CANDIDATE NAME

CLASS

INDEX NUMBER

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2020
Secondary 4

Tuesday

11 August 2020

ADDITIONAL MATHEMATICS
PAPER 1

4047/1

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figure, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

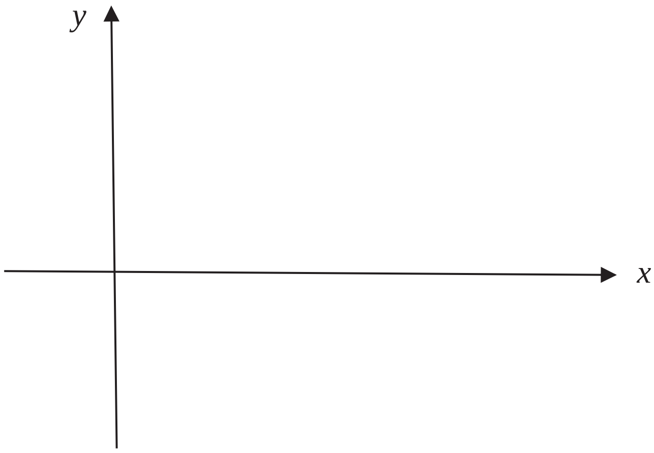
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} .$$

$$a^2 = b^2 + c^2 - 2bc \cos A .$$

$$\Delta = \frac{1}{2} bc \sin A .$$

1. **(i)** On the same diagram, sketch the curves $y^2 = 81x$ and $y = \dfrac{4}{\sqrt{x}}$ for $x > 0$. **[2]**



(ii) The parabola $y^2 = 81x$ intersects the curve $y = \dfrac{4}{\sqrt{x}}$ at the point A.

Find the equation of normal to the curve $y = \dfrac{4}{\sqrt{x}}$ at A. **[5]**

2. The equation of a polynomial is given by $g(x) = 2x^3 - x^2 + 8x - 4$.

(i) Find the remainder when $g(x)$ is divided by $2x$. **[1]**

(ii) Explain why $g(x) = 0$ has only one real root. **[4]**

2. **(iii)** Express $\dfrac{34}{2x^3 - x^2 + 8x - 4}$ in partial fractions. **[4]**

3. Do not use a calculator in this question.

A cylinder with radius $(2+\sqrt{3})$ cm has a volume of $\pi(26+4\sqrt{48})$ cm^3, find the height of the cylinder in the form $m+n\sqrt{3}$, where m and n are integers. **[4]**

4. It is given that $y = \frac{x^2}{2}(3-2x)^5$.

(i) Obtain an expression for $\frac{dy}{dx}$. **[2]**

(ii) Determine the set of values of x for which y is increasing. **[2]**

(iii) A point P moves along the curve $y = \frac{x^2}{2}(3-2x)^5$ in such a way that the y-coordinate of P is decreasing at a rate of 0.05 units per second.
Find the rate of increase of the x-coordinate of P when $x = 1$. **[2]**

5. The function f is defined, for $0° \le x \le 360°$, by $f(x) = a - b\sin cx$, where a, b and c are positive constants.

(a) Given that the period is 240º and the greatest and the least values of $f(x)$ are 7 and -1 respectively, find the values of a, b and c. **[3]**

(b) State the amplitude of $f(x)$. **[1]**

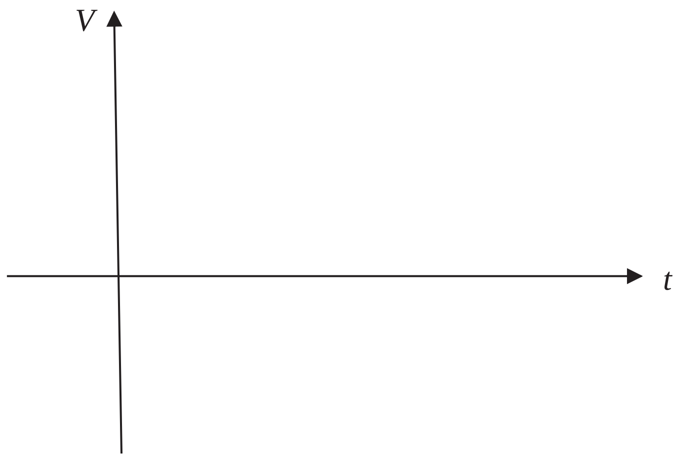
(c) Sketch the graph of $y = a - b\sin cx$. **[2]**

6. At the beginning of 2020, Mr Lim bought an antique watch for \$ 8000. It was believed that the value of the antique watch will increase continuously with time such that its value increased 10% after every 3 years.

(i) Find the value of the antique watch after 15 years, correct to the nearest dollar. **[1]**

(ii) Write down an expression for $\$V$, the value of the antique watch, after Mr Lim owned it for t years. **[1]**

(iii) Sketch the graph of V against t . **[1]**



(iv) Using your answer in part (ii), find the year that Mr Lim's watch first appreciates to \$50 000. **[3]**

7. **(a)** Given that $\int_a^b (\sqrt{3x+2})\ dx = 5$, where a and b are positive constants.

(i) Evaluate $\int_b^a (\sqrt{12x+8})\ dx$ **[2]**

(ii) Explain why $\int_a^b (\sqrt{3x+2})\ dx$ cannot be found when a = -1. **[1]**

(b) Solve the equation $\dfrac{\log_2 x}{\log_x 8} = 3$. **[4]**

8. **(a)** Without using a calculator, and showing all your working, find the exact value of $\cos\frac{\pi}{4}\cos\frac{5\pi}{12}-\sin\frac{\pi}{4}\sin\frac{5\pi}{12}$ **[2]**

(b) Given that θ is reflex and $\cos\theta = p$, where $p > 0$.

Express the following in terms of p.

(i) $\tan\theta$ **[2]**

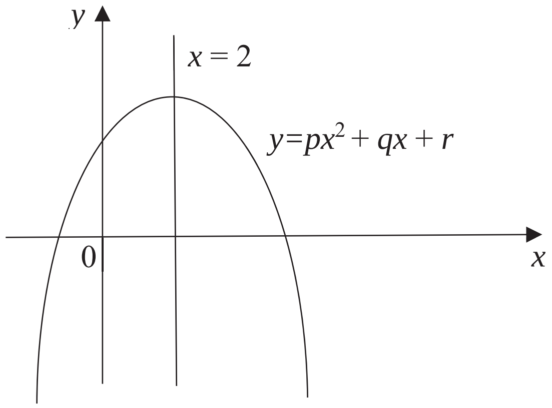
(ii) $\sec(\frac{\pi}{2}-\theta)$ **[2]**

9. **(i)** Write down the general term in the binomial expansion of $\left(2x-\frac{1}{3x^3}\right)^6$. **[1]**

(ii) Explain, showing your working clearly, if there is a term independent of x in the expansion. **[2]**

(iii) Using the general term, or otherwise, determine the term independent of x in the binomial expansion of $\left(3x^2+1+\frac{2}{x^2}\right)\left(2x-\frac{1}{3x^3}\right)^6$. **[4]**

10.



The diagram above shows part of the graph of $y = px^2 + qx + r$.
The equation of line of symmetry of the curve is $x = 2$.
Determine the conditions for each of the following expressions, justifying your answer.

(i) $q^2 - 4pr$ **[2]**

(ii) $\dfrac{dy}{dx}$ when $x = 2$ **[2]**

(iii) q **[2]**

11. The points $P(-6, 1)$, $Q(4, 5)$ and $R\ (-3, 8)$ lie on a circle C.

(i) Find the gradient of each of the lines PR and QR. **[1]**

(ii) What do your answers to part **(i)** imply about the line PQ relative to the circle C ?

Show your working clearly. **[2]**

(iii) Find the equation of the circle C. **[3]**

(iv) Write down the exact coordinates of the lowest point of the circle *C*. **[1]**

(v) Find the equation of tangent to the circle *C* at *P*. **[3]**

12. A curve $y = \mathrm{f}(x)$ is such that $\mathrm{f}''(x) = 24\sin 4x - 12\cos 2x$ and has a stationary point $\left(\frac{\pi}{4}, 1\right)$. Show that $\mathrm{f}''(x) + 4\,\mathrm{f}(x) = k\sin px + q$, where *k*, *p* and *q* are constants to be determined. **[6]**

End of Paper

CANDIDATE NAME	

CLASS		INDEX NUMBER	

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2020
Secondary 4

Thursday	**ADDITIONAL MATHEMATICS**	**4047/02**
13 August 2020	**PAPER 2**	**2 hours 30 minutes**

Candidates answer on the Question Paper.
No Additional Materials are required.

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The total number of marks for this paper is 100.

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$.

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Identities

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$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 Find the largest integer value of k for which the line $y=1-x$ does not intersect the curve $y=x^2+(k+2)x+1-k$. [4]

2 Given that the roots of $3x^2 + 2x = 7$ are α and β, find the quadratic equation whose roots are $\alpha + \frac{2}{\alpha}$ and $\beta + \frac{2}{\beta}$. [5]

3 **(a)** Sketch the graph of $y = |x^2 - 6x + 5|$ for $0 \le x \le 7$. [3]

(b) Solve the equation $|x^2 - 6x + 5| = 4$. [3]

4 **(a)** **(i)** Prove the identity $\dfrac{\sec A - \operatorname{cosec} A}{\cot A + \tan A} \equiv \sin A - \cos A$. [3]

(ii) Hence find all the angles between $0°$ and $180°$ which satisfy the equation $\left(\dfrac{2\sec A - 2\operatorname{cosec} A}{\cot A + \tan A}\right)^2 = 1$. [3]

(b) Find all the angles between 0 and 4 which satisfy $3\sec^2 y + 5\tan y = 5$. [4]

5 **(a)** **(i)** Find $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(xe^{3-2x}\right)$. [2]

(ii) Hence find $\int 4xe^{3-2x}\ \mathrm{d}x$. [3]

(b) The equation of a curve is $y = \dfrac{\ln x}{(3x+1)^2}$. Find the equation of tangent to the curve at $x = 1$. [4]

(c) A curve is such that $\dfrac{dy}{dx} = \dfrac{2}{3x+1} + \dfrac{3}{\sqrt{3x+1}}$. The curve passes through the point $(0, 3)$, find the equation of the curve. [4]

6 **(a)** Using the substitution $u = 5^x$, solve the equation $5^{x+2} - 25^{x+0.5} = 2(5^{x+1})$, giving your answer correct to 2 decimal places. [4]

(b) Solve the simultaneous equations: [4]

$$\log_4(2x + y) = \frac{1}{2} - \log_4 3$$

$$(3\sqrt{3})^x = 27(\sqrt{3})^y$$

7 The table shows experimental values of two variables, x and y, which are connected by an equation of the form, $e^y = ax^b$, where a and b are constants.

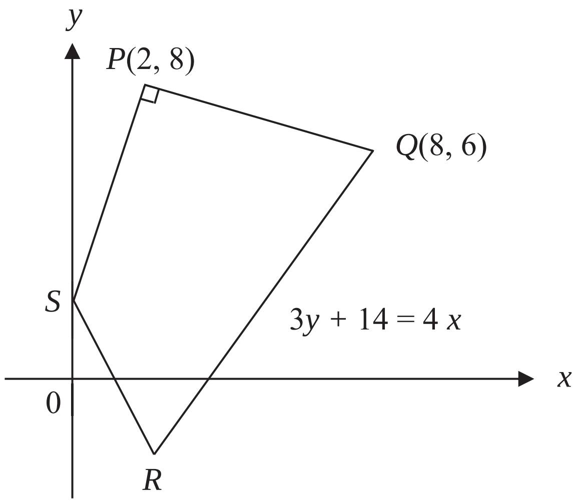
x	2	3	5	7	10
y	2.28	2.94	3.69	4.30	4.88

(i) Plot y against $\ln x$ for the given data and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of a and of b. [4]

(iii) Use your graph to estimate the value of x when $e^y = x^5$. [2]

8 Solutions to this question by accurate drawing will not be accepted.



The diagram shows a quadrilateral $PQRS$ in which the point P is (2, 8) and the point Q is (8, 6). The point R lies on the perpendicular bisector of PQ and the point S lies on the y-axis. The equation of QR is $3y+14=4x$ and angle $SPQ = 90^{\circ}$. Find

(i) the coordinates of S, [3]

(ii) the equation of the perpendicular bisector of PQ, [2]

(iii) the coordinates of R. [3]

(iv) the area of *PQRS*. [2]

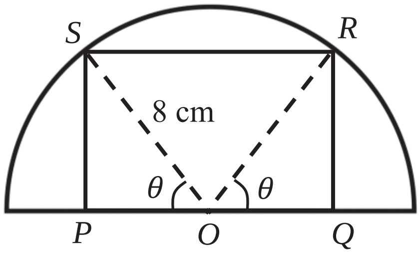
9 A particle moves in a straight line so that its velocity, v m/s, is given by $v = 2t^2 - 8t + 6$, where t is the time in seconds after the start of motion. At $t = 2$, the displacement of the particle from a fixed point O is 1 m. Find

(i) the times when the particle is instantaneously at rest, [2]

(ii) the minimum velocity of the particle and explain the significance of the answer obtained, [3]

(iii) the average speed travelled by the particle in the first 5 seconds. [4]

10



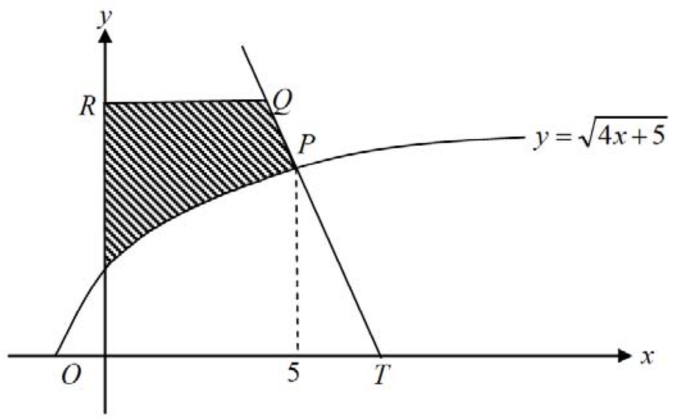
The diagram shows a metal wire, *PQRS*, placed on top of a semicircle metal sheet, with centre *O* and radius 8 cm. $\angle POS = \angle QOR = \theta$ radians.

(a) Show that the length of the metal wire *PQRS*, *L* cm, is given by $L = 16\sin\theta + 32\cos\theta$. [2]

(b) Find the value of θ for which *L* is stationary. [3]

(c) Hence, find the maximum value of *L*. [2]

11

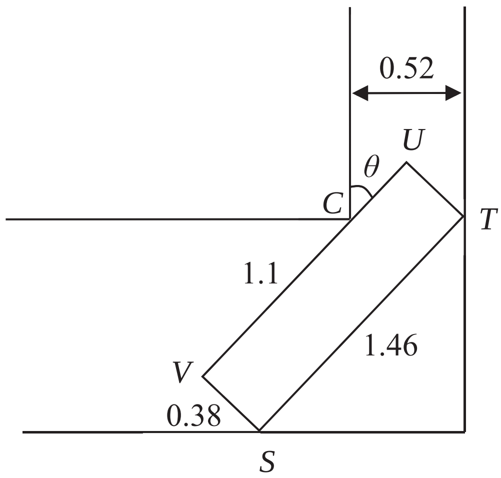


The diagram shows part of the curve $y=\sqrt{4x+5}$, which intersects with the line $x = 5$ at the point P. The normal to the curve at P meets the x-axis at the point T. RQ is parallel to the x-axis and $PT : PQ = 2 : 1$. Find

(a) the coordinates of P and of Q, [5]

(b) the area of the shaded region. [3]

12



The diagram shows two straight corridors meeting at right angles. Corridor A has a width of 0.52 m. $STUV$ is a rectangular block of length 1.46 m and breadth 0.38 m. The block touches the walls of the corridor at S and T, and also touches the inner corner of the corridor at C, where VC = 1.1 m. The angle between CU and the wall at C is θ, where $0° < \theta < 90°$.

(i) Show that $19\cos\theta + 18\sin\theta = 26$. [3]

(ii) Express $19\cos\theta + 18\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

(iii) **Hence**, find the values of θ. [3]

(iv) State the largest value of $19\cos\theta + 18\sin\theta$ and its corresponding value of θ. [2]

~ END OF PAPER ~

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PRELIMINARY EXAMINATION 2020
Secondary 4

Tuesday	**ADDITIONAL MATHEMATICS**	**4047/1**
11 August 2020	**PAPER 1** **(Markscheme)**	**2 hours**

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ALGEBRA

Quadratic Equation

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$.

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Identities

$$\sin^2 A + \cos^2 A = 1$$

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$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

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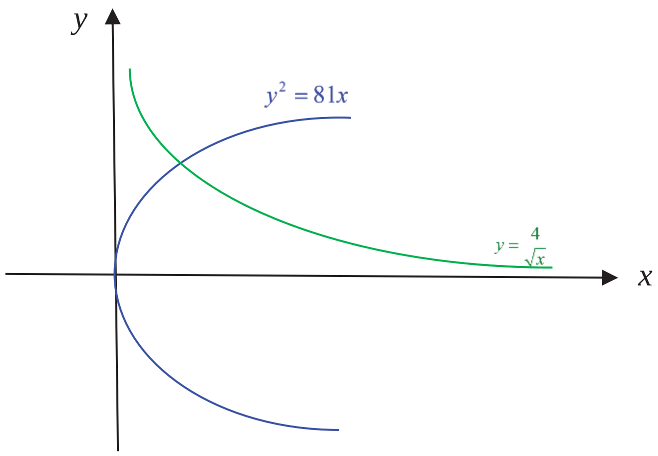
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

1. **(a)** On the same diagram, sketch the curves $y^2 = 81x$ and $y = \dfrac{4}{\sqrt{x}}$ for $x > 0$. **[2]**



(b) The parabola $y^2 = 81x$ intersects the curve $y = \dfrac{4}{\sqrt{x}}$ at the point A.

Find the equation of normal to the curve $y = \dfrac{4}{\sqrt{x}}$ at A. **[5]**

$$\left(\frac{4}{\sqrt{x}}\right)^2 = 81x$$

$$x^2 = \frac{16}{81}$$

$$x = \frac{4}{9} \quad or \quad x = -\frac{4}{9} \ (NA \because x > 0) \qquad \text{M1A1}$$

$$y = 6$$

$$\therefore A(\frac{4}{9}, 6)$$

$$y = \frac{4}{\sqrt{x}} = 4x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = -2x^{-\frac{3}{2}} = \frac{-2}{\sqrt{x^3}} \qquad \text{M1}$$

Gradient of tangent $= \dfrac{-27}{4}$

Gradient of normal $= \dfrac{4}{27}$ A1

Equation of normal: $y - 6 = \dfrac{4}{27}\left(x - \dfrac{4}{9}\right)$

$$\therefore y = \frac{4}{27}x + 5\frac{227}{243} \quad or \quad 243y = 36x + 1442 \qquad \text{A1}$$

2. The equation of a polynomial is given by $g(x) = 2x^3 - x^2 + 8x - 4$.

(i) Find the remainder when $g(x)$ is divided by $2x$. **[1]**

Remainder = $g(0)$ = -4 B1

(ii) Explain why $g(x) = 0$ has only one real root. **[4]**

$g(\frac{1}{2}) = 2(\frac{1}{8}) - \frac{1}{4} + 4 - 4 = 0$

$(2x - 1)$ is a factor of $g(x)$ M1

$g(x) = 2x^3 - x^2 + 8x - 4$

$g(x) = x^2(2x-1) + 4(2x-1)$

M1 *[Accept methods Fact grouping, comp coeff or Long Division]*

$\therefore g(x) = (2x-1)(x^2+4)$ A1

For $x^2 + 4$, D = 0-4(1)(4) = -16 < 0

No real roots.

$(2x - 1)$ is the only factor of $g(x)$. M1

(iii) Express $\dfrac{34}{2x^3 - x^2 + 8x - 4}$ in partial fractions. **[4]**

$$\frac{34}{2x^3 - x^2 + 8x - 4} = \frac{34}{(2x-1)(x^2+4)}$$

$$Let \quad \frac{34}{(2x-1)(x^2+4)} = \frac{A}{2x-1} + \frac{Bx+C}{x^2+4}$$ M1

$$34 = A(x^2+4) + (Bx+C)(2x-1)$$

Put $x = \frac{1}{2}$: $34 = A(\frac{1}{4} + 4)$

$A = 8$ A1

Put $x = 0$: $34 = 32 - C$

$C = -2$ A1

Comparing coeff of x^2 : $8 + 2B = 0$

$B = -4$ A1

$$\therefore \frac{34}{2x^3 - x^2 + 8x - 4} = \frac{8}{2x-1} + \frac{-4x-2}{x^2+4} \quad or \quad \frac{8}{2x-1} - \frac{4x+2}{x^2+4}$$

[Deduct 1 mark if this statement not shown]

3. **Do not use a calculator in this question.**

A cylinder with radius $(2+\sqrt{3})$ cm has a volume of $\pi(26+4\sqrt{48})$ cm^3, find the height of the cylinder in the form $m+n\sqrt{3}$, where m and n are integers. **[4]**

Let the height of the cylinder be *h*.

$\pi(2+\sqrt{3})^2 h = \pi(26+4\sqrt{48})$

$(4+4\sqrt{3}+3)h = (26+4\sqrt{48})$

$h = \dfrac{26+4\sqrt{48}}{7+4\sqrt{3}}$ M1

$h = \dfrac{26+4\sqrt{48}}{7+4\sqrt{3}} \times \dfrac{7-4\sqrt{3}}{7-4\sqrt{3}}$ M1

$= \dfrac{(26+16\sqrt{3})(7-4\sqrt{3})}{49-48}$

$= 182+112\sqrt{3}-104\sqrt{3}-192$ M1A1

$= 8\sqrt{3}-10$

4. It is given that $y = \frac{x^2}{2}(3-2x)^5$.

(i) Obtain an expression for $\frac{dy}{dx}$. **[2]**

$$\frac{dy}{dx} = \frac{x^2}{2}\left[5(3-2x)^4(-2)\right] + (3-2x)^5(x) \quad \text{M1}$$

$$= -5x^2(3-2x)^4 + x(3-2x)^5$$

$$= (3-2x)^4(3x-7x^2) \quad \text{A1}$$

(ii) Determine the set of values of x for which y is increasing. **[2]**

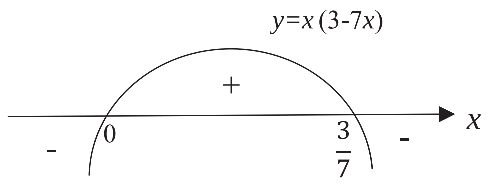
For y to be increasing, $\frac{dy}{dx} > 0$

$(3-2x)^4(3x-7x^2) > 0$ M1

$3x - 7x^2 > 0$

$x(3-7x) > 0$

$\therefore 0 < x < \frac{3}{7}$ A1



(iii) A point P moves along the curve $y = \frac{x^2}{2}(3-2x)^5$ in such a way that the y-coordinate of P is decreasing at a rate of 0.05 units per second. Find the rate of increase of the x-coordinate of P when $x = 1$. **[2]**

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$\frac{dy}{dt} = (3-2x)^4(3x-7x^2) \times \frac{dx}{dt} \quad \text{M1}$$

Subst. $x = 1$: $-0.05 = -4\frac{dx}{dt}$

$$\therefore \frac{dx}{dt} = 0.0125 \ units/s \quad \text{A1}$$

5. The function f is defined, for $0° \le x \le 360°$, by $f(x) = a - b\sin cx$, where a, b and c are positive constants.

(a) Given that the period is 240º and the greatest and the least values of $f(x)$ are 7 and -1 respectively, find the values of a, b and c. **[3]**

$$\frac{360°}{c} = 240°$$

B1

$$\therefore c = \frac{3}{2}$$

$a - b = -1$ --------- (1)

$a + b = 7$ --------- (2)

(1) + (2) : $2a = 6$

$a = 3$ B1

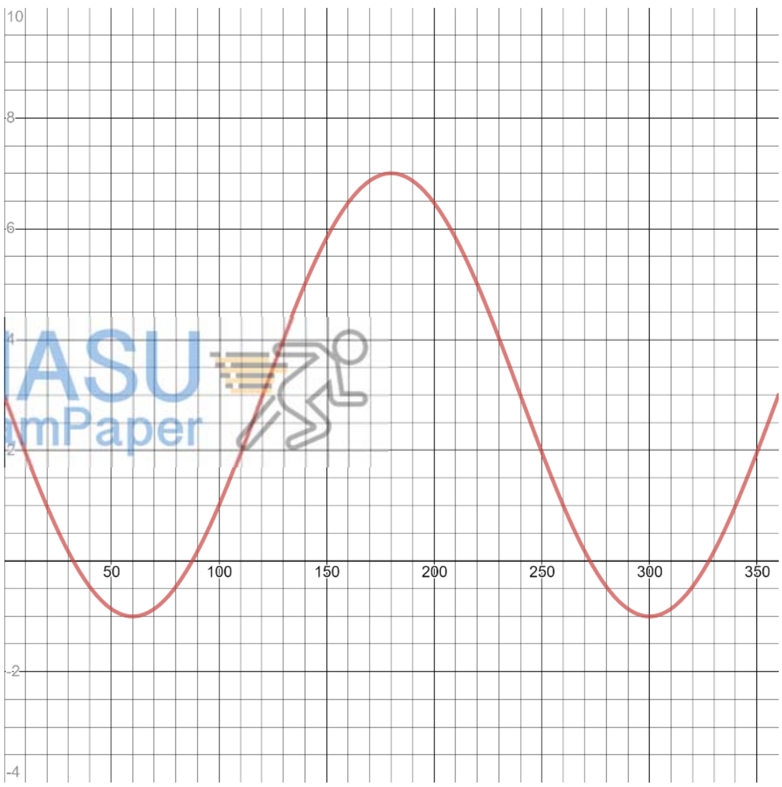
$b = 4$ B1

$$\therefore f(x) = 3 - 4\sin\frac{3}{2}x$$

(b) State the amplitude of $f(x)$. **[1]**

Amplitude = 4 B1

(c) Sketch the graph of $y = a - b\sin cx$. **[2]**



D1 : Correct shape

D1 : Correct End points & Turning points

6. At the beginning of 2020, Mr Lim bought an antique watch for \$ 8000. It was believed that the value of the antique watch will increase continuously with time such that its value increased 10% after every 3 years.

(i) Find the value of the antique watch after 15 years, correct to the nearest dollar. **[1]**

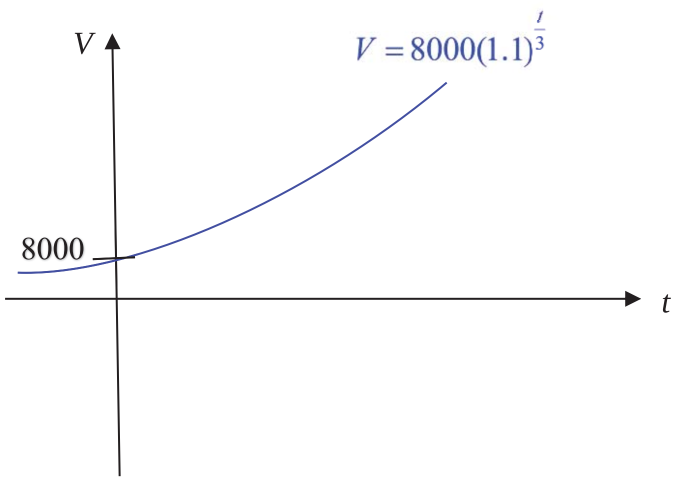
$$\text{Value of watch} = 8000(1.1)^{\frac{15}{3}}$$

$$= \$12884 \text{ (nearest dollar)} \qquad \text{A1}$$

(ii) Write down an expression for \$$V$, the value of the antique watch, after Mr Lim owned it for t years. **[1]**

$$V = 8000(1.1)^{\frac{t}{3}} \qquad \text{A1}$$

(iii) Sketch the graph of V against t . **[1]**



(iv) Using your answer in part (ii), find the year that Mr Lim's watch first appreciates to \$50 000. **[3]**

$$8000(1.1)^{\frac{t}{3}} = 50000$$

$$(1.1)^{\frac{t}{3}} = 6.25$$

$$\frac{t}{3}\lg 1.1 = \lg 6.25$$

$$\frac{t}{3} = \frac{\lg 6.25}{\lg 1.1} \qquad \text{M1A1}$$

$$\therefore t = \frac{3\lg 6.25}{\lg 1.1} = 57.68$$

$$\text{Year} = 2020 + 57 = 2077 \qquad \text{A1}$$

7. **(a)** Given that $\int_a^b (\sqrt{3x+2})\ dx = 5$, where *a* and *b* are positive constants.

(i) Evaluate $\int_b^a (\sqrt{12x+8})\ dx$ **[2]**

$$\int_b^a (\sqrt{12x+8})\ dx = \int_b^a (2\sqrt{3x+2})\ dx \qquad \text{M1}$$

$$= -2\int_a^b (\sqrt{3x+2})\ dx$$
$$= -2\times 5 \qquad \text{A1}$$
$$= -10$$

(ii) Explain why $\int_a^b (\sqrt{3x+2})\ dx$ cannot be found when $a = -1$. **[1]**

$$\int_{-1}^b (\sqrt{3x+2})\ dx = \left[\frac{(3x+2)^{\frac{3}{2}}}{\frac{3}{2}(3)}\right]_{-1}^b$$

Since $(3x+2)^{\frac{3}{2}}$ is only defined when $3x + 2 > 0$, i.e. $x > -\frac{2}{3}$,

$\int_{-1}^b (\sqrt{3x+2})\ dx$ cannot be evaluated.

(b) Solve the equation $\dfrac{\log_2 x}{\log_x 8} = 3$. **[4]**

$$\log_2 x = 3\log_x 8$$

$$\log_2 x = 3\left(\frac{\log_2 8}{\log_2 x}\right) \qquad \text{M1}$$

$$(\log_2 x)^2 = 9$$
$$\log_2 x = 3 \ \ or \ \ \log_2 x = -3 \qquad \text{M1}$$

$$x = 2^3 \ \ or \ \ x = 2^{-3}$$
$$\therefore x = 8 \ \ or \ \ x = \frac{1}{8} \qquad \text{A2}$$

8. (a) Without using a calculator, and showing all your working, find the exact value of

$\cos\frac{\pi}{4}\cos\frac{5\pi}{12}-\sin\frac{\pi}{4}\sin\frac{5\pi}{12}$ **[2]**

$$\cos\frac{\pi}{4}\cos\frac{5\pi}{12}-\sin\frac{\pi}{4}\sin\frac{5\pi}{12}=\cos\left(\frac{\pi}{4}+\frac{5\pi}{12}\right)$$ M1

$$=\cos\left(\frac{2\pi}{3}\right)$$ A1

$$=-\frac{1}{2}$$

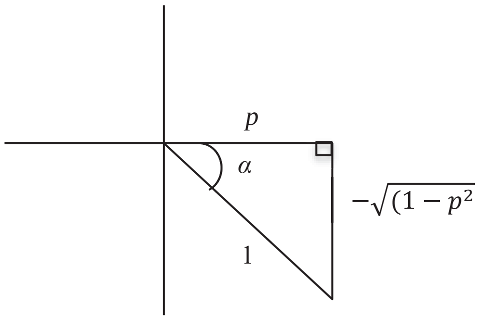
(b) Given that θ is reflex and $\cos\theta = p$, where $p > 0$.

Express the following in terms of p.

(i) $\tan\theta$ **[2]**

θ is in 4th quadrant.

$$\tan\theta=-\frac{\sqrt{1-p^2}}{p}$$ A1



M1

(ii) $\sec(\frac{\pi}{2}-\theta)$ **[2]**

$$\sec(\frac{\pi}{2}-\theta)=\frac{1}{\cos(\frac{\pi}{2}-\theta)}$$

$$=\frac{1}{\sin\theta}$$ M1

$$=-\frac{1}{\sqrt{1-p^2}} \quad or \quad -\frac{\sqrt{1-p^2}}{1-p^2}$$ A1

9. **(i)** Write down the general term in the binomial expansion of $\left(2x-\dfrac{1}{3x^3}\right)^6$. **[1]**

General term = $\binom{6}{r}(2x)^{6-r}\left(-\dfrac{1}{3x^3}\right)^r$ B1

(ii) Explain, showing your working clearly, if there is a term independent of x in the expansion. **[2]**

$6-r-3r=0$

$r=\dfrac{3}{2}$ M1

Since r is **<u>not a natural number/positive integer</u>**, there is **<u>no</u>** term independent of x. M1

(i) Using the general term, or otherwise, determine the term independent of x in the

binomial expansion of $\left(3x^2+1+\dfrac{2}{x^2}\right)\left(2x-\dfrac{1}{3x^3}\right)^6$. **[4]**

For coeff of x^{-2} : $6-4r=-2$

$r=2$

coeff of $x^{-2}=\binom{6}{2}(2)^{6-2}\left(-\dfrac{1}{3}\right)^2=26\dfrac{2}{3}$

For coeff of x^2 : $6-4r=2$

$r=1$

coeff of $x^2=\binom{6}{1}(2)^{6-1}\left(-\dfrac{1}{3}\right)^1=-64$

M1A1 for either correct method & answer

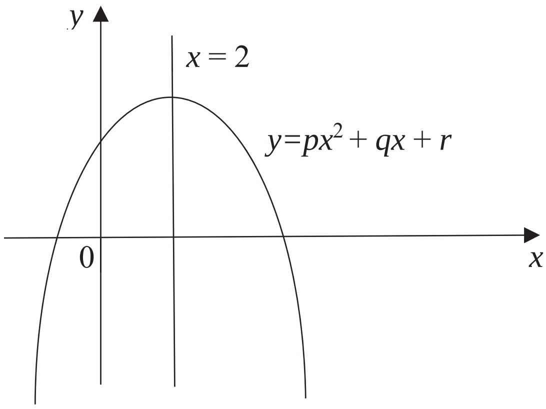
Otherwise Method : $\left(2x-\dfrac{1}{3x^3}\right)^6=(2x)^6+\binom{6}{1}(2x)^5\left(-\dfrac{1}{3x^3}\right)+\binom{6}{2}(2x)^4\left(-\dfrac{1}{3x^3}\right)^2+....$ M1A1

$=64x^6-64x^2+26\dfrac{2}{3}x^{-2}-....$

$\left(3x^2+1+\dfrac{2}{x^2}\right)\left(2x-\dfrac{1}{3x^3}\right)^6=\left(3x^2+1+\dfrac{2}{x^2}\right)\left(...-64x^2+26\dfrac{2}{3}x^{-2}+....\right)$

Term independent of x = $3(26\dfrac{2}{3})+2(-64)=-48$ A1

10.



The diagram above shows part of the graph of $y = px^2 + qx + r$.
The equation of line of symmetry of the curve is $x = 2$.
Determine the conditions for each of the following expressions, justifying your answer.

(i) $q^2 - 4pr$ **[2]**

(ii) $\dfrac{dy}{dx}$ when $x = 2$ **[2]**

(iii) q **[2]**

(i) **[Method 1 :]** Since the curve cuts the x-axis at two real and distinct points,
[Method 2 :] Since the equation $px^2 + qx + r = 0$ has two real and distinct roots,
[Method 3 :] Since $p < 0$, $r > 0$, $-4pr > 0$ *M1*
$\therefore D = q^2 - 4pr > 0$. A1

(ii) Since the line of symmetry is $x = 2$, the point on the curve where $x = 2$ is a stationary/turning point. M1
$\therefore \dfrac{dy}{dx} = 0$. A1

(iii) $\dfrac{dy}{dx} = 2px + q$
When $x = 2$, $4p + q = 0$
$q = -4p$
[Method 1 :] Since the curve has a maximum point, $p < 0$
[Method 2 :] $\dfrac{d^2y}{dx^2} = 2p < 0$ ($\because$ max pt), $p < 0$. M1

$\therefore q > 0$. A1

11. The points $P(-6, 1)$, $Q(4, 5)$ and $R(-3, 8)$ lie on a circle C.

(i) Find the gradient of each of the lines PR and QR. **[1]**

Grad. of $PR = \dfrac{8-1}{-3+6} = \dfrac{7}{3}$

Grad. of $QR = \dfrac{8-5}{-3-4} = -\dfrac{3}{7}$ B1 *[Award only if both grads correct]*

(ii) What do your answers to part **(i)** imply about the line PQ relative to the circle C ?

Show your working clearly.

[2]

Since (Grad. of PR) × (Grad. of QR) = -1, $\angle PRQ = 90°$.

Using **right-angle in a semi-circle** property, PQ is the **diameter** of the circle C.

M1 A1

(iii) Find the equation of the circle C. **[3]**

Mid-point of $PQ = \left(\dfrac{-6+4}{2}, \dfrac{1+5}{2}\right) = (-1, 3)$ M1

Radius $= \sqrt{(-1+6)^2+(3-1)^2} = \sqrt{29}$ B1

Equation of circle C : $(x+1)^2+(y-3)^2 = 29$ or $x^2+2x+y^2-6y-19=0$ A1

(iv) Write down the exact coordinates of the lowest point of the circle C. **[1]**

Lowest point $= (-1,\ 3-\sqrt{29})$ or $(-1,\ 3-\dfrac{\sqrt{116}}{2})$ B1

(v) Find the equation of tangent to the circle C at P. **[3]**

Grad. of line joining centre and $P = \dfrac{3-1}{-1+6} = \dfrac{2}{5}$

Grad. of tangent at $P = -\dfrac{5}{2}$ B1

Equation of tangent at P : $y-1 = -\dfrac{5}{2}(x+6)$ M1

$\therefore y = -\dfrac{5}{2}x - 14$ *or* $2y = -5x-28$ A1

12. A curve $y = \mathrm{f}(x)$ is such that $\mathrm{f}''(x) = 24\sin 4x - 12\cos 2x$ and has a stationary point $\left(\frac{\pi}{4}, 1\right)$. Show that $\mathrm{f}''(x) + 4\,\mathrm{f}(x) = k\sin px + q$, where *k*, *p* and *q* are constants to be determined. **[6]**

$\mathrm{f}''(x) = 24\sin 4x - 12\cos 2x$

$f'(x) = \int (24\sin 4x - 12\cos 2x)dx$

$f'(x) = \frac{-24\cos 4x}{4} - \frac{12\sin 2x}{2} + C_1,\ \text{where } C_1 \text{ is an arb. const.}$ M1

$f'(x) = -6\cos 4x - 6\sin 2x + C_1$

$Subst.\ f'\left(\frac{\pi}{4}\right) = 0$

$-6\cos\pi - 6\sin\frac{\pi}{2} + C_1 = 0$

$6 - 6 + C_1 = 0$

$\therefore C_1 = 0$

$\therefore\ f'(x) = -6\cos 4x - 6\sin 2x$ A1

$f(x) = \frac{-6\sin 4x}{4} + \frac{6\cos 2x}{2} + C_2,\ \text{where } C_2 \text{ is an arb. const.}$

$f(x) = \frac{-3\sin 4x}{2} + 3\cos 2x + C_2$ M1

$Subst.\ f\left(\frac{\pi}{4}\right) = 1$

$-\frac{3}{2}\sin 2\pi + 3\cos\frac{\pi}{2} + C_2 = 1$

$\therefore C_2 = 1$

$\therefore\ f(x) = \frac{-3\sin 4x}{2} + 3\cos 2x + 1$ A1

$\mathrm{f}''(x) + 4\,\mathrm{f}(x) = 24\sin 4x - 12\cos 2x - 6\sin 4x + 12\cos 2x + 4$

$\mathrm{f}''(x) + 4\,\mathrm{f}(x) = 18\sin 4x + 4$ A2

$\therefore k = 18,\ \ p = 4\ \ and\ \ q = 4$

[Award A1 if either k, p or q is wrong]

End of Paper

CANDIDATE NAME

CLASS

INDEX NUMBER

METHODIST GIRLS' SCHOOL

Founded in 1887

PRELIMINARY EXAMINATION 2020
Secondary 4

Thursday

13 August 2020

ADDITIONAL MATHEMATICS
PAPER 2

4047/02

2 hours 30 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST
Write your class, index number and name in the spaces at the top of this page.
Write in dark blue or black pen
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figure, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Find the largest integer value of k for which the line $y = 1 - x$ does not intersect the curve $y = x^2 + (k+2)x + 1 - k$. [4]

$y = 1 - x$
$y = x^2 + (k+2)x + 1 - k$

subs (1) into (2),
$1 - x = x^2 + (k+2)x + 1 - k$
$x^2 + (k+3)x - k = 0$ [1]

$b^2 - 4ac < 0$
$(k+3)^2 - 4(1)(-k) < 0$ [1]
$k^2 + 6k + 9 + 4k < 0$
$k^2 + 10k + 9 < 0$

$(k+1)(k+9) < 0$

Ans: $-9 < k < -1$ [1]

Largest $k = -2$ [1]

2 Given that the roots of $3x^2+2x=7$ are α and β, find the quadratic equation whose roots are $\alpha+\dfrac{2}{\alpha}$ and $\beta+\dfrac{2}{\beta}$. [5]

sum of roots, $\alpha+\beta=\dfrac{-2}{3}$

product of roots, $\alpha\beta=\dfrac{-7}{3}$ both correct - [1]

sum of new roots, $\alpha+\dfrac{2}{\alpha}+\beta+\dfrac{2}{\beta}$

$$=\alpha+\beta+2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$$

$$=\alpha+\beta+2\left(\frac{\alpha+\beta}{\alpha\beta}\right)$$

$$=-\frac{2}{3}+2\left[\frac{\left(-\frac{2}{3}\right)}{\left(-\frac{7}{3}\right)}\right]$$

$$=-\frac{2}{21}$$ [1]

product of new roots, $\left(\alpha+\dfrac{2}{\alpha}\right)\left(\beta+\dfrac{2}{\beta}\right)$

$$=\alpha\beta+\frac{2\alpha}{\beta}+\frac{2\beta}{\alpha}+\frac{4}{\alpha\beta}$$

$$=\alpha\beta+2\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)+\frac{4}{\alpha\beta}$$

$$=\alpha\beta+2\left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right)+\frac{4}{\alpha\beta}$$

$$=\alpha\beta+2\left[\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}\right]+\frac{4}{\alpha\beta}$$

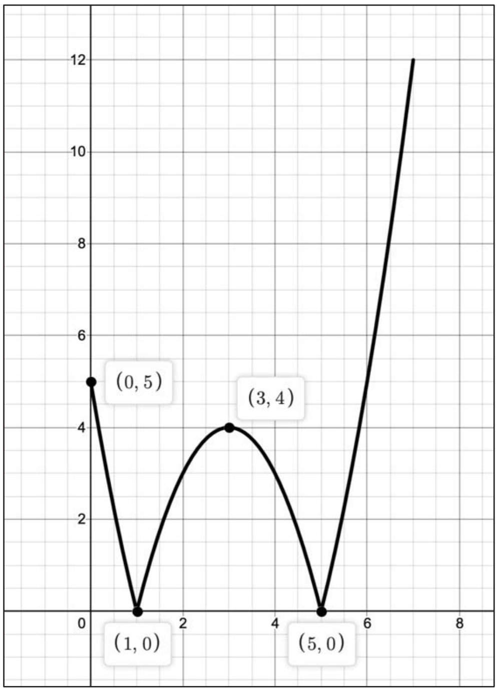
$$=-\frac{7}{3}+2\left[\frac{\left(-\frac{2}{3}\right)^2-2\left(-\frac{7}{3}\right)}{-\frac{7}{3}}\right]+\frac{4}{\left(-\frac{7}{3}\right)}$$ [1]

$$=\frac{-59}{7}$$ [1]

quadratic equation is $x^2-\left(-\dfrac{2}{21}\right)x+\left(-\dfrac{59}{7}\right)=0$

$21x^2+2x-177=0$ [1]

3 **(a)** Sketch the graph of $y = \left|x^2 - 6x + 5\right|$ for $0 \le x \le 7$. [3]



shape – [1]

(3,4), (1,0), (5,0) – [1]

end points – [1]

(b) Solve the equation $\left|x^2 - 6x + 5\right| = 4$. [3]

$\left|x^2 - 6x + 5\right| = 4$

$x^2 - 6x + 5 = 4$ or $x^2 - 6x + 5 = -4$ [1]

$x^2 - 6x + 1 = 0$ $\qquad$ $x^2 - 6x + 9 = 0$

$x = \dfrac{6 \pm \sqrt{(-6)^2 - 4(1)(-1)}}{2}$ $\qquad$ $(x-3)^2 = 0$

$x = 0.172$ or 5.83 $\qquad$ $x = 3$ [2]

4 **(a)** **(i)** Prove the identity $\dfrac{\sec A - \operatorname{cosec} A}{\cot A + \tan A} \equiv \sin A - \cos A$. [3]

LHS,

$$\frac{\sec A - \operatorname{cosec} A}{\cot A + \tan A}$$

$$= (\sec A - \operatorname{cosec} A) \div (\cot A + \tan A)$$

$$= \left(\frac{1}{\cos A} - \frac{1}{\sin A}\right) \div \left(\frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}\right) \quad [1]$$

$$= \left(\frac{\sin A - \cos A}{\sin A \cos A}\right) \div \left(\frac{\cos^2 A + \sin^2 A}{\sin A \cos A}\right)$$

$$= \left(\frac{\sin A - \cos A}{\sin A \cos A}\right) \div \left(\frac{1}{\sin A \cos A}\right) \quad [1]$$

$$= \left(\frac{\sin A - \cos A}{\sin A \cos A}\right) \times \left(\frac{\sin A \cos A}{1}\right)$$

$$= \sin A - \cos A \quad [1]$$

$$= \text{RHS}$$

(ii) Hence find all the angles between $0°$ and $180°$ which satisfy the equation $\left(\dfrac{2\sec A - 2\operatorname{cosec} A}{\cot A + \tan A}\right)^2 = 1$. [3]

$$\left(\frac{2\sec A - 2\operatorname{cosec} A}{\cot A + \tan A}\right)^2 = 1$$

$$4(\sin A - \cos A)^2 = 1 \quad [1]$$

$$4(\sin^2 A - 2\sin A\cos A + \cos^2 A) = 1$$

$$4 - 4\sin 2A = 1$$

$$\sin 2A = \frac{3}{4} \quad [1]$$

basic angle = 48.5904…

$$2A = \alpha,\ 180° - \alpha$$

$$A = 24.3°,\ 65.7° \quad [1]$$

(b) Find all the angles between 0 and 4 which satisfy $3\sec^2 y + 5\tan y = 5$. [4]

$3\sec^2 y + 5\tan y = 5$

$3\left(1+\tan^2 y\right) + 5\tan y - 5 = 0$ [1]

$3\tan^2 y + 5\tan y - 2 = 0$

$(3\tan y - 1)(\tan y + 2) = 0$ [1]

$\tan y = \dfrac{1}{3}$ or $\tan y = -2$

$\alpha = 0.32175...$ $\alpha = 1.1071...$

$y = \alpha,\ \pi + \alpha$ $y = \pi - \alpha,\ 2\pi - \alpha$

$y = 0.322,\ 3.46$ $y = 2.03$ [2]

5 **(a)** **(i)** Find $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(xe^{3-2x}\right)$. [2]

$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(xe^{3-2x}\right) = e^{3-2x}(1) + x\left(-2e^{3-2x}\right)$ [1]

$= e^{3-2x} - 2xe^{3-2x}$ [1]

(ii) Hence find $\int 4xe^{3-2x}\ \mathrm{d}x$. [3]

$\int\left(e^{3-2x} - 2xe^{3-2x}\right)\ \mathrm{d}x = xe^{3-2x} + c$ [1]

$\int e^{3-2x}\ \mathrm{d}x - \int 2xe^{3-2x}\ \mathrm{d}x = xe^{3-2x} + c$

$\int e^{3-2x}\ \mathrm{d}x - xe^{3-2x} - c = \int 2xe^{3-2x}\ \mathrm{d}x$

$\int 2xe^{3-2x}\ \mathrm{d}x = \dfrac{e^{3-2x}}{-2} - xe^{3-2x} + c_1$ [1]

$\int 4xe^{3-2x}\ \mathrm{d}x = -e^{3-2x} - 2xe^{3-2x} + 2c_1$ [1]

(b) The equation of a curve is $y=\dfrac{\ln x}{(3x+1)^2}$. Find the equation of tangent to the curve at $x=1$. [4]

$$y=\frac{\ln x}{(3x+1)^2}$$

$$\frac{dy}{dx}=\frac{(3x+1)^2\left(\frac{1}{x}\right)-(\ln x)\left[2(3x+1)(3)\right]}{(3x+1)^4} \qquad [1]$$

at $x=1$, $y=0$

$$\frac{dy}{dx}=\frac{(4)^2(1)-(\ln 1)\left[6(4)\right]}{(4)^4}=\frac{1}{16} \qquad [1]$$

equation of tangent,

$$y-0=\frac{1}{16}(x-1) \qquad [1]$$

$$16y=x-1 \qquad [1]$$

(c) A curve is such that $\dfrac{dy}{dx}=\dfrac{2}{3x+1}+\dfrac{3}{\sqrt{3x+1}}$. The curve passes through the point $(0,3)$, find the equation of the curve. [4]

$$\frac{dy}{dx}=\frac{2}{3x+1}+\frac{3}{\sqrt{3x+1}}$$

$$y=\int\left[\frac{2}{3x+1}+\frac{3}{\sqrt{3x+1}}\right]dx$$

$$y=\int\left(\frac{2}{3x+1}\right)dx+\int\left(\frac{3}{\sqrt{3x+1}}\right)dx$$

$$y=\frac{2}{3}\int\left(\frac{3}{3x+1}\right)dx+\int 3(3x+1)^{-\frac{1}{2}}\,dx$$

$$y=\frac{2}{3}\ln(3x+1)+3\left[\frac{2(3x+1)^{\frac{1}{2}}}{1(3)}\right]+c \qquad [2]$$

$$y=\frac{2}{3}\ln(3x+1)+2\sqrt{3x+1}+c$$

at $(0,3)$,

$$3=\frac{2}{3}\ln 1+2\sqrt{1}+c \qquad [1]$$

$$c=1$$

$$y=\frac{2}{3}\ln(3x+1)+2\sqrt{3x+1}+1 \qquad [1]$$

6 **(a)** Using the substitution $u = 5^x$, solve the equation $5^{x+2} - 25^{x+0.5} = 2(5^{x+1})$, giving your answer correct to 2 decimal places. [4]

$5^{x+2} - 25^{x+0.5} = 2(5^{x+1})$

$5^x \times 5^2 - 5^{2(x+0.5)} = 2(5^x \times 5^1)$

$25 \times 5^x - 5^{2x+1} = 10(5^x)$ [1]

$15 \times 5^x - 5^{2x} \times 5^1 = 0$

let $u = 5^x$,

$15u - 5u^2 = 0$

$5u(3-u) = 0$

$u = 0$ or $u = 3$ [1]

$5^x = 0$ $5^x = 3$

(NA) $x = \dfrac{\lg 3}{\lg 5} = 0.68$ [2]

(b) Solve the simultaneous equations: [4]

$$\log_4(2x+y) = \frac{1}{2} - \log_4 3$$
$$(3\sqrt{3})^x = 27(\sqrt{3})^y$$

$\log_4(2x+y) + \log_4 3 = \dfrac{1}{2}$

$\log_4 3(2x+y) = \dfrac{1}{2}$

$3(2x+y) = 4^{\frac{1}{2}}$

$6x + 3y = 2$ [1]

$(3\sqrt{3})^x = 27(\sqrt{3})^y$

$(3^{1.5})^x = 3^3\left(3^{0.5}\right)^y$

$3^{1.5x} = 3^{3+0.5y}$

$1.5x = 3 + 0.5y$

$3x = 6 + y$

$y = 3x - 6$ [1]

$6x + 3(3x-6) = 2$

$15x = 20$

$x = \dfrac{4}{3}$ [1]

$y = -2$ [1]

7 The table shows experimental values of two variables, x and y, which are connected by an equation of the form, $e^y = ax^b$, where a and b are constants.

x	2	3	5	7	10
y	2.28	2.94	3.69	4.30	4.88

(i) Plot y against $\ln x$ for the given data and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of a and of b. [4]

(iii) Use your graph to estimate the value of x when $e^y = x^5$. [2]

Candidate Name .... $y = 1.6096x + 1.15528$ .... Centre Number | Index Number

Subject .... Paper ....

Question No. ....

ln x	0.693	1.099	1.609	1.946	2.303
y	2.28	2.94	3.69	4.30	4.88

i) $e^y = ax^b$

$y = \ln(ax^b)$

$y = b\ln x + \ln a$

ii) $\ln a = 1.15$

$a = 3.16 \quad (3.00 - 3.32)$

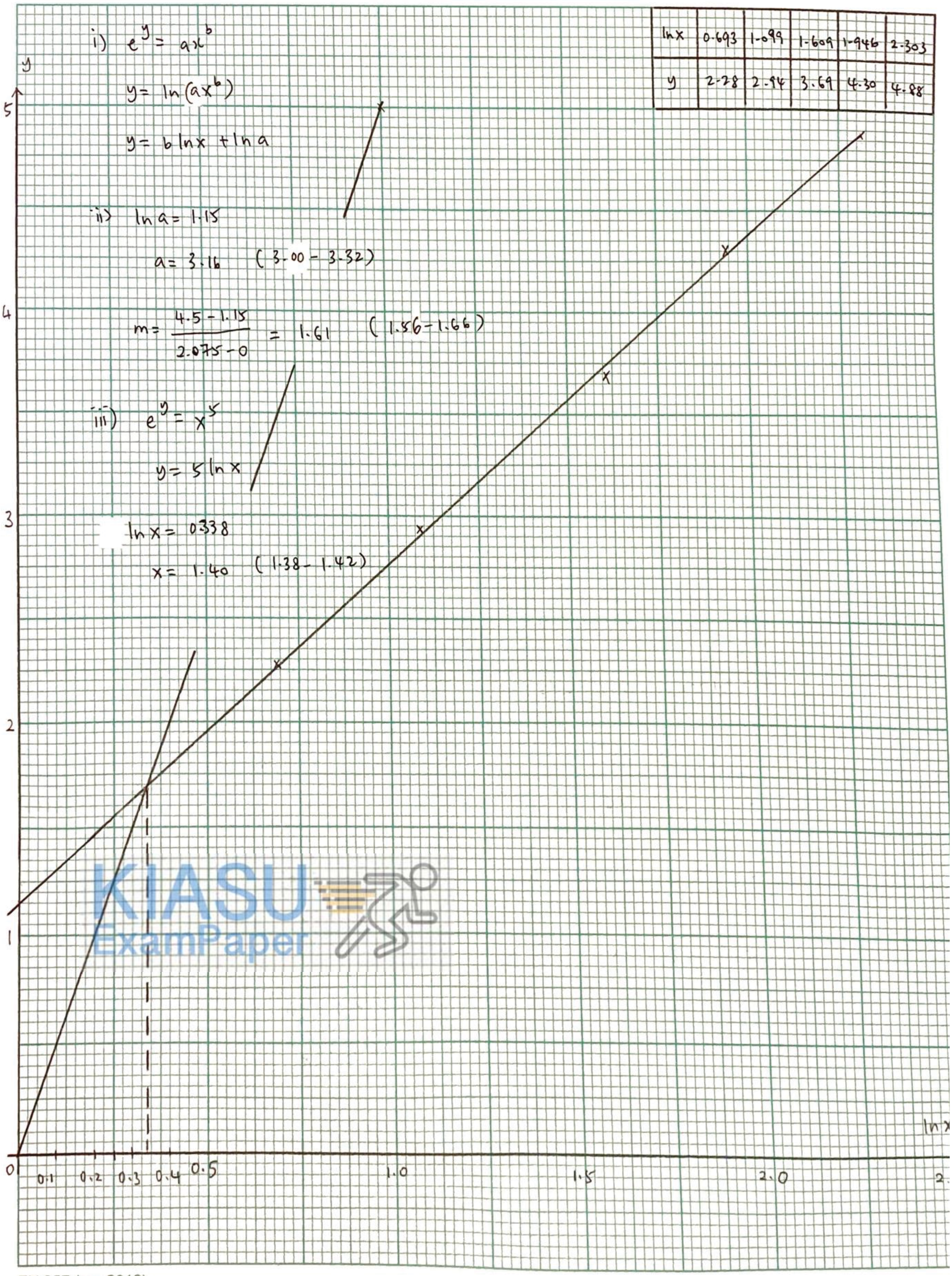
$m = \frac{4.5 - 1.15}{2.075 - 0} = 1.61 \quad (1.56 - 1.66)$

iii) $e^y = x^5$

$y = 5\ln x$

$\ln x = 0.338$

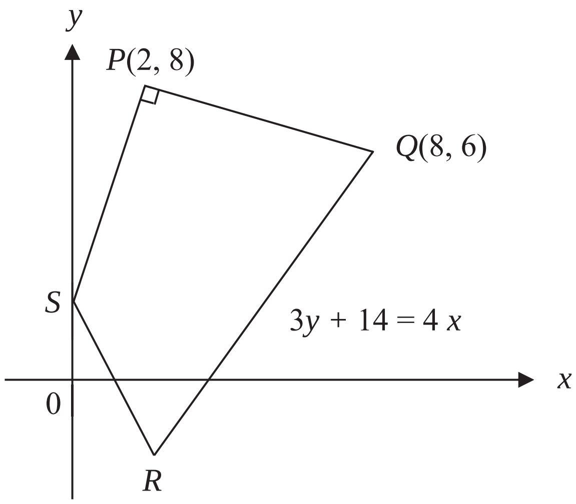
$x = 1.40 \quad (1.38 - 1.42)$



KIASU ExamPaper

EX 257 (rev 2012)

8 **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows a quadrilateral *PQRS* in which the point *P* is (2, 8) and the point *Q* is (8, 6). The point *R* lies on the perpendicular bisector of *PQ* and the point *S* lies on the *y*-axis. The equation of *QR* is $3y+14=4x$ and angle $SPQ = 90^\circ$. Find

(i) the coordinates of *S*, [3]

$$m_{PQ}=\frac{8-6}{2-8}=-\frac{1}{3}$$

$m_{SP}=3$ [1]

let $S(0,y)$,

$$m_{SP}=\frac{8-y}{2-0}$$

$3=\dfrac{8-y}{2}$ [1]

$6=8-y$

$y=2$

$S(0,2)$ [1]

(ii) the equation of the perpendicular bisector of *PQ*, [2]

mid-point of *PQ* $=\left(\dfrac{2+8}{2},\dfrac{8+6}{2}\right)=(5,7)$ [1]

$m=3$

equation of perpendicular bisector,

$y-7=3(x-5)$

$y=3x-8$ [1]

(iii) the coordinates of R. [3]

$y = 3x - 8$
$3y + 14 = 4x$

$3(3x-8)+14 = 4x$ [1]
$9x - 24 + 14 = 4x$
$5x = 10$
$x = 2$ [1]
$y = -2$

$R(2,-2)$ [1]

(iv) the area of *PQRS*. [2]

$$\text{area } = \frac{1}{2}\begin{vmatrix} 2 & 0 & 2 & 8 & 2 \\ 8 & 2 & -2 & 6 & 8 \end{vmatrix}$$ [1]

$$= \frac{1}{2}\left[(4+12+64)-(4-16+12)\right] = 40 \text{ units}^2$$ [1]

9 A particle moves in a straight line so that its velocity, v m/s, is given by $v = 2t^2 - 8t + 6$, where t is the time in seconds after the start of motion. At $t = 2$, the displacement of the particle from a fixed point O is 1 m. Find

(i) the times when the particle is instantaneously at rest, [2]

at $v = 0$,
$2t^2 - 8t + 6 = 0$ [1]
$2(t^2 - 4t + 3) = 0$
$2(t-1)(t-3) = 0$
$t = 1$ or $t = 3$ [1]

(ii) the minimum velocity of the particle and explain the significance of the answer obtained, [3]

$a = 4t - 8$
at $a = 0$, $t = 2$ [1]

min. velocity $= 2(4) - 8(2) + 6 = -2$ m/s [1]

The particle is moving in the opposite direction. [1]

(iii) the average speed travelled by the particle in the first 5 seconds. [4]

$s = \int (2t^2 - 8t + 6)\ \mathrm{d}t$
$s = 2\left(\frac{t^3}{3}\right) - 8\left(\frac{t^2}{2}\right) + 6t + c$ [1]

at $t = 2$, $s = 1$,
$1 = 2\left(\frac{8}{3}\right) - 8\left(\frac{4}{2}\right) + 6(2) + c$
$c = -\frac{1}{3}$
$s = \frac{2t^3}{3} - 4t^2 + 6t - \frac{1}{3}$ [1]

at $t = 0$, $s = -\frac{1}{3}$
at $t = 1$, $s = 2\frac{1}{3}$
at $t = 2$, $s = -\frac{1}{3}$
at $t = 5$, $s = 13$

Average speed $= \dfrac{\frac{1}{3} + \left(2\frac{1}{3} \times 2\right) + \left(\frac{1}{3} \times 2\right) + 13}{5} = 3.73$ m/s [2]

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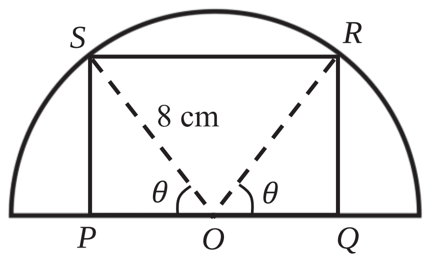

The diagram shows a metal wire, *PQRS*, placed on top of a semicircle metal sheet, with centre *O* and radius 8 cm. $\angle POS = \angle QOR = \theta$ radians.

(a) Show that the length of the metal wire *PQRS*, *L* cm, is given by $L = 16\sin\theta + 32\cos\theta$. [2]

$OP = OQ = 8\cos\theta$
$SP = RQ = 8\sin\theta$

$L = 2(8\sin\theta) + 4(8\cos\theta)$ [1]
$L = 16\sin\theta + 32\cos\theta$ [1]

(b) Find the value of θ for which *L* is stationary. [3]

$L = 16\sin\theta + 32\cos\theta$
$\frac{dL}{d\theta} = 16\cos\theta - 32\sin\theta$ [1]
at $\frac{dL}{d\theta} = 0$,

$16\cos\theta = 32\sin\theta$ [1]
$\frac{1}{2} = \tan\theta$
$\theta = 0.463647... = 0.464$ [1]

(c) Hence, find the maximum value of *L*. [2]

$$\frac{d^2L}{d\theta^2} = -16\sin\theta - 32\cos\theta$$

at $\theta = 0.463647...$, $\frac{d^2L}{d\theta^2} < 0$, max. *L* [1]
$L = 16\sin 0.463647... + 32\cos 0.463647...$
$L = 35.777... = 35.8$ [1]

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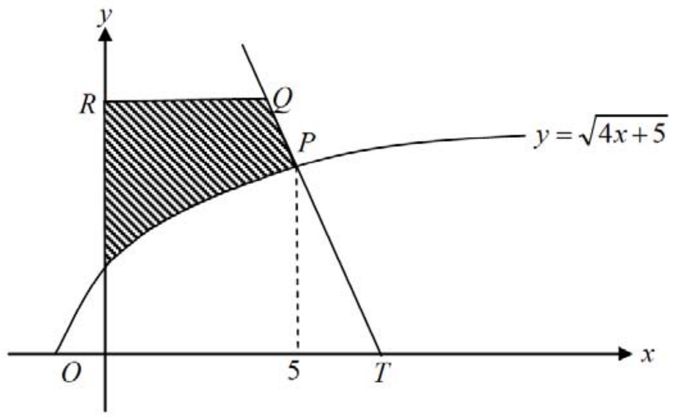


The diagram shows part of the curve $y=\sqrt{4x+5}$, which intersects with the line $x = 5$ at the point P. The normal to the curve at P meets the x-axis at the point T. RQ is parallel to the x-axis and $PT : PQ = 2 : 1$. Find

(a) the coordinates of P and of Q, [5]

$y=\sqrt{4x+5}$

$\frac{dy}{dx}=\frac{1}{2}(4x+5)^{-\frac{1}{2}}(4)$ [1]

$\frac{dy}{dx}=\frac{2}{\sqrt{4x+5}}$

at $x=5$, $y=5$

$\frac{dy}{dx}=\frac{2}{5}$

$P(5,5)$ [1]

let $T(x,0)$

$m_{PT}=\frac{5-0}{5-x}$

$-\frac{5}{2}=\frac{5}{5-x}$ [1]

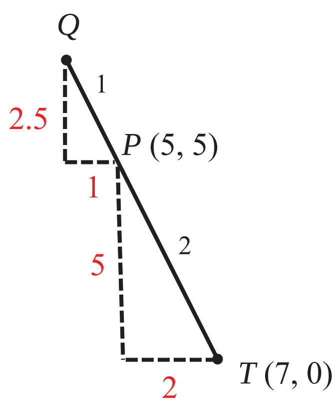
$-2=5-x$

$x=7$

$T(7,0)$ [1]

$Q(5-1,5+2.5)$

$=Q(4,7.5)$ [1]



(b) the area of the shaded region. [3]

area = square (*OP* diagonal) + trapezium – area under the curve

$$\text{area } = (5\times5)+\frac{1}{2}(4+5)(2.5)-\int_0^5 (4x+5)^{\frac{1}{2}}\ \mathrm{d}x \qquad [1]$$

$$=36.25-\left[\frac{2(4x+5)^{\frac{3}{2}}}{3(4)}\right]_0^5 \qquad [1]$$

$$=36.25-\frac{1}{6}\left(25^{\frac{3}{2}}-5^{\frac{3}{2}}\right)$$

$$=17.280...=17.3\text{ units}^2 \qquad [1]$$

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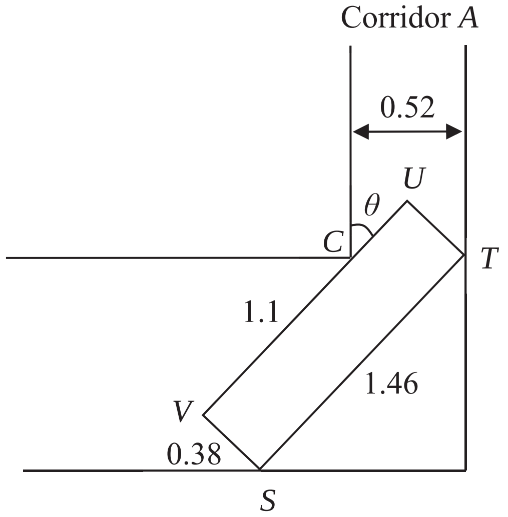
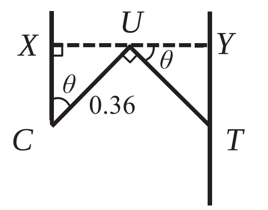


The diagram shows two straight corridors meeting at right angles. Corridor A has a width of 0.52 m. $STUV$ is a rectangular block of length 1.46 m and breadth 0.38 m. The block touches the walls of the corridor at S and T, and also touches the inner corner of the corridor at C, where $VC = 1.1$ m. The angle between CU and the wall at C is θ, where $0° < \theta < 90°$.

(i) Show that $19\cos\theta + 18\sin\theta = 26$. [3]



corridor A, $CU = 0.36$, $UT = 0.38$

$XY = XU + UY$

$0.52 = 0.36\sin\theta + 0.38\cos\theta$ [2]

$\times 50)$ $19\cos\theta + 18\sin\theta = 26$ [1]

(ii) Express $19\cos\theta + 18\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

$R = \sqrt{19^2 + 18^2} = \sqrt{685}$ [1]

$\tan\alpha = \frac{18}{19}$

$\alpha = 43.4518...$ [1]

$19\cos\theta + 18\sin\theta = \sqrt{685}\cos(\theta - 43.5°)$ [1]

(iii) **Hence**, find the values of θ. [3]

$19\cos\theta + 18\sin\theta = 26$

$\sqrt{685}\cos(\theta - 43.4518...°) = 26$

$\cos(\theta - 43.4518...°) = \dfrac{26}{\sqrt{685}}$ [1]

$\theta - 43.4518... = 6.5819...,\ -6.5819\ldots$

$\theta = 50.033...,\ 36.869\ldots$

$\theta = 50.0°,\ 36.9°$ [2]

(iv) State the largest value of 19cos θ + 18sin θ and its corresponding value of θ. [2]

$19\cos\theta + 18\sin\theta = \sqrt{685}\cos(\theta - 43.5°)$

Largest value = $\sqrt{685} \times 1 = 26.2$ [1]

$\theta - 43.4518... = 0°$

$\theta = 43.5°$ [1]

~ END OF PAPER ~