Name		( )	Class	

南华中学

NAN HUA HIGH SCHOOL

PRELIMINARY EXAMINATION 2020

Subject	**:**	**Additional Mathematics**
Paper	**:**	**4047/01**
Level	**:**	**Secondary Four Express**
Date	**:**	**25 August 2020**
Duration	**:**	**2 hours**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correcting fluid / tape.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

For Examiner's Use

This paper consists of 25 printed pages and 1 blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

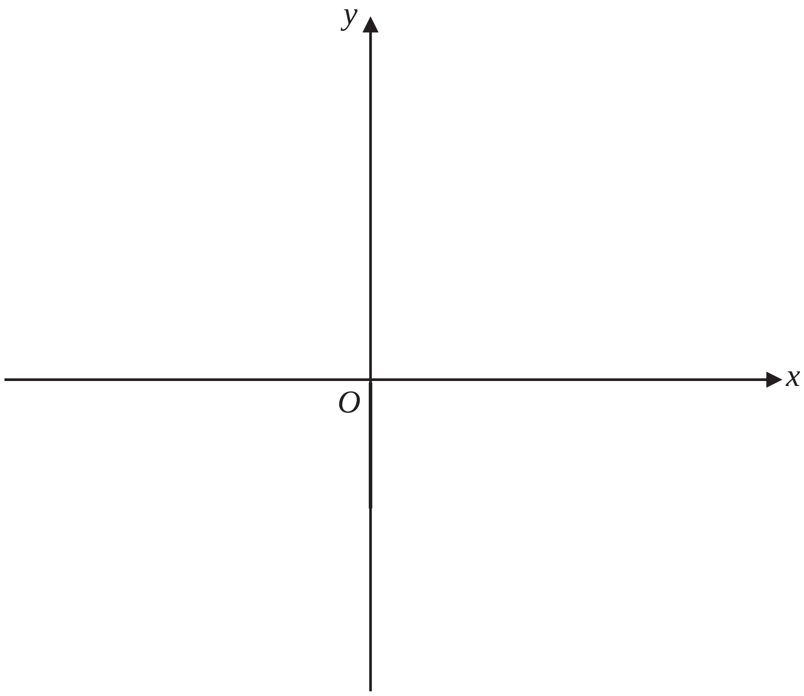
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 **Without using a calculator**, express $\text{cosec}^2 75°$ in the form of $a + b\sqrt{3}$ where a and b are integers. [4]

2 A curve has equation $y^2 = 9x$.

(i) Sketch the graph of $y^2 = 9x$. [1]



(ii) The curves $y^2 = 9x$ and $y = kx^{-\frac{3}{2}}$ meet at point M where $x = 4$. Find the value(s) of k. [3]

3 Given $A = \tan^{-1}\left(-\frac{3}{4}\right)$, find, **without using a calculator**, the value of $\sin\left(A - \frac{\pi}{2}\right)$. [3]

4 Given that $p + q = k + 1$ and $pq = 1$, where k is a constant, find the range of values of k for which p and q are distinct and real numbers. [4]

5 A function is defined by the equation $y = \mathrm{f}(x)$ such that $\mathrm{f}'(x) = \dfrac{1}{x-2} + e^{-2x}$, for $x > 2$.

(i) Given that the curve passes through the point $\left(3, -\dfrac{1}{e^6}\right)$, find an expression for $\mathrm{f}(x)$. [4]

(ii) Explain, with reasons, whether the function $f'(x)$ is increasing or decreasing. [3]

6 The equation of a curve is $y = mx^2 + (m+n)x + n$, where m and n are constants, and $m > 0$.
Explain why the curve will not lie completely above the x-axis. [3]

7 **(a)** Solve the equation $2x^3 - 3x^2 - 3x + 4 = 0$. [4]

(b) It is given that $x-5$ is a factor of $\mathrm{p}(x)+1$, where $\mathrm{p}(x)$ is a polynomial. Find the remainder when $\mathrm{g}(x)=2x^3-\mathrm{p}(x)+5$ is divided by $x-5$. [2]

8 The roots of the quadratic equation $4x^2 - 2x + 1 = 0$ are $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$. Find a quadratic equation whose roots are $\dfrac{\beta}{\alpha^2}$ and $\dfrac{\alpha}{\beta^2}$. [5]

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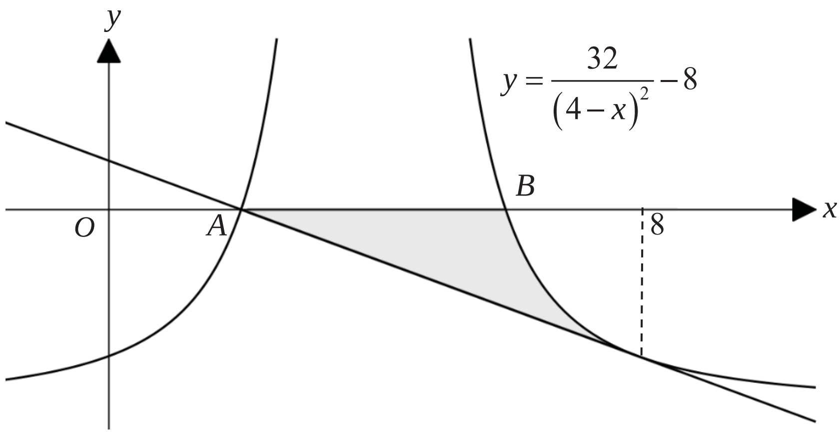
9 **(i)** Show that $\frac{d}{dx}\left(\frac{2\ln x}{x^3}\right) = \frac{2}{x^4} - \frac{6\ln x}{x^4}$. [3]

(ii) Integrate $\frac{\ln x}{x^4}$ with respect to x. [3]

10



The diagram shows part of the curve $y = \frac{32}{(4-x)^2} - 8$, intersecting the x-axis at A and B. The tangent to the curve at $x = 8$ meets the x-axis at A.

(i) Find the equation of the tangent. [4]

(ii) Find the area of the shaded region bounded by the tangent, the curve and the x-axis. [5]

11 The table shows experimental values of two variables *x* and *y*.

x	1.00	1.22	1.29	1.32	1.34
y	1	2	3	4	5

It is known that *x* and *y* are related by the equation $y = \dfrac{1}{ax^2 + b}$ for $x > 0$, where *a* and *b* are constants.

(i) Plot $x^2 y$ against y to obtain a straight line graph. [2]

11(i)

(ii) Use your graph to estimate the values of a and b. [4]

(iii) By drawing a suitable line on your graph, solve the equation $2.5 = \dfrac{1}{ax^2 + b}$. [2]

12 A circle has equation $x^2 + y^2 - 20x + 10y + 100 = 0$. Find

(i) the radius and coordinates of the centre of the circle, [3]

(ii) the equations of the tangents from the origin to the circle, [5]

(iii) **without solving simultaneous equations**, the coordinates of the points at which these tangents meet the curve. [3]

13 **(i)** Solve the equation $2\cos 2A - 3\sin A - 1 = 0$ for $0 \le A \le 2\pi$. [4]

(ii) On the same axes sketch, for $0 \le x \le 4\pi$, the graphs of

$$y = 2 + 3\sin\frac{x}{2} \text{ and } y = 2\cos x + 1.$$ [4]

(iii) Explain how the solutions of the equation in part **(i)** could be used to find the x-coordinates of the points of intersection of the graphs of part **(ii).** [2]

~ End of Paper ~

Name	( )	Class	

南华中学

NAN HUA HIGH SCHOOL

PRELIMINARY EXAMINATION 2020

Subject	**:**	**Additional Mathematics**
Paper	**:**	**4047/02**
Level	**:**	**Secondary Four Express**
Date	**:**	**31 August 2020**
Duration	**:**	**2 hours 30 minutes**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correcting fluid / tape.

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The total number of marks for this paper is 100.

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Binomial Theorem

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\text{Area of } \Delta ABC = \frac{1}{2}bc\sin A$$

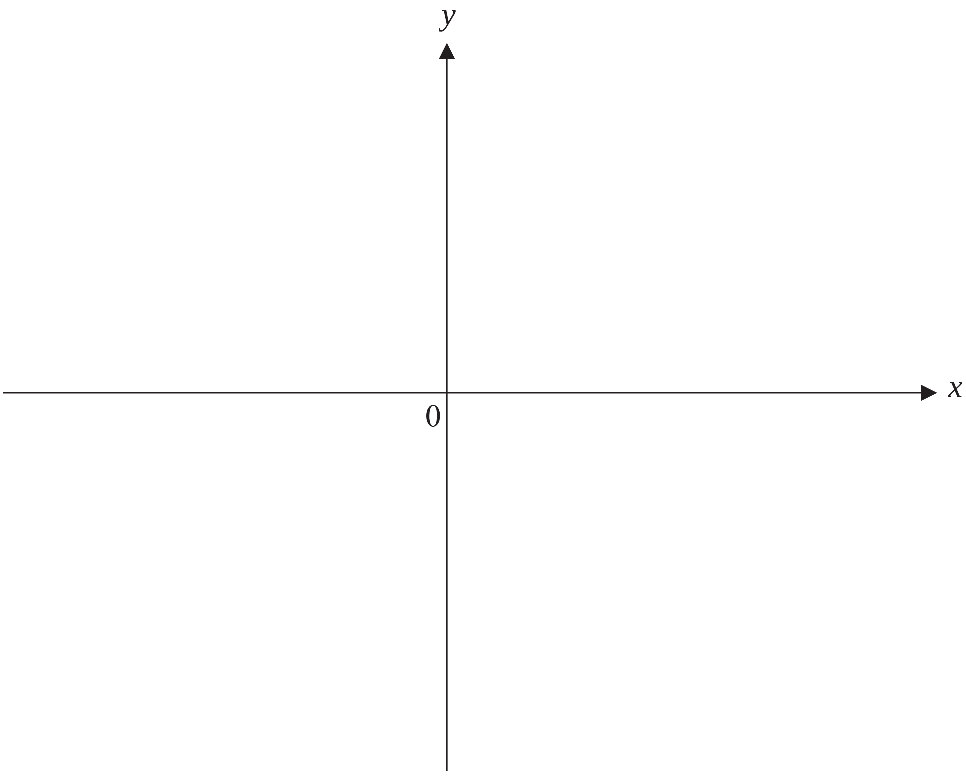
1 (a) Solve the equation $2^{6x}\left(1+\dfrac{7}{2^{3x}}\right)=30$. [5]

(b) Solve the equation $\log_x 2=\log_7\sqrt{7}+3\log_2 x$. [5]

(c) The graph of $y = \log_p(x-1)$ passes through the points with coordinates (2, q) and (5, 1).

(i) Determine the value of each of the constants p and q. [2]

(ii) Sketch the graph of $y = \log_p(x-1)$. [2]



2 An investigation on the growth of bacteria *A* in nutrient solution is conducted. Samples of nutrient solution containing bacteria *A* are tested at different time intervals. Results from the test show that the number of bacteria present, *P*, in *t* hours, is given by the equation $P = 300e^{kt} + 450$, where *k* is a constant. The number of bacteria doubles after one day.

(i) Find the number of the bacteria at the start of the experiment. [1]

(ii) Find the value of *k*. [2]

(iii) The number of bacteria *B*, in *t* hours, in another nutrient solution, is modelled by the equation $Q = \dfrac{164000}{20 + e^{10-0.3t}}$. Explain, with justification, the largest possible number of bacteria *B* that can be present in the nutrient solution over time. [2]

3 (a) (i) Express $\dfrac{3x^3+x^2+5x+15}{(x^2+3)(x+1)}$ in partial fractions. [5]

(ii) Differentiate $\ln\left(x^2+3\right)$ with respect to *x*. [1]

(iii) Hence find $\int \frac{3x^3+x^2+5x+15}{\left(x^2+3\right)\left(x+1\right)}\ dx$. [3]

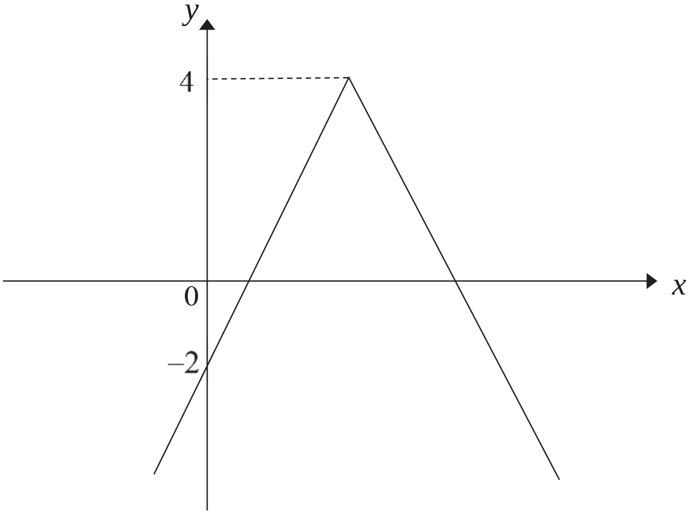
(b) The points (3, 6) and $(3, -1)$ lie on the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$ respectively. [2]
$\mathrm{f}'(x) = \mathrm{g}'(x)$ for all real values of x. Showing your working clearly, express $\mathrm{g}(x)$ in terms of $\mathrm{f}(x)$.

4 (i) Given that the constant term in the binomial expansion of $\left(3x-\frac{n}{x^2}\right)^6$ is 4860, find the value of the positive constant *n*. [4]

(ii) Using the value of n found in part (i), find the constant term in the expansion of [3]

$$\left(1+\frac{2}{x^3}\right)\left(3x-\frac{n}{x^2}\right)^6.$$

5 (a) The diagram below shows part of the graph of $y = a - 2|x - b|$, where a and b are constants. It is given that the y-intercept of the graph is -2 and the maximum value of y is 4.



(i) State the value of a. [1]

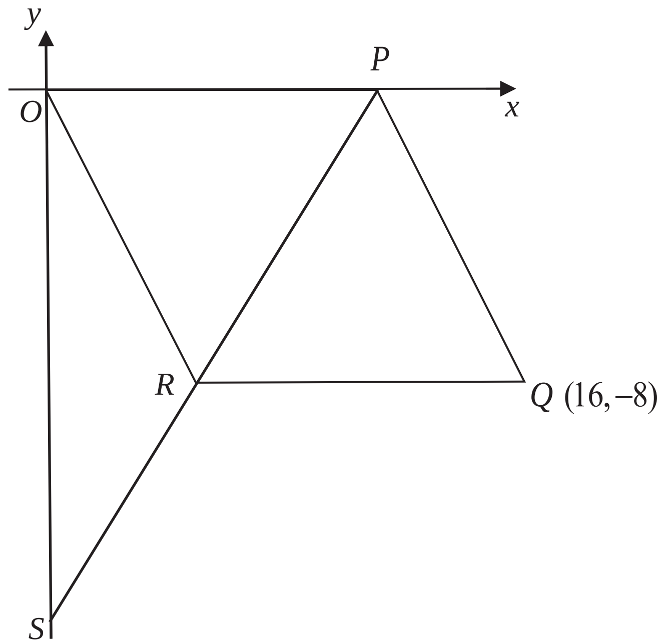
(ii) Find the value of b. [2]

(iii) State the value of k for which the equation $a - 2|x - b| = k$ has only one solution. [1]

(iv) State the range of values of *m* for which the equation $a-2|x-b|=mx-1$ has exactly two distinct solutions. [2]

(b) Solve the equation $2|x-2|=|6-3x|-8$. [4]

6



The diagram shows a rhombus *OPQR* in which point *O* is $(0,0)$ and point *Q* is $(16,-8)$. The point *P* lies on the *x*-axis. The line *PR* produced intersects the *y*-axis at *S*.

(i) Show that the equation of *PS* is $y = 2x - 20$. [3]

(ii) Given that T is the midpoint of OS, find the area of the quadrilateral $TSQP$. [3]

7 (i) Prove that $\dfrac{2\sin 2x - \sin 4x}{2\sin 2x + \sin 4x} = \tan^2 x$. [4]

(ii) Hence, find the exact value of $\int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{2\sin 2x - \sin 4x}{2\sin 2x + \sin 4x}\, dx$ [4]

8 (a) Given that $y = 3xe^{-2x}$, find

(i) $\dfrac{dy}{dx}$, [3]

(ii) the value of p if $p = e^{2x}\left(\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - 2y\right)$. [4]

(b) A curve has the equation $y = \ln\left(\frac{1-\cos x}{\sin x}\right)$.

(i) Show that $\frac{dy}{dx} = \text{cosec}\, x$. [5]

(ii) A point P moves along the curve such that $0 < x < \frac{\pi}{2}$. Find the exact value(s) of x when the rate of increase of y is twice the rate of increase of x. [3]

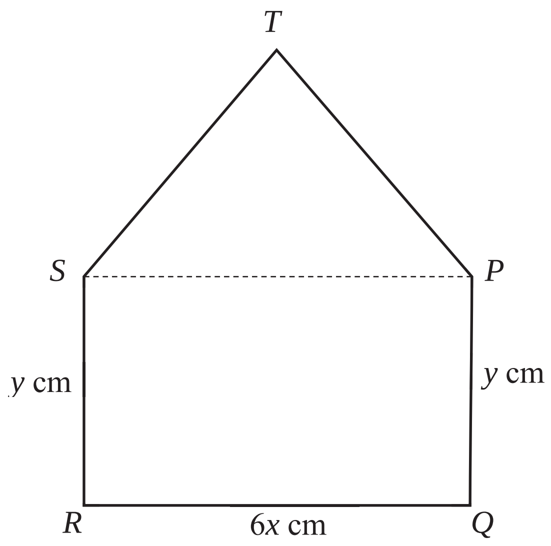
9 A theme park is building a new roller coaster. Part of the track of the roller coaster can be modelled after $H = 15\sin\theta + 8\cos\theta + 20$ where H m is the height above the ground level, $0^\circ \le \theta \le 360^\circ$.

(i) Express H in the form of $R\cos(\theta - \alpha) + k$ where k is a constant, $R > 0$ and α is an acute angle. [4]

The theme park is near an airbase. There are regulations forbidding buildings from exceeding a height of 50 m.

(ii) Explain if the design will exceed the stipulated height limit. [2]

10 The diagram shows a composite figure, *PQRST*. It consists of an equilateral triangle *TPS* and a rectangle *PQRS* with sides *y* cm and 6*x* cm. The perimeter of *PQRST* is 400 cm.



(i) Show that the area of the figure, $A = \left(9\sqrt{3} - 54\right)x^2 + 1200x$. [3]

(ii) Given that x can vary, find the value of x for which A has a stationary value. [3]

(iii) Determine if the value obtained in part (ii) will give a maximum or minimum area. [2]

11 A particle moves in a straight line so that its distance, s m from a fixed point X on the line is given by $s = -t^2 + 6t + 3$, for $t \leq 4$, where t is the time in seconds after passing through Y on the line. Find

(i) the distance XY, [1]

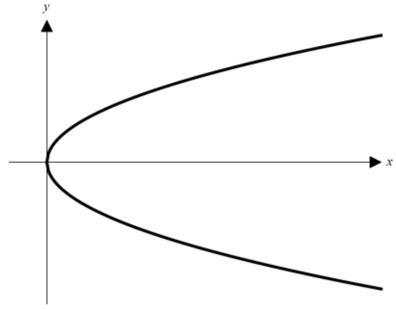
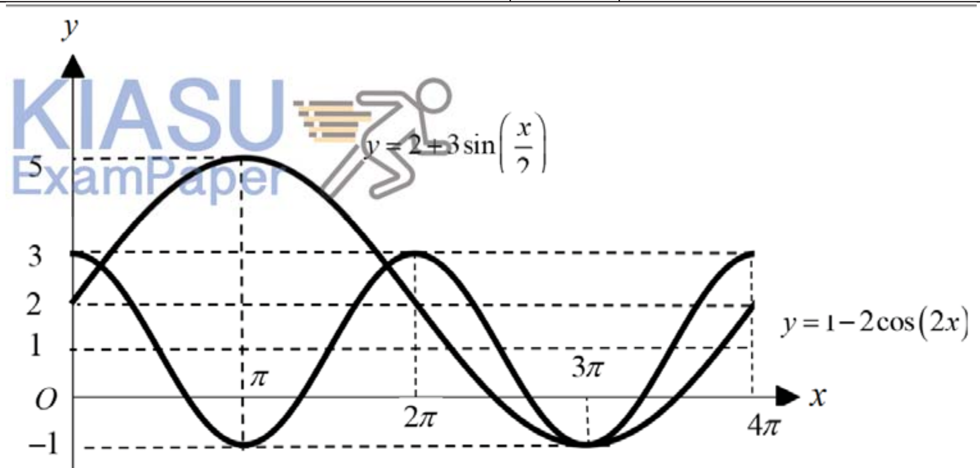
(ii) the total distance travelled by the particle in the first four seconds, [4]

(iii) the velocity of the particle when $t = 4$. [1]

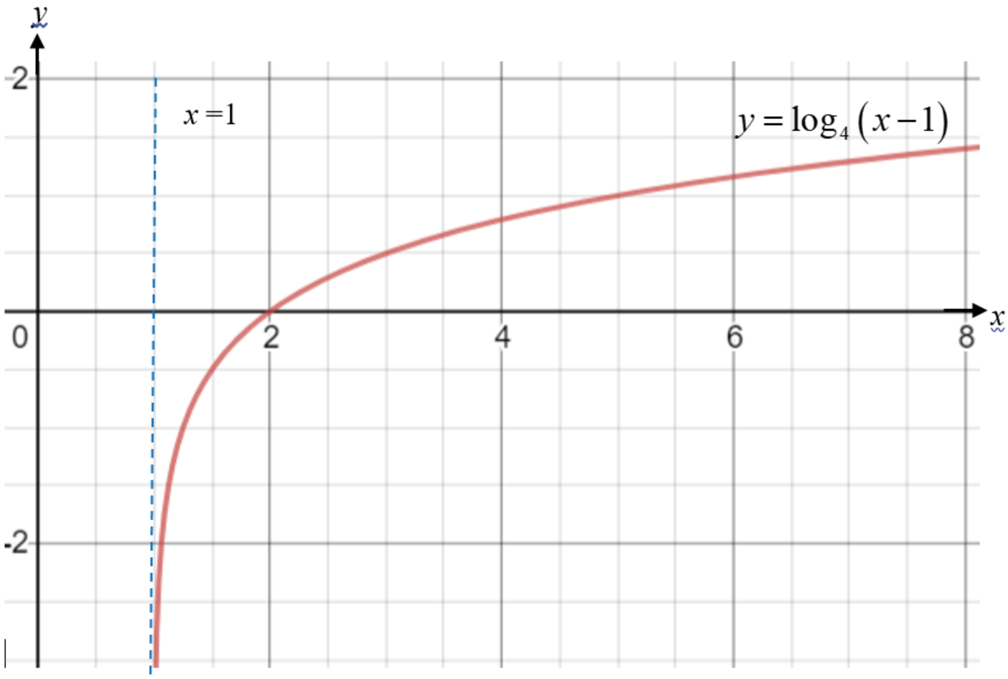
(iv) From $t = 4$, the acceleration of the particle changes to $a = -2t + k$, where k is a constant. The instantaneous velocity remained unchanged at $t = 4$. Given that the particle comes to rest at $t = 5$, show that $k = 11$. [4]

~End of Paper~

Answer Key

1	$8-4\sqrt{3}$ **2**	**2(i)**	
2(ii)	$k=\pm 48$	**3**	$-\frac{4}{5}$
4	$k<-3$ or $k>1$	**5(i)**	$\therefore \mathrm{f}(x)=\ln(x-2)-\frac{1}{2e^{2x}}-\frac{1}{2e^{6}}$
5(ii)	The function $\mathrm{f}'(x)=\frac{1}{x-2}+e^{-2x}$ is decreasing.		
7(a)	$\therefore x=1, x=1.69$ or -1.19	**7(b)**	256
8	$x^2+x+\frac{1}{4}=0$ or $4x^2+4x+1=0$	**9(ii)**	$\int \frac{\ln x}{x^4}\,dx=-\frac{1}{9x^3}-\frac{\ln x}{3x^3}+c$
10(i)	$y=-x+2$	**10(ii)**	10 unit^2
11(ii)	$\frac{1}{a}=-1\pm 0.1$ $a=-1$ (accept -1.11 to -0.909) $-\frac{b}{a}=\text{Gradient}$ $=2\pm 0.1$ $b=a\times gradient$	**11(iii)**	Accept from 1.24 to 1.28
12(i)	$(10,-5)$	**12(ii)**	$y=0$ or $y=-\frac{4}{3}x$
12(iii)	$(10,0)$ and $(6,-8)$	**13(i)**	$\therefore A=\frac{3\pi}{2}$, 0.253 or 2.89
13(ii)			
13(iii)	Replacing x with 2A.		

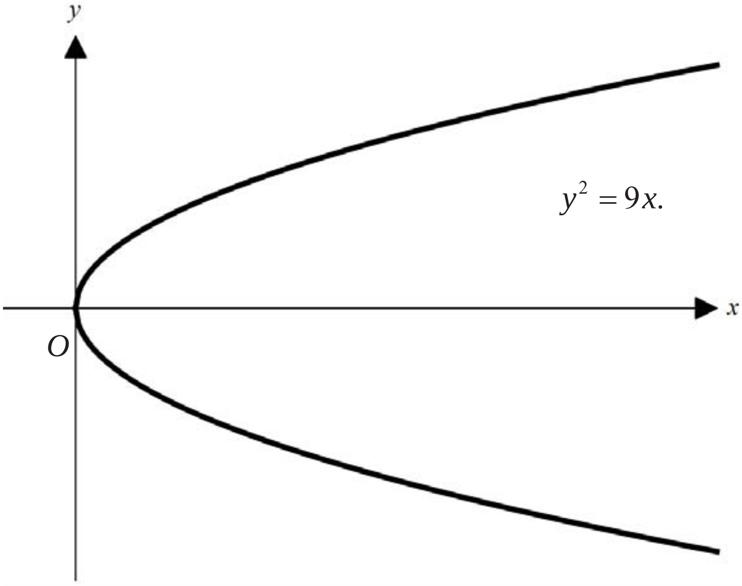
Answers

1(a)	0.528	1(b)	0.630, 1.41	1(ci)	$p = 4,\ q = 0$
1 (cii)					
2(i)	$P = 750$	2(ii)	$k = 0.0522$		
2(iii)	As t becomes very large, $e^{10-0.3t}$ approaches zero. $20 + e^{10-0.3t}$ approaches 20 $\frac{164000}{20+e^{10-0.3t}}$ approaches $\frac{164000}{20}$ … therefore 8200 **Thus, the largest possible number of bacteria B that can be present is 8200.**				
3(i)	$3 - \frac{4x}{x^2+3} + \frac{2}{x+1}$	3(ii)	$\frac{2x}{x^2+3}$	3(iii)	$3x + 2\ln(x+1) - 2\ln(x^2+3) + c$
3(b)	$g(x) = f(x) - 7$	4(i)	$n = 2$	4(ii)	-972
5a(i)	$a = 4$	5a(ii)	$b = 3$	5a(iii)	$k = 4$
5a(iv)	$-2 < m < \frac{5}{3}$	5(b)	-6, 10		
6(i)	$y = 2x - 20$	6(ii)	150	7(ii)	$\frac{4\sqrt{3}}{3} - \frac{\pi}{2}$
8a(i)	$3e^{-2x}(-2x+1)$	8a(ii)	$p = -9$	8b(ii)	$\frac{\pi}{6}$ rad
9(i)	$H = 17\cos(\theta - 61.9^\circ) + 20$	10(ii)	$x = 15.6$	10(iii)	$\frac{d^2A}{dx^2} = -76.8$, Maximum
11(i)	3	11(ii)	10	11(iii)	-2
11(iv)	11				

	Solutions
1	$\cot 75° = \dfrac{1-\tan 45° \tan 30°}{\tan 45° + \tan 30°}$ $= \dfrac{1-1\times\frac{\sqrt{3}}{3}}{1+\frac{\sqrt{3}}{3}}$ $= \dfrac{3-\sqrt{3}}{3+\sqrt{3}} \times \dfrac{3-\sqrt{3}}{3-\sqrt{3}}$ $= 2-\sqrt{3}$ $\operatorname{cosec}^2 75° = 1+\left(2-\sqrt{3}\right)^2$ $= 1+4+3-4\sqrt{3}$ $= 8-4\sqrt{3}$
	Alternative Solution 1: $\sin 75° = \sin 45° \cos 30° + \cos 45° \sin 30°$ $= \dfrac{\sqrt{2}}{2}\times\dfrac{\sqrt{3}}{2}+\dfrac{\sqrt{2}}{2}\times\dfrac{1}{2}$ $= \dfrac{\sqrt{6}+\sqrt{2}}{4}$ $\operatorname{cosec}^2 75° = \dfrac{16}{\left(\sqrt{6}+\sqrt{2}\right)^2}$ $= \dfrac{16}{8+4\sqrt{3}}$ $= \dfrac{4}{2+\sqrt{3}} \times \dfrac{2-\sqrt{3}}{2-\sqrt{3}}$ $= 8-4\sqrt{3}$
	Alternative Solution 2: $\operatorname{cosec}^2 75° = \dfrac{1}{\sin^2 75°}$ $= \dfrac{2}{1-\cos 150°}$ $= \dfrac{2}{1+\frac{\sqrt{3}}{2}}$ $= \dfrac{4}{2+\sqrt{3}} \times \dfrac{2-\sqrt{3}}{2-\sqrt{3}}$ $= 8-4\sqrt{3}$
	Total Marks: 4m

This paper consists of 12 printed pages.

2 **(i)**



(ii)

$y^2 = 9x$ ----(1)

$y = kx^{-\frac{3}{2}}$ ----(2)

Equating equations (1) and (2) gives,

$9x = \left(kx^{-\frac{3}{2}}\right)^2$

Since the curves meet at a point $x = 4$.

$36 = k^2 \times 4^{-3}$

$k = \pm 48$

Alternatively,

From (1), $y = \pm 3x^{\frac{1}{2}}$

$3\sqrt{x} = kx^{-\frac{3}{2}}$ or $-3\sqrt{x} = kx^{-\frac{3}{2}}$

Since the curves meet at a point $x = 4$.

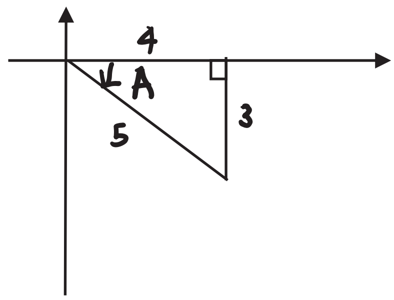
$12 = k \times 4^{-\frac{3}{2}}$ or $-12 = k \times 4^{-\frac{3}{2}}$

$k = 48$ $k = -48$

Total Marks: 4m

3

$\tan A = -\frac{3}{4}$



A is in the 4^{th} quadrant.

$$\sin\left(A-\frac{\pi}{2}\right)=\sin\left[-\left(\frac{\pi}{2}-A\right)\right]$$
$$=-\sin\left(\frac{\pi}{2}-A\right)$$
$$=-\cos A$$
$$=-\frac{4}{5}$$

Alternatively,

$$\sin\left(A-\frac{\pi}{2}\right)=\sin\left[-\left(\frac{\pi}{2}-A\right)\right]$$
$$=-\sin\left(\frac{\pi}{2}-A\right)$$
$$=-\left(\sin\frac{\pi}{2}\cos A-\cos\frac{\pi}{2}\sin A\right)$$
$$=-\left(1\times\frac{4}{5}-0\times-\frac{3}{5}\right)$$
$$=-\frac{4}{5}$$

Alternatively,

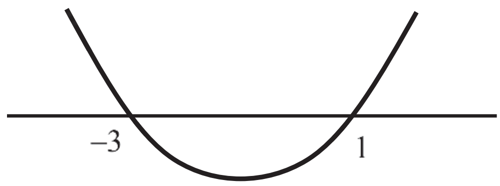
$$\sin\left(A-\frac{\pi}{2}\right)=\sin A\cos\frac{\pi}{2}-\cos A\sin\frac{\pi}{2}$$
$$=-\frac{3}{5}\times 0-\left(-\frac{4}{5}\right)\times 1$$
$$=-\frac{4}{5}$$

Total Marks: 3m

4

For the equation $x^2-(k+1)x+1=0$ to have 2 distinct and real roots p and q,

$$(k+1)^2-4\times 1>0$$
$$k^2+2k-3>0$$
$$(k+3)(k-1)>0$$



$k<-3$ or $k>1$

Total Marks: 4m

5	**(i)**	$f'(x)=\frac{1}{x-2}+e^{-2x}$ for $x>a$ $\int \frac{1}{x-2}+e^{-2x}\,dx=\ln(x-2)-\frac{e^{-2x}}{2}+c$, where c is an arbitrary constant At $\left(3,-\frac{1}{e^6}\right)$, $-e^{-6}=\ln(3-2)-\frac{e^{-2\times3}}{2}+c$ $-e^{-6}=-\frac{e^{-6}}{2}+c$ $c=-\frac{e^{-6}}{2}$ or -0.00124 $\therefore f(x)=\ln(x-2)-\frac{1}{2e^{2x}}-\frac{1}{2e^6}$ or $\therefore f(x)=\ln(x-2)-\frac{1}{2e^{2x}}-0.00124$
	(ii)	$f'(x)=\frac{1}{x-2}+e^{-2x}$ for $x>a$ $f''(x)=-\frac{1}{(x-2)^2}-2e^{-2x}$ Since $(x-2)^2>0$ and $e^{-2x}>0$ for $x>2$ $f''(x)=-\frac{1}{(x-2)^2}-2e^{-2x}<0$ Thus, the function $f'(x)=\frac{1}{x-2}+e^{-2x}$ is decreasing.
		Total Marks: 7m
6		For the curve $y=mx^2+(m+n)x+n$ to lie entirely above the x-axis, it would not have any x-intersections. $b^2-4ac=(m+n)^2-4mn$ $=m^2+2mn+n^2-4mn$ $=m^2-2mn+n^2$ $=(m-n)^2$ Since $(m-n)^2\geq0$ for all values of m and n, the curve would intersect with the x-axis. The curve will not lie completely above the x-axis.
		Total Marks: 3m

7	**(a)**	Let $\mathrm{f}(x) = 2x^3 - 3x^2 - 3x + 4$ $\mathrm{f}(1) = 2(1)^3 - 3(1)^2 - 3(1) + 4 = 0$ By Factor Theorem, $(x-1)$ is a factor of $\mathrm{f}(x)$. Let $2x^3 - 3x^2 - 3x + 4 = (x-1)(2x^2 + bx - 4)$ Compare the coefficient of x $-3 = -4 - b$ $b = -1$ $\mathrm{f}(x) = (x-1)(2x^2 - x - 4)$ $(x-1)(2x^2 - x - 4) = 0$ $x = 1, \text{ or } x = \dfrac{1 \pm \sqrt{1+32}}{4}$ $\therefore x = 1, x = 1.69 \text{ or } -1.19$
7	**(b)**	By Factor Theorem, $\mathrm{p}(5) + 1 = 0$ $\mathrm{p}(5) = -1$ By Remainder Theorem, $\mathrm{g}(5) = 2(5)^3 - \mathrm{p}(5) + 5$ $=256$ **Alternatively,** $\mathrm{g}(x) = 2(x)^3 + 6 - \mathrm{p}(x) - 1$ $\mathrm{g}(5) = 2(5)^3 + 6 - \mathrm{p}(5) - 1$ $=256$
		Total Marks: 6m

8	$\frac{1}{\alpha}\times\frac{1}{\beta}=\frac{1}{4}---(1)$ $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{1}{2}$ $\frac{\alpha+\beta}{\alpha\beta}=\frac{1}{2}$ From(1), $\alpha\beta=4$ $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{1}{2}$ $\frac{\alpha+\beta}{\alpha\beta}=\frac{1}{2}$ $\alpha+\beta=2$ Sum of roots is $\frac{\beta}{\alpha^2}+\frac{\alpha}{\beta^2}$ $=\frac{\alpha^3+\beta^3}{(\alpha\beta)^2}$ $=\frac{(\alpha+\beta)\left[(\alpha+\beta)^2-3\alpha\beta\right]}{(\alpha\beta)^2}$ $=\frac{2\left[(2)^2-3(4)\right]}{4^2}$ $=-1$ Product of roots is $\frac{\beta}{\alpha^2}\times\frac{\alpha}{\beta^2}=\frac{1}{\alpha\beta}$ $=\frac{1}{4}$ The quadratic equation $x^2+x+\frac{1}{4}=0$ or $4x^2+4x+1=0$
	Total Marks: 5m

9	**(i)**	$\frac{d}{dx}\left(\frac{2\ln x}{x^3}\right)=\frac{x^3\left(\frac{2}{x}\right)-2\ln x\left(3x^2\right)}{x^6}$ $=\frac{2x^2-6x^2\ln x}{x^6}$ $=\frac{2x^2}{x^6}-\frac{6x^2\ln x}{x^6}$ $=\frac{2}{x^4}-\frac{6\ln x}{x^4}$
	(ii)	$\int \frac{2}{x^4}-\frac{6\ln x}{x^4}\,dx=\frac{2\ln x}{x^3}+c$ where c is an arbitrary constant $\int \frac{2}{x^4}\,dx-6\int\frac{\ln x}{x^4}\,dx=\frac{2\ln x}{x^3}+c$ $6\int\frac{\ln x}{x^4}\,dx=\int\frac{2}{x^4}\,dx-\frac{2\ln x}{x^3}+c_1$ where c_1 is an arbitrary constant $=-\frac{2}{3x^3}-\frac{2\ln x}{x^3}+c_2,$ where c_2 is an arbitrary constant $\int\frac{\ln x}{x^4}\,dx=-\frac{1}{9x^3}-\frac{\ln x}{3x^3}+c_3,$ where c_3 is an arbitrary constant or $=-\frac{1+3\ln x}{9x^3}+c_3$
		Total Marks: 6m

10	**(i)**	$y=\dfrac{32}{(4-x)^2}-8$ $\dfrac{dy}{dx}=\dfrac{64}{(4-x)^3}$ when $x=8$, $\dfrac{dy}{dx}=-1 \Rightarrow$ gradient $=-1$ When $x=8$, $y=\dfrac{32}{(4-8)^2}-8=-6$ $\therefore (8,-6)$ Equation of tangent $y+6=-(x-8)$ $y=-x+2$
	(ii)	When $y=0$, $0=\dfrac{32}{(4-x)^2}-8$ $x=2$ or 6 $\therefore A(2,0)$ and $B(6,0)$ Area of shaded region $=\dfrac{1}{2}\times(8-2)\times 6-\left\|\int_6^8 \dfrac{32}{(4-x)^2}-8\ dx\right\|$ $=18\text{-}\left\|\left[\dfrac{32}{(4-x)}-8x\right]_6^8\right\|$ $=18\text{-}\left\|\left(\dfrac{32}{(4-8)}-64\right)-\left(\dfrac{32}{(4-6)}-48\right)\right\|$ $=18-\|-72+64\|-(2)+2\dfrac{5}{8}$ $=10$ unit^2
		Total Marks: 9m

11 **(i)**

x	1	1.22	1.29	1.32	1.34
y	1	2	3	4	5
$Y = x^2 y$	1	3.00	4.99	6.97	8.98
X=y	1	2	3	4	5

(ii)

$y = \dfrac{1}{ax^2 + b}$

$\left(ax^2 + b\right) y = 1$

$ax^2 y + by = 1$

$x^2 y = \dfrac{1}{a} - \dfrac{b}{a} y$

$\dfrac{1}{a} = -1 \pm 0.1$

$a = -1$ (accept -1.11 to -0.909)

$-\dfrac{b}{a} = \text{Gradient}$

$= \dfrac{8-2}{4.5-1.5}$

$= 2 \pm 0.1$

$b = a \times gradient$

(iii)

$2.5 = \dfrac{1}{ax^2 + b}$

Draw the vertical line $\text{X} = 2.5$

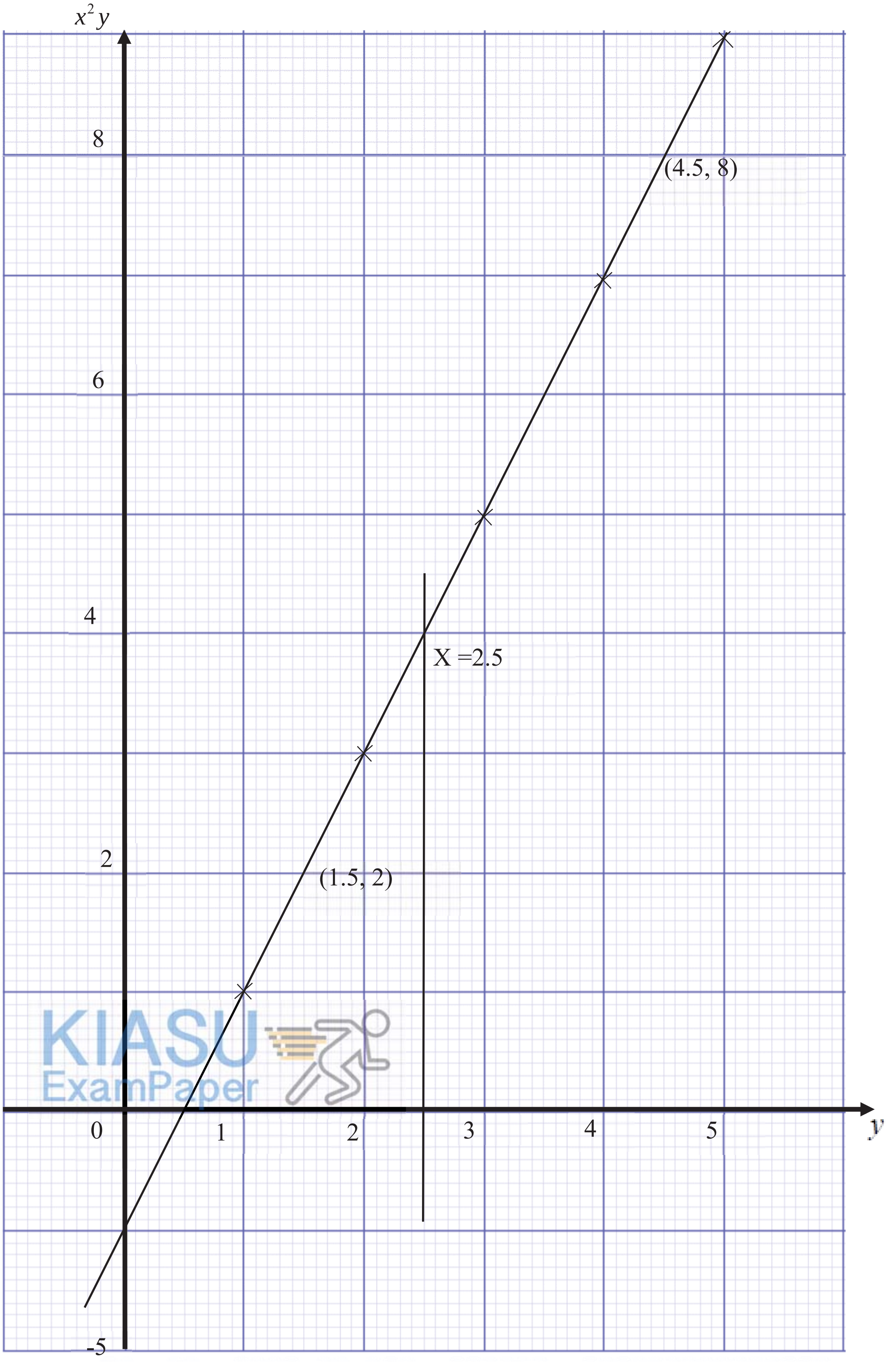
$x^2 y = 4$

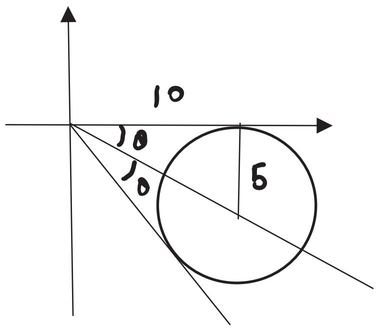
$x = \sqrt{\dfrac{4}{2.5}}$ $(x > 0)$

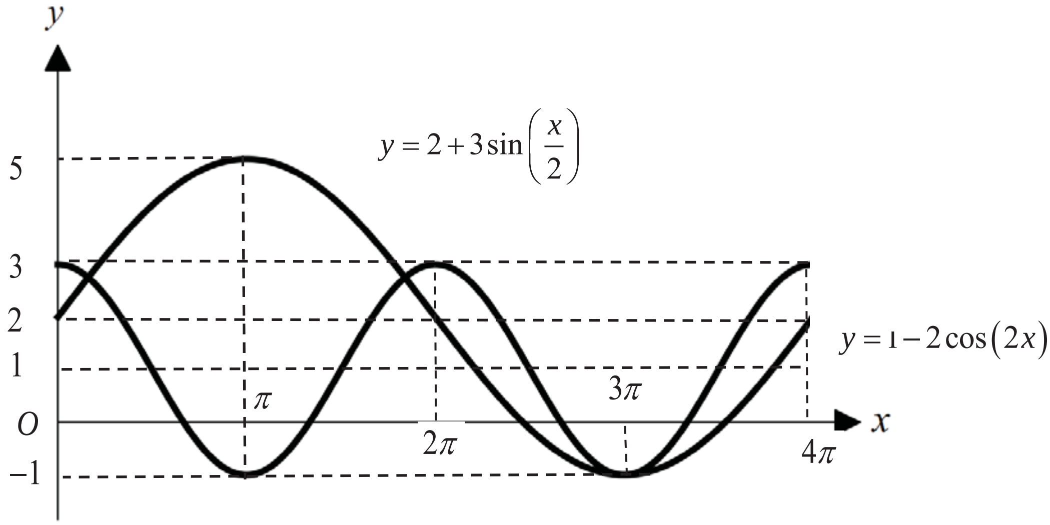
$= 1.26$

(accept from 1.24 to 1.28)

Total Marks: 8m


x^2y
8
6
4
2
-5
0
1
2
3
4
5
y
(4.5, 8)
X =2.5
(1.5, 2)
KIASU ExamPaper

12	**(i)**	$x^2+y^2-20x+10y+100=0$ $x^2-20x+10^2+y^2+10y+5^2=-100+10^2+5^2$ $(x-10)^2+(y+5)^2=25$ Radius of the circle =5 units Centre of the cirlce: is $(10,-5)$
	(ii)	Let the equation(s) of the tangent be $y=mx$ $x^2+(mx)^2-20x+10(mx)+100=0$ $(m^2+1)x^2+(10m-20)x+100=0$ $b^2-4ac=0$ $(10m-20)^2-4(m^2+1)\times 100=0$ $-3m^2-4m=0$ $m(-3m-4)=0$ $m=0$ or $m=-\frac{4}{3}$ Hence, equations of tangent are $y=0$ or $y=-\frac{4}{3}x$. **Alternatively,** Equation of the tangent $y=0$. $\tan\theta=\frac{1}{2}$ $\tan 2\theta=\frac{1}{1-\frac{1}{4}}$ (using double angle formula) $=\frac{4}{3}$  Hence the equation of the other tangent is $y=-\frac{4}{3}x$.
	(iii)	Since the tangent $y=0$ is x-axis, and tangent ⊥ radius, the point required is $(10,0)$ For the other tangent line $y=-\frac{4}{3}x$, let the point required be $\left(a,-\frac{4}{3}a\right)$ $\frac{-5-\left(-\frac{4}{3}a\right)}{10-a}=\frac{3}{4}$ $a=6$ Hence, the point required is $(6,-8)$.
		Total Marks: 11m

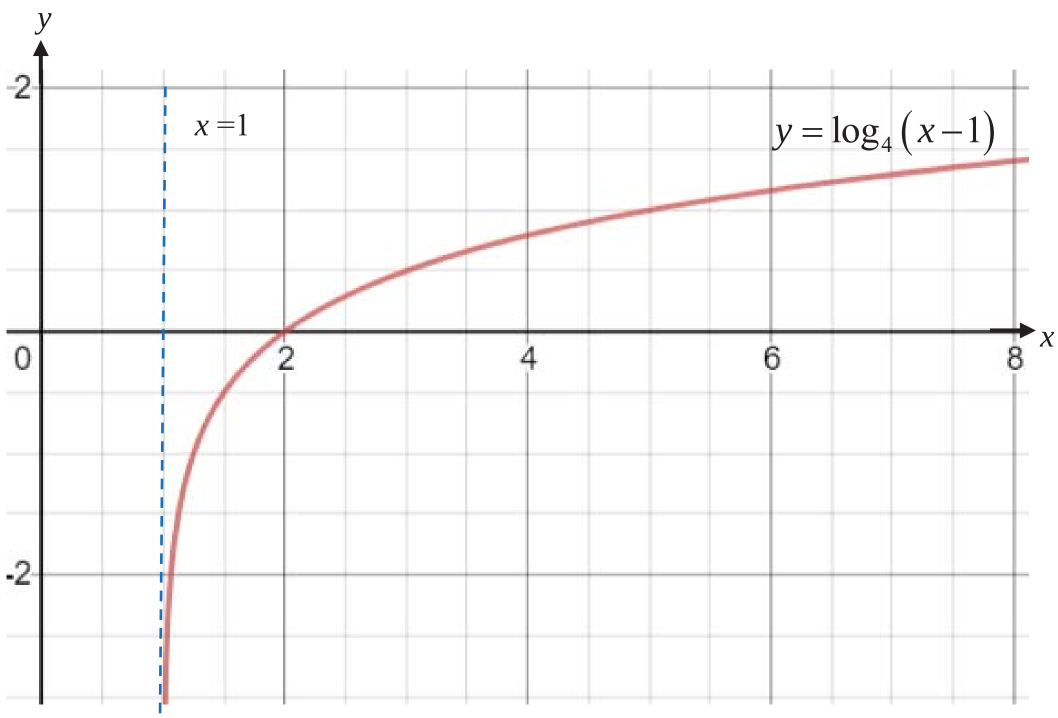
13	**(i)**	$2\cos 2A - 3\sin A - 1 = 0$ $2(1-2\sin^2 A) - 3\sin A - 1 = 0$ $-4\sin^2 A - 3\sin A + 1 = 0$ $(\sin A + 1)(-4\sin A + 1) = 0$ $\sin A = -1$ or $4\sin A + 1 = 0$ $A = \frac{3\pi}{2}$ or $\sin A = \frac{1}{4}$ basic $\angle$=0.25268 $A = 0.253,\ 2.89$ $\therefore A = \frac{3\pi}{2},\ 0.253$ or 2.89
	(ii)	
	(iii)	$2 + 3\sin\left(\frac{x}{2}\right) = 2\cos(x) + 1$ $2\cos(x) - 3\sin\left(\frac{x}{2}\right) - 1 = 0$ sub $x = 2A$ $2\cos 2A - 3\sin A - 1 = 0$ From Part(i), $A = \frac{3\pi}{2},\ 0.253$ or 2.89 $x = 3\pi, 0.506, 5.78$
		Total Marks: 10m

2020 S4 Prelim AM P2 Solution

Solutions

1	**(a)**	$2^{6x}+\frac{7\left(2^{6x}\right)}{2^{3x}} = 30$ $2^{6x}+7\left(2^{3x}\right)-30=0$ Let $u=2^{3x}$ $u^2+7u-30=0$ $(u-3)(u+10)=0$ $u-3=0$ OR $u+10=0$ $u=3$ $u=-10$ $2^{3x}=3$ $2^{3x}=-10$ (Reject, no solution) $3x=\frac{\lg 3}{\lg 2}$ $x=\frac{1}{3}\left(\frac{\lg 3}{\lg 2}\right)$ $x=0.528$ (3sf)
	(b)	$\log_x 2=\log_7\sqrt{7}+3\log_2 x$ $\frac{1}{\log_2 x}=\frac{1}{2}+3\log_2 x$ Let $u=\log_2 x$ $\frac{1}{u}=\frac{1}{2}+3u$ $2=u+6u^2$ $6u^2+u-2=0$ $(2u-1)(3u+2)=0$ $u=\frac{1}{2}$ OR $u=-\frac{2}{3}$ $\log_2 x=\frac{1}{2}$ $\log_2 x=-\frac{2}{3}$ $x=2^{\frac{1}{2}}$ $x=2^{-\frac{2}{3}}$ $x=1.41$ $x=0.630$ (3sf)

	c(i)	$y=\log_p(x-1)$
		Sub (5, 1)
		$1=\log_p(5-1)$
		$p=4$
		Sub $p=4$, $x=2$, $y=q$
		$q=\log_4(2-1)$
		$q=0$

	c(ii)	
2	**(i)**	When $t=0$, $P=300e^{k(0)}+450$ $P=750$
	(ii)	$1500=300e^{24k}+450$ $300e^{24k}=1050$ $e^{24k}=3.5$ $24k=\ln(3.5)$ $k=\dfrac{\ln(3.5)}{24}$ $k=0.0522$ (3sf)

	(iii)	As t becomes very large, $e^{10-0.3t}$ approaches zero. $20+e^{10-0.3t}$ approaches 20 $\frac{164000}{20+e^{10-0.3t}}$ approaches $\frac{164000}{20}$ … therefore 8200 Thus, the largest possible number of bacteria B that can be present is 8200.
3	**(i)**	$$\begin{array}{r} 3 \\ 3x^3-3x^2+3x+3\overline{\big)3x^3+x^2+5x+15} \\ -(3x^3+3x^2+9x+9) \\ \hline -2x^2-4x+6 \\ \hline \end{array}$$ $\frac{3x^3+x^2+5x+15}{(x^2+3)(x+1)}=3+\frac{(-2x^2-4x+6)}{(x^2+3)(x+1)}$ Let $\frac{-2x^2-4x+6}{(x^2+3)(x+1)}=\frac{Ax+B}{x^2+3}+\frac{C}{x+1}$ $-2x^2-4x+6=(Ax+B)(x+1)+C(x^2+3)$ Sub $x=-1$, $-2(-1)^2-4(-1)+6=C\left[(-1)^2+3\right]$ $C=2$ By comparison of coefficient, Constant: $6 = B + 3C$ $6 = B + 3(2)$ $B = 0$ x^2 term: $-2 = A + C$ $A = -4$ $\frac{3x^3+x^2+5x+15}{(x^2+3)(x+1)}=3-\frac{4x}{x^2+3}+\frac{2}{x+1}$

		Alternative Solution: $\frac{3x^3+x^2+5x+15}{(x^2+3)(x+1)}=3-\frac{2x^2+4x-6}{(x^2+3)(x+1)}$ Let $\frac{2x^2+4x-6}{(x^2+3)(x+1)}=\frac{Ax+B}{x^2+3}+\frac{C}{x+1}$ $2x^2+4x-6=(Ax+B)(x+1)+C(x^2+3)$ Sub $x=-1$, $2(-1)^2+4(-1)-6=C\left[(-1)^2+3\right]$ $C=-2$ By comparison of coefficient, Constant: $-6=B+3C$ $-6=B+3(-2)$ $B=0$ x^2 term: $2=A+C$ $2=A-2$ $A=4$ $\frac{3x^3+x^2+5x+15}{(x^2+3)(x+1)}=3-\frac{4x}{x^2+3}+\frac{2}{x+1}$
	(ii)	$\frac{d}{dx}\ln(x^2+3)=\frac{2x}{x^2+3}$
	(iii)	$\int\frac{3x^3+x^2+5x+15}{(x^2+3)(x+1)}\,dx$ $=\int 3+\frac{2}{x+1}-\frac{4x}{x^2+3}\,dx$ $=\int 3\,dx+\int\frac{2}{x+1}\,dx-\int\frac{4x}{x^2+3}\,dx$ $=\int 3\,dx+2\int\frac{1}{x+1}\,dx-2\int\frac{2x}{x^2+3}\,dx$ $=3x+2\ln(x+1)-2\ln(x^2+3)+c$

	(b)	Since $f'(x)=g'(x)$, $f(x)+c_1=g(x)+c_2$ $g(x)=f(x)+c_1-c_2$ Since (3, 6) is 7 units above $(3,-1)$ $g(x)=f(x)-7$
4	**(i)**	$T_{r+1}=\binom{6}{r}(3x)^{6-r}\left(-\frac{n}{x^2}\right)^r$ $T_{r+1}=\binom{6}{r}3^{6-r}(-n)^r x^{6-3r}$ $6-3r=0$ $r=2$ When $r = 2$, $\binom{6}{2}3^{6-2}(-n)^2=4860$ $n^2=4$ $n=-2$ (reject, since n is positive) $n=2$
	(ii)	For x^3 term in $\left(3x-\frac{n}{x^2}\right)^6$, $6-3r=3$ $r=1$ When $r = 1$, $T_2=\binom{6}{1}3^{6-1}(-2)^1 x^{6-3}$ $T_2=-2916x^3$ $\left(1+\frac{2}{x^3}\right)\left(3x-\frac{n}{x^2}\right)^6 = \left(1+\frac{2}{x^3}\right)\left(4860-2916x^3+...\right)$ $=(1)(4860)+\left(\frac{2}{x^3}\right)\left(-2916x^3\right)+...$ $=-972+...$ Constant term = -972

2020 S4 Prelim AM P2 Solution

5	**a(i)**	$a = 4$
	a(ii)	$y = a - \|2x - 2b\|$ $-2b = -6$ $b = 3$
	a(iii)	$k = 4$
	a(iv)	Gradient of left arm of graph = $\frac{5}{3}$ Gradient of right arm of graph = -2 $-2 < m < \frac{5}{3}$
	(b)	$2\|x-2\| = \|3x-6\| - 8$ $2\|x-2\| = \|3\|\|x-2\| - 8$ $\|x-2\| = 8$ $x - 2 = 8$ OR $x - 2 = -8$ $x = 10$ $x = -6$ <u>Check:</u> When $x = -6$, LHS = $2\|-6-2\| + 3 = 19$ RHS = $\|6 - 3(-6)\| - 5 = 19$ LHS = RHS When $x = 10$, LHS = $2\|10-2\| + 3 = 19$ RHS = $\|6 - 3(10)\| - 5 = 19$ LHS = RHS $x = 10$ OR $x = -6$
6	**(i)**	Gradient of OQ = $\frac{-8-0}{16-0}$ $= -\frac{1}{2}$ $OQ \perp PS$ $\therefore$ Gradient of $PS = 2$ Since $OPQR$ is a rhombus, $\left(\frac{0+16}{2}, \frac{0-8}{2}\right) = (8, -4)$ $y + 4 = 2(x - 8)$ $y = 2x - 20$ Hence, the equation is $y = 2x - 20$

	(ii)	Gradient of PR $=2$ (from part (i)) $y=2(10)+c$ $c=-20$ Equation of PR: $y=2x-20$ $\therefore S(0,-20)$ When y = 0, $0=2x-20$ $x=10$ $P(10,\ 0)$ $T(0,-10)$ Area of OPQS $=\frac{1}{2}\begin{vmatrix} 0 & 0 & 16 & 10 & 0 \\ -10 & -20 & -8 & 0 & -10 \end{vmatrix}$ = 150 units^2
7	**(i)**	LHS = $\frac{2\sin 2x-\sin 4x}{2\sin 2x+\sin 4x}$ $=\frac{2\sin 2x-2\sin 2x\cos 2x}{2\sin 2x+2\sin 2x\cos 2x}$ $=\frac{2\sin 2x(1-\cos 2x)}{2\sin 2x(1+\cos 2x)}$ $=\frac{1-\cos 2x}{1+\cos 2x}$ $=\frac{1-(1-2\sin^2 x)}{1+(2\cos^2 x-1)}$ $=\frac{2\sin^2 x}{2\cos^2 x}$ $=\tan^2 x$ = RHS

	(ii)	$\int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{2\sin 2x(1-\cos 2x)}{2\sin 2x(1+\cos 2x)}\,dx$ $=\int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} \tan^2 x\,dx$ $=\int_{-\frac{\pi}{6}}^{\frac{\pi}{3}} (\sec^2 x-1)\,dx$ $=[\tan x - x]_{-\frac{\pi}{6}}^{\frac{\pi}{3}}$ $=\left[\tan\left(\frac{\pi}{3}\right)-\frac{\pi}{3}\right]-\left[\tan\left(-\frac{\pi}{6}\right)+\frac{\pi}{6}\right]$ $=\sqrt{3}-\frac{\pi}{3}-\left(-\frac{1}{\sqrt{3}}+\frac{\pi}{6}\right)$ $=\sqrt{3}+\frac{1}{\sqrt{3}}-\frac{\pi}{2}$ $=\frac{4\sqrt{3}}{3}-\frac{\pi}{2}$
8	**a(i)**	$\frac{dy}{dx}=(3x)(-2e^{-2x})+(e^{-2x})(3)$ $\frac{dy}{dx}=3e^{-2x}(-2x+1)$
	a(ii)	$\frac{dy}{dx}=3e^{-2x}(-2x+1)$ $\frac{d^2y}{dx^2}=3e^{-2x}(-2)+(-2x+1)(3e^{-2x})(-2)$ $\frac{d^2y}{dx^2}=-6e^{-2x}(2-2x)$ $\frac{d^2y}{dx^2}=12e^{-2x}(x-1)$ $p=e^{2x}\left(\frac{d^2y}{dx^2}+\frac{dy}{dx}-2y\right)$ $p=e^{2x}\left[(12xe^{-2x}-12e^{-2x})+(3e^{-2x}-6xe^{-2x})-2(3xe^{-2x})\right]$ $p=12x-12+3-6x-6x$ $p=-9$

	b(i)	$y = \ln\left(\frac{1-\cos x}{\sin x}\right)$ $y = \ln(1-\cos x) - \ln(\sin x)$ $\frac{dy}{dx} = \frac{\sin x}{1-\cos x} - \frac{\cos x}{\sin x}$ $\frac{dy}{dx} = \frac{\sin^2 x - \cos x(1-\cos x)}{\sin x(1-\cos x)}$ $\frac{dy}{dx} = \frac{\sin^2 x - \cos x + \cos^2 x}{\sin x(1-\cos x)}$ $\frac{dy}{dx} = \frac{1-\cos x}{\sin x(1-\cos x)}$ $\frac{dy}{dx} = \frac{1}{\sin x}$ $\frac{dy}{dx} = \text{cosec } x$
		Alternative Solution $y = \ln\left(\frac{1-\cos x}{\sin x}\right)$ $\frac{dy}{dx} = \frac{\frac{d}{dx}\left(\frac{1-\cos x}{\sin x}\right)}{\left(\frac{1-\cos x}{\sin x}\right)}$ $\frac{dy}{dx} = \frac{\left[\frac{\sin x(\sin x)-(1-\cos x)(\cos x)}{\sin^2 x}\right]}{\left(\frac{1-\cos x}{\sin x}\right)}$ $\frac{dy}{dx} = \frac{\sin x(\sin x)-(1-\cos x)(\cos x)}{\sin x(1-\cos x)}$ $\frac{dy}{dx} = \frac{\sin^2 x - \cos x + \cos^2 x}{\sin x(1-\cos x)}$ $\frac{dy}{dx} = \frac{1-\cos x}{\sin x(1-\cos x)}$ $\frac{dy}{dx} = \frac{1}{\sin x}$ $\frac{dy}{dx} = \text{cosec } x$

	(ii)	$\frac{dy}{dt}=2\frac{dx}{dt}$ $\frac{dy}{dt}=\left(\frac{dy}{dx}\right)\left(\frac{dx}{dt}\right)$ $2\frac{dx}{dt}=\left(\frac{dy}{dx}\right)\left(\frac{dx}{dt}\right)$ $\frac{dy}{dx}=2$ $\text{cosec } x=2$ $\sin x=\frac{1}{2}$ $x=\frac{\pi}{6}$ rad
9	**(i)**	Let $8\cos\theta+15\sin\theta=R\cos(\theta-\alpha)$ $8\cos\theta+15\sin\theta=R\cos\theta\cos\alpha+R\sin\theta\sin\alpha$ By comparison, $R\cos\alpha=8$ ------ (1) $R\sin\alpha=15$ ------- (2) $(1)^2+(2)^2$: $R^2\cos^2\alpha+R^2\sin^2\alpha=8^2+15^2$ $R=\sqrt{8^2+15^2}$ $R=17$ (2) / (1): $\tan\alpha=\frac{15}{8}$ $\alpha=61.928^\circ$ $H=17\cos(\theta-61.9^\circ)+20$
	(ii)	$H=15\sin\theta+8\cos\theta+20$ $H=17\cos(\theta-61.9^\circ)+20$ $\cos(\theta-61.9^\circ)\le 1$ for all values of θ $17\cos(\theta-61.9^\circ)\le 17$ $17\cos(\theta-61.9^\circ)\le 17$ $17\cos(\theta-61.9^\circ)+20\le 37$ Since maximum height < 50 m, the design will NOT exceed the stipulated height limit.

10	**(i)**	Area of figure = $6xy+\frac{1}{2}(6x)(6x)\sin 60^{\circ}$ $A=6xy+9\sqrt{3}x^2$ ------ (1) erimeter of figure = $2y+3(6x)$ $400=2y+18x$ $y=200-9x$ ------ (2) Sub (2) into (1) $A=6x(200-9x)+9\sqrt{3}x^2$ $A=\left(9\sqrt{3}-54\right)x^2+1200x$ (Shown)
	(ii)	$\frac{dA}{dx}=2\left(9\sqrt{3}-54\right)x+1200$ $\frac{dA}{dx}=0$ $2\left(9\sqrt{3}-54\right)x+1200=0$ $x=-\frac{600}{9\sqrt{3}-54}$ $x=15.6$ (3sf)
	(iii)	$\frac{d^2A}{dx^2}=2\left(9\sqrt{3}-54\right)$ $\frac{d^2A}{dx^2}=-76.8$ $\frac{d^2A}{dx^2}<0$ Value obtained in (ii) is a maximum.
11	**(i)**	When t = 0, $XY=-(0)^2+6(0)+3$ $XY=3$ m

	(ii)	$v=-2t+6$ At instantaneous rest, $v = 0$ $-2t+6=0$ $t=3$ When $t=3$, $s=-(3)^2+6(3)+3$ $s=12$ m When $t=4$, $s=-(4)^2+6(4)+3$ $s=11$ m Total distance travelled = 9 + 1 =10 m
	(iii)	When $t=4$, $v=-2(4)+6$ $v=-2$ m/s
	(iv)	$a=-2t+k$ $v=\int(-2t+k)\,dt$ $v=-t^2+kt+c$ When $t = 4$, $-2=-(4)^2+4k+c$ $c=14-4k$ $v=-t^2+kt+14-4k$ When $t = 5$, $v = 0$, $0=-(5)^2+5k+14-4k$ $k=11$