Name:	Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR 2020
PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS **4047/01**
Paper 1 **25 August 2020**

Additional Materials: NIL **2 hours**

READ THESE INSTRUCTIONS FIRST
Write your name, register number and class in the spaces at the top of the page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
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The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **80**.

For Examiners' Use

Question	1	2	3	4	5	6	7	8	9	10	11	12	13
Marks													

Table of Penalties		Question Number	Parent's/Guardian's Signature	80
Presentation	–1			
Units	–1			
Significant Figures	–1			

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-r)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

1 Find the range of values of k for which the quadratic equation $(k+2)x^2 - 2kx + 1 = 0$ has real roots. [3]

2 A triangle ABC has an area of $\left(14 - 3\sqrt{2}\right)$ cm^2 and $AC = \left(5 - \frac{4}{\sqrt{2}}\right)$ cm. Express the perpendicular distance from B to AC in the form of $a + b\sqrt{2}$, where a and b are rational numbers. [4]

[Turn over

3 Given that $(1, h)$ is a solution of the simultaneous equations $\frac{1}{x}+\frac{2}{y}-3=0$ and $x+ky-6=0$, find the values of x and of y. [5]

4 Given that $y = \dfrac{e^{-2x}}{3x-5}$, find the range of values of x for which y is an increasing function. [5]

[Turn over

5 **(i)** Given that $\lg 2 = x$ and $\lg 3 = y$, find the value of $\log_5 24$ in terms of x and y. [4]

(ii) Express $4^{\ln 3} + 2^{\ln 9}$ in the form 2^m, where m is a constant. [2]

6 Express $\dfrac{x^4}{x^4 - 16}$ in partial fractions. [6]

[Turn over

7 Water flows out of a filled container of height 20 cm such that after t seconds, the depth of water in the container is h cm and the volume of water left in the container is given by $V = 10 + \frac{h^2(h+2)}{10}$. Given that the height of the water level is decreasing at a rate of $(4t-1)$ cm/s. find the rate of decrease of the volume of the water when $t = 2$. [6]

8 It is given that $\tan B = \frac{3}{4}$ where B is an acute angle. Without solving for B, find the exact value of

(i) $\cos 3B$, [4]

(ii) $\tan \frac{B}{2}$. *[Hint: Use double angle formula.]* [3]

[Turn over

9 **(i)** Sketch the graph of $f(x) = 3 - |2x - 3|$ for $-2 \le x \le 7$, indicating the coordinates of the intercepts, endpoints and vertex clearly. [3]

(ii) State least positive value of $\dfrac{1}{f(x)}$. [1]

(iii) State the range of values of m such that the line $y = mx + 2$ intersects the graph at 2 distinct points. [3]

[Turn over

10 A company wants to produce open cylindrical containers with a capacity of 6000 cm^3 each.

(i) Find the radius of the container if the company wants to minimize the amount of material used. [4]

(ii) For the value of the radius found in (i), show that the amount of material used is minimum. [2]

(iii) State an assumption made in your calculations above. [1]

[Turn over

11 A radioactive substance decays according to the equation $A = 15e^{-bt}$, where A is the mass in grams of radioactive substance remaining and t is the time in hours after the radioactive substance starts to decay.

(i) Given that there are 10 grams of radioactive substance remaining after 30 hours, verify that the value of b is 0.013516. [2]

(ii) Find the mass of radioactive substance left at the end of 1 week. [1]

(iii) Find the number of hours for the radioactive substance to decay to half of its original mass. [3]

(iv) Find the rate at which the substance decays at $t = 50$ hours. [2]

[Turn over

12 It is given that $f(x)=\frac{1}{2}-3\cos 2x$ for $0\le x\le 2\pi$ and $g(x)=2\sin\left(\frac{x}{2}\right)$ for $0\le x\le 2\pi$.

(i) State the period of $g(x)$. [1]

(ii) State the least and greatest values of $f(x)$. [1]

(iii) Sketch, on the same axes, the graphs of $y=f(x)$ and $y=g(x)$ for $0\le x\le 2\pi$. [4]

(iv) Hence, state the number of solutions of the equation $6\cos 2x + 4\sin\left(\frac{x}{2}\right) = 1$ for $0 \le x \le 2\pi$. [1]

(v) Determine the value of k for which the equation $\frac{1}{2} - 3\cos 2x = 2\sin\left(\frac{x}{2}\right) + k$ has one solution. [1]

[Turn over

13 **(i)** Find the coordinates of the foot of the perpendicular from $A(3,\ 11)$ to the line l, $2x+4y=25$. [4]

(ii) Hence, calculate the perpendicular distance from A to the line l. [2]

(iii) If B is the reflection of A in the line l, find the coordinates of B. [2]

END OF PAPER

Name:	Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR 2020
PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS **4047/02**
Paper 2 **28 August 2020**

Additional Materials: NIL **2 hours 30 minutes**

READ THESE INSTRUCTIONS FIRST
Write your name, register number and class in the spaces at the top of the page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
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The total of the marks for this paper is **100**.

For Examiners' Use

Question	1	2	3	4	5	6	7	8	9	10	11
Marks											

Table of Penalties		Question Number	Parent's/Guardian's Signature	
Presentation	–1			
Units	–1			
Significant Figures	–1			100

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

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Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-r)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

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$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
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$$\Delta = \frac{1}{2} bc \sin A$$

1 The equation of a polynomial is given by $P(x) = 2x^3 + cx^2 + 7$. The remainder when $P(x)$ is divided by $2x - 1$ is twice the remainder when $P(x)$ is divided by $x - 2$.

(i) Show that the value of c is -5. [3]

(ii) Using the value of c in part (i), explain why the equation $P(x) = 0$ has only one real root. [4]

[Turn over

2 A particle moves in a straight line so that after t seconds, its distance, s metres, from a fixed point O is given by $s = e^t - \frac{5}{2e^t} - 7$.

(i) Find the distance of the particle from O when $t = \ln 2$.
By comparing with the instant when the particle first moves off, determine if the particle is moving towards or away from O at $t = \ln 2$. [3]

(ii) Find the value of t when the velocity, v = 5.5 m/s. [4]

3 **(i)** Differentiate $x\cos^2 x$ with respect to x. [2]

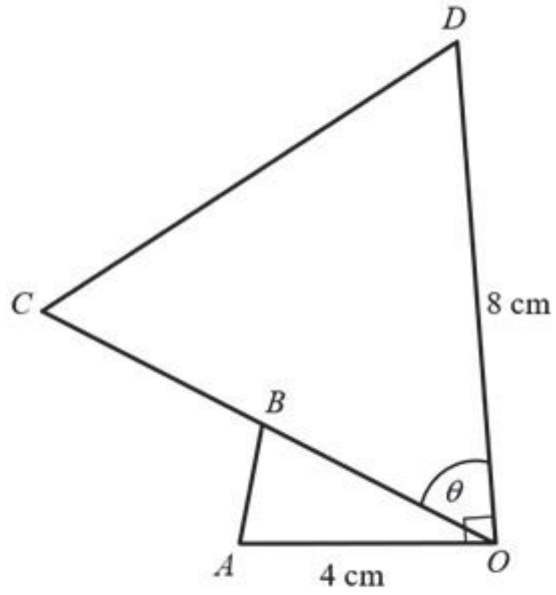
(ii) Hence show that $\int_0^{\frac{\pi}{4}} \frac{\sin^2 x + x\sin 2x}{2}\, dx = \frac{\pi}{16}$. [4]

[Turn over

4 **(a)** Find the range of values of k for which $3x^2+8x+k$ is always positive. [2]

(b) The equation $px^2+7qx+9p=0$ has equal roots. Given that p and q are positive, find the ratio $p:q$ and solve the equation. [4]

5 The diagram shows two isosceles triangles OAB and COD.
$OA = OB = 4$ cm, $OC = OD = 8$ cm, angle $AOD = \frac{\pi}{2}$ and angle $COD = \theta$.
The sum of the areas of triangles OAB and OCD is S cm^2.



(i) Show that S can be expressed in the form $p\sin\theta + q\cos\theta$, where p and q are constants. [3]

(ii) Express S in the form $R\sin(\theta+\alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.
Given that the sum of the areas of triangles OAB and OCD is 32 cm^2, find the value of θ. [5]

[Turn over

6 A rectangle of area A m^2 has sides measuring x m by $(px+q)$ m, where p and q are constants and x and A are variables. Values of x and A are given in the table below.

x	50	100	150	200	250
A	3800	10 500	20 400	33 000	48 500

(i) Using suitable variables, draw, on the grid opposite, a straight line graph and hence estimate the value of each of the constants p and q. [6]

(ii) On the same diagram, draw the straight line representing the equation $A = x^2$ and explain the significance of the value of x given by the point of intersection of the two lines. [3]

[Turn over

7 **(a)** Given that $\operatorname{cosec} x + \cot x = 4$ and x is acute, show that $\operatorname{cosec} x - \cot x = \frac{1}{4}$. [5]

Hence find the value of $\sec x$.

(b) Given that $\frac{1 + 2\cos^2\theta}{\sin^2\theta} = \frac{43}{3}$ for $90^o \le \theta \le 180^o$, find the value of $\frac{\sin\theta}{1 + \cos\theta}$, giving your answer in the form $\frac{1}{3}\left(a + b\sqrt{c}\right)$ where *a*, *b* and *c* are integers. [5]

8 The quadratic equation $2x^2 + 6x + k = 0$ has roots α and β.

(i) Find the value of $(\alpha - \beta)^2$ in terms of *k*. [3]

(ii) Find the smallest integer value of *k* if the equation has no *x*-intercept. [3]

(iii) Given that the value of *k* is 5, find a quadratic equation whose roots are

$\dfrac{\alpha^2}{\beta}$ and $\dfrac{\beta^2}{\alpha}$. [5]

[Turn over

9 (a) Find the coefficient of $\frac{1}{x^4}$ in $\left(2x-\frac{1}{x}\right)^{14}$. [3]

(b) **(i)** Write down, without simplifying, the fourth term in the binomial expansion of $\left(ax+\frac{b}{x}\right)^{n}$. [2]

(ii) If this term is a constant term, find the value of n. [2]

(iii) The fourth term of the binomial expansion is 160 and $a-b=1$ where both a and b are positive.
Use the value of n in part (ii) to calculate the value of a and of b. [4]

[Turn over

10 A circle with centre C, passes through the origin and has x-intercept of 4 at point A and y-intercept of 3 at point B.

(i) Explain why the centre of the circle is (2, 1.5). [1]

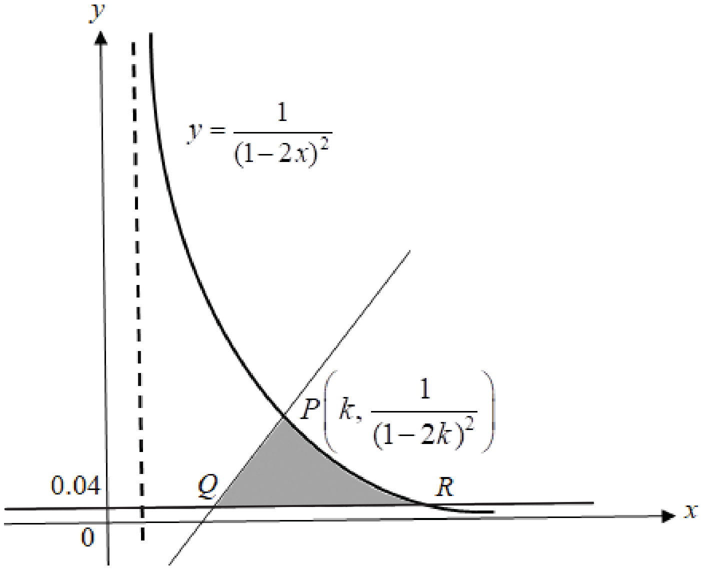
(ii) Determine the equation of the circle in the general form. [3]

(iii) OD is a diameter of the circle. Find the coordinates of D. [2]

(iv) The tangent at D cuts the x-axis at E. Find the area of triangle BCE. [5]

[Turn over

11



The diagram (not drawn to scale) shows part of the curve $y = \frac{1}{(1-2x)^2}$ passing through the point $P\left(k, \frac{1}{(1-2k)^2}\right)$, where k is a constant. The normal to the curve at P is perpendicular to $2y = 1 - x$ and the normal meets the horizontal line $y = 0.04$ at point Q. The curve meets the same horizontal line at the point R.

(i) State the equation of the asymptote denoted by dotted lines. [1]

(ii) Prove that $k = 1.5$. [4]

(iii) Show that the x-coordinates of Q and R are 1.395 and 3 respectively. [4]

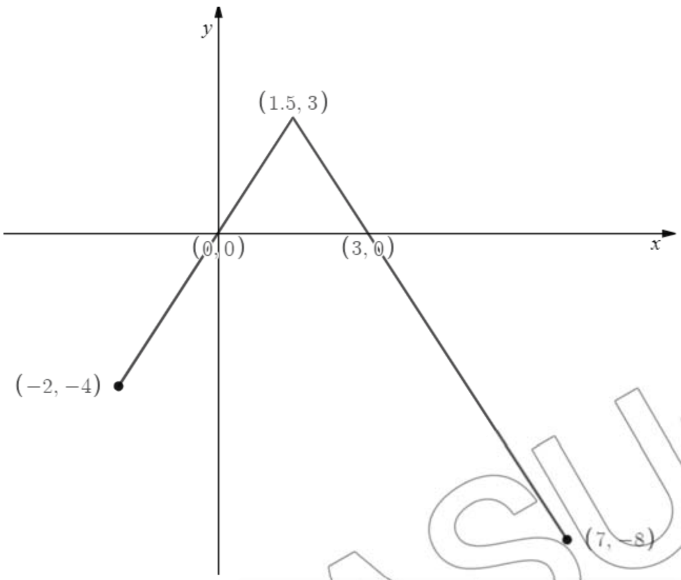
(iv) Find the area of the shaded region PQR, giving your answer to 2 significant figures. [5]

END OF PAPER

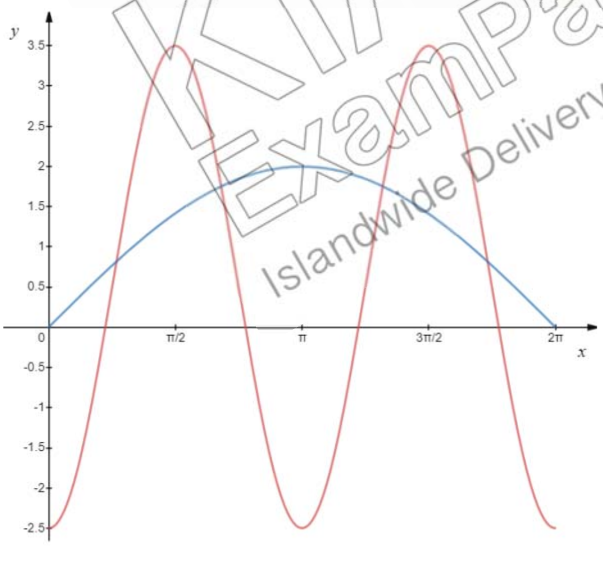
2020 CGS AM Prelim Paper 1

Qn	Answer Key		
1	$k \le -1$ or $k \ge 2$, $k \ne -2$	10(i)	12.4 cm or $\sqrt[3]{\frac{6000}{\pi}}$
2	$\left(\frac{116}{17}+\frac{26}{17}\sqrt{2}\right)$ cm	(iii)	The thickness of the container is negligible.
3	$x = 2$, $y = \frac{4}{5}$ $x = 1, y = 1$	11(ii)	1.55 g
4	$x < \frac{7}{6}$	(iii)	51.3 hours
5(i)	$\frac{3x+y}{1-x}$	(iv)	0.103g/h
(ii)	$2^{\ln 9+1}$ or $2^{\ln 9e}$ or $2^{2\ln 3+1}$	12(i)	4π
6	$\frac{x^4}{x^4-16} = 1+\frac{1}{2(x-2)}-\frac{1}{2(x+2)}-\frac{2}{x^2+4}$	(ii)	least value of $f(x) = -2.5$ greatest value of $f(x) = 3.5$
7	450.8 cm^3/s	(iv)	4
8(i)	$-\frac{44}{125}$	(v)	$k = -4.5$
(ii)	$\frac{1}{3}$	13(i)	(0.5, 6).
9(i)		(ii)	5.59 units
(ii)	$\frac{1}{3}$	(iii)	$B(-2,\ 1)$
(iii)	$-\frac{10}{7} \le m < \frac{2}{3}$		

9(i)



12 (iii)



2020 CGS AM Prelim Paper 2 Answer Key

Qn	Answer Key		
1(i)	$x=-1$ is the only root		
2(i)	6.25 m; towards *O*	(ii)	$t = 1.61$
3(i)	$-x\sin 2x+\cos^2 x$	(ii)	$\frac{\pi}{16}$
4(a)	$k>5\frac{1}{3}$	(b)	$p:q=7:6$; $x=-3$
5(i)	$S=32\sin\theta+8\cos\theta$	(ii)	$S=8\sqrt{17}\sin(\theta+0.245)$; 1.08
6(i)	Plot $\frac{A}{x}$ against x; $p = 0.59$ *(accept 0.59 – 0.62)*; $q = 46$ *(Accept 45 – 47)*		
(ii)	$x = 115$; square		
7(a)	$\sec x=\frac{17}{15}$	(b)	$\frac{1}{3}\left(7+2\sqrt{10}\right)$
8(i)	$9 - 2k$	(ii)	5
(iii)	$10x^2+18x+25=0$		
9(a)	–64 064	(b)(i)	$T_4=\binom{n}{3}(ax)^{n-3}\left(\frac{b}{x}\right)^3$
		(b)(ii)	$n = 6$
		(b)(iii)	$a = 2$; $b = 1$
10(i)	C lies on the perpendicular bisectors of *OA* and *OB*.	(ii)	$x^2+y^2-4x-3y=0$
(iii)	*D*(4, 3)	(iv)	1.6875 units
11(i)	$x = 0.5$	(iv)	0.10 units^2

Name: Worked Solutions	Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR 2020
PRELIMINARY EXAMINATION

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Paper 1 **25 August 2020**

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-r)\ldots(n-r+1)}{r!}$

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$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 Find the range of values of k for which the quadratic equation $(k+2)x^2 - 2kx + 1 = 0$ has real roots. [3]

Solution:

$(k+2)x^2 - 2kx + 1 = 0$

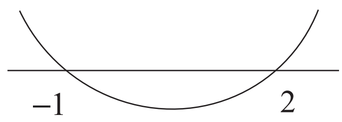
For the equation to have real roots, $b^2 - 4ac \geq 0$

$(-2k)^2 - 4(k+2)(1) \geq 0$ [M1]

$4k^2 - 4k - 8 \geq 0$

$k^2 - k - 2 \geq 0$

$(k-2)(k+1) \geq 0$ [M1]



[A1]

$k \leq -1$ or $k \geq 2$, $k \neq -2$

2 A triangle ABC has an area of $\left(14 - 3\sqrt{2}\right)$ cm^2 and $AC = \left(5 - \frac{4}{\sqrt{2}}\right)$ cm. Express the perpendicular distance from B to AC in the form of $a + b\sqrt{2}$, where a and b are rational numbers. [4]

Solution:

Let the perpendicular distance from B to AC be h.

$\frac{1}{2}\left(5 - \frac{4}{\sqrt{2}}\right)h = 14 - 3\sqrt{2}$ [M1]

$\left(\frac{5\sqrt{2} - 4}{\sqrt{2}}\right)h = 28 - 6\sqrt{2}$

$h = \frac{28\sqrt{2} - 12}{5\sqrt{2} - 4}$ [M1]

[M1] – Rationalising denominator

$= \frac{28\sqrt{2} - 12}{5\sqrt{2} - 4} \times \frac{5\sqrt{2} + 4}{5\sqrt{2} + 4}$

$= \frac{280 + 112\sqrt{2} - 60\sqrt{2} - 48}{50 - 16}$

$= \frac{232 + 52\sqrt{2}}{34}$

$= \left(\frac{116}{17} + \frac{26}{17}\sqrt{2}\right)$ cm [A1]

3 Given that $(1,h)$ is a solution of the simultaneous equations $\frac{1}{x}+\frac{2}{y}-3=0$ and $x+ky-6=0$, find the values of x and of y. [5]

Solution:

$$1+\frac{2}{h}-3=0$$
$$\frac{2}{h}=2$$
$$h=1$$

[M1] – Substitute $(1,h)$ into both equations.

$$1+k(1)-6=0$$
$$k=5$$

[A1] – Correct values of h and k.

$$\frac{1}{x}+\frac{2}{y}-3=0 \quad \text{--- (1)}$$
$$x+5y-6=0$$
$$x=6-5y \quad \text{--- (2)}$$

Sub (2) into (1):

$$\frac{1}{6-5y}+\frac{2}{y}-3=0$$

[M1]

$$\frac{y+2(6-5y)}{y(6-5y)}=3$$
$$12-9y=18y-15y^2$$
$$5y^2-9y+4=0$$

[M1]

$$(5y-4)(y-1)=0$$
$$y=\frac{4}{5} \quad \text{or} \quad 1$$
$$x=2 \quad \text{or} \quad 1$$

[A1]

4 Given that $y = \dfrac{e^{-2x}}{3x-5}$, find the range of values of x for which y is an increasing function. [5]

Solution:

$$y = \frac{e^{-2x}}{3x-5}$$

$$\frac{dy}{dx} = \frac{(3x-5)(-2e^{-2x}) - (e^{-2x})(3)}{(3x-5)^2}$$ [M1]

$$= \frac{-6xe^{-2x} + 10e^{-2x} - 3e^{-2x}}{(3x-5)^2}$$

$$= \frac{-6xe^{-2x} + 7e^{-2x}}{(3x-5)^2}$$

For y to be an increasing function,

$$\frac{-6xe^{-2x} + 7e^{-2x}}{(3x-5)^2} > 0$$ [M1]

Since $(3x-5)^2 > 0$ for all real values of x, [M1] – condition stated

$-6xe^{-2x} + 7e^{-2x} > 0$

$e^{-2x}(-6x+7) > 0$

Since $e^{-2x} > 0$,

$-6x + 7 > 0$ [M1] – condition stated

$x < \dfrac{7}{6}$ [A1]

5 **(i)** Given that $\lg 2 = x$ and $\lg 3 = y$, find the value of $\log_5 24$ in terms of x and y. [4]

Solution:

(i)

$$\log_5 24 = \frac{\lg 24}{\lg 5}$$ [M1]

$$= \frac{\lg(2^3 \times 3)}{\lg\left(\frac{10}{2}\right)}$$ [M1] – breaking into prime factors

$$= \frac{3\lg 2 + \lg 3}{\lg 10 - \lg 2}$$ [M1] – applying product or quotient law

$$= \frac{3x + y}{1 - x}$$ [A1]

(ii) Express $4^{\ln 3} + 2^{\ln 9}$ in the form 2^m, where m is a constant. [2]

Solution:

(ii)

$$4^{\ln 3} + 2^{\ln 9} = 2^{2\ln 3} + 2^{\ln 9}$$

$$= 2^{\ln 9} + 2^{\ln 9}$$ [M1] – $4^{\ln 3} = 2^{\ln 9}$

$$= 2(2^{\ln 9})$$

$$= 2^{\ln 9 + 1} \quad \text{or} \quad 2^{\ln 9e} \quad \text{or} \quad 2^{2\ln 3 + 1}$$ [A1]

6 Express $\dfrac{x^4}{x^4-16}$ in partial fractions. [6]

Solution:

$$\frac{x^4}{x^4-16}=\frac{x^4-16+16}{x^4-16}$$
$$=1+\frac{16}{x^4-16}$$
[B1] – or long division

$$x^4-16=(x^2-4)(x^2+4)$$
$$=(x-2)(x+2)(x^2+4)$$
[B1] – Factorise denominator completely

Let $\dfrac{16}{(x-2)(x+2)(x^2+4)}=\dfrac{A}{x-2}+\dfrac{B}{x+2}+\dfrac{Cx+D}{x^2+4}$. [B1]

Multiply throughout by $(x-2)(x+2)(x^2+4)$,

$16=A(x+2)(x^2+4)+B(x-2)(x^2+4)+(Cx+D)(x-2)(x+2)$

Let $x=2$,

$16=A(4)(8)$

$A=\dfrac{1}{2}$

Let $x=-2$,

$16=B(-4)(8)$

$B=-\dfrac{1}{2}$

Let $x=0$,

$16=\dfrac{1}{2}(2)(4)-\dfrac{1}{2}(-2)(4)+D(-2)(2)$

$D=-2$

Let $x=1$,

$16=\dfrac{1}{2}(3)(5)-\dfrac{1}{2}(-1)(5)+(C-2)(-1)(3)$

$C=0$

[M2] – solve for A, B, C using substitution or comparing coefficients. 2 marks if all correct, 1 mark if 2 correct.

$$\frac{x^4}{x^4-16}=1+\frac{1}{2(x-2)}-\frac{1}{2(x+2)}-\frac{2}{x^2+4}$$
[A1]

7 Water flows out of a filled container of height 20 cm such that after t seconds, the depth of water in the container is h cm and the volume of water left in the container is given by $V=10+\frac{h^2(h+2)}{10}$. Given that the height of the water level is decreasing at a rate of $(4t-1)$ cm/s. find the rate of decrease of the volume of the water when $t=2$. [6]

Solution:

$$V=10+\frac{h^2(h+2)}{10}$$
$$=10+\frac{h^3+2h^2}{10}$$
$$\frac{dV}{dh}=\frac{3h^2+4h}{10}$$ [B1]

$$\frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt}$$
$$=\left(\frac{3h^2+4h}{10}\right)\times\left(-(4t-1)\right)$$ [M1]

$$\frac{dh}{dt}=-(4t-1)$$
$$h=\int -4t+1 \text{ dt}$$ [M1]
$$=-2t^2+t+c$$
When $t=0$, $h=20$, $c=20$.
$$h=-2t^2+t+20$$ [M1]

When $t=2$, $h=14$ cm.

$$\frac{dV}{dt}=\left(\frac{3(14)^2+4(14)}{10}\right)\times\left(-(4(2)-1)\right)$$
$$=-450.8 \text{ cm}^3/\text{s}$$ [M1]

Rate of decrease of the volume of water when $t=2$ is $450.8 \text{ cm}^3/\text{s}$. [A1]

8 It is given that $\tan B = \dfrac{3}{4}$ where *B* is an acute angle. Without solving for *B*, find the exact value of

(i) $\cos 3B$, [4]

Solution:

(i)
$$\cos 3B = \cos(2B + B)$$
$$= \cos 2B \cos B - \sin 2B \sin B$$ [M1] – addition formula
$$= (2\cos^2 B - 1)(\cos B) - (2\sin B \cos B)(\sin B)$$ [M1] – double angle
$$= \left(2\left(\frac{4}{5}\right)^2 - 1\right)\left(\frac{4}{5}\right) - \left(2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)\right)\left(\frac{3}{5}\right)$$ [M1]
$$= -\frac{44}{125}$$ [A1]

(ii) $\tan \dfrac{B}{2}$. *[Hint: Use double angle formula.]* [3]

Solution:

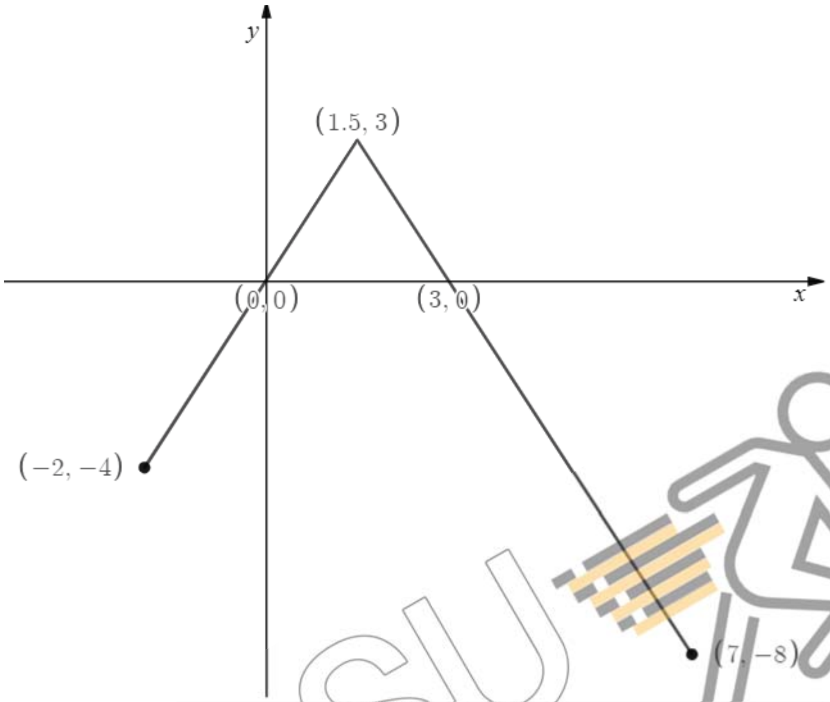
(ii)
$$\tan B = \frac{2\tan \frac{B}{2}}{1 - \tan^2 \frac{B}{2}}$$
$$\frac{3}{4} = \frac{2\tan \frac{B}{2}}{1 - \tan^2 \frac{B}{2}}$$ [M1]
$$3 - 3\tan^2 \frac{B}{2} = 8\tan \frac{B}{2}$$
$$3\tan^2 \frac{B}{2} + 8\tan \frac{B}{2} - 3 = 0$$
$$\left(3\tan \frac{B}{2} - 1\right)\left(\tan \frac{B}{2} + 3\right) = 0$$ [M1]
$$\tan \frac{B}{2} = \frac{1}{3} \quad \text{or} \quad -3$$

Since $\tan \dfrac{B}{2} > 0$, $\tan \dfrac{B}{2} = \dfrac{1}{3}$. [A1]

9 **(i)** Sketch the graph of $f(x)=3-|2x-3|$ for $-2\leq x\leq 7$, indicating the coordinates of the intercepts, endpoints and vertex clearly. [3]

Solution:

(i)



B1 – Shape
B1 – Intercepts and vertex
B1 - Endpoints

(ii) State least positive value of $\dfrac{1}{f(x)}$. [1]

Solution:

(ii) $\dfrac{1}{3}$ [B1]

(iii) State the range of values of m such that the line $y = mx + 2$ intersects the graph at 2 distinct points. [3]

Solution:

(iii) $m = \dfrac{2-(-8)}{0-7}$ [M1]

$= -\dfrac{10}{7}$

$m = \dfrac{3-2}{1.5-0}$ [M1]

$= \dfrac{2}{3}$

$-\dfrac{10}{7} \le m < \dfrac{2}{3}$ [B1]

10 A company wants to produce open cylindrical containers with a capacity of 6000 cm^3 each.

(i) Find the radius of the container if the company wants to minimize the amount of material used. [4]

Solution:

(i)
$$V = \pi r^2 h$$
$$\pi r^2 h = 6000$$
$$h = \frac{6000}{\pi r^2}$$ [M1]

$$A = \pi r^2 + 2\pi r h$$
$$= \pi r^2 + 2\pi r\left(\frac{6000}{\pi r^2}\right)$$ [M1]
$$= \pi r^2 + \frac{12000}{r}$$
$$\frac{dA}{dr} = 2\pi r - \frac{12000}{r^2}$$

For stationary value of A, $\frac{dA}{dr} = 0$ [M1]

$$2\pi r - \frac{12000}{r^2} = 0$$
$$r^3 = \frac{6000}{\pi}$$
$$r = 12.4 \text{ cm} \quad \text{or} \quad \sqrt[3]{\frac{6000}{\pi}}$$ [A1]

(ii) For the value of the radius found in (i), show that the amount of material used is minimum. [2]

Solution:

(ii)	$\frac{d^2A}{dr^2} = 2\pi + \frac{24000}{r^3}$ When $r^3 = \frac{6000}{\pi}$, $\frac{d^2A}{dr^2} = 2\pi + \frac{24000}{\left(\frac{6000}{\pi}\right)}$ $= 6\pi > 0$ Since $\frac{d^2A}{dr^2} > 0$, the amount of material used is minimum when r = 12.4 cm.	[M1] – accept first derivative test [A1] – $\frac{d^2A}{dr^2} > 0$ must be stated with justification shown. Must at least state $r>0$ if no substitution shown.

(iii) State an assumption made in your calculations above. [1]

Solution:

(iii)	The thickness of the container is negligible.	[B1]

11 A radioactive substance decays according to the equation $A = 15e^{-bt}$, where A is the mass in grams of radioactive substance remaining and t is the time in hours after the radioactive substance starts to decay.

(i) Given that there are 10 grams of radioactive substance remaining after 30 hours, verify that the value of b is 0.013516. [2]

Solution:

(i)

$$A = 15e^{-bt}$$

$$10 = 15e^{-30b}$$

$$e^{-30b} = \frac{10}{15}$$ [M1]

$$-30b = \ln\frac{10}{15}$$

$$b = -\frac{1}{30}\ln\frac{10}{15}$$

$$= 0.013516$$ [A1]

(ii) Find the mass of radioactive substance left at the end of 1 week. [1]

Solution:

(ii)

$$A = 15e^{-0.013516(7\times24)}$$

$$= 1.55\text{g}$$ [B1]

(iii) Find the number of hours for the radioactive substance to decay to half of its original mass. [3]

Solution:

(iii) When $t = 0$, $A = 15$. [B1] - as long students show understanding that $A = 15$ when $t = 0$.

$$A = 15e^{-0.013516t}$$

$$\frac{15}{2} = 15e^{-0.013516t}$$

$$e^{-0.013516t} = \frac{1}{2}$$ [M1]

$$-0.013516t = \ln\frac{1}{2}$$

$$t = 51.3 \text{ hours}$$ [A1]

(iv) Find the rate at which the substance decays at $t = 50$ hours. [2]

Solution:

(iv) $A = 15e^{-0.013516t}$

$\frac{dA}{dt} = -0.20274e^{-0.013516t}$ [M1]

When $t = 50$,

$\frac{dA}{dt} = -0.20274e^{-0.013516(50)}$

$= -0.103$

The rate of radioactive decay at $t = 50$ hours is 0.103g/h. [A1]

12 It is given that $f(x)=\frac{1}{2}-3\cos 2x$ for $0\le x\le 2\pi$ and $g(x)=2\sin\left(\frac{x}{2}\right)$ for $0\le x\le 2\pi$.

(i) State the period of $g(x)$. [1]

Solution:

(i) $\text{period}=\frac{2\pi}{\frac{1}{2}}$

$=4\pi$ [B1]

(ii) State the least and greatest values of $f(x)$. [1]

Solution:

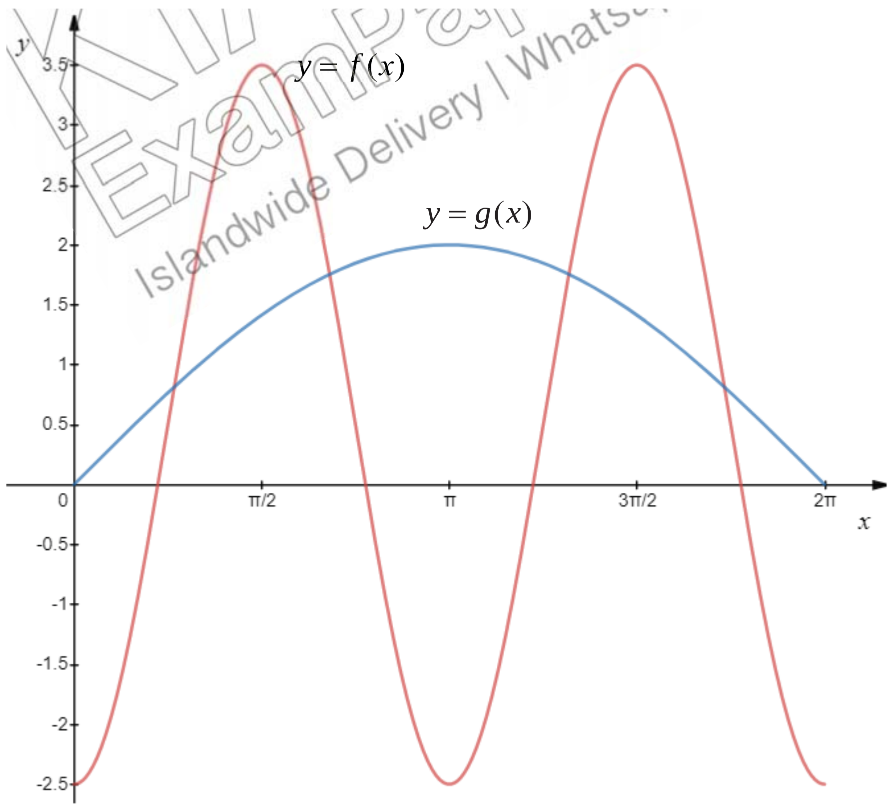
(ii) least value of $f(x)=-2.5$ [B1] – Both correct

greatest value of $f(x)=3.5$

(iii) Sketch, on the same axes, the graphs of $y=f(x)$ and $y=g(x)$ for $0\le x\le 2\pi$. [4]

Solution:

(iii)



For each graph:
B1 – shape
B1 – correct periods and max/min points

(iv) Hence, state the number of solutions of the equation $6\cos 2x + 4\sin\left(\frac{x}{2}\right) = 1$ for $0 \le x \le 2\pi$. [1]

Solution:

(iv) $6\cos 2x + 4\sin\left(\frac{x}{2}\right) = 1$

$$2\sin\frac{x}{2} = \frac{1}{2} - 3\cos 2x$$

Number of solutions = 4 [B1]

(v) Determine the value of k for which the equation $\frac{1}{2} - 3\cos 2x = 2\sin\left(\frac{x}{2}\right) + k$ has one solution. [1]

Solution:

(v) $k = -4.5$ [B1]

13 **(i)** Find the coordinates of the foot of the perpendicular from $A(3, 11)$ to the line l, $2x+4y=25$. [4]

Solution:

(i) $2x+4y=25$

$y=-\frac{1}{2}x+\frac{25}{4}$

Gradient of perpendicular = 2 [B1]

Equation of perpendicular line:

$y-11=2(x-3)$ [M1]

$y=2x+5$

$2x+4y=25$ --- (1)

$y=2x+5$ --- (2)

Sub (2) into (1):

$2x+4(2x+5)=25$ [M1]

$10x=5$

$x=0.5$

$y=6$

Coordinates are (0.5, 6). [A1]

(ii) Hence, calculate the perpendicular distance from A to the line l. [2]

Solution:

(ii) perpendicular distance from A to $l=\sqrt{(3-0.5)^2+(11-6)^2}$ [M1]

$=\sqrt{31.25}$

$=5.59$ units (3sf) [A1]

(iii) If B is the reflection of A in the line l, find the coordinates of B. [2]

Solution:

(iii) Let the coordinates of B be (x, y).

$\left(\frac{x+3}{2}, \frac{y+11}{2}\right)=(0.5,\ 6)$ [M1]

$\frac{x+3}{2}=0.5$ $\frac{y+11}{2}=6$

$x=-2$ $y=1$

$B(-2, 1)$ [A1]

Name: Worked Solutions	Register No.:	Class:

CRESCENT GIRLS' SCHOOL
SECONDARY FOUR 2020
PRELIMINARY EXAMINATION

ADDITIONAL MATHEMATICS **4047/02**
Paper 2 **28 August 2020**

Additional Materials: NIL **2 hours 30 minutes**

READ THESE INSTRUCTIONS FIRST
Write your name, register number and class in the spaces at the top of the page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the answer spaces provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is **100**.

For Examiners' Use

Question	1	2	3	4	5	6	7	8	9	10	11
Marks											

Table of Penalties		Question Number	Parent's/Guardian's Signature	100
Presentation	–1			
Units	–1			
Significant Figures	–1			

This document consists of **17** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-r)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

1 The equation of a polynomial is given by $P(x)=2x^3+cx^2+7$. The remainder when $P(x)$ is divided by $2x-1$ is twice the remainder when $P(x)$ is divided by $x-2$.

(i) Show that the value of c is -5. [3]

$P(x)=2x^3+cx^2+7$

$P\left(\frac{1}{2}\right)=2P(2)$ [M1]

$2\left(\frac{1}{2}\right)^3+c\left(\frac{1}{2}\right)^2+7=2\left[2(2)^3+c(2)^2+7\right]$

$\frac{1}{4}+\frac{c}{4}+7=46+8c$ [M1]

$\frac{31}{4}c=-\frac{155}{4} \Rightarrow c=-5$ [A1]

(ii) Using the value of c in part (i), explain why the equation $P(x)=0$ has only one real root. [4]

$P(x)=2x^3-5x^2+7$

$P(-1)=2(-1)^3-5(-1)^2+7$
$=0$ [M1]

By Factor Theorem, $x+1$ is a factor of $P(x)$.

$2x^3-5x^2+7=(x+1)(2x^2+px+7)$

Compare coeff of x: $0=7+p$ [M1] – long division, synthetic division

$p=-7$

$P(x)=0$

$(x+1)(2x^2-7x+7)=0$

$x=-1$ or $x=\frac{7\pm\sqrt{(-7)^2-4(2)(7)}}{2(2)}$ [M1] – or find discriminant

$=\frac{7\pm\sqrt{-7}}{4}$ (no real root)

$\therefore$ there is only one real root, i.e. $x=-1$ in the equation $P(x)=0$. [A1] – with appropriate conclusion

[Turn over

2 A particle moves in a straight line so that after t seconds, its distance, s metres, from a fixed point O is given by $s = e^t - \frac{5}{2e^t} - 7$.

(i) Find the distance of the particle from O when $t = \ln 2$.
By comparing with the instant when the particle first moves off, determine if the particle is moving towards or away from O at $t = \ln 2$. [3]

$s = e^t - \frac{5}{2e^t} - 7$

When $t = \ln 2$, $s = e^{\ln 2} - \frac{5}{2e^{\ln 2}} - 7$

$= -6.25$ m

The distance of the particle from O is 6.25 m. [B1]

When $t = 0$, $s = -8.5$ m. The particle is moving towards O at $t = \ln 2$. [B1][B1]

(ii) Find the value of t when the velocity, $v = 5.5$ m/s. [4]

$s = e^t - \frac{5e^{-t}}{2} - 7$

$v = \frac{ds}{dt} = e^t + \frac{5e^{-t}}{2}$ [M1]

$e^t + \frac{5}{2e^t} = 5.5$

Let $y = e^t$

$2y^2 - 11y + 5 = 0$ [M1]

$(2y-1)(y-5) = 0$

$y = 0.5$ or 5 [A1] – Both correct

$e^t = 0.5$ or $e^t = 5$

$t = \ln 0.5$ or $t = \ln 5$

$= -0.693$ (rejected) or $t = 1.61$ [A1] – must reject neg value

$\therefore t = 1.61$

3 **(i)** Differentiate $x\cos^2 x$ with respect to x. [2]

$\frac{d}{dx}x\cos^2 x = x(2)(\cos x)(-\sin x)+(\cos^2 x)(1)$ [M1] – product rule

$= -x\sin 2x+\cos^2 x$ [A1]- Accept $-2x\sin x\cos x + \cos^2 x$

(ii) Hence show that $\int_0^{\frac{\pi}{4}} \frac{\sin^2 x + x\sin 2x}{2}\,dx = \frac{\pi}{16}$. [4]

$\frac{d}{dx}x\cos^2 x = -x\sin 2x+\cos^2 x$

$\frac{d}{dx}x\cos^2 x = -x\sin 2x+1-\sin^2 x$ [M1] -- $\cos^2 x = 1-\sin^2 x$

$\sin^2 x + x\sin 2x = 1-\frac{d}{dx}x\cos^2 x$

$\int_0^{\frac{\pi}{4}} \frac{\sin^2 x + x\sin 2x}{2}\,dx = \int_0^{\frac{\pi}{4}} \frac{1}{2}\,dx - \frac{1}{2}\left[x\cos^2 x\right]_0^{\frac{\pi}{4}}$ [M1]

$= \frac{1}{2}\left[x - x\cos^2 x\right]_0^{\frac{\pi}{4}}$ [M1]

$= \frac{1}{2}\left[\frac{\pi}{4}-\frac{\pi}{4}(\frac{1}{2})-0\right]$

$= \frac{\pi}{16}$ [A1]— must show substitution

[Turn over

4 **(a)** Find the range of values of k for which $3x^2+8x+k$ is always positive. [2]

$3x^2+8x+k>0$
$b^2-4ac<0$
$8^2-4(3)k<0$ [M1]
$k>5\frac{1}{3}$ [A1]

(b) The equation $px^2+7qx+9p=0$ has equal roots. Given that p and q are positive, find the ratio $p:q$ and solve the equation. [4]

$px^2+7qx+9p=0$ has equal roots, $b^2-4ac=0$

$(7q)^2-4(p)(9p)=0$ [M1]

$49q^2-36p^2=0$

$(7q+6p)(7q-6p)=0$ [M1]

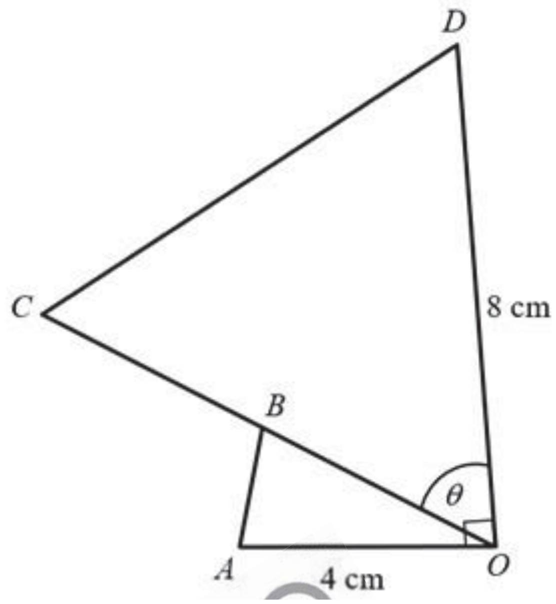
$\frac{p}{q}=-\frac{7}{6}$ (rejected since p and q are positive) or $\frac{p}{q}=\frac{7}{6}$

$p:q=7:6$ [A1] – Must reject neg value

$px^2+7qx+9p=0$
$x^2+7\left(\frac{q}{p}\right)x+9=0$
$x^2+7\left(\frac{6}{7}\right)x+9=0$
$x^2+6x+9=0$
$(x+3)^2=0 \Rightarrow x=-3$ [B1]

Alt mtd:
Let the root be α.
Sum of roots, $2\alpha=-\frac{7q}{p}$
$\alpha=-7\left(\frac{6}{7}\right)$
$=-3$

5 The diagram shows two isosceles triangles OAB and COD.
$OA = OB = 4$ cm, $OC = OD = 8$ cm, angle $AOD = \frac{\pi}{2}$ and angle $COD = \theta$.
The sum of the areas of triangles OAB and OCD is S cm^2.



(i) Show that S can be expressed in the form $p\sin\theta + q\cos\theta$, where p and q are constants. [3]

$S = \frac{1}{2}(8)^2 \sin\theta + \frac{1}{2}(4)^2 \sin\left(\frac{\pi}{2} - \theta\right)$ [M1] – Using $\frac{1}{2}ab\sin C$

Since $\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$, [M1] – need to specify

$S = 32\sin\theta + 8\cos\theta$ [A1]

(ii) Express S in the form $R\sin(\theta + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.
Given that the sum of the areas of triangles OAB and OCD is 32 cm^2, find the value of θ. [5]

$32\sin\theta + 8\cos\theta = R\sin(\theta + \alpha)$

$R = \sqrt{32^2 + 8^2}$ [M1]
$= 8\sqrt{17}$

$\alpha = \tan^{-1}\left(\frac{8}{32}\right)$ [M1]

$= 0.24498$

$S = 8\sqrt{17}\sin(\theta + 0.245)$ [A1]
$8\sqrt{17}\sin(\theta + 0.24498) = 32$

$\sin(\theta + 0.24498) = \frac{4}{\sqrt{17}}$ [M1]

Basic angle = 1.3258
$\theta = 1.3258 - 0.24498$
$= 1.08$ [A1]

[Turn over

6 A rectangle of area A m^2 has sides measuring x m by $(px+q)$ m, where p and q are constants and x and A are variables. Values of x and A are given in the table below.

x	50	100	150	200	250
A	3800	10 500	20 400	33 000	48 500

(i) Using suitable variables, draw, on the grid opposite, a straight line graph and hence estimate the value of each of the constants p and q. [6]

$A = x(px+q)$

$\frac{A}{x} = px+q$ [M1]

Plot $\frac{A}{x}$ against x

x	50	100	150	200	250
$\frac{A}{x}$	76	105	136	165	194

[B1] – Table of values

Appropriate scale with labelled axes [B1]

All points plotted correctly and joined with best-fit straight line [B1]

From the graph,

Gradient, $p = \frac{180-100}{225-90}$

$= 0.59$ [B1] Accept 0.58 – 0.62

$q = 46$ [B1] Accept 45 to 47

(ii) On the same diagram, draw the straight line representing the equation $A = x^2$ and explain the significance of the value of x given by the point of intersection of the two lines. [3]

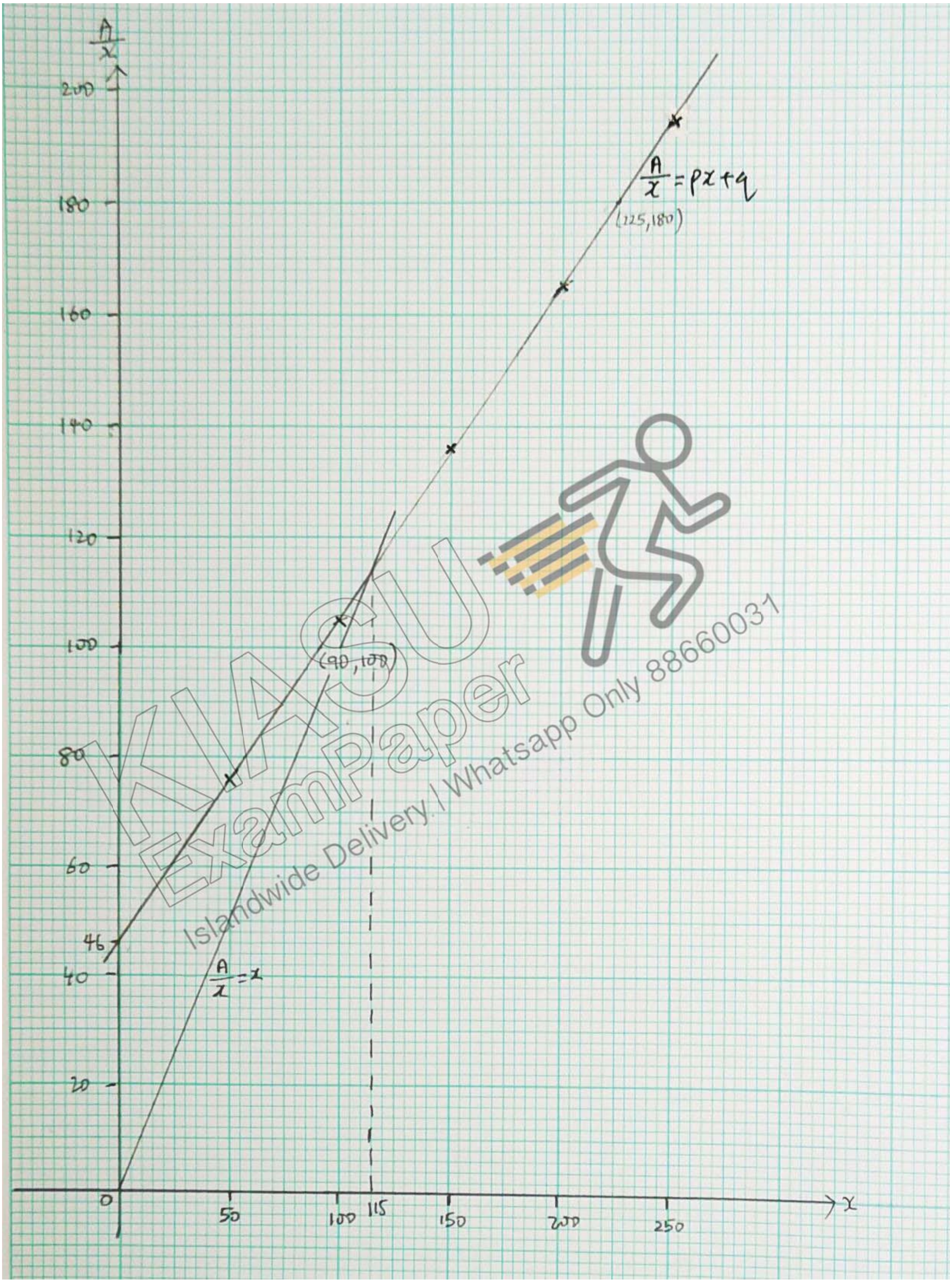
$A = x^2$

Insert $\frac{A}{x} = x$

Correct line inserted and equation labelled [B1]

The intersection of the graphs $\frac{A}{x} = px+q$ and $\frac{A}{x} = x$ give <u>$x = 115$</u> [B1] – Accept 110 – 120

which happens when the <u>rectangle is a square</u>. [B1]



A/x
200
180
160
140
120
100
80
60
46
40
20
0
50
100
115
150
200
250
x
A/x = px + q
(125, 180)
(90, 100)
A/x = x

[Turn over

Alternative Method

(i) $A = x(px+q)$

$\frac{A}{x^2} = \frac{q}{x} + p$ [M1]

Plot $\frac{A}{x^2}$ against $\frac{1}{x}$

$\frac{1}{x}$	0.020	0.010	0.0067	0.0050	0.0040
$\frac{A}{x^2}$	1.52	1.05	0.907	0.825	0.776

[B1] – Table of values

Appropriate scale with labelled axes [B1]

All points plotted correctly and joined with best-fit straight line [B1]

From the graph,

Gradient, $q = \frac{1.35 - 0.776}{0.01625 - 0.0040}$

$= 46.9$ [B1] Accept 45 to 47

$p = 0.59$ [B1] Accept 0.58 – 0.62

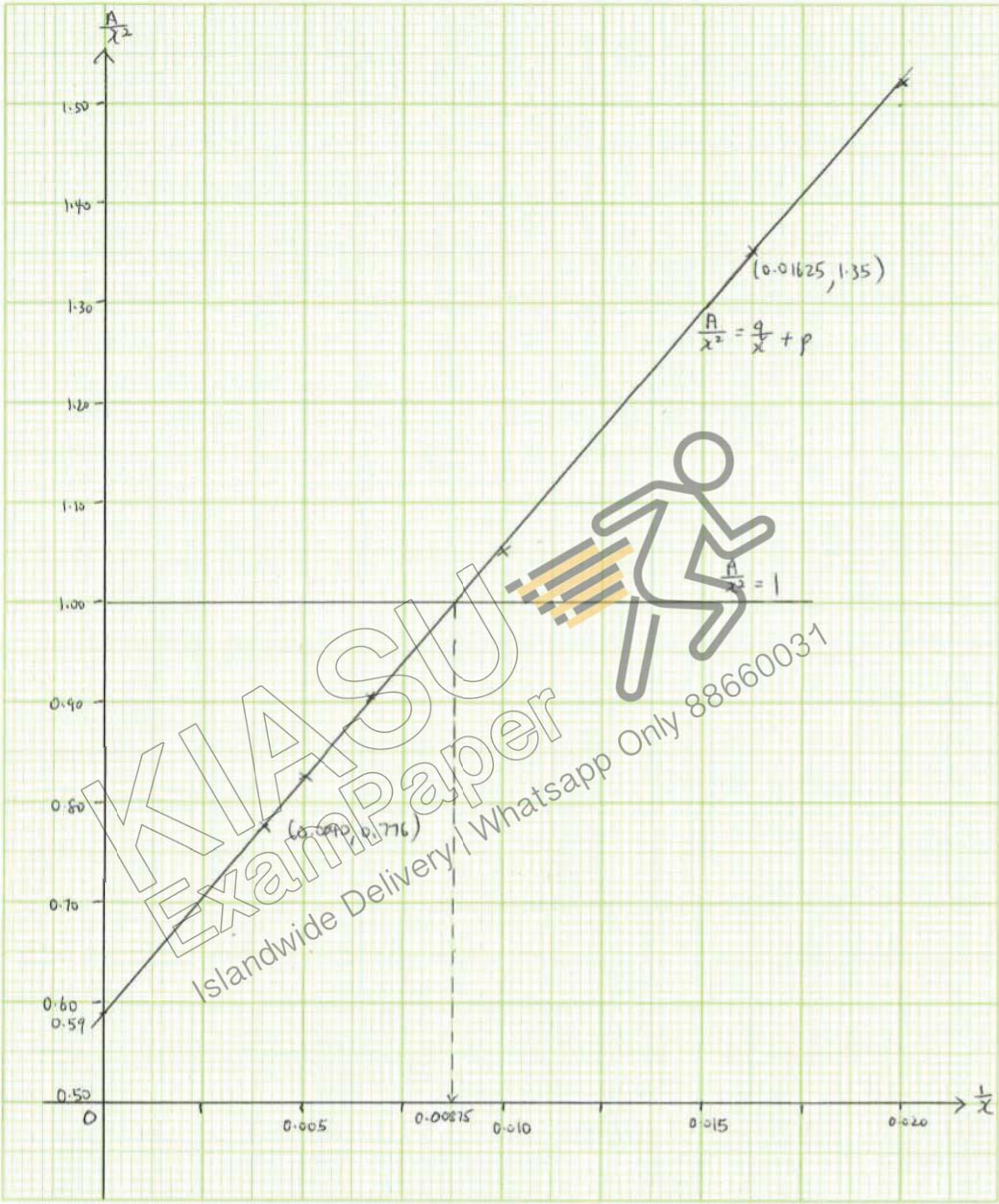
(ii) $A = x^2$

$\Rightarrow \quad \frac{A}{x^2} = 1$

Correct line inserted and equation labelled [B1]

The intersection $\frac{A}{x^2} = \frac{q}{x} + p$ and $\frac{A}{x^2} = 1$ give $x = \frac{1}{0.0875} = \underline{114}$ [B1] – Accept 110 – 120

which happens when the <u>rectangle is a square</u>. [B1]



$\frac{A}{x^2}$
1.50
1.40
1.30
1.20
1.10
1.00
0.90
0.80
0.70
0.60
0.59
0.50
0
0.005
0.00875
0.010
0.015
0.020
$\frac{1}{x}$
(0.01625, 1.35)
$\frac{A}{x^2} = \frac{q}{x} + p$
$\frac{A}{x^2} = 1$
(0.0040, 0.776)

7 **(a)** Given that $\operatorname{cosec} x + \cot x = 4$ and x is acute, show that $\operatorname{cosec} x - \cot x = \frac{1}{4}$.
Hence find the value of $\sec x$. [5]

$\operatorname{cosec} x + \cot x = 4$
$(\operatorname{cosec} x + \cot x)(\operatorname{cosec} x - \cot x) = 4(\operatorname{cosec} x - \cot x)$ [M1]
$\operatorname{cosec}^2 x - \cot^2 x = 4\operatorname{cosec} x - 4\cot x$
$(1 + \cot^2 x) - \cot^2 x = 4(\operatorname{cosec} x - \cot x)$ [M1]
$\operatorname{cosec} x - \cot x = \frac{1}{4}$ (shown) [A1]

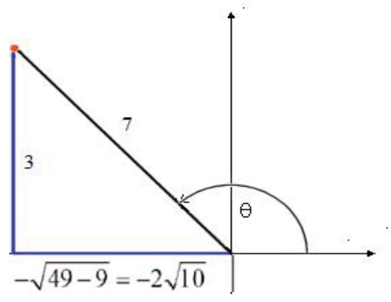
$\operatorname{cosec} x + \cot x = 4$ ----- (1)
$\operatorname{cosec} x - \cot x = \frac{1}{4}$ ----- (2)
(1) + (2) : $2\operatorname{cosec} x = \frac{17}{4}$ [M1] -- Solve simultaneous eqns

$\operatorname{cosec} x = \frac{17}{8} \Rightarrow \sin x = \frac{8}{17}$
Hence $\cos x = \frac{15}{17} \Rightarrow \sec x = \frac{17}{15}$ [A1]

(b) Given that $\frac{1 + 2\cos^2\theta}{\sin^2\theta} = \frac{43}{3}$ for $90^o \le \theta \le 180^o$, find the value of $\frac{\sin\theta}{1+\cos\theta}$,
giving your answer in the form $\frac{1}{3}\left(a + b\sqrt{c}\right)$ where a, b and c are integers. [5]

$\frac{1 + 2\cos^2\theta}{\sin^2\theta} = \frac{43}{3}$
$3 + 6\cos^2\theta = 43\sin^2\theta$
$3 + 6(1 - \sin^2\theta) = 43\sin^2\theta$ [M1]
$49\sin^2\theta = 9$
$\sin\theta = \frac{3}{7}$ or $\sin\theta = -\frac{3}{7}$ (rejected since $90^o \le \theta \le 180^o$) [A1]

$\cos\theta = -\frac{2\sqrt{10}}{7}$ [B1]



$\frac{\sin\theta}{1+\cos\theta} = \frac{\frac{3}{7}}{1 + \left(-\frac{2\sqrt{10}}{7}\right)}$

$= \frac{3}{7 - 2\sqrt{10}} \times \frac{7 + 2\sqrt{10}}{7 + 2\sqrt{10}}$ [M1]

$= \frac{1}{3}\left(7 + 2\sqrt{10}\right)$ [A1]

Alternative method for 7(a)

$\operatorname{cosec} x + \cot x = 4$

$\frac{1}{\sin x} + \frac{\cos}{\sin x} = 4$

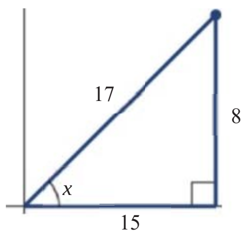
$1 + \cos x = 4\sin x$

Squaring both sides and applying $\sin^2 x = 1 - \cos^2 x$

$1 + 2\cos x + \cos^2 x = 16\left(1 - \cos^2 x\right)$ [M1]

$17\cos^2 x + 2\cos x - 15 = 0$

$(17\cos x - 15)(\cos x + 1) = 0$ [M1]



$\cos x = \frac{15}{17}$ or $\cos x = -1$ (NA since x is acute)

$\cos ecx = \frac{1}{\sin x} = \frac{17}{8}$

$\cot x = \frac{1}{\tan x} = \frac{15}{8}$

$\operatorname{cosec} x - \cot x = \frac{17}{8} - \frac{15}{8} = \frac{1}{4}$ (shown) [A1] – with correct workings above, must reject –1

$\sec x = \frac{1}{\cos x}$ [M1]—with sketch or working

$= \frac{17}{15}$ [A1]

8 The quadratic equation $2x^2 + 6x + k = 0$ has roots α and β.

(i) Find the value of $(\alpha - \beta)^2$ in terms of k. [3]

Sum of roots, $\alpha + \beta = -3$

Product of roots, $\alpha\beta = \dfrac{k}{2}$ [B1]

$(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$

$= (\alpha + \beta)^2 - 4\alpha\beta$ [M1]

$= (-3)^2 - 4\left(\dfrac{k}{2}\right)$

$= 9 - 2k$ [A1]

(ii) Find the smallest integer value of k if the equation has no x-intercept. [3]

$2x^2 + 6x + k = 0$ has no real roots, $b^2 - 4ac < 0$

$36 - 4(2)(k) < 0$ [M1]

$k > 4.5$ [A1]

Smallest integer value of k is 5. [B1]

(iii) Given that the value of k is 5, find a quadratic equation whose roots are

$\dfrac{\alpha^2}{\beta}$ and $\dfrac{\beta^2}{\alpha}$. [5]

New sum of roots, $\dfrac{\alpha^2}{\beta} + \dfrac{\beta^2}{\alpha} = \dfrac{\alpha^3 + \beta^3}{\alpha\beta}$

$= \dfrac{(\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)}{\alpha\beta}$ [M1]

$= \dfrac{(\alpha + \beta)\left[(\alpha + \beta)^2 - 3\alpha\beta\right]}{\alpha\beta}$ [M1]

$= \dfrac{(-3)\left[(-3)^2 - 3(2.5)\right]}{2.5}$

$= -\dfrac{9}{5}$ [A1]

New product of roots, $\dfrac{\alpha^2}{\beta} . \dfrac{\beta^2}{\alpha} = \dfrac{5}{2}$ [B1]

Quadratic equation is $x^2 - \left(-\dfrac{9}{5}\right)x + \dfrac{5}{2} = 0$

$10x^2 + 18x + 25 = 0$ [B1]

[Turn over

9 (a) Find the coefficient of $\frac{1}{x^4}$ in $\left(2x-\frac{1}{x}\right)^{14}$. [3]

$\left(2x-\frac{1}{x}\right)^{14}$

$T_{r+1}=\binom{14}{r}(2x)^{14-r}\left(-\frac{1}{x}\right)^{r}$ [M1]

$=\binom{14}{r}(2)^{14-r}(-1)^{r}(x)^{14-2r}$

$14-2r=-4$ [M1]

$2r=18 \Rightarrow r=9$

Coefficient of $\frac{1}{x^4}$ is $=\binom{14}{9}(2)^{14-9}(-1)^{9}$

$= -64\,064$ [A1]

(b) **(i)** Write down, without simplifying, the fourth term in the binomial expansion of $\left(ax+\frac{b}{x}\right)^{n}$. [2]

$\left(ax+\frac{b}{x}\right)^{n}$

4th term $\Rightarrow r = 3$ [B1]

$T_4=\binom{n}{3}(ax)^{n-3}\left(\frac{b}{x}\right)^{3}$ [B1]

(ii) If this term is a constant term, find the value of *n*. [2]

$T_4 = \frac{n(n-1)(n-2)}{6}\left(a\right)^{n-3}(b)^3\left(x\right)^{n-6}$ [M1] -- x^{n-6}

Constant term, $n-6=0$

$n=6$ [A1]

(iii) The fourth term of the binomial expansion is 160 and $a-b=1$ where both *a* and *b* are positive.
Use the value of *n* in part (ii) to calculate the value of *a* and of *b*. [4]

$T_4 = \frac{6(5)(4)}{6}\left(a\right)^{6-3}(b)^3 = 160$ [M1]

$20a^3b^3 = 160$

$ab = 2$ ------------- (1)

$a = 1+b$ -------- (2) [M1] – Solve simultaneous eqns

Sub (2) into (1): $b(1+b) = 2$

$b^2+b-2=0$

$(b+2)(b-1)=0$

$b=-2$ (rejected since *b* > 0) or *b* = 1 [A1] – must reject neg value

Hence *a* = 2 [A1]

[Turn over

10 A circle with centre C, passes through the origin and has x-intercept of 4 at point A and y-intercept of 3 at point B.

(i) Explain why the centre of the circle is (2, 1.5). [1]

$A = (4, 0)$ and $B = (0, 3)$. The centre C must <u>lie on the perpendicular bisectors of OA and OB</u>.
Hence $C = (2, 1.5)$. [B1]

(ii) Determine the equation of the circle in the general form. [3]

Radius of circle, $OC = \sqrt{2^2 + \left(\frac{3}{2}\right)^2}$ [M1]

$= \frac{5}{2}$ units

Equation of circle is $(x-2)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{25}{4}$ [M1]

$x^2 - 4x + 4 + y^2 - 3y + \frac{9}{4} = \frac{25}{4}$

$x^2 + y^2 - 4x - 3y = 0$ [A1]

(iii) OD is a diameter of the circle. Find the coordinates of D. [2]

$\left(\frac{0+a}{2}, \frac{0+b}{2}\right) = \left(2, \frac{3}{2}\right)$ [M1]

Therefore, $D(4, 3)$. [A1]

(iv) The tangent at D cuts the x-axis at E. Find the area of triangle BCE. [5]

Gradient of $OD = \dfrac{3}{4}$

Gradient of tangent at D is $-\dfrac{4}{3}$ [B1]

Equation of tangent at D is $y-3=-\dfrac{4}{3}(x-4)$ [M1]

$$y=-\frac{4}{3}x+\frac{25}{3}$$

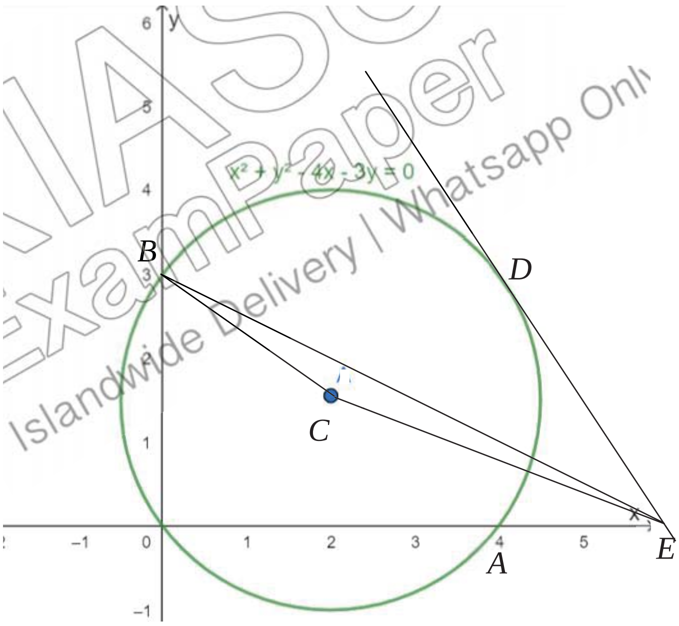
At E, $y=0$

$0=-\dfrac{4}{3}x+\dfrac{25}{3} \Rightarrow x=\dfrac{25}{4}$. Hence $E(6.25, 0)$ [A1]

Area of triangle $BCE = \dfrac{1}{2}\begin{vmatrix} 0 & 2 & 6.25 & 0 \\ 3 & 1.5 & 0 & 3 \end{vmatrix}$ [M1]

$$= \frac{1}{2}(18.75-6-9.375)$$

$= 1.6875$ units^2 [A1] -- exact



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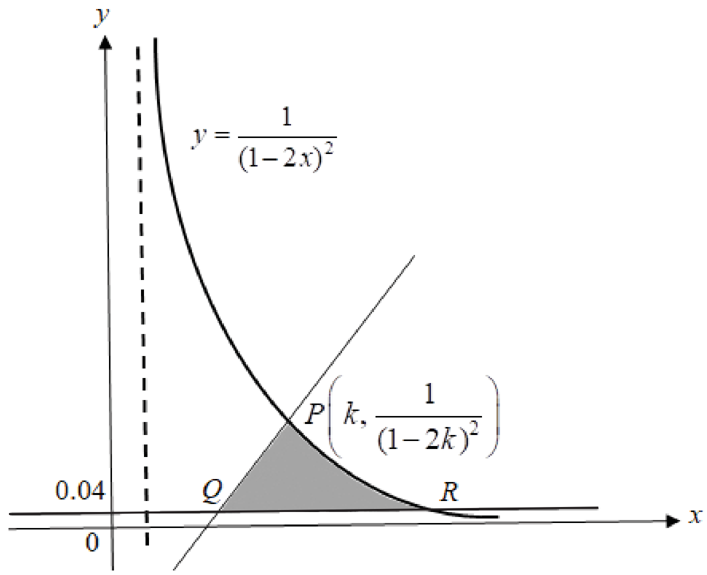

The diagram (not drawn to scale) shows part of the curve $y=\frac{1}{(1-2x)^2}$ passing through the point $P\left(k, \frac{1}{(1-2k)^2}\right)$, where k is a constant. The normal to the curve at P is perpendicular to $2y=1-x$ and the normal meets the horizontal line $y=0.04$ at point Q. The curve meets the same horizontal line at the point R.

(i) State the equation of the asymptote denoted by dotted lines. [1]

$x=0.5$ [B1]

(ii) Prove that $k=1.5$. [4]

$y=(1-2x)^{-2}$

$\frac{dy}{dx}=(-2)(1-2x)^{-3}(-2)$ [M1]

$=\frac{4}{(1-2x)^3}$ [A1]

$2y=1-x$

$y=-\frac{1}{2}x+\frac{1}{2}$

When $x=k$, $\frac{dy}{dx}=-\frac{1}{2}$

$\frac{4}{(1-2k)^3}=-\frac{1}{2}$ [M1]

$(1-2k)^3=-8$

$1-2k=-2$

$k=1.5$ (shown) [A1]

(iii) Show that the *x*-coordinates of *Q* and *R* are 1.395 and 3 respectively. [4]

$y = \dfrac{1}{[1-2(1.5)]^2} = \dfrac{1}{4}$

Therefore, $P\left(1\frac{1}{2}, \frac{1}{4}\right)$ [B1]

Gradient of $PQ = 2$

Equation of PQ is $y - \frac{1}{4} = 2(x - \frac{3}{2})$ [M1]

$y = 2x - \frac{11}{4}$

At Q, $0.04 = 2x - \frac{11}{4}$ [M1]

$x = 1.395$ — x-coordinate of Q is 1.395.

At R, $0.04 = \dfrac{1}{(1-2x)^2}$

$(1-2x)^2 = 25$

$1-2x = 5$ or $1-2x = -5$

$x = -2$ (rejected as $x > 0$) or $x = 3$ — x-coordinate of Q is 3. [A1]

(iv) Find the area of the shaded region *PQR*, giving your answer to 2 significant figures. [5]

Area of shaded region $PQR = \frac{1}{2}(1.5-1.395)(0.25-0.04) + \int_{1.5}^{3} (1-2x)^{-2} - 0.04\, dx$

[M1] -- Δ area [M1] -- integral

$= 0.011025 + \left[\dfrac{(1-2x)^{-1}}{(-1)(-2)} - 0.04x\right]_{1.5}^{3}$ [M1]

$= 0.011025 + \left[\dfrac{1}{2(1-2x)} - 0.04x\right]_{1.5}^{3}$

$= 0.011025 + \left(-\frac{1}{10} - 0.12\right) - \left(-\frac{1}{4} - 0.06\right)$ [M1]

$= 0.10$ units^2 (to 2 s.f.) [A1]

END OF PAPER

2020 CGS AM Prelim Paper 2 Answer Key

Qn	Answer Key		
1(i)	$x = -1$ is the only root		
2(i)	6.25 m; towards *O*	(ii)	$t = 1.61$
3(i)	$-x\sin 2x + \cos^2 x$	(ii)	$\frac{\pi}{16}$
4(a)	$k > 5\frac{1}{3}$	(b)	$p : q = 7 : 6$; $x = -3$
5(i)	$S = 32\sin\theta + 8\cos\theta$	(ii)	$S = 8\sqrt{17}\sin(\theta + 0.245)$; 1.08
6(i)	Plot $\frac{A}{x}$ against x; $p = 0.59$ *(accept 0.59 – 0.62)*; $q = 46$ *(Accept 45 – 47)*		
(ii)	$x = 115$; square		
7(a)	$\sec x = \frac{17}{15}$	(b)	$\frac{1}{3}\left(7 + 2\sqrt{10}\right)$
8(i)	$9 - 2k$	(ii)	5
(iii)	$10x^2 + 18x + 25 = 0$		
9(a)	$-64\,064$	(b)(i)	$T_4 = \binom{n}{3}(ax)^{n-3}\left(\frac{b}{x}\right)^3$
		(b)(ii)	$n = 6$
		(b)(iii)	$a = 2$; $b = 1$
10(i)	C lies on the perpendicular bisectors of *OA* and *OB*.	(ii)	$x^2 + y^2 - 4x - 3y = 0$
(iii)	*D*(4, 3)	(iv)	1.6875 units
11(i)	$x = 0.5$	(iv)	0.10 units^2