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CONVENT OF THE HOLY INFANT JESUS SECONDARY
Preliminary Examination in preparation for
the General Certificate of Education Ordinary Level 2020

CANDIDATE NAME

CLASS

REGISTER NUMBER

ADDITIONAL MATHEMATICS 4047/01

Paper 1

2 September 2020

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 The function $f(x)$ is defined by $f(x)=\dfrac{5x+2}{2x-3}$ for all values of x, $x\neq\dfrac{3}{2}$.

Determine, with working, whether f(x) is an increasing function or a decreasing function. [3]

2 The value of a newly mined diamond increases by half its value every 10 years. It is given that V_o is the value of the diamond at a particular time and V is its value t years later. Find the value of the constant m in the relationship $V=V_o e^{mt}$. [3]

3 It is given that $\lg\left(a+2\right)=\log_{100}(b-1)$.

(i) Express b in terms of a. [3]

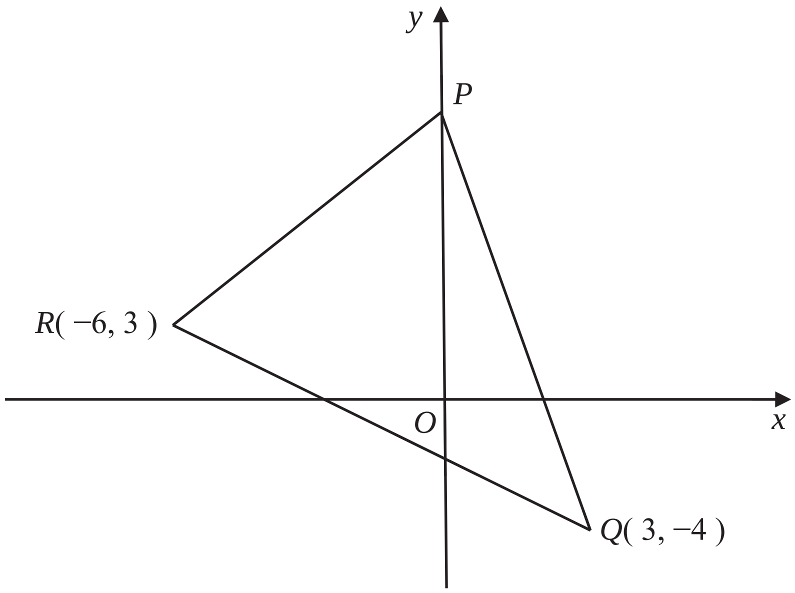
(ii) Given that $b \geq 10$, find the range of values for a. [3]

4 The equation of a curve is $y = kx\sqrt{2x+3}$ where k is a constant.

(i) Obtain an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in the form $\frac{ak(x+b)}{\sqrt{2x+3}}$ where a and b are integers. [3]

(ii) A point moves along the curve in such a way that when $x = 3$, the rate of increase of y with respect to time is thrice the rate of increase of x with respect to the time. Find the value of k. [2]

5



In the diagram, PQR is an isosceles triangle in which $PQ = QR$ and the coordinates of Q and R are (3, −4) and (−6, 3) respectively. P is a point on the y-axis.

(i) Find the coordinates of P. [3]

(ii) Find the equation of the line which passes through Q and the midpoint of PR. [3]

6 **(i)** Prove that $\dfrac{\cos^3\theta+\sin^3\theta}{\cos\theta+\sin\theta}=\dfrac{1}{2}(2-\sin 2\theta)$. [3]

(ii) Hence solve the equation $\dfrac{\cos^3\theta+\sin^3\theta}{\cos\theta+\sin\theta}=\dfrac{5}{2}-2\sin^2 2\theta$ for $0° \le \theta \le 180°$. [4]

7 **(i)** Sketch the graph of $y = |2x - 5|$. [2]

(ii) Find the x-coordinates of the points of intersection of the graph of $y = |2x - 5|$ and the line $y = 12 - x$. [2]

(iii) Determine the range of values of m such that $|2x - 5| = -|3x| + m$ has one or more solutions. [1]

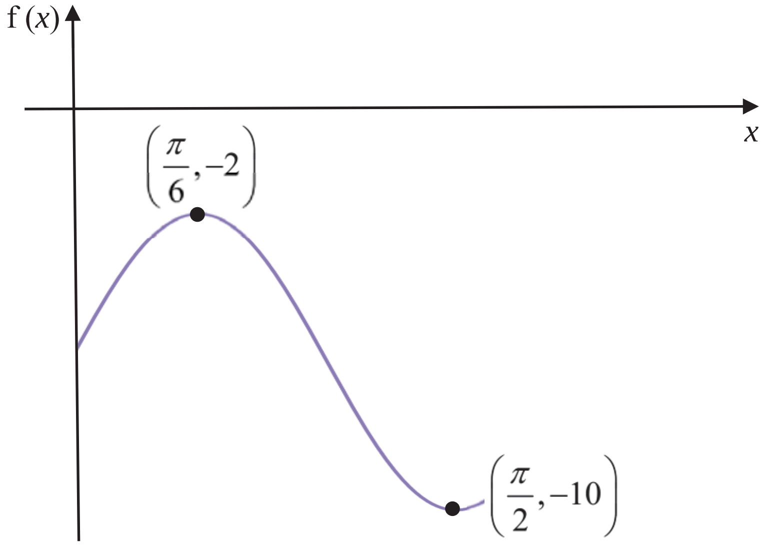
8 **(i)** Find the range of values of p such that $y = px^2 - 4x + p$ lies entirely above the x-axis. [4]

(ii) Explain clearly why the line $y = x + 2k$ will intersect the curve $2y^2 - x^2 = 8$ at two distinct points for all real values of k. [5]

9 **(i)** On the same diagram, sketch the curves $y=\frac{1}{9}x^{\frac{5}{2}}$ and $y=9x^{\frac{1}{2}}$ for $x \geq 0$. [2]

(ii) Show that the tangent to the curve $y=9x^{\frac{1}{2}}$ at the point of intersection of the two curves passes through the point $(-1, 12)$. [5]

10 **(a)**



The diagram shows part of the curve $f(x) = 4\sin ax - b$.

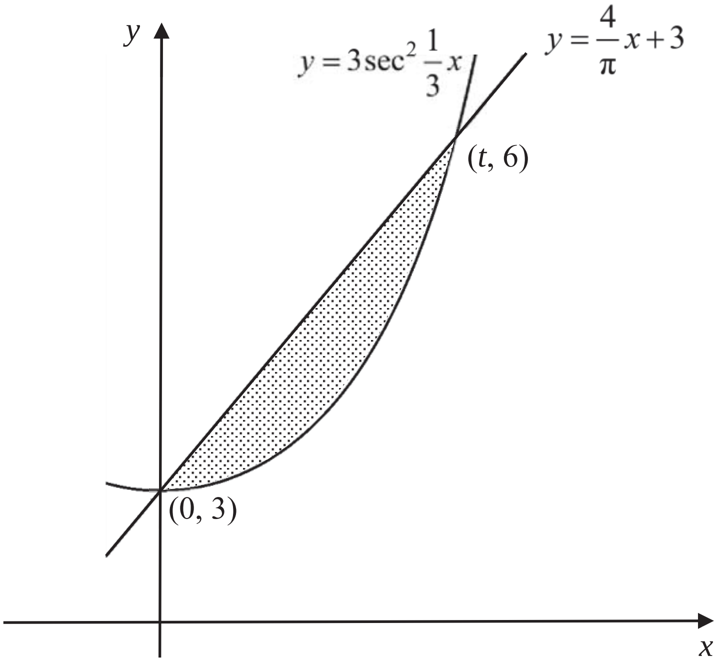
The coordinates of the turning points shown are $\left(\frac{\pi}{6}, -2\right)$ and $\left(\frac{\pi}{2}, -10\right)$.

Find the value of a and of b. [2]

(b) Given that $g(x) = 2\sin 3x - 5$, express $g(x)$ as a cubic function in terms of $\sin x$. [5]

[Turn over

11



The diagram shows part of the curve $y = 3\sec^2 \frac{1}{3}x$ and the straight line $y = \frac{4}{\pi}x + 3$.
The line intersects the curve at $(t, 6)$ and $(0, 3)$.

(i) Find the value of t, in terms of π. [1]

(ii) Hence find the area of the shaded region, expressing your answer in terms of π. [4]

12 A particle moves in a straight line so that, t seconds, after leaving a fixed point O, its velocity, v cm/s, is given by $v = 6t^2 - (5k - 4)t + 8 - 13h$ where h and k are constants. The acceleration of the particle, 3 seconds after leaving O is 30 cm/s^2.

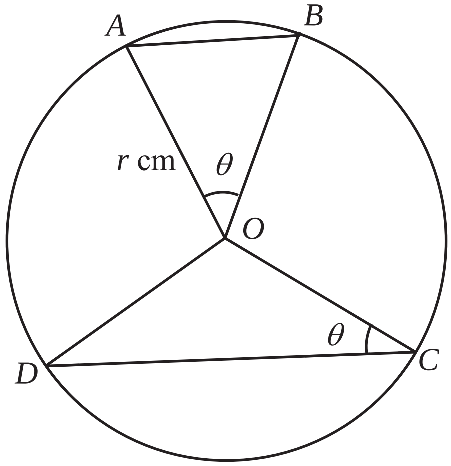
(i) Show that $k = 2$. [2]

The displacement of the particle from O is -500 cm, 2 seconds after leaving O.

(ii) Find the time when the particle returns to O. [5]

(iii) Explain clearly why the particle changes its direction only once. [2]

13



The diagram shows a circular mosaic tile of radius r cm and centre O. The design consists of two triangles OAB and OCD.
Angle AOB = angle OCD = θ radians.

(i) Show that the total area, A cm^2, of triangles OAB and OCD is given by

$$A=\frac{1}{2}r^2\left(\sin\theta+\sin 2\theta\right).$$ [3]

(ii) Given that θ can vary, find the value of θ for which A is a maximum. [5]
(You are not required to show that A is a maximum)

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CANDIDATE NAME	
CLASS	
REGISTER NUMBER	

ADDITIONAL MATHEMATICS 4047/02

Paper 2 **14 September 2020**

Candidates answer on the Question Paper. **2 hours and 30 minutes**
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

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The total number of marks for this paper is 100.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

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Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

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$$\sin 2A = 2 \sin A \cos A$$

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Formulae for $\Delta\, ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 The function f is defined by $f(x) = xe^{-x} + ke^{\frac{3}{5}x}$, where k is a constant.
Given that $f'(0) = 5$, find the value of k. [4]

2 Given that $y = 3^x$, solve the equation $6(3^{-2x}) - 1 = 3^{-x}$. [4]

[Turn over

3 **(i)** By using long division, explain how $x-3$ is a factor of $2x^3-13x^2+24x-9$. [3]

(ii) Express $\dfrac{5x^2-30x+10}{2x^3-13x^2+24x-9}$ as the sum of three partial fractions. [5]

Continuation of working space for question 3(ii)

(iii) Hence find $\int \frac{10x^2 - 60x + 20}{2x^3 - 13x^2 + 24x - 9}\,\mathrm{d}x$. [4]

4 **(i)** Differentiate $\left(3x+1\right)\ln(3x+1)-3x$ with respect to x. [3]

(ii) Hence evaluate $\int_0^2 \ln(3x+1)^2\,\mathrm{d}x$. [4]

5 The third term in the binomial expansion of $\left(x-\frac{3}{x}\right)^n$, where n is a positive integer, is kx^8.

(i) Show that $n=12$. [2]

(ii) Find the value of k. [1]

(iii) Find the coefficient of x^8 in the expansion of $\left(x-\frac{3}{x}\right)^{12}\left(2+x^2\right)$. [4]

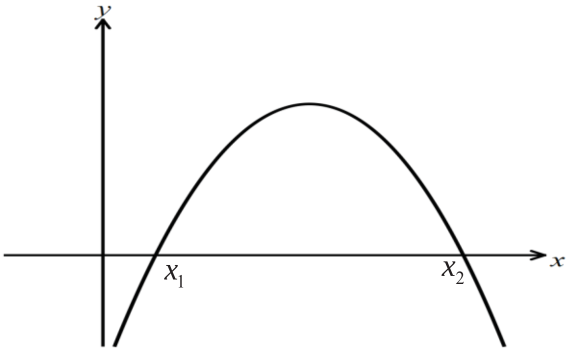
[Turn over

6 The roots of the quadratic equation $x^2 - 3x - 2 = 0$ are α and β.

(i) Show that $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = -\dfrac{13}{2}$. [3]

(ii) Find a quadratic equation with roots $\alpha - \dfrac{1}{\alpha}$ and $\beta - \dfrac{1}{\beta}$. [5]

7



The quadratic curve $y = -x^2 + bx - 1$, where b is a positive integer, intersects the x-axis at x_1 and x_2 as shown in the diagram.

(i) Explain, showing your working clearly, why the smallest value of b is 3. [3]

(ii) Using $b = 3$, find, without the use of a calculator, the exact value of $\frac{x_1}{x_2}$,

leaving your answer in the form $\frac{p + q\sqrt{5}}{r}$, where p, q and r are integers. [4]

[Turn over

8 **(a)** A circle, C_1, has equation $x^2 + y^2 + 4x - 10y - 20 = 0$.

Find the radius and the coordinates of the centre of C_1. [3]

(b) A circle, C_2, passes through the points $A(2,0)$ and $B(8,0)$. The centre of C_2 lies above the x-axis. The y-axis is tangent to C_2.

(i) Show that the y-coordinate of the centre of C_2 is 4 and hence find the equation of the circle, C_2. [4]

Continuation of working space for question 8b(i)

(ii) Find the coordinates of the point on C_2 which is furthest from $A(2,0)$. [2]

9 **(i)** Show that $4\cos 4x - 3\sin 4x$ can be expressed in the form $R\cos(4x+\alpha)$,
where R and α are constants, $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$. [4]

(ii) Given that $\mathrm{f}(x) = \mathrm{e}^{-3x}\sin 4x$, show that $\mathrm{f}'(x)$ can be written in the form
$\mathrm{f}'(x) = R\mathrm{e}^{-3x}\cos(4x+\alpha)$ where R and α are the constants found in part **(i)**. [3]

(iii) Hence, find the smallest positive value of x for which the curve $y = \mathrm{f}(x)$ has a stationary point. [3]

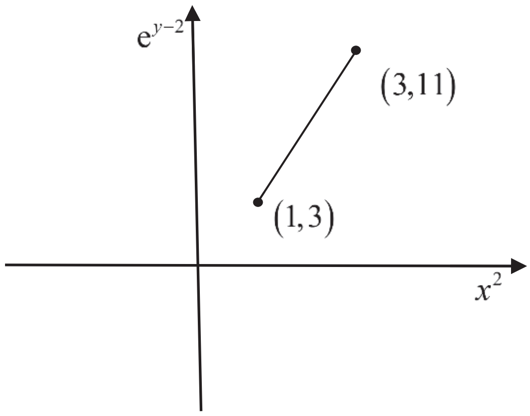
[Turn over

10 The point $P(1,-2)$ lies on the curve $y=\mathrm{f}(x)$ such that $\mathrm{f}'(x)=\dfrac{6x^3+9}{x^2}-\dfrac{9}{2}\sqrt{x}$, $x>0$.

(i) Find an expression for $\mathrm{f}(x)$. [4]

(ii) Find the equation of the normal to the curve $y=\mathrm{f}(x)$ at the point $P(1,-2)$ and express it in the form $ax+by+c=0$ where a , b and c are integers. [3]

11 **(a)**



The diagram shows part of a straight line graph obtained by plotting e^{y-2} against x^2.

(i) Given that the line passes through the points $(1,3)$ and $(3,11)$, express y in terms of x. [3]

(ii) Explain clearly why the range of values of x for which the equation found in part **(i)** is not defined for $-\frac{1}{2} \leq x \leq \frac{1}{2}$. [3]

[Turn over

11 **(b)** A new machine is used to measure the surface area of a solid, A cm^2 with a length of x cm. It is known that A and x are related by the equation, $A = px + qx^2$, where p and q are constants. The table below shows corresponding values of A and x. One of the values of A is believed to be inaccurate.

x (cm)	2	4	6	8	10
A (cm^2)	42	148	318	552	700

(i) Draw the graph of $\frac{A}{x}$ plotted against x, using a scale of 1 cm for 1 unit on the x-axis and a scale of 1 cm for 5 units on the $\frac{A}{x}$ axis. [3]

(ii) Use the graph to estimate the value of each of the constants p and q. [3]

(iii) Identify the inaccurate value of A and suggest a reason why this may be inaccurate. [2]

12 The curve $y = \frac{a}{2x+1} + 9x - 5$, where a is a constant, has two stationary points.

(i) Given that $\left(-\frac{1}{6}, y\right)$ is one of the stationary points, find y. [5]

(ii) Find the coordinates of the other stationary point. [3]

(iii) Find an expression for $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ and hence determine the nature of these stationary points. [3]

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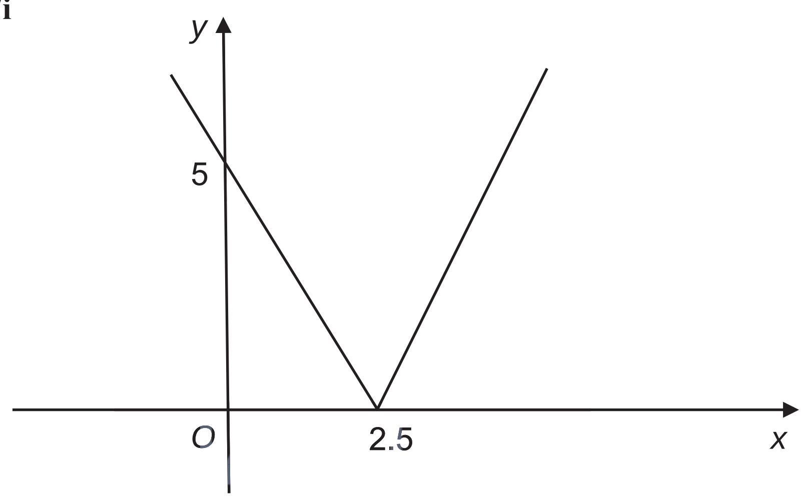
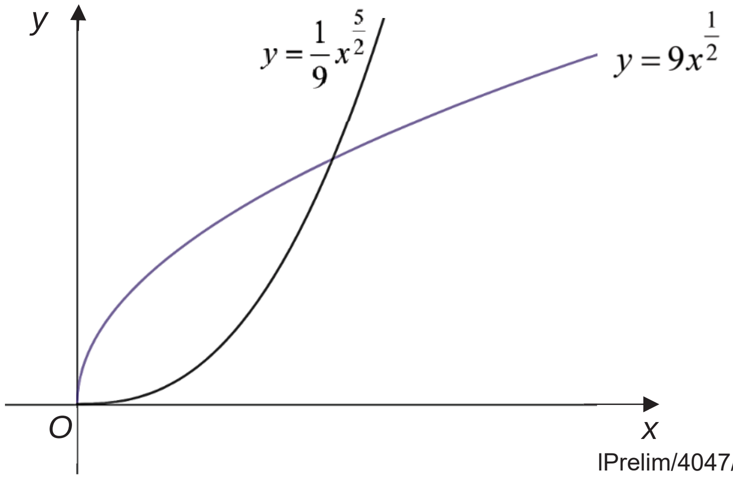
Answers 4E/5N Prelim 2020 AMath Paper 1

No.	Answers	No.	Answers
1	$f'(x) = \dfrac{-19}{(2x-3)^2}$ $\therefore$ Decreasing Function	8i	$p > 2$
2	$m = 0.0405$	8ii	$b^2 - 4ac > 32$ Since $b^2 - 4ac$ is always positive for all real values of k, the line will intersect the curve at 2 distinct pts.
3i	$b = (a+2)^2 + 1$	10i	$a = 3 \quad b = 6$
3ii	$a \geq 1$ or $a \leq -5$ (reject)	10ii	$g(x) = -8\sin^3 x + 6\sin x - 5$
4i	$\dfrac{dy}{dx} = \dfrac{3k(x+1)}{\sqrt{2x+3}}$	11i	$t = \dfrac{3\pi}{4}$
4ii	$k = \dfrac{3}{4}$	11ii	$\left(\dfrac{27\pi}{8} - 9\right)$ units^2
		12ii	12 seconds
5i	$P(0, 7)$	12iii	When $v = 0$, $t = 7$ or $t = -6$ (reject) Since the particle is at instantaneous rest when $t = 7$, the particle changes its direction only once.
5ii	$2y = -3x + 1$		
6ii	$x = 45°,\ 114.3°,\ 155.7°$		
7ii	$x = \dfrac{17}{3}$ or $x = -7$	13ii	$\theta = 0.936$ rad
7iii	$m \geq 5$		

7i



9i



Answers to A Math 2020 Prelim P2

1. $k = \dfrac{20}{3}$

2. $x = 0.631$

3. (ii) $\dfrac{5x^2 - 30x + 10}{2x^3 - 13x^2 + 24x - 9} = \dfrac{-3}{5(2x-1)} + \dfrac{14}{5(x-3)} - \dfrac{7}{(x-3)^2}$

   (iii) $\dfrac{-3}{5}\ln(2x-1) + \dfrac{28}{5}\ln(x-3) + \dfrac{14}{(x-3)} + c$

4. (i) $3\ln(3x+1)$ (ii) 5.08

5. (ii) 594 (iii) -4752

6. (ii) $\therefore$ equation is $x^2 - \dfrac{9}{2}x + 4 = 0$ or $2x^2 - 9x + 8 = 0$

7. (ii) $\dfrac{7 - 3\sqrt{5}}{2}$

8. (a) Radius = 7 units and centre is $(-2, 5)$

   (b) (i) Equation of C_2 is $(x-5)^2 + (y-4)^2 = 25$ (ii) Coordinates are $(8, 8)$

9. (i) $4\cos 4x - 3\sin 4 = 5\cos(4x + 0.644)$ (ii) $5e^{-3x}\cos(4x + 0.644)$ (iii) $x = 0.232$

10. (i) $f(x) = 3x^2 - 3x^{\frac{3}{2}} - \dfrac{9}{x} + 7$ (ii) $2x + 21y + 40 = 0$

11. (a) (i) $y = \ln(4x^2 - 1) + 2$

    (b) (ii) $p = 5$, $q = 8$ (ii) Inaccurate $A = 700$

12. (i) when $x = -\dfrac{1}{6}$, $y = \dfrac{2}{\left(2\left(-\dfrac{1}{6}\right)+1\right)} + 9\left(-\dfrac{1}{6}\right) - 5 = -\dfrac{7}{2}$ (ii) coordinates are $\left(-\dfrac{5}{6}, -\dfrac{31}{2}\right)$

    (iii)

$$\frac{d^2y}{dx^2} = 8(2x+1)^{-3}.2$$
$$= \frac{16}{(2x+1)^3}$$

$\left(-\dfrac{1}{6}, -\dfrac{7}{2}\right)$ is a minimum point, $\left(-\dfrac{5}{6}, -\dfrac{31}{2}\right)$ is a maximum point.