\pi < \alpha < \frac{3\pi}{2}$, find, in terms of p,

(i) $\tan(-\alpha)$ [1]

(ii) $\tan\left(\frac{\pi}{2} - \alpha\right)$ [1]

(iii) $\sec\alpha$ [2]

2



The diagram above shows an inverted circular cone of radius 4 cm and height of 20 cm. Liquid leaks out through a small hole in the vertex at a constant rate of 5 cm^3/s. Find the rate at which the height of the liquid in the cone is decreasing when the height of liquid is 12 cm. [4]

3 The number of tulips, y, that have bloomed in a huge flower bed can be modelled by the equation $y = (2 + 5t)(900 - t^2)$, where t is the number of days that have passed from the day that the seeds of the tulips have been planted.

(i) Using this model, find the number of seeds that were initially planted. [1]

(ii) Find the rate of increase in the number of tulips that have bloomed when $t = 10$. [2]

(iii) Adam commented that the model will not be valid after 30 days. Is he correct? Explain your answer. [2]

4 Given that $y = (1 + 3x)e^{3x}$, find the value of the constant k such that

$$\frac{d^2y}{dx^2} - 3\left(\frac{dy}{dx}\right) - 9y = kxe^{3x}\ .$$ [4]

5 The equation of a curve is $y = x^3 + kx^2 - 3x + 1$, where k is a constant.

(a) Find, in terms of k, the gradient of tangent to the curve at the point $x = 1$. [2]

(b) Explain why the curve has two stationary points, for any real values of k. [3]

(c) If $k = -4$, find the range of values of x for which y is a decreasing function. [2]

6 The number of cells, $N(t)$, in an experiment can be modelled by a function $N(t) = 50000(1.6^{0.5t})$ where t is the number of hours that have passed from the start of the experiment.

(i) Find the number of cells present at the start of the experiment. [1]

(ii) Find the time taken for the number of cells to be ten times the initial number of cells, giving your answer to the nearest hour. [3]

(iii) If the number of cells at time $t = t_2$ is double the number of cells at the time $t = t_1$, prove that the difference between the two timings ($t_2 - t_1$) is approximately 2.95 when converted to 3 significant figures. [3]

7 **(i)** Write down and simplify the first 3 terms in the expansion of $(1 - 3x)^5$ in ascending powers of x. [2]

(ii) Given that the coefficient of x^2 in the expansion of $(1 - 3x)^5(1 + 5x)^n$ is 90, find the value of n, where n is a positive integer. [5]

8 The line L_1 has equation $y = ax + 5$. Line L_1 intersects the curve $y = x^2 + 2x + 4$ at M, the turning point of the curve.

(i) Show that $a = 2$. [2]

(ii) Given that L_1 meets the curve $y = x^2 + 2x + 4$ at another point N, find the equation of a line parallel to L_1 and passing through midpoint of M and N. [3]

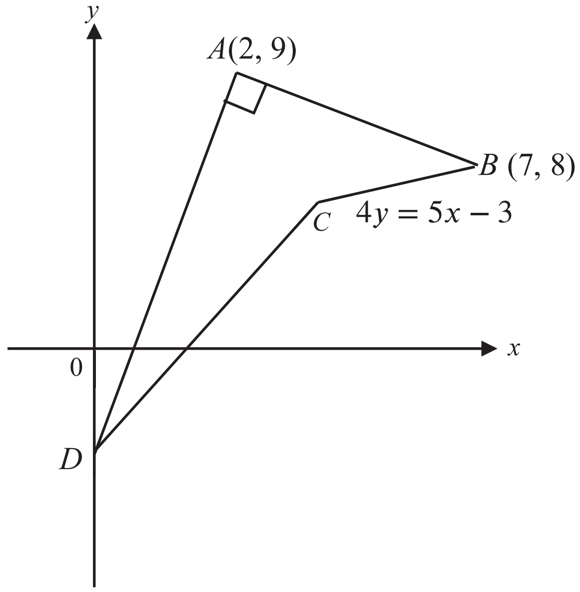
9 Two particles, A and B, leave the origin O at the same time and travel along the positive x-axis. Particle A starts with a velocity of 10 m/s and moves with a constant acceleration of $\frac{2}{3}$ m/s^2. Particle B starts from rest and its acceleration at time, t s, is given by $\left(1+\frac{t}{3}\right)$ m/s^2.

(i) Express the velocity, v_A m/s, and the displacement, s_A m, of particle A in terms of t. [3]

(ii) Express the velocity, v_B m/s, and the displacement, s_B m, of particle B in terms of t. [3]

(iii) Hence, find the distance from O when A and B meet after leaving O. [4]

10 Solutions to this question by accurate drawing will not be accepted.



The diagram above shows a quadrilateral $ABCD$ where the point C lies on the perpendicular bisector of AB and the point D lies on y-axis. The equation of the line BC is $4y = 5x - 3$. Given the coordinates of A and B are (2, 9) and (7, 8) respectively, find

(i) the equation of AD, [2]

(ii) the equation of perpendicular bisector of AB, [2]

(iii) the coordinates of C, [2]

[Turn over

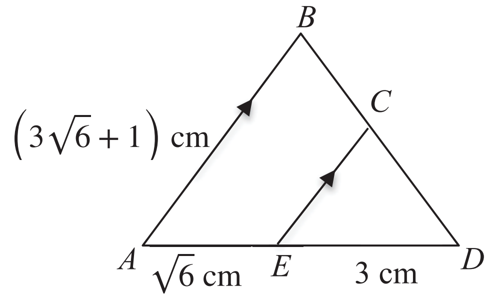
(iv) the area of quadrilateral $ABCD$. [2]

11 Solve the following equations.

(a) $3^{x+2} = \frac{10}{3} - 3^{2x+1}$, [2]

(b) $e^{3x} + 2e^{x} = 3e^{2x}$. [3]

(c)



The diagram above shows a pair of similar triangles *ABD* and *ECD* where *AB* is parallel to *EC*. Given that $AB = \left(3\sqrt{6}+1\right)$ cm, $AE = \sqrt{6}$ cm and $ED = 3$ cm, express the length of *EC* in the form $\left(a\sqrt{6}+b\right)$ cm where a and b are rational numbers. [3]

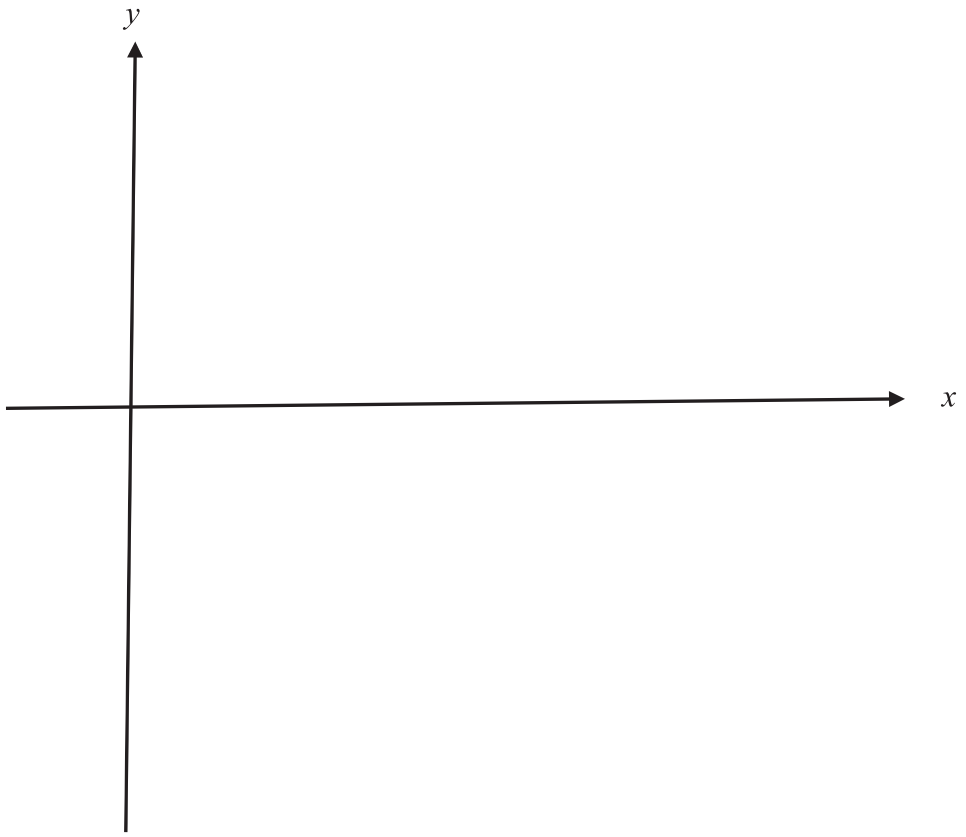
12 It is given that $y_2 = \frac{\cos 2x}{2}$ and $y_1 = \sin x - 1$.

(i) State the amplitude and period, in degrees of y_1, [2]

For the interval of $0° \leq x \leq 360°$,

(ii) solve the equation $y_1 = y_2$, [3]

(iii) sketch, on the same diagram, the graphs of y_1 and y_2 , [4]



(i) find the set of values of x for which $y_1 - y_2 < 0$. [2]

End of Paper

SCHOOL OF SCIENCE AND TECHNOLOGY, SINGAPORE

SECONDARY 4
PRELIMINARY EXAMINATION
ADDITIONAL MATHEMATICS
Paper 2 4047/2

16 September 2020 (Wednesday) **2 hours 30 minutes**

CANDIDATE NAME	

CLASS		INDEX NUMBER		

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The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your answer scripts securely together.

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For Examiner's Use		
Q1	5	
Q2	5	
Q3	6	
Q4	5	
Q5	8	
Q6	8	
Q7	9	
Q8	10	
Q9	10	
Q10	10	
Q11	11	
Q12	13	
Total		/100

This document consists of **26** printed pages including the cover page.

The total number of marks for this paper is **100.**

Mathematical Formulae

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$$\left(a+b\right)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n *is a positive integer and* $\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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$$\sin^2 A + \cos^2 A = 1$$

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Formulae for $\Delta\ ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

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1 **(i)** Differentiate $x^2 \ln 3x$ with respect to x. [2]

(ii) Hence find $\int x \ln 3x \, dx$. [3]

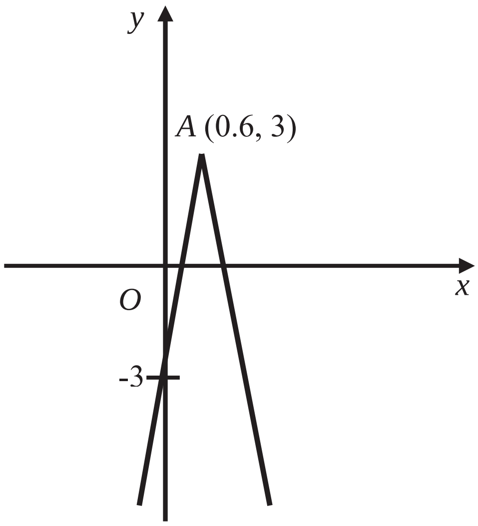
2 Express $\dfrac{x^3+1}{\left(x^2+1\right)\left(x-2\right)}$ in partial fractions. [5]

3 The quadratic equation $3x^2 + 2x + 1 = 0$ has roots α and β.

(i) Show that the value of $\alpha^3 + \beta^3$ is $\frac{10}{27}$. [3]

(ii) Find a quadratic equation whose roots are $\frac{1}{\alpha^3}$ and $\frac{1}{\beta^3}$. [3]

4



The diagram shows part of the graph $y = r - q|3 - px|$, where p, q and r are positive constants. The graph has a vertex at A (0.6, 3) and y-intercept of -3.

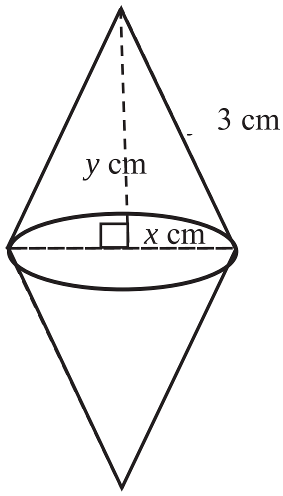
(i) Determine the values of p, q and r. [3]

(ii) State the value or range of values of k such that $k = r - q|3 - px|$ has

(a) 1 solution, [1]

(b) 2 solutions. [1]

5 A buoy is formed by two identical right circular cones of sheet iron joined by its bases with a radius of x cm. The buoy has a vertical height of y cm and a slant height of 3 cm.



(i) Express y in terms of x. [1]

(ii) Given that x can vary, find the exact value of x for which the volume, V, of the buoy is stationary. [4]

(iii) Determine with reasons whether this value of V is a maximum or minimum. [2]

(iv) Find the exact surface area of the buoy when V is stationary, leaving your answer in terms of π . [1]

6 The equation of a polynomial is given by $\mathrm{p}(x) = 2x^3 + 2ax^2 - x^2 + ax - a$ where a is a constant.

(i) Find the remainder when $\mathrm{p}(x)$ is divided by $(x + 1)$. [1]

(ii) Show that $(2x - 1)$ is a factor of $\mathrm{p}(x)$. [2]

(iii) Showing your working clearly, factorise $p(x)$ completely, leaving your answer in terms of a. [2]

(iv) Find the range of values of a for which the equation $p(x) = 0$ has only one real root. [3]

7 The table below shows the data obtained from an experiment on the vertical motion based on the oscillation of a spring with different masses attached to it.

Mass, x kg	0.02	0.03	0.04	0.05	0.15
Frequency of oscillations, y	16	13	11.4	10	6

It is known that the mass, x kg, and the frequency of oscillations per second, y, are related by the equation $xy^2 = k$, where k is a constant.

(a) Plot y^2 against $\frac{1}{x}$ and draw a straight line graph. [3]

(b) Use your graph to estimate

(i) the frequency of oscillations when a mass of 0.08 kg is attached to the spring, [1]

(ii) the mass which produces 15 oscillations per second, [1]

(iii) the value of k. [1]

(c) When the spring is replaced by a second spring, the relation between y and x is represented by $y^2 = \frac{2}{x} + 80$.

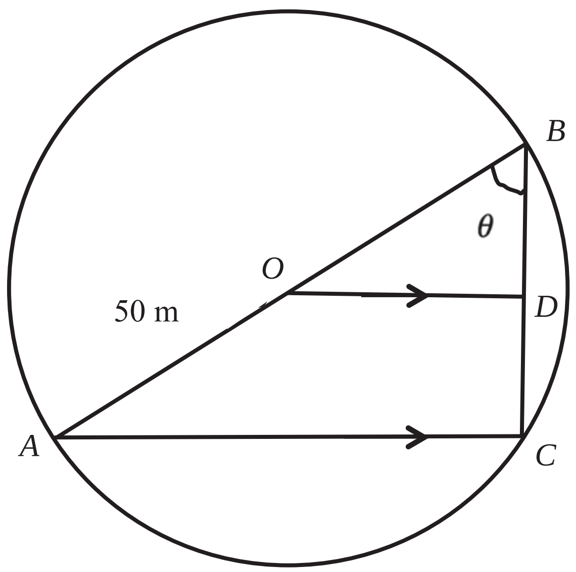
(i) On the same diagram, draw the line representing the second spring. [1]

(ii) Hence, explain how to find the mass which produces the same frequency of oscillations by both springs. [2]

8 **(i)** Prove that $\dfrac{\sec A + \tan A - 1}{1 - \sec A + \tan A} \equiv \dfrac{1 + \sin A}{\cos A}$. [5]

(ii) Hence solve the equation $\dfrac{\sec A + \tan A - 1}{1 - \sec A + \tan A} = 3\cos A$ for $0 < A < 2\pi$. [5]

9



The diagram shows a circular field with centre O and radius 50 m.
A, B and C are points on the circumference of the field and angle $ABC = \theta$. D is a point on BC such that OD is parallel to AC.
The trapezium $AODC$ is the jogging path of a man.

(i) Show that $BD = DC = 50\cos\theta$. [2]

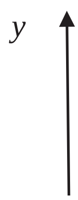
(ii) Show that the perimeter, L m, of the jogging path $AODC$ can be expressed in the form $p+q\cos\theta+r\sin\theta$, where p, q and r are constants to be found. [3]

(iii) Express L in the form of $p+R\cos(\theta-\alpha)$ where $R>0$

and $0^\circ < \alpha < 90^\circ$. [3]

(iv) Hence state the maximum perimeter of the jogging path *AODC*.
Find the value of θ at which this occurs. [2]

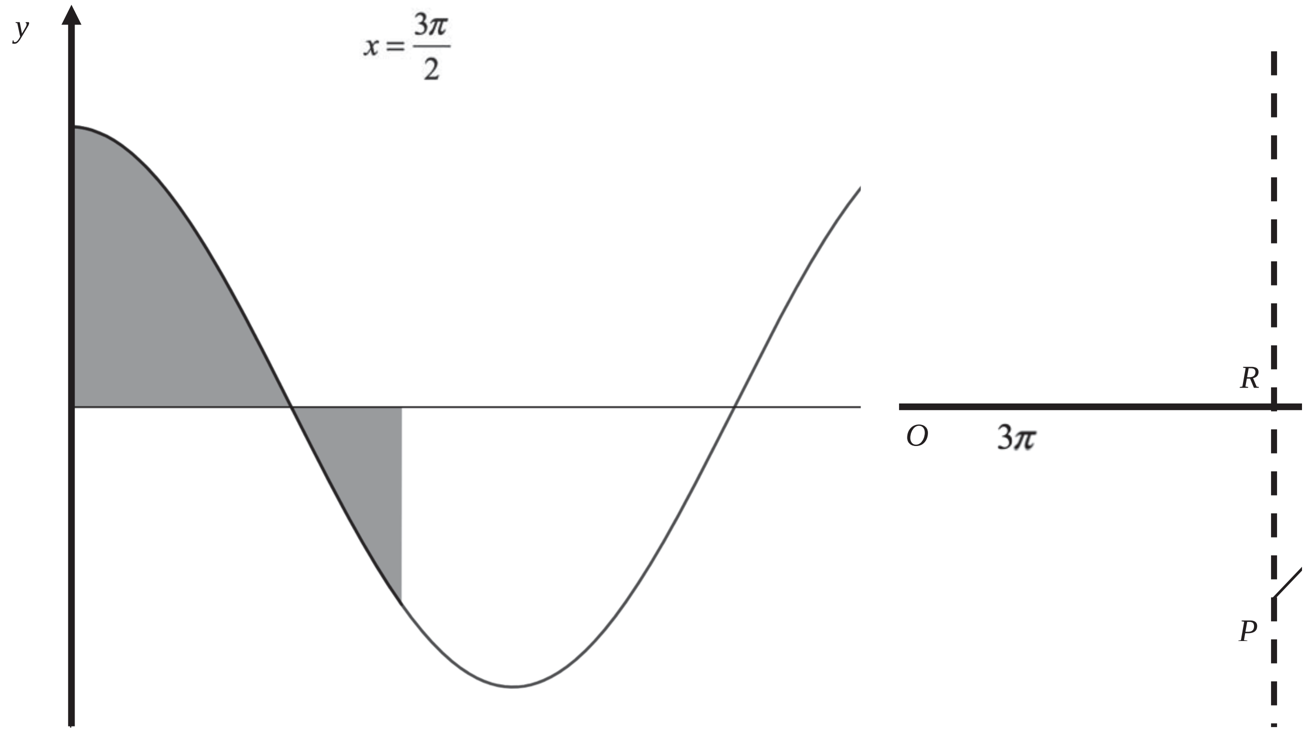
10 **(a)** Sketch the graph of $y = \log_3 x$. [2]



(b) Express $\log_9 4 + \log_3(x-4) = 2\log_3 x$ as a quadratic equation in x and explain why there are no real solutions. [4]

(c) Given that $\log_b(x^2y) = m$ and $\log_b(x^3y) = n$, express, $\log_b x$ and $\log_b y$ in terms of m and n. [4]

11



The diagram shows part of the curve $y = 4\cos\left(\frac{x}{2}\right)$ that meets the x-axis at $x = \pi$ and $x = 3\pi$. The line $x = \frac{3\pi}{2}$ meets the x-axis at R and the curve at P. The normal to the curve at P meets the x-axis at Q.

(i) Find the equation of the normal at P, expressing your answer in exact form. [4]

[Turn Over

(ii) Find the exact coordinates of Q. [2]

(iii) Find the exact area of the shaded region. [5]

12 A circle, C_1 has equation $x^2 + y^2 + 8x - 12y + 16 = 0$.

(i) Find the radius and the coordinates of the centre of C_1. [3]

(ii) The lowest point on the circle is A. Explain why A lies on the x-axis. [1]

A second circle, C_2 , has a diameter PQ. The point P has coordinates (-1, 3) and the equation of the tangent to C_2 at Q is $2y = x - 18$.

(iii) Find the equation of the diameter PQ and hence the coordinates of Q. [4]

(iv) Find the equation of the circle, C_2. [3]

[Turn Over

(v) Determine whether the circles C_1 and C_2 intersect each other. [2]

END OF PAPER

SECONDARY 4
PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS
Paper 1 4047/01

14 SEP 2020 (Monday) 2 hours

CANDIDATE NAME: **Marking Scheme**

CLASS

INDEX NUMBER

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$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

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$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Given that $\tan\alpha = p$, where $\pi < \alpha < \frac{3\pi}{2}$, find, in terms of p ,

(i) $\tan(-\alpha)$ [1]

$$\tan(-\alpha) = -\tan\alpha$$
$$= -p$$

(ii) $\tan\left(\frac{\pi}{2} - \alpha\right)$ [1]

$$\tan\left(\frac{\pi}{2} - \alpha\right) = \frac{1}{\tan\alpha}$$

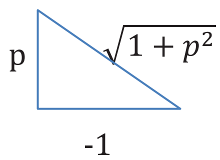
$$= \frac{1}{p}$$

(iii) $\sec\alpha$ [2]

$$\sec\alpha = \frac{1}{\cos\alpha}$$

$$= \frac{1}{\frac{-1}{\sqrt{1+p^2}}} \quad \text{or} \quad = 1 \div \frac{(-1)}{\sqrt{1+p^2}}$$

$$= -\sqrt{1+p^2}$$



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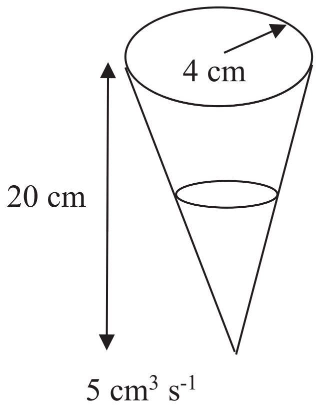


The diagram above shows a inverted circular cone of radius 4 cm and height of 20 cm. Liquid leaks out through a small hole in the vertex the constant rate of 5 cm^3/s At what rate is the height of the liquid in the cone decreasing when the height of the liquid is 12 cm? [4]

$\frac{dV}{dt} = -5\ cm^3/s$

$V = \frac{1}{3}\pi r^2 h$

$\frac{r}{4} = \frac{h}{20}$

$=> r = \frac{1}{5}h$

$V = \frac{1}{3}\pi\left(\frac{1}{5}h\right)^2 h$

$=> V = \frac{\pi}{75}h^3$

$\frac{dV}{dh} = \frac{\pi}{25}h^2$ differentiate $V = \frac{\pi}{75}h^3$]

$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$

$-5 = \frac{\pi}{25}(12)^2\left(\frac{dh}{dt}\right)$ substitute the correct values into the connected rate of change formula]

$\frac{dh}{dt} = \frac{-5\times 25}{144\pi} = \frac{-125}{144\pi}$ or $-0.276\ cm^3/s\ (3\ s.f)$

The rate at which the depth of the liquid is decreasing is 0.276 cm^3/s.

OR $\frac{125}{144\pi}$ cm^3/s.

3 The number of tulips, y, that bloomed in a huge flower bed can be modelled by the equation $y = (2 + 5t)(900 - t^2)$, where t is the number of days that have passed from the day that the seeds of the tulips have been planted.

(i) Using this model, find the number of seeds that were initially planted. [1]

When $t = 0$,
$y = (2 + 0)(900)$
$= 1800$ tulips

(ii) Find the rate of increase in the number of tulips that have bloomed when $t = 10$. [2]

$\frac{dy}{dt} = (2 + 5t)(-2t) + (900 - t^2)5$ correct differentiation apply product rule

$= -4t - 10t^2 + 4500 - 5t^2$
$= -15t^2 - 4t + 4500$

When $t = 10$,
$\frac{dy}{dt} = -15(100) - 40 + 4500$
$= 2960\ tulips/day$

OR
Alternatively. by direct differentiation after
y= $(2 + 5t)(900 - t^2)$
$= 1800 - 2t^2 + 4500t - 5t^3$
$\frac{dy}{dt} = -15t^2 - 4t + 4500$
[M1 for differentiation of $1800 - 2t^2 + 4500t - 5t^3$]
When $t = 10$,
$\frac{dy}{dt} = 15(100) - 40 + 4500$
$= 2960\ tulips/day$

(iii) Adam commented that the model will not be a valid after 30 days. Is he correct? Explain your answer.

$y = (2 + 5t)(900 - t^2)$ [2]
If t > 30 , $(900 - t^2) < 0$ which implies that y which represents the number of tulips will be negative. NEED to explain that number of tulips must be positive or cannot be negative]
So, the model will not be valid so Adam is correct.

4 Given that $y = (1 + 3x)e^{3x}$, find the value of k such that

$$\frac{d^2y}{dx^2} - 3\left(\frac{dy}{dx}\right) - 9y = kxe^{3x} .$$ [4]

$y = (1 + 3x)e^{3x} = e^{3x} + 3xe^{3x}$

$\frac{dy}{dx} = (1 + 3x)(3e^{3x}) + (e^{3x})(3)$ do correct differentiation of y

$= 3e^{3x}(1 + 3x + 1) = 3e^{3x}(2 + 3x)$ *or* $6e^{3x} + 9xe^{3x}$

$\frac{d^2y}{dx^2} = 3e^{3x}(3) + (2 + 3x)\ (9e^{3x})$ obtain the 2nd derivative in any form

$= 9e^{3x} + (2 + 3x)\ (9e^{3x})$

$= 9e^{3x}(1 + 2 + 3x)$

$= 9e^{3x}(3 + 3x)$ *or* $27e^{3x} + 27xe^{3x}$

$\frac{d^2y}{dx^2} - 3\left(\frac{dy}{dx}\right) - 9y$

$= 27e^{3x} + 27xe^{3x} - 18e^{3x} - 27xe^{3x} - 9(e^{3x} + 3xe^{3x})$

correct substitution into LHS of equation]

$= -27xe^{3x}$

$=> k = -27$

5 The equation of a curve is $y = x^3 + kx^2 - 3x + 1$, where k is a constant.

(a) Find, in terms of k, the gradient of the tangent to the curve y at the point $x = 1$. [2]

$\frac{dy}{dx} = 3x^2 + 2kx - 3.$

When $x = 1$, $\frac{dy}{dx} = 3x + 2k - 3$

$= 2k$.

(b) Explain why the curve has two stationary points, for any real values of k. [3]

$\frac{dy}{dx} = 3x^2 + 2kx - 3 = 0$ [M1 for understanding that $\frac{dy}{dx} = 0$ gives the stationary points]

Discriminant = $(2k)^2 - 4(3)(-3)$. DO NOT PUT '$>$ ' here
$= 4k^2 + 36 > 0$ since $4k^2 > 0$ Discriminant > 0]

Therefore, the curve y has two stationary points. {Remember to conclude]

ALTERNATIVE SOLUTIONS

$\frac{dy}{dx} = 3x^2 + 2kx - 3$............Eq 1

At $\frac{dy}{dx} = 0,$

$x = \frac{-2k \pm \sqrt{4k^2 - 4(3)(-3)}}{2(3)}$ [showing directly there are 2 roots by Eq 1 with any method]

$x = \frac{-k + \sqrt{(k^2+9)}}{3}$ OR $\frac{-k - \sqrt{(k^2+9)}}{3}$

Therefore, there are 2 solutions.

(c) If $k = -4$, find the range of values of x for which y is a decreasing function.

[2]

$3x^2 - 8x - 3 < 0.$ $\qquad [\frac{dy}{dx} < 0\ ]$

$(3x + 1)(x - 3) < 0.$

$-\frac{1}{3} < x < 3.$

6 The number of cells in an experiment can be modelled by a function $N(t) = 50000(1.6^{0.5t})$ where t is the number of hours that have passed from the start of the experiment.

(i) Find the number of cells present at the start of the experiment. [1]

When $t = 0$, number of cells $N(0) = 50\,000\,(1.6^0)$
$= 50\,000$

(ii) Find the time taken for the number of cells to be ten times the initial number of cells giving your answer to the nearest hour. [3]

$500\,000 = 50\,000\,(1.6^{0.5t})$
$10 = 1.6^{0.5t}$
$\lg 10 = 0.5t \lg 1.6$
$0.5t = \frac{lg10}{lg1.6}$ [
$$t = \frac{1}{lg1.6} \times \frac{1}{0.5}$$
$= 9.798\ h$
$t = 10$ hr (to the nearest hr)

(iii) If the number of cells at time $t = t_2$ is double the number of cells at the time $t = t_1$, prove that the difference between the two timings $(t_2 - t_1)$ is approximately 2.95 when converted to 3 significant figures. [3]

$N(t) = 50\,000\,(1.6^{0.5t})$

{DO NOT LET t_1 be any Specific Values, Need to prove in General}

$N(t_2) = 2N(t_1)$
$50\,000\,(1.6^{0.5t_2}) = 2(50000)1.6^{0.5t_1}$
$(1.6^{0.5t_2}) = 2(1.6^{0.5t_1})$

$$\left.\begin{aligned} \frac{1.6^{0.5t_2}}{1.6^{0.5t_1}} &= 2 \\ lg1.6^{0.5(t_2-t_1)} &= lg2 \\ 0.5(t_2 - t_1) &= \frac{lg2}{lg1.6} \end{aligned}\right\} \text{manipulation of exp equation to log equation}$$

$(t_2 - t_1) = \frac{1}{0.5}\left(\frac{lg2}{lg1.6}\right)$
$= \frac{2lg2}{lg1.6}$ = 2.95 (correct to 3 significant figures)

7 **(i)** Write down, and simplify, the first 3 terms in the expansion of $(1-3x)^5$ in ascending powers of x. [2]

$(1-3x)^5$
$= 1 + 5(-3x) + \binom{5}{2}(-3x)^2 + \ldots\ldots$
[M1 for applying Binomial Theorem for Expansion]

$= 1 - 15x + 90x^2 + \ldots\ldots$

,

(ii) Given that the coefficient of x^2 in the expansion of $(1-3x)^5(1+5x)^n$ is 90, find the value of n where n is a positive integer. [5]

$(1+5x)^n = 1 + \binom{n}{1}5x + \binom{n}{2}(5x)^2 + \ldots\ldots$
$= 1 + 5nx + \frac{25n(n-1)}{2}x^2 + \ldots\ldots$

$(1-3x)^5(1+5x)^n = (1-15x+90x^2)\left[1 + 5nx + \frac{25n(n-1)}{2}x^2\right]$
[simplify $(1+5x)^n$

Comparing coefficient of x^2,

$\frac{25n(n-1)}{2} - 75n + 90 = 90$ [compare the coefficient of x^2]
$\Rightarrow \frac{25(n^2-n)}{2} - 75n = 0$
$\Rightarrow 25(n^2-n) - 150n = 0$ DO NOT USE TRIAL AND ERROR SHOW WORKING
$25n^2 - 175n = 0$

$\Rightarrow 25n(n-7) = 0$
$n = 7$; $n = 0$ $(reject)$

8 : line L_l has equation $y = ax + 5$. Line L_l intersects the curve $y = x^2 + 2x + 4$ at urning point M.

Show that $a = 2$. [2]

: $x^2 + 2x + 4$

— $= 2x + 2$

$+ 2 = 0$

$2x = -2$

$= -1.$ [M1 for finding $x = 1$].

$+ 5 = x^2 + 2x + 4$ ....eq 1

$x = -1$ into eq1

$+ 5 = 1 - 2 + 4$

$= 2$ (Shown)

ernative solutions: Or x are form

: $x^2 + 2x + 4.$ OR

— $= 2x + 2$

$+ 2 = 0$

$2x = -2$

$= -1.$ [M1].

: $(-1)^2 + 2(-1) + 4$

$- 2 + 4$

$-1, 3)$

$x = -1, y = 3$ into $y = ax + 5$

: $a(-1) + 5$

: $5 - 3$

Given that L_l meets the curve $y = x^2 + 2x + 4$ at another point N. Find the equation of a line parallel to L_l and passing through the midpoint of M and N. [3]

: $x^2 + 2x + 4$

$+ 2x + 4 = 2x + 5.$

$= 1$

: ± 1

en $x = 1, y = 7$

$1, 7)$ $M(-1, 3)$

$idpt = \left(\frac{1+(-1)}{2}, \frac{7+3}{2}\right) = (0, 5)$

iation of parallel to L_l and passing through $(0, 5)$ is

$$y - 5 = 2(x - 0)$$

$$y = 2x + 5.$$

9 Two particles, A and B leave the origin O at the same time and travel along the positive x-axis. Particle A starts with a velocity of 10 m/s and moves at a constant acceleration of $\frac{2}{3}$ m/s^2. Particle B starts from rest and its acceleration at time, t s, is $\left(1+\frac{t}{3}\right)$ m/s^2.

(i) Express the velocity, v_A m/s and the displacement, s_A m of particle A in terms of t. [3]

A

$v_A = 10$

$a = \frac{2}{3}$

B

$v_B = 0$

$a = 1 + \frac{1}{3}t$

$v_A = \frac{2}{3}t + c_1$

$10 = 0 + c_1$

$\therefore v_A = \frac{2}{3}t + 10$

$s_A = \frac{2}{3}\left(\frac{t^2}{2}\right) + 10t + c_2$

$s_A = \frac{1}{3}t^2 + 10t + c_2$

when $t = 0$, $s = 0 \Rightarrow c_2 = 0$

$s_A = \frac{1}{3}t^2 + 10t$

(ii) Express the velocity, v_B m/s and the displacement, s_B m of particle B in terms of t. [3]

$v_B = t + \frac{1}{3}\left(\frac{t^2}{2}\right) + c_3$

$v_B = t + \frac{1}{6}t^2 + c_3$

$c_3 = 0$

$v_B = t + \frac{1}{6}t^2$

when $t = 0$, $s = 0 \Rightarrow c_3 = 0$

$s_B = \frac{t^2}{2} + \frac{1}{6}\left(\frac{t^3}{3}\right) + c_4$

when $t = 0$, $s = 0 \Rightarrow c_4 = 0$

$s_B = \frac{t^2}{2} + \frac{1}{18}t^3$

(iii) Hence, find the distance from O when A and B meet after leaving O. [4]

$\frac{1}{3}t^2 + 10t = \frac{t^2}{2} + \frac{1}{18}t^3$ [M1 for equating $s_A = s_B$]

$$\frac{1}{18}t^3 + \frac{t^2}{2} - \frac{1}{3}t^2 - 10t = 0$$

$t^3 + 9t^2 - 6t^2 - 180t = 0$

$t^3 + 3t^2 - 180t = 0$ [M1 for solving the cubic equation]

$t(t^2 + 3t - 180) = 0$

$t(t - 12)(t + 15) = 0$

$t = 0, 12\ or - 15\ (reject)$ [A1]

when $t = 12$s

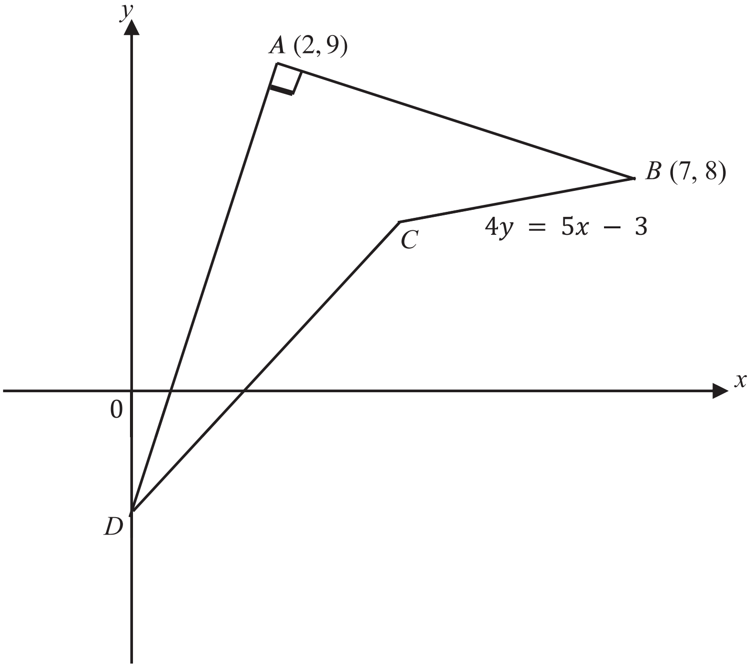
$s_A = \frac{1}{3}(12)^2 + 10(12)$
$= 168$ m

or

$s_B = \frac{(12)^2}{2} + \frac{1}{18}(12)^3$
$= 168$ m

[A1 for substituting into either s_A or s_B]

10 Solutions to this question by accurate drawing will not be accepted.



The diagram above shows a quadrilateral *ABCD* where the point *C* lie on the perpendicular bisector of *AB* and the point *D* lies on *y*-axis. The equation of the line *BC* is $4y = 5x - 3$. Given the coordinates of *A* and *B* are (2, 9) and (7,8) respectively. Find [2]

(i) the equation of *AD*,

Gradient $AB = \frac{8-9}{7-2} = -\frac{1}{5}$

Gradient $AD = 5$ [M1 finding the gradient of *AD* and deducing the gradient *of AD*]

$$Equation\ of AD\ is\ y - 9 = 5(x-2)$$
$$y - 9 = 5x - 10$$
$$y = 5x - 1 \text{ [A1]}$$

(ii) the equation of perpendicular bisector of *AB*, [2]

Gradient of perpendicular = 5

Midpoint of $AB = \left(\frac{2+7}{2}, \frac{9+8}{2}\right) = \left(\frac{9}{2}, \frac{17}{2}\right)$ [M1 for finding the coordinate of the midpoint]

Equation of perpendicular bisector is

$$y - \frac{17}{2} = 5\left(x - \frac{9}{2}\right)$$
$$y - \frac{17}{2} = 5x - \frac{45}{2}$$
$$y = 5x - 14. \quad \text{[A1]}$$

(iii) the coordinates of *C* , [2]

$y = 5x - 14$ ----------(1)

$4y = 5x - 3$----------(2)

(2)−(1), $4y - y = (5x - 3) - (5x - 14)$ [M1 for solving Simultaneously Equations]

$$3y = -3 + 14$$
$$\therefore \ y = \frac{11}{3} \text{ or } 3\frac{2}{3}$$
$$\therefore \ x = \frac{53}{15} \text{ or } 17\frac{2}{3}$$

$\therefore \ C\left(17\frac{2}{3}, 3\frac{2}{3}\right).$ [A1]

(v) the area of the quadrilateral *ABCD*. [2]

$$\textit{Area of quadrilateral } ABCD = \frac{1}{2}\begin{vmatrix} 0 & 17\frac{2}{3} & 7 & 2 & 0 \\ -1 & 3\frac{2}{3} & 8 & 9 & -1 \end{vmatrix}$$

$= 89.2$ units^2 or $\frac{266}{3}$ units^2 or $88\frac{2}{3}$ units^2 [A1]

[M1 for applying 'shoe-lace' method. Coordinates are listed in the anti-clockwise direction.]

11 Solve the following equations.

(a) $3^{x+2} = \frac{10}{3} - 3^{2x+1}$. [2]

$(3^x)3^2 = \frac{10}{3} - (3^{2x})(3^1)$ [M1 for application of indices law seen]

$let\ u = 3^x$

$9u = \frac{10}{3} - 3u^2$

$27u = 10 - 9u^2$

$9u^2 + 27u - 10 = 0$

$(3u - 1)(3u + 10) = 0$

$u = \dfrac{1}{3} \quad or \quad u = -\dfrac{10}{3}\ (reject)$

$3^x = \frac{1}{3} \Rightarrow x = -1$ [A1]

(b) $e^{3x} + 2\,e^x = 3e^{2x}$ [3]

$e^{3x} - 3e^{2x} + 2\,e^x = 0$

$e^x(\,e^{2x} - 3e^x + 2) = 0$

[M1 for factorisation or forming quadratic equation in e^x]

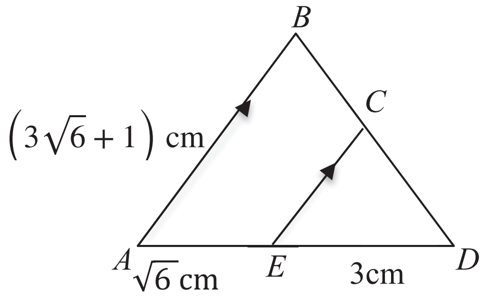
$e^x(e^x - 1)(e^x - 2) = 0.$

$e^x = 0\ or\ e^x = 1\ \ or\ e^x = 2$ [M1 for finding $e^x = 1$ and $e^x = 2$ and rejecting $e^x = 0$.]

(reject)

therefore, $x = 0$ or $x = \ln 2$ or 0.693. [A1]

(c)



The diagram above shows a pair of similar triangles *ABD* and *ECD* where *AB* is parallel to *EC*. Given that $AB = \left(3\sqrt{6}+1\right)$ cm , $AE = \sqrt{6}$ cm and ED =3 cm, express the length of *EC* in the form ($a\sqrt{6}+b$) cm where a and b are rational numbers. [3]

$\frac{EC}{1+3\sqrt{6}} = \frac{3}{3+\sqrt{6}}$ [M1 for applying properties of similar triangles]

$EC = \dfrac{3}{3+\sqrt{6}}\left(1+3\sqrt{6}\right)$

$= \frac{3(1+3\sqrt{6})(3-\sqrt{6})}{(3+\sqrt{6})(3-\sqrt{6})}$ [M1 for rationalization]

$= 9\sqrt{6} - 18 + 3 - \sqrt{6}$

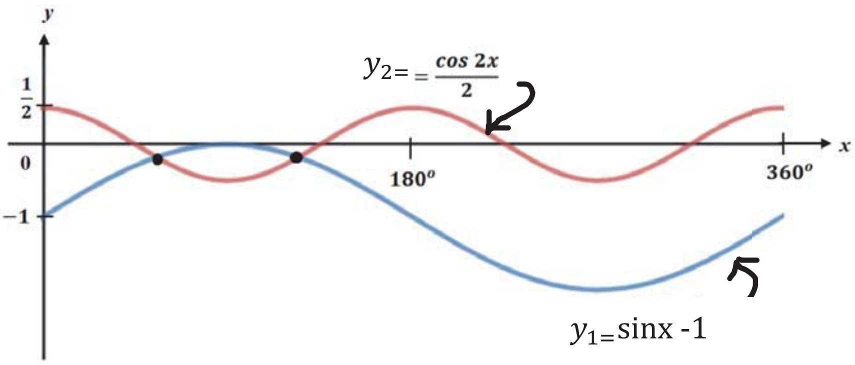
$= 8\sqrt{6} - 15$ [A1]

12	It is given that $y_2 = \frac{\cos 2x.}{2}$ and $y_1 = \sin x - 1$. **(i)** State the amplitude and period, in degrees of y_1, Amplitude = 1 and period = 360° For the interval of $0° \leq x \leq 360°$, **(ii)** solve the equation $y_1 = y_2$, $\frac{\cos 2x}{2} = \sin x - 1$ $\cos 2x = 2\sin x - 2$ $1\text{-}2 \sin^2 x = 2\sin x - 2$ [M1 for application of double angle formula $\cos 2x = 1\text{-}2 \sin^2 x$] $2 \sin^2 x + 2\sin x - 3 = 0$ $\sin x = 0.8228$ or $\sin x = -1.8229$ (reject) [M1 for obtaining sinx= 0.8228] $x = 55.4°$ or $124.6°$. [A1]	[2] [3]

[2

[3

(iii) sketch, on the same diagram, the graphs of y_1 and y_2 , [4] [4]



M1 -for correct shape for $y_1 = \frac{\cos 2x}{2}$ and A1 correct graph drawn
M1 -for correct shape for $y_2 = \sin x - 1$ and A1 correct graph drawn ,
(correctly translated).

(iv) find the set of values of x for which $y_1 - y_2 < 0$. [2] [2

$y_1 - y_2 < 0 \Longrightarrow y_1 < y_2$

From the graph and part (ii), the set of values of x is $x < 55.4°$ or $x >124.6°$
[B1 for 55.4° and B1 for 124.6°]

End of Paper

SECONDARY 4
PRELIMINARY EXAMINATION
ADDITIONAL MATHEMATICS
Paper 2 4047/2

16 September 2020 (Wednesday) **2 hours 30 minutes**

CANDIDATE NAME	**Solutions**

CLASS		INDEX NUMBER		

READ THESE INSTRUCTIONS FIRST
Do not turn over the page until you are told to do so.
Write your name, class and index number in the spaces above.
Write in dark blue or black pen in the writing papers provided.

You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

INFORMATION FOR CANDIDATES
Answer **all** the questions in the space provided.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your answer scripts securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **100.**

For Examiner's Use		
Q1	5	
Q2	5	
Q3	6	
Q4	5	
Q5	8	
Q6	8	
Q7	9	
Q8	10	
Q9	10	
Q10	10	
Q11	11	
Q12	13	
Total	/100	

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n *is a positive integer and* $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\mathrm{cos}ec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for Δ *ABC*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 **(i)** Differentiate $x^2 \ln 3x$ with respect to x. [2]

$$\frac{dy}{dx} = x^2\left(\frac{3}{3x}\right) + \ln 3x(2x)$$
$$= x + 2x\ln 3x$$

– apply product rule

(ii) Hence find $\int x \ln 3x \, dx$. [3]

$$\int x + 2x\ln 3x dx = x^2 \ln 3x + C$$
$$\int x dx + \int 2x \ln 3x dx = x^2 \ln 3x + C$$
$$\int 2x \ln 3x dx = x^2 \ln 3x - \frac{x^2}{2} + C$$
$$\int x \ln 3x dx = \frac{1}{2}x^2 \ln 3x - \frac{x^2}{4} + D$$

– apply anti-differentiation

– separating terms to integrate and integration of x

2 Express $\dfrac{x^3+1}{(x^2+1)(x-2)}$ in partial fractions. [5]

$$\frac{x^3+1}{x^3-2x^2+x-2}=1+\frac{2x^2-x+3}{(x^2+1)(x-2)}$$

– Use long division to convert improper to proper rational function

Let

$$\frac{2x^2-x+3}{(x^2+1)(x-2)}=\frac{Ax+B}{x^2+1}+\frac{C}{x-2}$$

– Decomposition of partial fractions into its correct form

$$2x^2-x+3=(Ax+B)(x-2)+C(x^2+1)$$

By Substitution:

At $x = 2$,

$$2(2)^2-2+3=C(5)$$

$$C=\frac{9}{5}$$

At $x = 0$,

$$3=-2B+C$$

$$2B=\frac{9}{5}-3$$

$$B=-\frac{3}{5}$$

By comparing coefficient of x^2,

$$A+C=2$$

$$A=2-\frac{9}{5}=\frac{1}{5}$$

$$\frac{x^3+1}{(x^2+1)(x-2)}=1+\frac{x-3}{5(x^2+1)}+\frac{9}{5(x-2)}$$

3 The quadratic equation $3x^2+2x+1=0$ has roots α and β.

(i) Show that the value of $\alpha^3+\beta^3$ is $\frac{10}{27}$. [3]

$$3x^2+2x+1=0$$
$$x^2+\frac{2}{3}x+\frac{1}{3}=0$$
$$\alpha+\beta=-\frac{2}{3}$$
$$\alpha\beta=\frac{1}{3}$$

– for obtaining sum and product of roots α and β.

$$\alpha^3+\beta^3$$
$$=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)$$
$$=\frac{-2}{3}\left[(\alpha+\beta)^2-3\alpha\beta\right]$$
$$=\frac{-2}{3}\left[\left(\frac{-2}{3}\right)^2-1\right]$$
$$=\frac{-2}{3}\left(\frac{-5}{9}\right)$$
$$=\frac{10}{27}$$

– for re-expressing $\alpha^3+\beta^3$ into $\alpha+\beta$ and $\alpha\beta$. Acceptable too:

$$(\alpha+\beta)^3$$
$$=\alpha^3+3\alpha^2\beta+2\alpha\beta^2+\beta^3$$
$$=(\alpha+\beta)^3+3\alpha\beta(\alpha+\beta)$$

(ii) Find a quadratic equation whose roots are $\frac{1}{\alpha^3}$ and $\frac{1}{\beta^3}$. [3]

$$\frac{1}{\alpha^3}+\frac{1}{\beta^3}$$
$$=\frac{\beta^3+\alpha^3}{\alpha^3\beta^3}$$
$$=\frac{\frac{10}{27}}{\frac{1}{27}}$$
$$=10$$

– for finding sum of roots

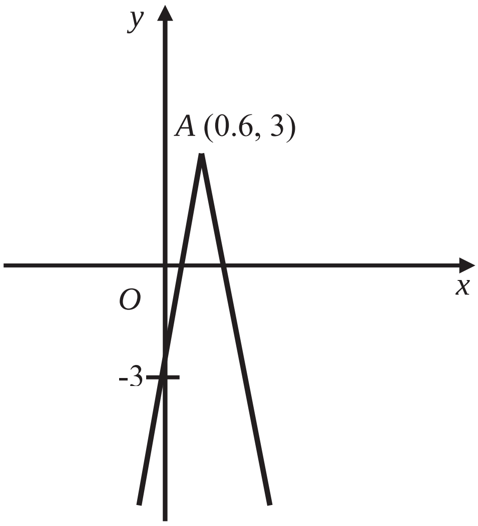
$$\frac{1}{\alpha^3}\left(\frac{1}{\beta^3}\right)$$
$$=\frac{1}{\alpha^3\beta^3}$$
$$=\frac{1}{\left(\frac{1}{3}\right)^3}$$

– for finding product of roots

$$=27$$

$$x^2-10x+27=0$$

4



The diagram shows part of the graph $y=r-q|3-px|$, where p, q and r are positive constants. The graph has a vertex at A (0.6, 3) and y-intercept of -3.

(i) Determine the values of p, q and r. [3]

At vertex,

$$3-px=0$$
$$px=3$$
$$x=\frac{3}{p}$$
$$\frac{3}{p}=0.6$$
$$p=5$$
$$r=3$$

At the y-intercept of -3, $x=0$,

$-3 = 3 - q|3|$

$3q = 6$

$q = 2$

(ii) State the value or range of values of k such that $k = r - q|3 - px|$ has

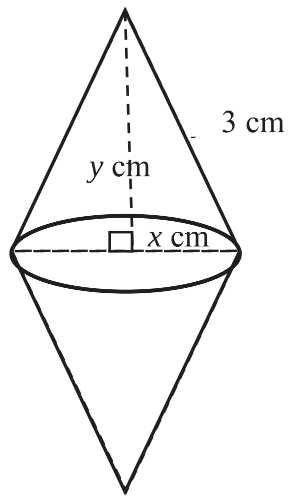
(a) 1 solution, [1]

$k = 3$

(b) 2 solutions. [1]

$k < 3$

5 A buoy is formed by two identical right circular cones of sheet iron joined by its bases with a radius of x cm. The buoy has a vertical height of y cm and a slant height of 3 cm.



(i) Express y in terms of x. [1]

$y^2 + x^2 = 3^2$

$y = \pm\sqrt{9 - x^2}$ (-ve rejected as vertical height is positive)

$y = \sqrt{9 - x^2}$

– applying Pythagoras' Theorem to obtain y in terms of x

(ii) Given that x can vary, find the exact value of x for which the volume, V, of the buoy is stationary. [4]

$$V = \frac{2}{3}\pi x^2 y$$

$$= \frac{2}{3}\pi x^2 \sqrt{9-x^2}$$

-expressing volume of buoy in terms of x

$$\frac{dV}{dx} = \frac{2}{3}\pi x^2\left[\frac{1}{2}\left(9-x^2\right)^{\frac{-1}{2}}\left(-2x\right)\right] + \sqrt{9-x^2}\left(\frac{4\pi}{3}x\right)$$

-applying product rule to find dV/dx

$$= \frac{-2\pi x^3}{3\sqrt{9-x^2}} + \sqrt{9-x^2}\left(\frac{4\pi}{3}x\right)$$

$$= \frac{1}{3\sqrt{9-x^2}}\left[-2\pi x^3 + 4\pi x(9-x^2)\right]$$

$$= \frac{1}{3\sqrt{9-x^2}}\left[36\pi x - 6\pi x^3\right]$$

At stationary point,

$$\frac{dV}{dx} = 0$$

-identifying that dV/dx = 0 at stationary point

$$\frac{1}{3\sqrt{9-x^2}}\left[36\pi x - 6\pi x^3\right] = 0$$

$$\left[36\pi x - 6\pi x^3\right] = 0$$

$$x(6-x^2) = 0$$

$$x = 0, x = \pm\sqrt{6}$$

x=0 rejected;
-ve Rejected as radius x is positive

(iii) Determine with reasons whether this value of V is a maximum or minimum. [2]

Using first derivative test,

x	$\sqrt{6}^-$	$\sqrt{6}$	$\sqrt{6}^+$
$\frac{dV}{dx}$	>0	0	<0

Volume is maximum.

Using second derivative test,

$$\frac{d^2V}{dx^2} = \frac{2\pi}{3}\left[\frac{(18-9x^2)\sqrt{9-x^2} - (18x-3x^3)\frac{1}{2}\left(9-x^2\right)^{-\frac{1}{2}}(-2x)}{9-x^2}\right]$$

When

$$x = \sqrt{6}$$

$$\frac{d^2V}{dx^2} = -43.5 < 0$$

Volume is maximum

-either first or second derivative test – conclusion that volume is maximum

(iv) Find the exact surface area of the buoy when V is stationary, leaving your answer in terms of π. [1]

Surface area

$$= 2\pi(x)(3)$$
$$= 6\sqrt{6}\pi cm^2$$

6 The equation of a polynomial is given by $p(x) = 2x^3 + 2ax^2 - x^2 + ax - a$ where a is a constant.

(i) Find the remainder when $p(x)$ is divided by $(x + 1)$. [1]

$$p(-1)$$
$$= 2(-1)^3 + 2a(-1)^2 - (-1)^2 + a(-1) - a$$
$$= -2 + 2a - 1 - a - a$$
$$= -3$$

Remainder = -3

(ii) Show that $(2x - 1)$ is a factor of $p(x)$. [2]

$$p\left(\frac{1}{2}\right)$$
$$= 2\left(\frac{1}{2}\right)^3 + 2a\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + a\left(\frac{1}{2}\right) - a$$
$$= \frac{1}{4} + \frac{a}{2} - \frac{1}{4} + \frac{a}{2} - a = 0$$

– for finding p(1/2) or using long division/synthetic division

Since remainder = 0, $(2x - 1)$ is a factor of $p(x)$.

-conclusion stating remainder = 0, therefore (2x-1) is a factor of p(x).

(iii) By showing clearly your working, factorise $p(x)$. [2]

	2	2a – 1	a	-a
0.5		1	a	a
	2	2a	2a	0

– Using synthetic division/long division

Alternatively, by long division

$$\begin{array}{r} x^2 + ax + a \\ 2x-1\overline{)2x^3 + (2a-1)x^2 + ax + a} \\ 2x^3 - \quad x^2 \\ \hline 2ax^2 + ax \\ 2ax^2 - ax \\ \hline 2ax + a \\ 2ax - a \end{array}$$

$$2x^3 + 2ax^2 - x^2 + ax - a = (2x-1)(x^2 + ax + a)$$

(iv) Find the range of values of a for which the equation $\mathrm{p}(x) = 0$ has only one real root. [3]

$(2x-1)(x^2 + ax + a) = 0$

For p(x) = 0 to have only one real root,

$(a)^2 - 4(1)(a) < 0$ -apply discriminant < 0

$a^2 - 4a < 0$ – solve inequality by factorising

$a(a-4) < 0$

$0 < a < 4$

7 The table below shows the data obtained from an experiment on the vertical motion based on the oscillation of a spring with different masses attached to it.

Mass, x kg	0.02	0.03	0.04	0.05	0.15
Frequency of oscillations, y	16	13	11.4	10	6

It is known that the mass, x kg, and the frequency of oscillations per second, y, are related by the equation $xy^2 = k$, where k is a constant.

(a) Plot y^2 against $\frac{1}{x}$ and draw a straight line graph. [3]

$y^2 = \frac{k}{x}$

$\frac{1}{x}$	50	33.3	25	20	6.67
y^2	256	169	129.96	100	36

– table of values
– 5 plotted points
– straight line that passes through the points

(b) Use your graph to estimate

(i) the frequency of oscillations when a mass of 0.08 kg is attached to the spring, [1]

$x = 0.08$

$\frac{1}{x} = 12.5$

From graph, read off corresponding value of y^2
Acceptable range $60 < y^2 < 70$

$y^2 = 65$

$y = 8.06$

– Acceptable range $7.75 < y < 8.37$

(ii) the mass which produces 15 oscillations per second, [1]

$y = 15$

$y^2 = 225$

$\frac{1}{x} = 44$

From graph, read off corresponding value of $1/x$
Acceptable range $43 < 1/x < 45$

$x = 0.0227$

- Acceptable range $0.0233 < x < 0.0222$

(iii) the value of k. [1]

$k = \frac{230-0}{45-0} = 5.11$

– Acceptable range
$5 < k < 5.22$

(c) When the spring is replaced by a second spring, the relation between y and x is represented by $y^2 = \frac{2}{x} + 80$.

(i) On the same diagram, draw the line representing the second spring. [1]

[B1] – drawing the line, needs to pass through vertical intercept

y^2	80	130	180
1/x	0	25	50

(ii) Hence, explain how to find the mass which produces the same frequency of oscillations by both springs. [2]

Both the lines intersect at (25, 130).
Acceptable range of $1/x$-coord $24<1/x<26$; Acceptable range of y^2-coord $125 < y^2 < 135$

The intersection point indicates the mass that produces the same frequency of oscillations by both springs = 1/25 = 0.0400g

– Acceptable range
1/24 < mass < 1/26
0.0417g < mass < 0.0385g

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8 **(i)** Prove that $\dfrac{\sec A+\tan A-1}{1-\sec A+\tan A}\equiv\dfrac{1+\sin A}{\cos A}$. [5]

LHS

$=\dfrac{\sec A+\tan A-(\sec^2 A-\tan^2 A)}{1-\sec A+\tan A}$ -using trigo identity $\sec^2 A-\tan^2 A=1$

$=\dfrac{\sec A+\tan A-(\sec A+\tan A)(\sec A-\tan A)}{1-\sec A+\tan A}$ -applying $a^2-b^2=(a+b)(a-b)$

$=\dfrac{(\sec A+\tan A)[1-(\sec A-\tan A)]}{1-\sec A+\tan A}$ -factoring out sec A + tan A

$=\sec A+\tan A$ -division of 1- sec A + tan A

$=\dfrac{1}{\cos A}+\dfrac{\sin A}{\cos A}$ -conversion of sec A & tan A in terms of sin A & cos A

$=\dfrac{1+\sin A}{\cos A}$

(ii) Hence solve the equation $\dfrac{\sec A+\tan A-1}{1-\sec A+\tan A}=3\cos A$ for $0<A<2\pi$. [5]

$\dfrac{1+\sin A}{\cos A}=3\cos A$ -use result from trigo identity in 8(i)

$1+\sin A=3\cos^2 A$

$1+\sin A=3(1-\sin^2 A)$ -use of identity $\sin^2 A+\cos^2 A=1$ to form quadratic equation in trigo function

$1+\sin A=3-3\sin^2 A$

$3\sin^2 A+\sin A-2=0$

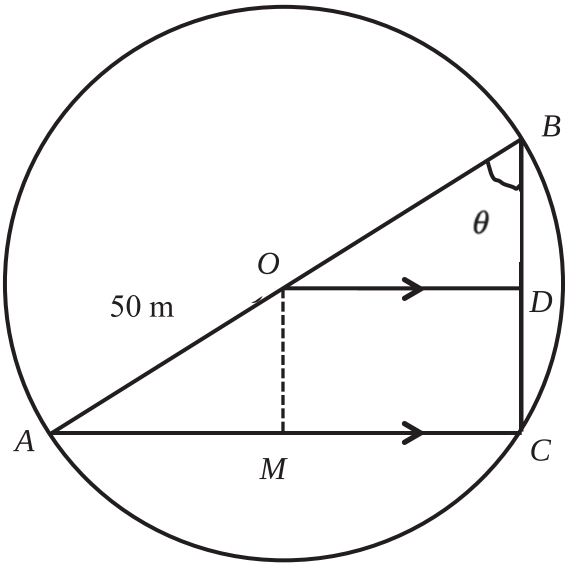
$(3\sin A-2)(\sin A+1)=0$

$\sin A=\dfrac{2}{3},\qquad \sin A=-1$ -solve to obtain values for sin *A*

Basic angle = 0.72973

$A=0.730, 2.41 \quad A=\dfrac{3\pi}{2}$

9



The diagram shows a circular field with centre O and radius 50 m.
A, B and C are points on the circumference of the field and angle $ABC = \theta$. D is a point on BC such that OD is parallel to AC.
The trapezium $AODC$ is the jogging path of a man.

(i) Explain why $BD = DC = 50\cos\theta$. [2]

Method 1

Angle BDO = Angle OMA = 90°
Angle OBD = Angle AOM = θ
Triangle OBD is similar to Triangle AOM.

– Identify similar triangles to use corresponding lengths

$$\frac{OA}{BO} = \frac{OM}{BD} = 1$$

$OM = BD$

Since $OM = DC$, $DC = BD$.
Using triangle OBD,

$$\cos\theta = \frac{BD}{50}$$

$BD = 50\cos\theta$

– use trigo ratios to find BD/BC

Method 2

Angle ACB = 90° (right angle in a semicircle)

$$\cos\theta = \frac{BC}{100}$$

$BC = 100\cos\theta$

-identifying right angle and applying trigo ratio to find BC

Angle ODB = 90° (corresponding angles since OD is parallel to AC)

$\cos\theta = \dfrac{BD}{50}$

-identifying corresponding right angle and applying trigo ratio to find BD

$BD = 50\cos\theta$

$DC = 100\cos\theta - 50\cos\theta = 50\cos\theta$

$\therefore BD = DC$

(ii) Show that the perimeter, L m, of the jogging path *AODC* can be expressed in the form $p + q\cos\theta + r\sin\theta$, where p, q and r are constants to be found. [3]

$\sin\theta = \dfrac{AC}{100}$

$AC = 100\sin\theta$

$\sin\theta = \dfrac{OD}{50}$

$OD = 50\sin\theta$

– find AC using trigo ratio

– find OD using trigo ratio

$L = 50 + 50\cos\theta + 100\sin\theta + 50\sin\theta$

$= 50 + 50\cos\theta + 150\sin\theta$

(iii) Express L in the form of $p + R\cos(\theta - \alpha)$ where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

$L = 50 + 50\cos\theta + 150\sin\theta$

$= 50 + \sqrt{50^2 + 150^2}\cos(\theta - \alpha)$

$= 50 + \sqrt{25000}\cos(\theta - 71.6^\circ)$

$\tan\alpha = \dfrac{150}{50} = 3$

$\alpha = 71.6^\circ$

– Obtaining $\tan\alpha$
– Finding α

– Obtaining [illegible]/158 (to 3sf)

(iv) Hence state the maximum perimeter, L m, of the jogging path *OACD*. Find the value of θ at which this occurs. [2]

Maximum $L = 50 + \sqrt{25000} = 208$

Occurs when

$\cos(\theta - 71.6^\circ) = 1$

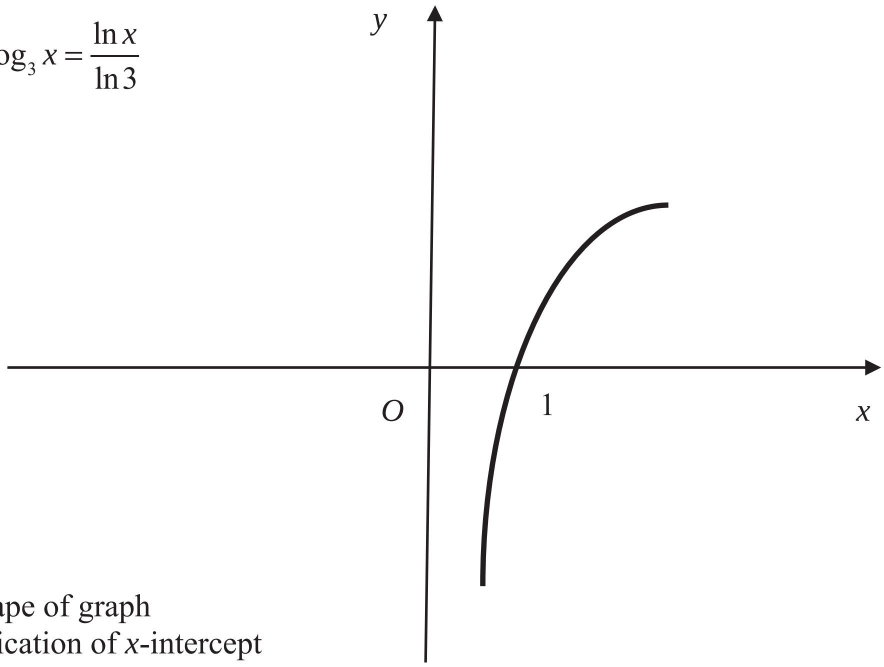
$\theta - 71.6^\circ = 0^\circ$

$\theta = 71.6^\circ$

-equating max of $\cos(\theta - 71.6^\circ) = 1$ to find θ.

10 **(a)** Sketch the graph of $y = \log_3 x$. [2]

$y = \log_3 x = \dfrac{\ln x}{\ln 3}$



– shape of graph
– indication of x-intercept

(b) Express $\log_9 4 + \log_3(x-4) = 2\log_3 x$ as a quadratic equation in x and explain why there are no real solutions. [4]

$$\frac{\log_3 4}{\log_3 9} + \log_3(x-4) = \log_3 x^2$$

– apply change of base and power law

$$\frac{\log_3 4}{2} + \log_3(x-4) = \log_3 x^2$$

$$\log_3 4^{\frac{1}{2}} + \log_3(x-4) = \log_3 x^2$$

$$4^{\frac{1}{2}}\left(x-4\right) = x^2$$

– apply product law

$$2x - 8 = x^2$$

$$x^2 - 2x + 8 = 0$$

– obtain quadratic equation

Discriminant

$$= \left(-2\right)^2 - 4(1)(8)$$

$$= -28 < 0$$

– explain why no real solutions using discriminant

There are no real solutions.

(c) Given that $\log_b(x^2y)=m$ and $\log_b(x^3y)=n$, express, $\log_b x$ and $\log_b y$ in terms of m and n. [4]

$\log_b\left(x^2y\right)=m$

$\log_b x^2+\log_b y=m$

$2\log_b x+\log_b y=m---(1)$

- apply product and power laws to obtain equation (1)

$\log_b\left(x^3y\right)=n$

$\log_b x^3+\log_b y=n$

$3\log_b x+\log_b y=n---(2)$

– apply product and power laws to obtain equation (2)

$(2)-(1)$

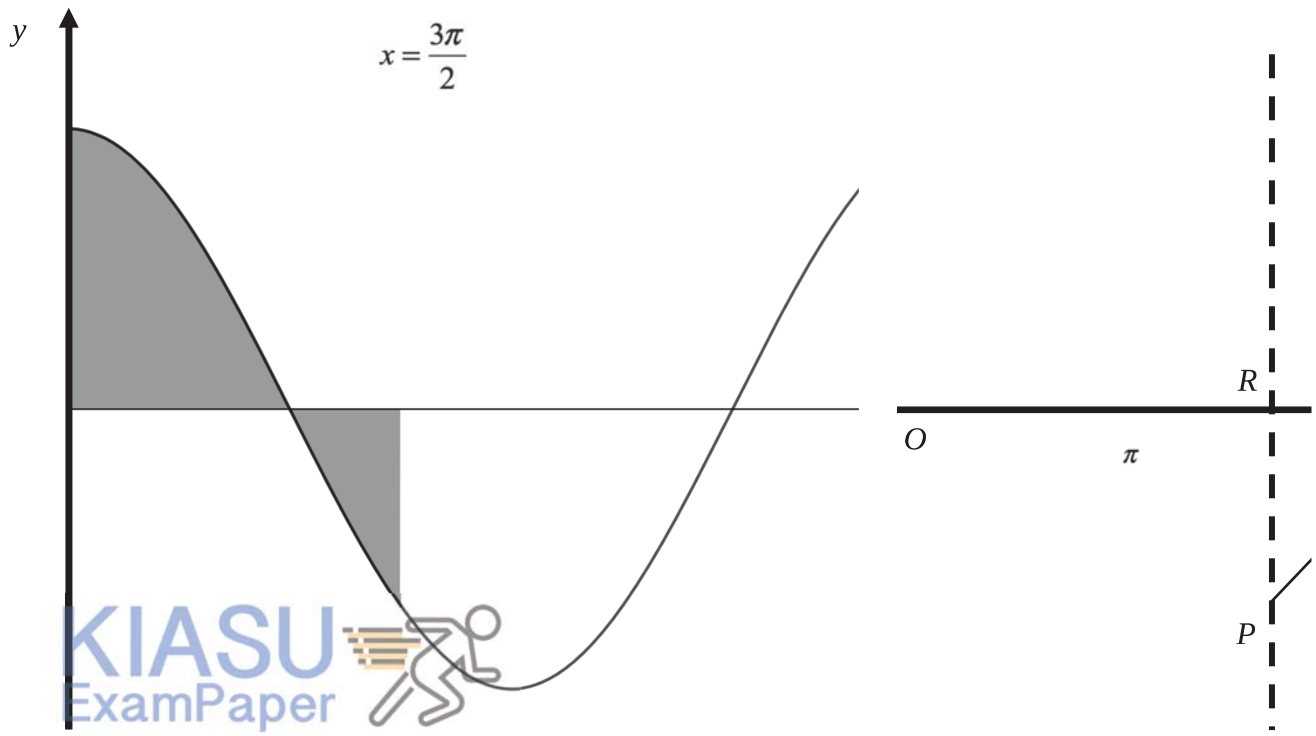
$\log_b x=n-m$

$\log_b y$

$=m-2(n-m)$

$=3m-2n$

11



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The diagram shows part of the curve $y=4\cos\left(\frac{x}{2}\right)$ that meets the x-axis at $x=\pi$ and $x=3\pi$. The line $x=\frac{3\pi}{2}$ meets the x-axis at R and the curve at P. The normal to the curve at P meets the x-axis at Q.

(i) Find the equation of the normal at P, expressing your answer in exact form. [4]

$$y = 4\cos\left(\frac{x}{2}\right)$$

$$\frac{dy}{dx} = -4\sin\left(\frac{x}{2}\right)\left(\frac{1}{2}\right) = -2\sin\left(\frac{x}{2}\right)$$

-Differentiating to find gradient of tangent

At $x = \frac{3\pi}{2}$,

gradient of tangent

$$= -2\sin\left(\frac{3\pi}{4}\right)$$

$$= -2\left(\frac{1}{\sqrt{2}}\right)$$

$$= -\sqrt{2}$$

Gradient of normal $= \frac{1}{\sqrt{2}}$

– evaluation of gradient of normal

Coordinate of P:

$$y = 4\cos\left(\frac{x}{2}\right)$$

$$= 4\cos\left(\frac{3\pi}{4}\right)$$

$$= 4\left(\frac{-\sqrt{2}}{2}\right)$$

$$= -2\sqrt{2}$$

$$P\left(\frac{3\pi}{2}, -2\sqrt{2}\right)$$

– finding coordinate of P

Equation of normal:

$$y - \left(-2\sqrt{2}\right) = \frac{1}{\sqrt{2}}\left(x - \frac{3\pi}{2}\right)$$

$$y = \frac{x}{\sqrt{2}} - \frac{3\pi}{2\sqrt{2}} - 2 \quad 2$$

– finding equation of normal

OR

$$y = \frac{\sqrt{2}}{2}x - \frac{3\sqrt{2}}{4}\pi - \frac{4}{\sqrt{2}}$$ (equivalent forms are acceptable)

(ii) Find the exact coordinates of Q. [2]

At $y = 0$,

$$\frac{x}{\sqrt{2}} - \frac{3\pi}{2\sqrt{2}} - 2\sqrt{2} = 0$$

$$\frac{x}{\sqrt{2}} = 2\sqrt{2} + \frac{3\pi}{2\sqrt{2}}$$

$$x = 4 + \frac{3\pi}{2}$$

$$Q(4 + \frac{3\pi}{2}, 0)$$

(iii) Find the exact area of the shaded region. [5]

Area

$$= \int_0^{\pi} 4\cos\left(\frac{x}{2}\right)dx + \left|\int_{\pi}^{\frac{3\pi}{2}} 4\cos\left(\frac{x}{2}\right)dx\right|$$

– Expression or function to represent area for $0 \leq x \leq \pi$
– Expression or function to represent area below x-axis

$$= \left[\frac{4\sin\left(\frac{x}{2}\right)}{\frac{1}{2}}\right]_0^{\pi} + \left|\left[\frac{4\sin\left(\frac{x}{2}\right)}{\frac{1}{2}}\right]_{\pi}^{\frac{3\pi}{2}}\right|$$

-Integration of trig function

$$= 8\sin\frac{\pi}{2} + \left|8\sin\frac{3\pi}{4} - 8\sin\frac{\pi}{2}\right|$$

– evaluation of limits

$$= 8 + \left|8\left(\frac{\sqrt{2}}{2}\right) - 8\right|$$

$$= 8 + \left|4\sqrt{2} - 8\right|$$

$$= 16 - 4\sqrt{2} units^2$$

– area in exact form

12 A circle, C_1 has equation $x^2+y^2+8x-12y+16=0$.

(i) Find the radius and the coordinates of the centre of C_1. [3]

$$x^2+8x+y^2-12y+16=0$$

$$(x+4)^2-16+(y-6)^2-36+16=0$$

- completing the square

$$(x+4)^2+(y-6)^2=6^2$$

Radius = 6
Centre = (-4, 6)

(ii) The lowest point on the circle is A. Explain why A lies on the x-axis. [1]
Since the circle has centre at (-4, 6) with radius 6, the lowest point on the circle is (-4, 0). Therefore, A lies on the x-axis.

A second circle, C_2 , has a diameter PQ. The point P has coordinates (-1, 3) and the equation of the tangent to C_2 at Q is $2y=x-18$.

(iii) Find the equation of the diameter PQ and hence the coordinates of Q. [4]

$$2y=x-18$$

$$y=\frac{x}{2}-9$$

Gradient of tangent = ½
Gradient of diameter = -2
Equation of diameter PQ:

– find gradient of diameter PQ

$$y-3=-2(x+1)$$

$$y=-2x+1$$

To find coord of Q, find intersection between equation of tangent and equation of diameter

$$-2x+1=\frac{x}{2}-9$$

– find intersection between equation of tangent and equation of diameter

$$2.5x=10$$

$$x=4$$

$$y=-7$$

$$Q(4,-7)$$

(iv) Find the equation of the circle, C_2. [3]

Length of PQ

$$= \sqrt{(-1-4)^2 + (3+7)^2}$$
$$= \sqrt{25+100}$$
$$= \sqrt{125}$$

Radius $= \dfrac{\sqrt{125}}{2}$

– finding length of diameter to obtain radius

Midpoint of PQ

$$= \left(\frac{-1+4}{2}, \frac{3-7}{2}\right)$$
$$= \left(\frac{3}{2}, -2\right)$$

– finding center of circle

Equation of the circle, C_2,

$$\left(x-\frac{3}{2}\right)^2 + (y+2)^2 = \left(\frac{\sqrt{125}}{2}\right)^2$$
$$\left(x-\frac{3}{2}\right)^2 + (y+2)^2 = \frac{125}{4}$$

(v) Determine whether the circles C_1 and C_2 intersect each other. [2]

Distance between centre of circles C_1 and C_2

$$= \sqrt{\left(-4-\frac{3}{2}\right)^2 + (6+2)^2}$$
$$= 9.71$$

– computing distance between centre of circles and sum of radii

Sum of radii of circles C_1 and C_2

$$= \frac{\sqrt{125}}{2} + 6$$
$$= 11.6$$

Since distance between centre of circles C_1 and C_2 < sum of radii, both circles C_1 and C_2 intersect each other.

END OF PAPER