CEDAR GIRLS' SECONDARY SCHOOL
Preliminary Examination
Secondary Four

CANDIDATE NAME | Sec 4 (   ) Reg. No: (   )

CENTRE NUMBER | | | | |

INDEX NUMBER | | | | |

ADDITIONAL MATHEMATICS
Paper 1

4047/01
14 September 2020

2 hours

Candidates answer on the Question Paper.

Additional Materials: NIL

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 75.

For Examiner's Use
75

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

Answer **all** the questions.

1 **(a)** It is given that $\left(2-\sqrt{5}\right)^2-\frac{20}{3+\sqrt{5}}=p+q\sqrt{5}$, where p and q are integers.

Find the value of p and of q. [4]

(b) Find the values of x and y which satisfy the equations

$$4^{x+y}\times 2^{-3y}=1\ ,$$

$$\frac{81^x}{3^{3x+2y}}=\frac{1}{27}.$$

[4]

2 **(a)** Show that the curve $y = 3x^2 - 10x + 2$ lies entirely above the line $y = -4x - 11$. [2]

(b) The equation of a curve is $y = -x^2 - 6x + 3$. Find the range of values of the constant m for which the line $y = mx + 4$ meets the curve at 2 distinct points. [4]

3 **(a)** Given that $\lg z = k$, find an expression, in terms of k, for $\log_z(10z)$. [3]

(b) Solve the equation $\log_4(2-x)-1=\log_{16}(x+1)+\log_4\left(\frac{1}{16}\right)$. [5]

4 **(a)** Find the term independent of x in the expansion of $\left(x-\dfrac{1}{2x^3}\right)^{16}$. [3]

(b) **(i)** Write down and simplify the first three terms in the expansion, in ascending powers of x, of $\left(1-\frac{x}{3}\right)^n$, where n is a positive integer. [2]

(ii) The first three terms in the expansion of $\left(m+x-x^2\right)\left(1-\frac{x}{3}\right)^n$ are $4-7x+kx^2$. Find the value of each of the constants m, n and k. [5]

5 **(i)** Prove that $(\sec A + \tan A)^2 = \dfrac{1+\sin A}{1-\sin A}$. [4]

(ii) Hence solve the equation $(\sec A + \tan A)^2 = \dfrac{1}{2}$ for $0° < A < 720°$. [3]

6 **(a)** It is given that $y=(x+3)(2x-1)^5$.

(i) Obtain an expression for $\frac{dy}{dx}$ in the form $(ax+b)(2x-1)^4$, where a and b are integers. [2]

(ii) Determine the range of values of x for which y is a decreasing function. [3]

(b) It is given that $f(x)=x^3+px^2+qx-7$ where p and q are integers.
The only values of x for which f(x) is an increasing function of x are those values for which $x<-2$ or $x>6$.
Find the value of p and of q. [3]

(c) The equation of a curve is $y = \frac{3\ln(2x)}{x}, x > 0$. Given that y is decreasing at a constant rate of 36 units per second, find the rate of change of x when $x = 0.5$. [3]

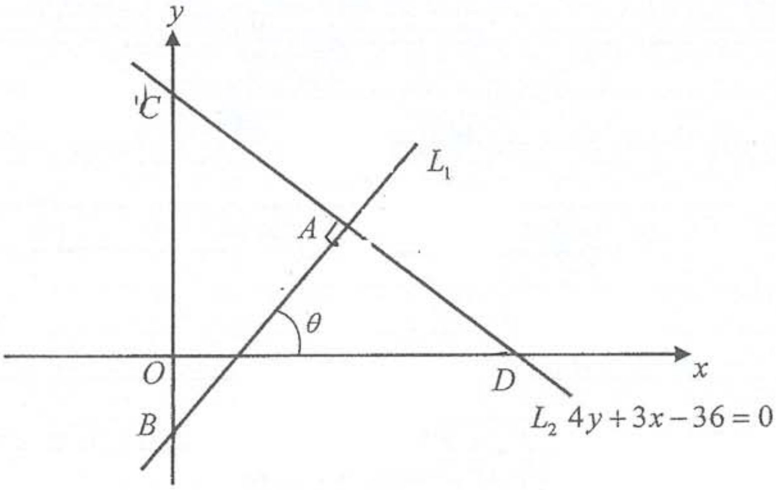
7 A particle moves in a straight line so that, t seconds after leaving a fixed point O, its velocity, v m/s, is modelled by $v = 24t(t-1) - 48$.

(i) Find the acceleration of the particle when it comes to instantaneous rest. [3]

(ii) Explain why the velocity of the particle is never less than -54 m/s. [2]

(iii) Find the total distance travelled by the particle in the interval $t = 0$ to $t = 5$. [4]

8



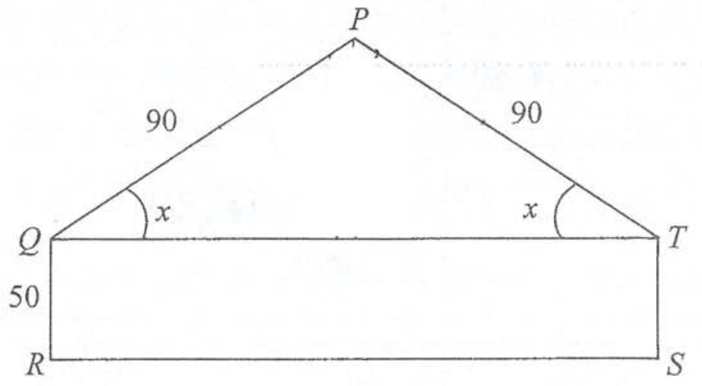
The diagram shows two lines L_1 and L_2 intersecting at point A.
Line L_1 makes an acute angle θ with the x-axis and intersects the y-axis at point B.
Line L_2 has equation $4y+3x-36=0$.
L_2 intersects the y-axis at point C and intersects the x-axis at point D.

(i) Given that $\tan\theta = \frac{4}{3}$, explain why L_1 is perpendicular to L_2. [2]

(ii) Given further that line L_1 is the perpendicular bisector of line CD, find the coordinates of point B. [3]

(iii) Find the area of triangle ABD. [2]

9



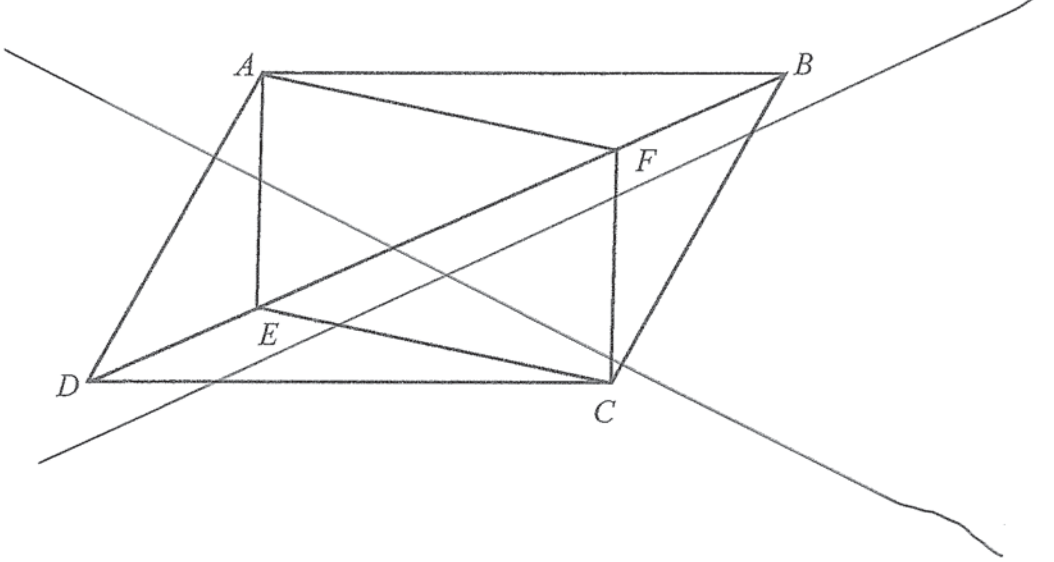
A farmer designed a plot of land, $PQRST$, consisting of a rectangle, $QRST$ and an isosceles triangle PQT to plant his crops.
It is given that $PQ = 90\,\text{m}$, $QR = 50\,\text{m}$, angle $PQT =$ angle $PTQ = x$ radians.

(i) Given that the area of the plot of land $PQRST$ is $A\ \text{m}^2$, show that $A = 4050 \sin 2x + 9000 \cos x$. [3]

(ii) Given that x can vary, find the value of x which will provide the farmer with the largest possible area to plant his crops. [6]

10 In the diagram below, $ABCD$ is a parallelogram.
The points E and F lie on the diagonal DB such that $DE : EF : FB = 1 : 2 : 1$.

Prove that the quadrilateral $AFCE$ is a parallelogram. [5]



End of Paper

CEDAR GIRLS' SECONDARY SCHOOL
Preliminary Examination
Secondary Four

CANDIDATE NAME	Sec 4 ( ) Reg. No: ( )

CENTRE NUMBER		INDEX NUMBER	

ADDITIONAL MATHEMATICS
4047/02

Paper 2

15th September 2020

2 hours 30 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

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Write in dark blue or black pen.
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At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

For Examiner's Use
/100

This document consists of **20** printed pages.

[Turn over

Mathematical Formulae

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

Answer **all** the questions.

1 **(i)** Sketch the graph of $y = e^x$. [2]

(ii) By inserting an appropriate straight line on the same sketch in **(i)**, state the number of solution(s) for the equation $\frac{1}{2}x = \ln\sqrt{3x+2}$. [2]

2 **(a)** Given that $\sin A < 0$ and $\cos A = \frac{2}{3}$, find the exact value of

(i) $\sin 2A - \cos 2A$, [3]

(ii) $\cos \frac{1}{2} A$. [2]

(b) The height of water tides in Singapore, h metres, is modelled by the equation $h = 1.1\sin kt + 1.8$, where k is a constant and t is the time in hours after midnight.

The average time difference between two successive high tides is 12.5 hours.

(i) Explain why this model suggests that the highest tide is 2.9 m. [2]

(ii) Show that the value of k is $\frac{4\pi}{25}$. [2]

3 **(a)** Solve the equation $5|2-3x|-3|6x-4|=-1+6x$. [4]

(b) When the function $\mathrm{f}(x)\equiv 2x^3+ax^2+bx+6$ is divided by x^2+x-2, the remainder is $-8x+4$.

(i) Show that $a=1$ and $b=-13$. [3]

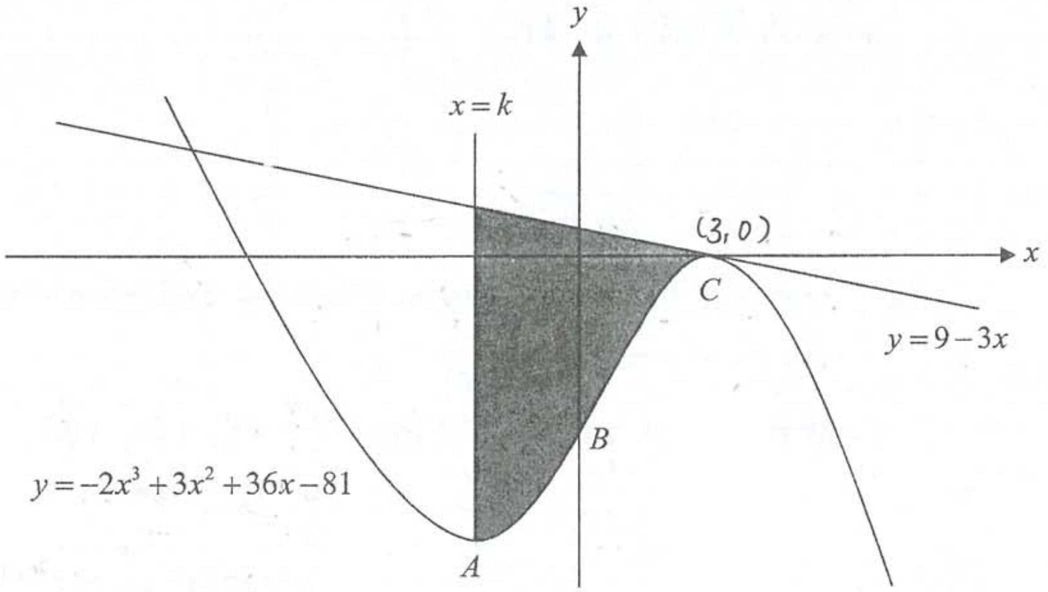
(ii) Show that $(x-2)$ is a factor of f(x) and hence, factorise f(x) completely. [3]

(iii) By using a suitable substitution, solve the equation $6z^3 - 13z^2 + z + 2 = 0$. [3]

4 **(i)** Sketch, on the same axes, the graphs of $y = |\cos 2x|$ and $y = \dfrac{x}{2\pi}$ for interval $0 \le x \le 2\pi$. [4]

(ii) Hence, state the number of solution(s) to the equation $|2\pi \cos 2x| = x$ for $0 \le x \le 2\pi$. [2]

5 The diagram shows part of the curve $y=-2x^3+3x^2+36x-81$ meeting the y-axis at point B and the x-axis at point C. The tangent to the curve at point C is given by $y=9-3x$. The curve has a minimum point at A and a maximum point at C.



(i) Find the coordinates of the points A, B and of C. [5]

(ii) A vertical line $x=k$ passes through point A.
Find the exact area of the shaded region bounded by the curve, the line $x=k$ and the line $y=9-3x$. [3]

6 (a) The roots of the quadratic equation $2x^2+5x-4=0$ are α^2 and β^2.

(i) Find the value of $\alpha^2+\beta^2$ and of $\alpha^2\beta^2$. [2]

(ii) Hence, find the quadratic equation, with integer coefficients, whose roots are $\alpha^2+\frac{1}{\beta^2}$ and $\beta^2+\frac{1}{\alpha^2}$. [3]

(b) A circle has a diameter AB. The point A has coordinates $(-4, 4)$ and the equation of the tangent to the circle at B is given by $4y + 3x = 54$.

(i) Find the equation of the diameter AB and hence, the coordinates of B. [3]

(ii) Find the radius and the coordinates of the centre of the circle. [2]

(iii) Determine whether the point $(3, 6)$ lies inside, on or outside the circle. [2]

7 The mass, m grams, of a radioactive subtance, t hours after observations began, is given by $m = m_0 e^{-kt}$, where m_0 and k are constants. The table below shows corresponding values of m and t.

m (grams)	55.8	38.9	27.2	19.0
t (hours)	12	24	36	48

(i) Draw the graph of $\ln m$ plotted against t, using a scale of 2 cm for 10 units on the t-axis and a scale of 2 cm for 0.5 units on the $\ln m$-axis. [3]

(ii) Use the graph to estimate the value of each of the constants m_0 and k. [5]

(iii) Use your graph to estimate the time taken for the substance to lose half of its original mass. [3]

8 **(a)** Differentiate $2\sin^3(4x)-\cos^8\left(\frac{1}{2}x\right)$ with respect to x. [3]

(b) It is given that $g(x)=x^3+ax^2+bx+5$, where a and b are constants. $P(-1,10)$ is a turning point of g(x).

(i) Find the value of a and of b. [4]

(ii) Determine if P is a maximum point. [3]

9 A curve is such that $\frac{d^2y}{dx^2} = 8e^{-2x}$. Given that $\frac{dy}{dx} = 9$ when $x = 0$, and the curve passes through the point $(\ln 2, 15\ln 2)$, find the equation of the curve. [5]

10 **(a)** Given that $\int_1^4 \mathrm{f}(x)\,\mathrm{d}x = 3$ and $\int_4^9 \mathrm{f}(x)\,\mathrm{d}x = 8$, find

(i) $\int_1^4 3\mathrm{f}(x)\,\mathrm{d}x - \int_9^4 2\mathrm{f}(x)\,\mathrm{d}x$, [2]

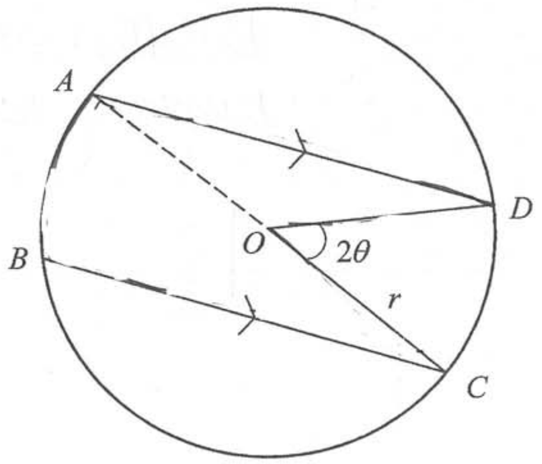
(ii) $\int_4^9 5\mathrm{f}(x) - \sec^2 3x + 2x\,\mathrm{d}x$. [2]

(b) **(i)** Express $\dfrac{2-5x+8x^2}{(3x+2)(x^2+4)}$ in partial fractions. [4]

(ii) Differentiate $\ln\left(x^2+4\right)$ with respect to x. [1]

(iii) Hence, find $\int_1^2 \frac{2-5x+8x^2}{(3x+2)(x^2+4)}\mathrm{d}x + \int_1^2 \frac{3+4x}{(x^2+4)}\mathrm{d}x$, leaving your answer in the form of $a\ln\frac{8}{5}$, where a is a rational number. [5]

11 In the circle with centre O, the points A, B, C and D lie on the circumference such that AOC is a straight line. It is given that AD is parallel to BC. The circle has a radius of r cm and angle $DOC = 2\theta$, where $0° < \theta < 90°$.



(i) Show that the perimeter of pentagon $ABCOD$, P cm, is given by $P = 2r\left(2\cos\theta + \sin\theta + 1\right)$. [3]

(ii) Express $2\cos\theta + \sin\theta$ in the form of $R\cos(\theta - \alpha)$. [3]

(iii) Hence, find in terms of r, the maximum perimeter and the corresponding value of θ. [2]

☺ *End of Paper* ☺

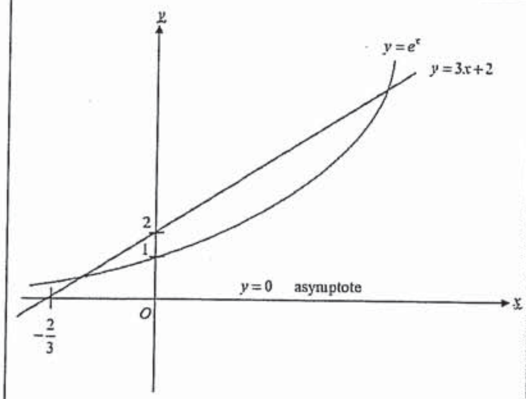
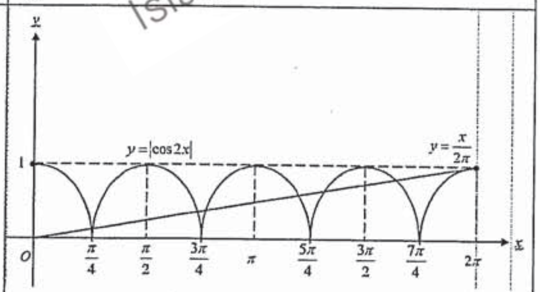
CEDAR GIRLS' SECONDARY SCHOOL
SECONDARY 4 ADDITIONAL MATHEMATICS
Answer Key for 2020 Preliminary Examination

PAPER 4047/1

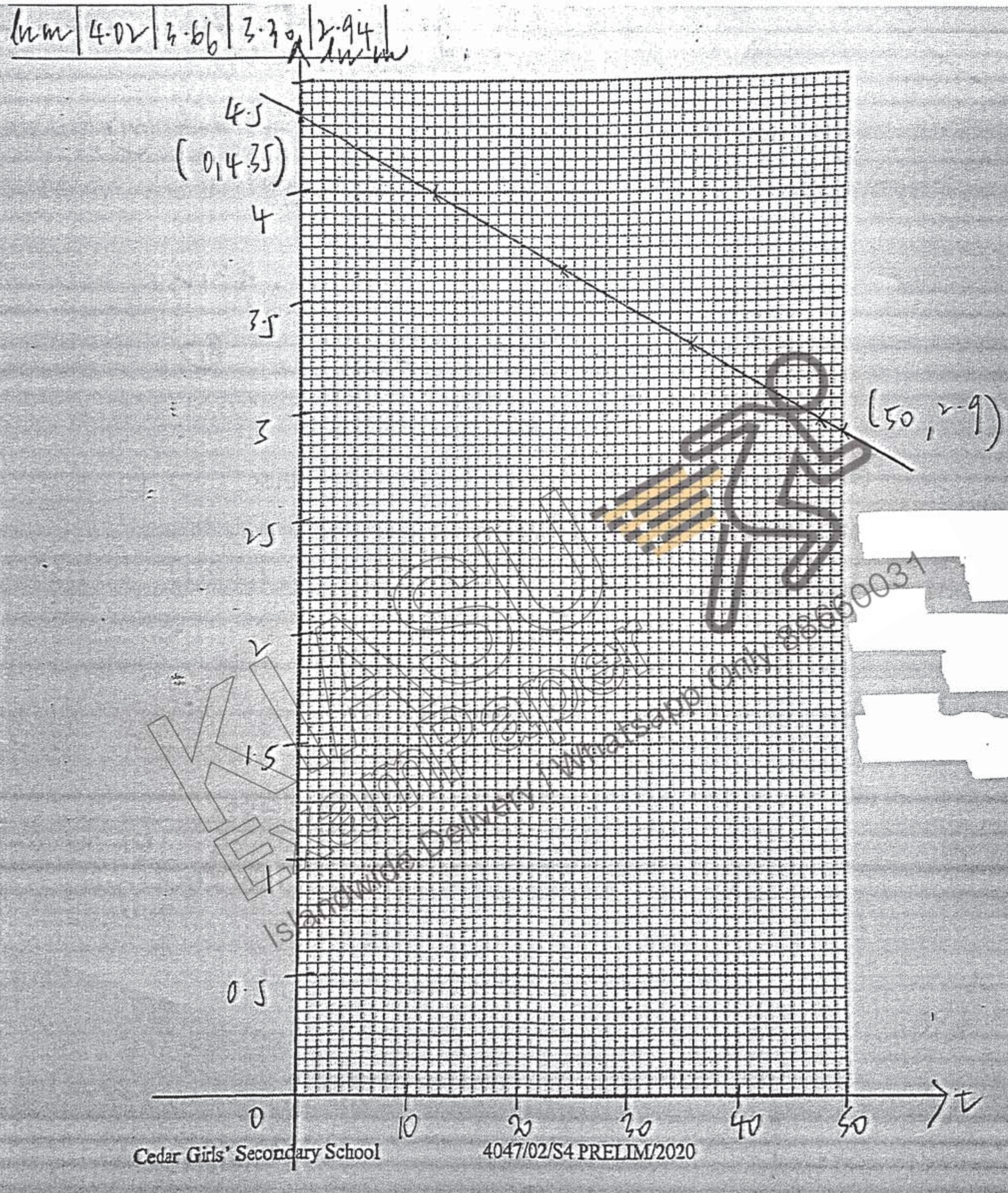
1(a)	$p=-6, q=1$	6(a)(i)	$(2x-1)^4(12x+29)$
1(b)	$x=1, y=2$	6(a)(ii)	$x<-\frac{29}{12}$
2(a)	Show discriminant >0 and curve opens upwards	6(b)	$p=-6,\ q=-36$
2(b)	$m<-8$ or $m>-4$	6(c)	-3 units per second
3(a)	$\frac{1}{k}+1$	7(i)	72 m/s^2
3(b)	1.60	7(ii)	$v=24(t^2-t-2)$ $=24\left(t-\frac{1}{2}\right)^2-54$ Since $\left(t-\frac{1}{2}\right)^2\geq 0$, $24\left(t-\frac{1}{2}\right)^2-54\geq -54$
4(a)	$\frac{455}{4}$		
4(b)(i)	$1-\frac{n}{3}x+\frac{n(n-1)}{18}x^2+\ldots$		
4(b)(ii)	$m=4,\ n=6,\ k=\frac{11}{3}$	7(iii)	620 m
5(i)	$(\sec A+\tan A)^2$ $=\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)^2$ $=\left(\frac{1+\sin A}{\cos A}\right)^2$ $=\frac{(1+\sin A)^2}{1-\sin^2 A}$ $=\frac{1+\sin A}{1-\sin A}$	8(i)	Gradient of L_1 = $\tan\theta=\frac{4}{3}$, Gradient of L_2 = $-\frac{3}{4}$ Gradient of $L_1\times$ Gradient of L_2 $=\frac{4}{3}\times-\frac{3}{4}=-1$ Since product of gradients of L_1 and L_2 is -1, L_1 is perpendicular to L_2.
5(ii)	$199.5°, 340.5°, 559.5°, 700.5°$	8(ii)	$B(0,-3.5)$
9(ii)	0.503(3 sf)	8(iii)	37.5 sq units
9(i)	Length $QT=2(90\cos x)=180\cos x$ Area of rectangle, $QRST=50\times180\cos x=9000\cos x$ Area of triangle $PQT=\frac{1}{2}(90)(90)\sin(180°-2x)$ $=4050\sin 2x$ Therefore, $A=4050\sin 2x+9000\cos x$ (shown)		

CEDAR GIRLS' SECONDARY SCHOOL
SECONDARY 4 ADDITIONAL MATHEMATICS
Answer Key for 2020 Prelim Examination Paper 2

PAPER 4047/2

1(i)			
(ii)	Draw $y = 3x + 2$ From the graph, there are two solutions.		
2(a)(i)	$\frac{1-4\sqrt{5}}{9}$	**2(a)(ii)**	$-\frac{\sqrt{30}}{6}$
2(b)(i)	Highest tide occurs when $\sin kt = 1$ Highest tide $= 1.1(1) + 1.8 = 2.9$ m (shown)		
2(b)(ii)	period = 12.5 hours $\frac{2\pi}{k} = 12.5$ $k = \frac{2\pi}{12.5} = \frac{4\pi}{25}$ (shown)		
3(a)	$x = -\frac{1}{3}$		
3(b)(ii)	$f(x) = (x-2)(2x-1)(x+3)$		
3(b)(iii)	$\frac{1}{z} = x, z = \frac{1}{2}, -\frac{1}{3}, 2$		
4(i)			
4(ii)	8 solutions		
5(i)	$A(-2,-125)$ $B(0,-81)$ $C(3,0)$		
5(ii)	350 units^2		
6(a)(i)	$\alpha^2+\beta^2 = -\frac{5}{2}$ $\alpha^2\beta^2 = -2$	**6(a)(ii)**	$4x^2 - 5x - 2 = 0$

6(b)(i)	$y = \frac{4}{3}x + \frac{28}{3}$; $B(2,-12)$		
6(b)(ii)	Radius = 5 units; Centre $(-1,8)$		
6(b)(iii)	$\sqrt{(3+1)^2+(6-8)^2} = 4.47$ units 4.47 units< radius of circle the point (3, 6) lies inside the circle		
7(ii) **7(iii)**	$k = 0.029$; $m_0 = 77.5$ (3 s.f.) $t = 24$h		
8(a)	$24\sin^2 4x \cdot \cos 4x + 4\cos^7\left(\frac{1}{2}x\right)\sin\left(\frac{1}{2}x\right.$		
8(b)(i)	$a = -3, b = -9$		
8(b)(ii)	P is a maximum point		
9	$y = 2e^{-2x} + 13x + \ln 4 - \frac{1}{2}$		
10(a)(i)	25	**10(a)(ii)**	106 (3 s.f.)
10(b)(i)	$\frac{2-5x+8x^2}{(3x+2)(x^2+4)} = \frac{2}{3x+2} + \frac{2x-3}{x^2+4}$		
10(b)(ii)	$\frac{d}{dx}\ln(x^2+4) = \frac{2x}{x^2+4}$		
10(iii)	$\frac{11}{3}\ln\frac{8}{5}$		
11(i)	$\cos\theta = \frac{AM}{r}$ $\sin\theta = \frac{OM}{r}$ $AM = r\cos\theta$ $OM = r\sin\theta$ $AD = 2r\cos\theta$ $AB = 2r\sin\theta$ perimeter of pentagon $=2r\sin\theta + 2(2r\cos\theta) + 2r$ $= 2r(2\cos\theta + \sin\theta + 1)$ (shown)		
11(ii)	$2\cos\theta + \sin\theta = \sqrt{5}\cos(\theta - 26.6°)$ (1 d.p.)		
11(iii)	maximum perimeter = $2r(\sqrt{5}+1)$ cm corresponding $\theta = 26.6°$ (1 d.p.)		


ln m | 4.02 | 3.66 | 3.30 | 2.94
ln m
4.5
(0, 4.35)
4
3.5
3
(50, 2.9)
2.5
2
1.5
1
0.5
0
10
20
30
40
50
t