

NAME: _____ ()

CLASS: 4 ()



**ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2020**

S4

ADDITIONAL MATHEMATICS

4047/01

Paper 1

1 September 2020 Tuesday

2 hours

Candidates answer on the Question Paper

Additional Material: 1 Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters and glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 80.

For Examiners' Use

Examiners' Use					
Question	Marks	Question	Marks	Table of Penalties	
1		7			
2		8		Units	
3		9		Presentation	
4		10		Accuracy	
5		11		Total:	
6		12			
Parent's Name & Signature:				<div></div> <div>80</div>	
Date:					

This paper consists of **18** printed pages.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \triangle = \frac{1}{2}ab \sin C$$

1 The roots of the quadratic equation $2x^2 - 6x + 1 = 0$ are α and β .

(i) Find the value of $\alpha^2 + \beta^2$. [3]

(ii) Find the value of $\frac{\alpha^2 + \beta^2}{\alpha\beta}$. [1]

(iii) Form a quadratic equation with roots $\frac{\alpha}{\beta} + 2$ and $\frac{\beta}{\alpha} + 2$. [3]

[Turn over]

- 2 The mass, m grams, of a radioactive substance, present at time t days after first being observed, is given by the formula $m = 30 e^{-0.025t}$.

(i) Find the mass remaining after 30 days. [2]

(ii) Find the number of days required for the mass to drop to half of its initial value. Give your answer correct to the nearest integer. [2]

(iii) State the value m approaches when t becomes large. [1]

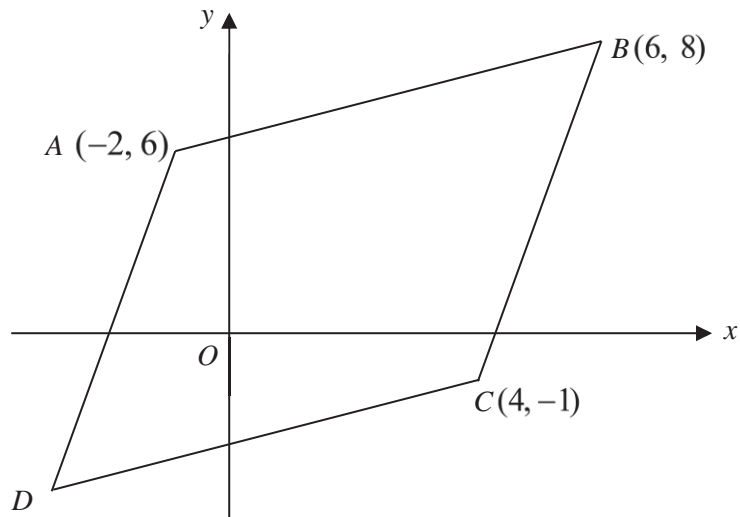
- 3 (a) Find, in ascending powers of x , the first three terms in the expansion of $(2-x)^7$. [2]

Hence, find the value of the constant a for which the coefficient of x^2 in the expansion of $(a-x)(2-x)^7$ is 616. [3]

- (b) In the expansion of $\left(x^2 - \frac{1}{2x^4}\right)^n$ in descending powers of x , the sixth term is independent of x . Find the value of n and the term independent of x . [4]

[Turn over]

4 Solutions to this question by accurate drawing will not be accepted.



The diagram shows a parallelogram $ABCD$ in which $A(-2, 6)$, $B(6, 8)$ and $C(4, -1)$ are the coordinates of its vertices. Find the

(i) equation of AD , [2]

(ii) coordinates of D , [2]

(iii) equation of the perpendicular bisector of the line AD , [2]

(iv) area of the parallelogram $ABCD$, [2]

(v) acute angle the line AB makes with the y -axis. [2]

[Turn over

5 **(i)** Show that $\frac{d}{dx}(\tan^3 5x) = 15 \sec^4 5x - 15 \sec^2 5x$. [3]

(ii) Use your answers to part **(i)**, find $\int \sec^4 5x \, dx$. [4]

- 6 (i) Given that $y = \frac{3x}{\sqrt{5-4x}}$, express $\frac{dy}{dx}$ in the form $\frac{ax+b}{\sqrt{(5-4x)^n}}$ where a , b and n are real constants. [4]

- (ii) Hence find the equation of the normal to the curve $y = \frac{3x}{\sqrt{5-4x}}$ at the point on the curve where $x = 1$. [2]

[Turn over]

7 (i) On the same diagram, sketch the graphs of $y = 16x^{\frac{5}{3}}$ and $y = \frac{9}{\sqrt[3]{x}}$ for $x > 0$. [3]

(ii) Find the x – coordinate of the intersection point of the two graphs. [2]

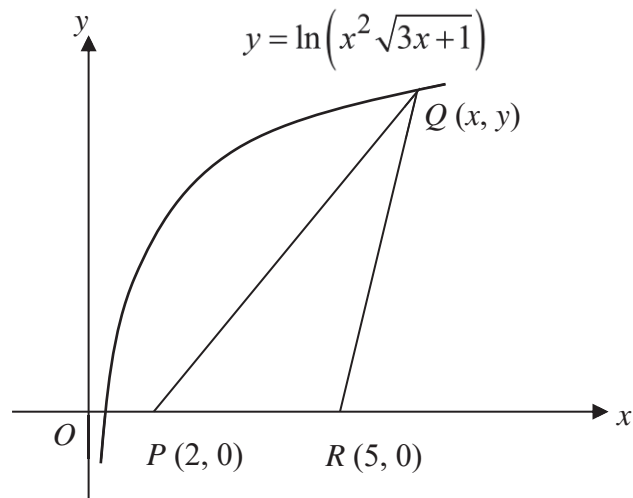
8 Given that $y = \operatorname{cosec} x \tan x$,

(i) show that $\frac{dy}{dx} = \sin x \sec^2 x$, and [2]

(ii) determine where y is decreasing for $0 \leq x \leq 2\pi$. [2]

[Turn over]

- 9 The diagram shows the curve $y = \ln(x^2 \sqrt{3x+1})$ and three points $P(2, 0)$, $Q(x, y)$ and $R(5, 0)$. The point $Q(x, y)$ lies on the curve.

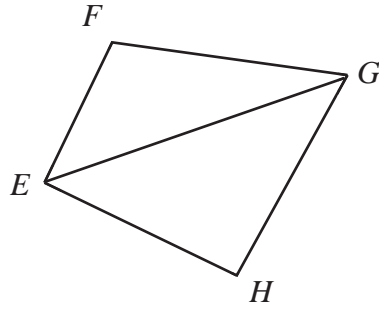


- (i) Show that the area, A units², of the triangle PQR is given by

$$A = 3 \ln x + \frac{3}{4} \ln(3x+1). \quad [2]$$

- (ii) Given that x is increasing at a rate of 0.2 units/s, find the rate at which the area, A , is changing at the instant when $x = 15$ units. [3]

[Turn over



$EFGH$ is a plot of land that comprises two smaller plots, triangle EFG and triangle EGH . EF and EH are perpendicular, angle $FEG = \theta$, $EH = 42$ m, $EG = 55$ m and $EF = 48$ m.

- (i) Show that the area, $A \text{ m}^2$, of $EFGH$ can be expressed as $A = 1320 \sin \theta + 1155 \cos \theta$. [2]

- (ii) Express A in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

(iii) Find the value of θ if the area is 1231 m^2 .

[2]

[Turn over

- 11** **(a)** The variables x and y are related in such a way that when $\frac{x}{y}$ is plotted against $\frac{1}{x}$, a straight line is obtained. The line passes through $(2, 9)$ and $(5, 3)$. Find an expression for y in terms of x . [4]

- (b) The table shows experimental values of two variables, x and y .

x	2	4	6	8
y	8.48	5.99	4.90	4.24

It is known that x and y are related by the equation $x^n y = k$, where n and k are constants. Draw a suitable straight line graph to represent the above data and use it to estimate the values of n and k .

[6]

[Turn over

- 12 Solve the equation $3\operatorname{cosec}^2 x \sin x = 5(\cos x + \sin x)$, giving the principal values of x , in radians. [5]

End of paper

NAME: _____ ()

CLASS: 4 ()



**ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2020**

S4

ADDITIONAL MATHEMATICS**4047/02**

Paper 2

28 August 2020 Friday**2 hours 30 minutes**

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the space at the top of this page.

Write in dark blue or black pen.

You may use a HB pencil for any diagrams or graphs.

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The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 100.

For Examiners' Use

Question	Marks	Question	Marks	Table of Penalties	
1		7			
2		8		Units	
3		9		Presentation	
4		10		Accuracy	
5		11		Total:	
6		12			
Parent's Name & Signature:				<div style="border: 1px solid black; width: 100%; height: 100%; position: relative;"> 100 </div>	
Date:					

This document consists of **18** printed pages.**[Turn over**

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Binomial expansion

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\dots(n-r+1)}{r!}$

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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area of } \Delta = \frac{1}{2}ab \sin C$$

- 1 (a) The equation of a curve is $y = mx^2 + (m-3)x + m$, where m is a constant.

Find the range of values of m for which the curve lies completely above the x -axis. [5]

- (b) Given that $y = ax^2 - 4x + c$ is always negative, give an example of values of a and c which satisfy the condition. [2]

- 2 (a) Given that $2x^4 + 3x^3 + ax^2 - 9x + 9 = (x^2 - 1)(x - 2)Q(x) - 3x^2 + bx + c$ is an identity, state, with reason, the degree of $Q(x)$. [1]
- (b) Find the value of a , of b and of c . [5]

- (c) Hence, find the remainder when $2x^4 + 3x^3 + ax^2 - 9x + 9$ is divided by $(3x - 1)$. [1]

- 3 (a) Given that $p = 3^x$ and $q = 3^y$, express $\log_3 \frac{p^7 q}{243}$ in terms of x and y . [4]

- (b) Given that $\log_2 x - \log_x x^2 = \frac{1}{3} - \log_8 2x$, find the value of x , leaving your answer in index form. [4]

- 4 (a) Without using a calculator, express $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{15}+\sqrt{2}}$ in the form of $a\sqrt{10}+b\sqrt{3}$. [4]

- (b) Without the use of a calculator, solve the equation $\sqrt[3]{27^x} - 81^{x+1} = 0$. [3]

- 5 (a) (i) Given the curve $y = -|3x - x^2| + 4$, find the x -coordinates of the points where the curve meets the x -axis. [2]

- (ii) Sketch the curve $y = -|3x - x^2| + 4$, giving the coordinates of the turning point and of the points where the curve meets the axes. [3]

- (b) Explain why there are only two solutions to the equation $-|3x - x^2| = k - 4$ for $k < 1.75$. [2]

- (c) Determine the maximum value for m for which the line $y = mx + 1$ intersects the graph of $y = -|3x - x^2| + 4$ at three points. [1]

- 6 (i) Express $\frac{7x+11}{(x-1)(x+2)^2}$ in partial fractions. [4]

- (ii) Hence, find $\int \frac{7x+11}{2(x-1)(x+2)^2} dx$. [3]

- 7 A piece of wire which has a fixed length of k cm long is bent to form a rectangle. Show that the area of the rectangle is a maximum when it is a square. [5]

8 Given a circle with the equation $(2x+5)(x+2)+(2y+1)(y-5)=0$,

(i) Express the equation of the circle in standard form.

[5]

- (ii) Find the length of the chord when the line $y = -2x$ cuts the circle.

[5]

9 (a) (i) Prove the identity $\sin x \cos x + \cot x \cos^2 x = \cot x$. [4]

(ii) Hence, solve $\sin 3x \cos 3x + \cot 3x \cos^2 3x = 1$ for $0 \leq x \leq \pi$. [3]

- (b) (i) On the same axes, sketch the graphs of $y = 3 \sin x - 1$ and $y = \tan \frac{x}{2}$ for $0 \leq x \leq 2\pi$. [5]

- (ii) Hence, state the number of solutions of $3 \sin x - 1 = \tan \frac{x}{2}$ for $0 \leq x \leq 2\pi$. [1]

- 10** Two particles, A and B , leave a point O at the same time and travel in the same direction along the same straight line.
Particle A starts with a velocity of 9 m/s and moves with a constant acceleration of 2 m/s^2 .
Particle B starts from rest and moves with an acceleration of $a \text{ m/s}^2$, where $a = 1 + \frac{t}{3}$ and t seconds is the time since leaving O . Find
- (a) an expression for the velocity of each particle in terms of t , [3]

- (b) an expression for the displacement of each particle in terms of t , [3]

(c) the distance from O at which particle B collides with A , [3]

(d) the speed of each particle at the point of collision. [2]

11 Given that $\cos A = p$ and that A is acute, express the following in terms of p .

(i) $\sin 2A$

[3]

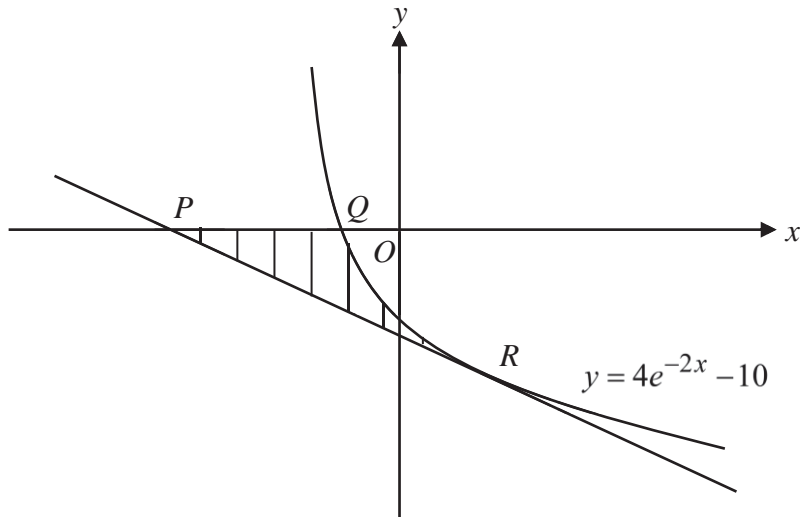
(ii) $\cos \frac{A}{2}$

[3]

- 12** The diagram shows the curve, $y = 4e^{-2x} - 10$. The curve crosses the x -axis at Q . The line PR is a tangent to the curve at R and intersects the x -axis at P . The x -coordinate of R is $\ln 2$.

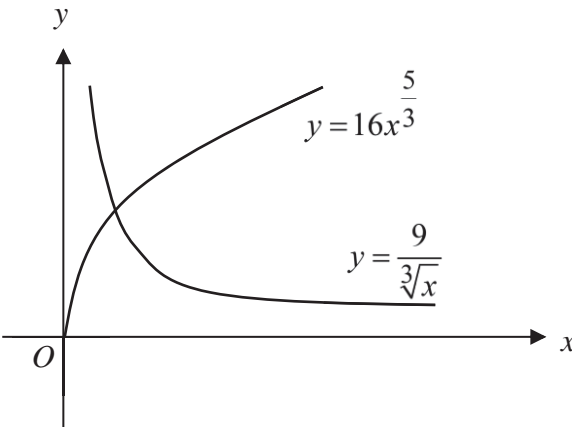
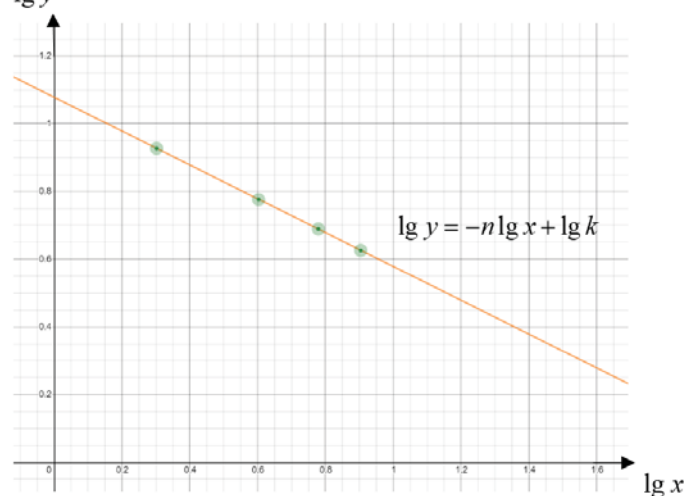
Find the area of the shaded region, PQR , which is the region enclosed by curve, the x -axis and the line PR correct to 3 significant figures.

[11]

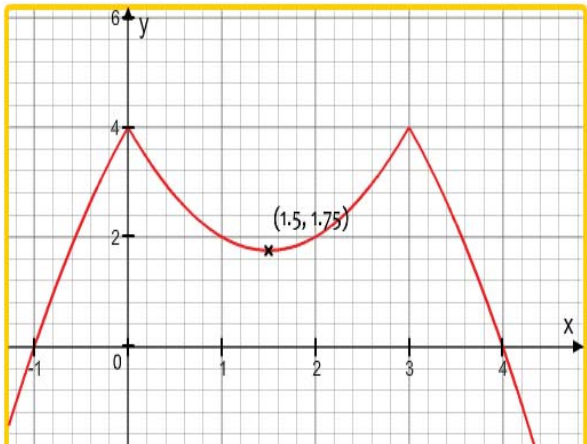
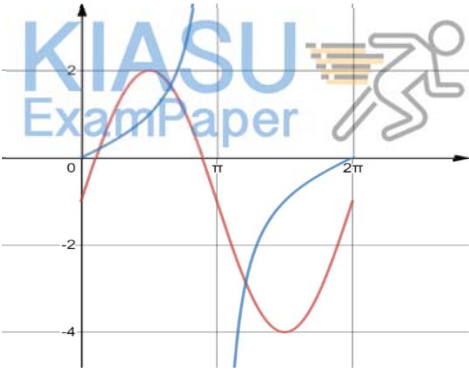


Continuation of working space for Question 12.

Answer key for AM P1

1	(i) $\alpha^2 + \beta^2 = 8$ (ii) $\frac{\alpha^2 + \beta^2}{\alpha\beta} = 16$ (iii) $x^2 + 20x + 37 = 0$	2	(i) The remaining mass after 30 days is 14.2g. (ii) The number of days required is 28 days. (iii) As $t \rightarrow \infty$, $30e^{-0.025t} \rightarrow 0$, the value of m approaches 0.
3	(a) $a = \frac{1}{4}$ (b) $-\frac{3003}{32}$ or $-93\frac{27}{32}$	4	(i) $y = \frac{9}{2}x + 15$ (ii) $(-4, -3)$ (iii) $y = -\frac{2}{9}x + \frac{5}{6}$ (iv) 68 units ² (v) 76.0° (1 d.p.)
5	(ii) $\frac{1}{15}\tan^3 5x + \frac{1}{5}\tan 5x + C_2$		
6	(i) $\frac{15-6x}{\sqrt{(5-4x)^3}}$ (ii) $9y + x = 28$	7	(i) 
8	(ii) For a decreasing function, $\frac{dy}{dx} < 0$ For $0 \leq x \leq 2\pi$, $\sec^2 x > 0$, $x \neq \frac{\pi}{2}, \frac{3\pi}{2}$, $\sin x < 0$ for $\pi < x < 2\pi$ Hence, y is decreasing for $\pi < x < 2\pi$, $x \neq \frac{3\pi}{2}$		(ii) $x = \frac{3}{4}$ ($x > 0$)
9	(ii) The area is increasing at 0.0498 units ² /s.		
10	(ii) $A \approx \sqrt{3076425} \sin(\theta + 41.2^\circ)$ or $A \approx 165\sqrt{113} \sin(\theta + 41.2^\circ)$ (iii) $\theta = 3.4^\circ$		
12	$\cot x = -\frac{1}{3}$ $\cot x = 2$ $\tan x = -3$ or $\tan x = \frac{1}{2}$ $x = -1.25$ $x = 0.464$	11	(i) $y = \frac{x^2}{13x-2}$ (ii)  $k = 12.0$ (11.5–12.6) $n = 0.500$ (0.45–0.55)

Answer key for AM paper 2

1	(a) $m > 1$ (b) $a = -1$ and $c = -5$ or $a = -2$ and $c = -2$ or $a = -5$ and $c = -1$ or any pairs of values that fulfill the above criteria	2	(a) Since degree of dividend = degree of quotient + degree of divisor, degree of $Q(x) = 1$ ALTERNATIVE: Must multiply with x^3 to give degree 4 in the polynomial on the left. (b) $a = -19$, $b = -6$, $c = -5$ (c) $\frac{326}{81}$
3	(a) $7x + y - 5$ (b) $x = 2^{\frac{3}{2}}$ or $x = 8^{\frac{1}{2}}$ or $x = 64^{\frac{1}{4}}$	5	(a)(i) $x = 4$ or $x = -1$ (ii)
4	(a) $\frac{4}{13}\sqrt{10} - \frac{7}{13}\sqrt{3}$ (b) $x = -\frac{4}{3}$		
6	(i) $\frac{2}{x-1} - \frac{2}{x+2} + \frac{1}{(x+2)^2}$ (ii) $\ln(x-1) - \ln(x+2) - \frac{1}{2(x+2)} + c$		
7	$x = \frac{k}{4}$, $\frac{d^2 A}{dx^2} = -2 < 0$ Therefore, since the stationary value occurs when the sides of the rectangle are $\frac{k}{4}$ cm, and it is a maximum value, the maximum area of the rectangle occurs when it is a square.	<p>(b)</p> $- 3x - x^2 = k - 4$ $- 3x - x^2 + 4 = k$ $y = k$ <p>$y = k$ is a horizontal line and when k is lesser than 2.25, it will be below the turning point and so it will only intersect the curve at the two outer arms thereby giving two solutions only.</p> <p>(c) $m = 1$</p>	
8	(i) $\left(x + \frac{9}{4}\right)^2 + \left(y - \frac{9}{4}\right)^2 = \frac{61}{8}$ (ii) 5.14 units (to 3 s.f.)		
9	(a) (ii) $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}$ rad (b)(i)		
	(ii) From the sketch, the two functions intersect at three points. Hence there are three solutions for the equation for $0 \leq x \leq 2\pi$		
10	(a) $v_B = t + \frac{1}{6}t^2$ (b) $s_B = \frac{1}{2}t^2 + \frac{1}{18}t^3$ (c) 486 m (d) 72 m/s		

11	(i) $2p\sqrt{1-p^2}$ (ii) $\sqrt{\frac{p+1}{2}}$	12	$R(\ln 2, -9) \quad P\left(\ln 2 - \frac{9}{2}, 0\right) \quad Q\left(-\frac{1}{2} \ln \frac{5}{2}, 0\right)$ Area = 13.2 units ²
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**ANGLICAN HIGH SCHOOL
SECONDARY FOUR
PRELIMINARY EXAMINATIONS 2020**



ADDITIONAL MATHEMATICS

4047/01

Paper 1

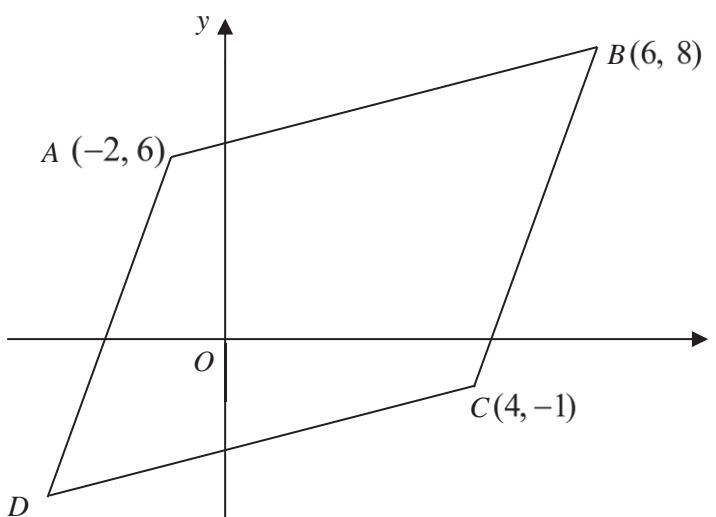
**1 September 2020 Tuesday
2 hours**

Marking Scheme

1	<p>The roots of the quadratic equation $2x^2 - 6x + 1 = 0$ are α and β.</p> <p>(i) Find the value of $\alpha^2 + \beta^2$. [3]</p> <p>(ii) Find the value of $\frac{\alpha^2 + \beta^2}{\alpha\beta}$. [1]</p> <p>(iii) Form a quadratic equation with roots $\frac{\alpha}{\beta} + 2$ and $\frac{\beta}{\alpha} + 2$. [3]</p>
1(i)	<div> $\alpha + \beta = -\left(\frac{-6}{2}\right) = 3$ $\alpha\beta = \frac{1}{2}$ $(\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$ $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $\alpha^2 + \beta^2 = (3)^2 - 2\left(\frac{1}{2}\right)$ $\alpha^2 + \beta^2 = 8$ </div> <div> <p>M1 for both roots</p> <p>M1</p> <p>A1</p> </div>
(ii)	<div> $\frac{\alpha^2 + \beta^2}{\alpha\beta} = 8 \div \frac{1}{2} = 16$ </div> <div> <p>B1</p> </div>
(iii)	<div> $\frac{\alpha}{\beta} + 2 + \frac{\beta}{\alpha} + 2 = \frac{\alpha^2 + \beta^2}{\alpha\beta} + 4$ $\frac{\alpha}{\beta} + 2 + \frac{\beta}{\alpha} + 2 = 16 + 4 = 20$ $\left(\frac{\alpha}{\beta} + 2\right)\left(\frac{\beta}{\alpha} + 2\right) = 1 + \frac{2\alpha}{\beta} + \frac{2\beta}{\alpha} + 4$ $\left(\frac{\alpha}{\beta} + 2\right)\left(\frac{\beta}{\alpha} + 2\right) = 5 + \frac{2\alpha^2 + 2\beta^2}{\alpha\beta}$ $\left(\frac{\alpha}{\beta} + 2\right)\left(\frac{\beta}{\alpha} + 2\right) = 5 + 2(16) = 37$ $x^2 - (20)x + 37 = 0$ $x^2 - 20x + 37 = 0$ </div> <div> <p>M1 for sum of roots</p> <p>M1 for product of roots</p> <p>A1</p> </div>

2	<p>The mass, m grams, of a radioactive substance, present at time t days after first being observed, is given by the formula $m = 30 e^{-0.025t}$.</p> <p>(i) Find the mass remaining after 30 days. [2]</p> <p>(ii) Find the number of days required for the mass to drop to half of its value at $t = 0$. Give your answer correct to the nearest integer. [2]</p> <p>(iii) State the value m approaches when t becomes large. [1]</p>
2(i)	<div> $m = 30 e^{-0.025(30)}$ $= 14.171$ $= 14.2$ The remaining mass after 30 days is 14.2g. </div> <div> M1 A1 – 0 mark for omission of unit in answer </div>
(ii)	<div> $15 = 30 e^{-0.025t}$ $e^{-0.025t} = \frac{1}{2}$ $-0.025t = \ln \frac{1}{2}$ $t = 27.726$ $t = 28$ The number of days required is 28 days. </div> <div> B1 A1 </div>
(iii)	<div> As $t \rightarrow \infty$, $30 e^{-0.025t} \rightarrow 0$, the value of m approaches 0. </div> <div> A1 </div>

3	<p>(a) Find, in ascending powers of x, the first three terms in the expansion of $(2-x)^7$. [2]</p> <p>Hence, find the value of the constant a for which the coefficient of x^2 in the expansion of $(a-x)(2-x)^7$ is 616. [3]</p> <p>(b) In the expansion of $\left(x^2 - \frac{1}{2x^4}\right)^n$ in descending powers of x, the sixth term is independent of x. Find the value of n and the term independent of x. [4]</p>
3(a)(i)	<div> <div> $(2-x)^7 = 2^7 - \binom{7}{1}(2^6)x + \binom{7}{2}(2^5)x^2 + \dots$ $= 128 - 448x + 672x^2 + \dots$ </div> <div> $(a-x)(2-x)^7$ $= (a-x)(128 - 448x + 672x^2 + \dots)$ $= \dots + 672ax^2 + 448x^2 + \dots$ </div> <div> <p>coefficient of x^2: $672a + 448 = 616$</p> $a = \frac{1}{4}$ </div> </div> <div> <div>M1</div> <div>A1</div> <div>M1</div> <div>M1</div> <div>A1</div> </div>
(ii)	<div> $\left(x^2 - \frac{1}{2x^4}\right)^n$ $T_6 = \binom{n}{5} (x^2)^{n-5} \left(-\frac{1}{2x^4}\right)^5$ $= \binom{n}{5} (x^{2n-10}) \left(-\frac{1}{2}\right)^5 x^{-20}$ $= \binom{n}{5} (x^{2n-30}) \left(-\frac{1}{2}\right)^5$ </div> <div> <p>As it is independent of x</p> $2n - 30 = 0$ $n = 15$ </div> <div> <p>Value of the term = $\binom{15}{5} \left(-\frac{1}{2}\right)^5$</p> $= -\frac{3003}{32} \text{ or } -93\frac{27}{32}$ </div> <div> <div>M1</div> <div>A1</div> <div>M1</div> <div>A1</div> </div>

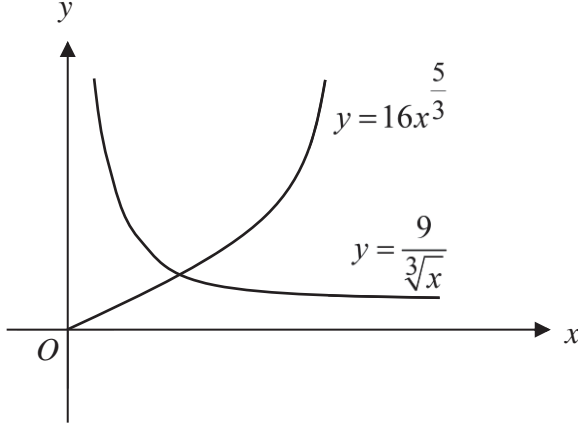
4	<p>Solutions to this question by accurate drawing will not be accepted.</p>  <p>The diagram shows a parallelogram $ABCD$ in which $A(-2, 6)$, $B(6, 8)$ and $C(4, -1)$ are the coordinates of its vertices. Find the</p> <p>(i) equation of AD, [2] (ii) coordinates of D, [2] (iii) equation of the perpendicular bisector of the line AD, [2] (iv) area of the parallelogram $ABCD$, [2] (v) acute angle the line AB makes with the y-axis. [2]</p>
4(i)	<p>Gradient of AD = Gradient of BC</p> $= \frac{8+1}{6-4}$ $= \frac{9}{2}$ <p>Equation of AD :</p> $\frac{y-6}{x+2} = \frac{9}{2}$ $y = \frac{9}{2}x + 15$ <p>M1</p> <p>A1</p>
(ii)	<p>Coordinates of $D = (-2-2, 6-9)$</p> $= (-4, -3)$ <p>M1</p> <p>A1</p>

(iii)	<p>Gradient of Perpendicular Bisector $= -\frac{2}{9}$</p> <p>Midpoint of $AD = \left(\frac{-2+(-4)}{2}, \frac{-3+6}{2} \right)$</p> <p>$= \left(-3, \frac{3}{2} \right)$</p> <p>Equation of Perpendicular Bisector</p> <p>$y - \frac{3}{2} = -\frac{2}{9}(x+3)$</p> <p>$y = -\frac{2}{9}x + \frac{5}{6}$</p>	<p>M1</p> <p>A1</p>
(iv)	<p>Area of $ABCD$</p> <p>$= \frac{1}{2} \begin{vmatrix} -2 & -4 & 4 & 6 & -2 \\ 6 & -3 & -1 & 8 & 6 \end{vmatrix}$</p> <p>$= \frac{1}{2} [(6+4+32+36) - (-24-12-6-16)]$</p> <p>$= 68 \text{ units}^2$</p>	<p>M1</p> <p>clockwise and -68 (max 1 mark)</p> <p>A1</p>
(v)	<p>Angle made by line AB with y-axis</p> <p>$= \tan^{-1} \left(\frac{6+2}{8-6} \right)$</p> <p>$= 76.0^\circ \text{ (1 d.p.)}$</p>	<p>M1</p> <p>A1 accept both degrees and radians</p>

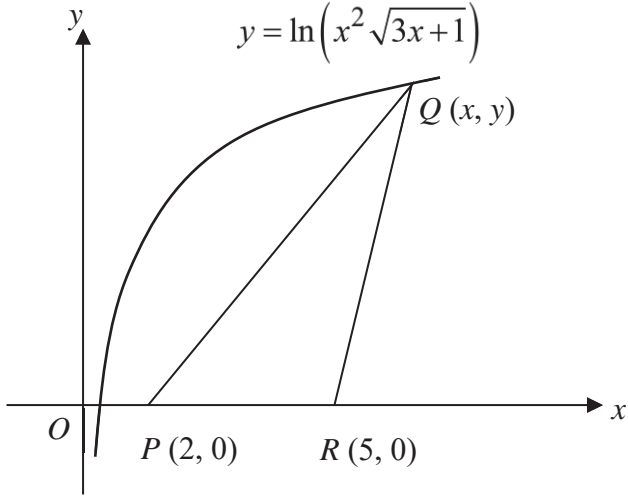
5	<p>(i) Show that $\frac{d}{dx}(\tan^3 5x) = 15 \sec^4 5x - 15 \sec^2 5x$. [3]</p> <p>(ii) Use your answers to part (i), find $\int \sec^4 5x \, dx$. [4]</p>	
(i)	<p>$\frac{d}{dx}(\tan^3 5x) = 3(\tan^2 5x)(\sec^2 5x)(5)$</p> <p>$= 15(\tan^2 5x)(\sec^2 5x)$</p> <p>$= 15(\sec^2 5x - 1)(\sec^2 5x)$</p> <p>$= 15 \sec^4 5x - 15 \sec^2 5x$</p>	<p>M1 – power</p> <p>M1 – chain rule</p> <p>differentiate $\tan 5x$</p> <p>M1</p>

(ii)	$\frac{d}{dx}(\tan^3 5x) = 15 \sec^4 5x - 15 \sec^2 5x$ <p>Integrate w.r.t. x</p> $\tan^3 5x + C_1 = \int (15 \sec^4 5x - 15 \sec^2 5x) dx$ $\tan^3 5x + C_1 = 15 \int \sec^4 5x dx - 15 \int \sec^2 5x dx$ $\tan^3 5x + C_1 = 15 \int \sec^4 5x dx - 15 \left(\frac{\tan 5x}{5} \right)$ $\tan^3 5x + 3 \tan 5x + C_1 = 15 \int \sec^4 5x dx$ $\int \sec^4 5x dx = \frac{1}{15} \tan^3 5x + \frac{1}{5} \tan 5x + C_2$	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>If everything is C, minus 1 mark for overall presentation</p>
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6	<p>(i) Given that $y = \frac{3x}{\sqrt{5-4x}}$, express $\frac{dy}{dx}$ in the form $\frac{ax+b}{\sqrt{(5-4x)^n}}$ where a, b and n are real constants. [4]</p> <p>(ii) Hence find the equation of the normal to the curve $y = \frac{3x}{\sqrt{5-4x}}$ at the point on the curve where $x = 1$. [2]</p>
6(i)	<div style="display: flex; justify-content: space-between;"> <div style="width: 65%;"> $y = \frac{3x}{\sqrt{5-4x}}$ $\frac{dy}{dx} = \frac{\sqrt{5-4x} \frac{d}{dx}(3x) - (3x) \frac{d}{dx} \sqrt{5-4x}}{(\sqrt{5-4x})^2}$ $= \frac{\sqrt{5-4x}(3) - (3x) \frac{1}{2}(5-4x)^{-\frac{1}{2}}(-4)}{5-4x}$ $= \frac{\sqrt{5-4x}(3) + \frac{(3x)2}{\sqrt{5-4x}}}{5-4x}$ $= \frac{\sqrt{5-4x}(3) + \frac{6x}{\sqrt{5-4x}}}{5-4x} \times \frac{\sqrt{5-4x}}{\sqrt{5-4x}}$ $= \frac{(5-4x)(3) + 6x}{(5-4x)\sqrt{5-4x}}$ $= \frac{15-6x}{\sqrt{(5-4x)^3}}$ </div> <div style="width: 30%; text-align: center;"> <p>M1 – Quotient Rule M1 – Chain Rule</p> <p>M1</p> <p>A1</p> </div> </div>
(ii)	<div style="display: flex; justify-content: space-between;"> <div style="width: 65%;"> $x = 1, y = \frac{3(1)}{\sqrt{5-4(1)}} = 3$ $\text{Gradient of the tangent} = \frac{15-6(1)}{\sqrt{(5-4(1))^3}}$ $= 9$ $\text{Gradient of the normal} = -\frac{1}{9}$ <p>Equation of normal</p> $y - 3 = -\frac{1}{9}(x - 1)$ $y = -\frac{1}{9}x + \frac{28}{9}$ $9y + x = 28$ </div> <div style="width: 30%; text-align: center;"> <p>M1 – find gradient of tangent and normal</p> <p>A1</p> </div> </div>

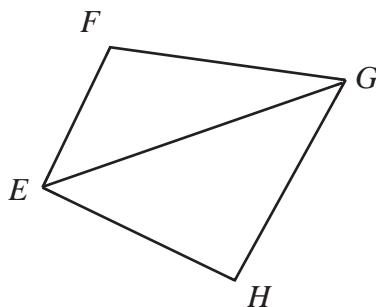
7	<p>(i) On the same diagram, sketch the graphs of $y = 16x^{\frac{5}{3}}$ and $y = \frac{9}{\sqrt[3]{x}}$ for $x > 0$. [3]</p> <p>(ii) Find the x – coordinate of the intersection point of the two graphs. . [2]</p>	
		<p>B1 – For shape of $y = \frac{9}{\sqrt[3]{x}}$</p> <p>B1 – For asymptotes</p> <p>B1 – For shape of $y = 16x^{\frac{5}{3}}$ and pass through origin</p>
(ii)	$\frac{9}{\sqrt[3]{x}} = 16x^{\frac{5}{3}}$ $\frac{9}{16} = x^{\frac{5}{3}} \times \sqrt[3]{x}$ $x^{\frac{5}{3}} \times x^{\frac{1}{3}} = \frac{9}{16}$ $x^2 = \frac{9}{16}$ $x = \frac{3}{4} \quad (x > 0)$	<p>M1</p> <p>A1</p>

	$y = \csc x \tan x$ $= \frac{\tan x}{\sin x}$ $\frac{dy}{dx} = \frac{\sin x \sec^2 x - \tan x \cos x}{\sin^2 x}$ $= \frac{\sin x \sec^2 x - \sin x}{\sin^2 x}$ $= \frac{\sin x (\sec^2 x - 1)}{\sin^2 x}$ $= \frac{\sin x (\tan^2 x)}{\sin^2 x}$ $= \frac{1}{\sin x} \left(\frac{\sin^2 x}{\cos^2 x} \right)$ $= \frac{1}{\cos^2 x} (\sin x)$ $= \sin x \sec^2 x$	<p>M1</p> <p>M1</p>
(ii)	<p>For a decreasing function, $\frac{dy}{dx} < 0$</p> <p>For $0 \leq x \leq 2\pi$, $\sec^2 x > 0$, $x \neq \frac{\pi}{2}, \frac{3\pi}{2}$, $\sin x < 0$ for $\pi < x < 2\pi$</p> <p>Hence $\frac{dy}{dx} < 0$ for $\pi < x < 2\pi$, $x \neq \frac{3\pi}{2}$</p>	<p>B1</p> <p>B1 - if didn't write $x \neq \frac{3\pi}{2}$, its acceptable.</p>

9	<p>The diagram shows the curve $y = \ln(x^2\sqrt{3x+1})$ and three points $P(2, 0)$, $Q(x, y)$ and $R(5, 0)$. The point $Q(x, y)$ lies on the curve.</p>  <p>(i) Show that the area, A units², of the triangle PQR is given by $A = 3 \ln x + \frac{3}{4} \ln(3x+1)$ [2]</p> <p>(ii) Given that x is increasing at a rate of 0.2 units/s, find the rate at which the area, A, is changing at the instant when $x = 15$ units. [3]</p>
9(i)	$A = \frac{1}{2} \times PR \times y = \frac{1}{2} \times 3 \times \ln(x^2\sqrt{3x+1})$ $= \frac{3}{2} \ln(x^2\sqrt{3x+1})$ $A = \frac{3}{2} (\ln x^2 + \ln \sqrt{3x+1})$ $= \frac{3}{2} \left(2 \ln x + \ln(3x+1)^{\frac{1}{2}} \right)$ $= \frac{3}{2} \left(2 \ln x + \frac{1}{2} \ln(3x+1) \right)$ $= 3 \ln x + \frac{3}{4} \ln(3x+1)$ <p>M1</p> <p>M1 – apply the laws for logarithms</p>
(ii)	$\frac{dx}{dt} = 0.2 \text{ m/s and } x = 15$ $A = 3 \ln x + \frac{3}{4} \ln(3x+1)$ $\frac{dA}{dx} = 3 \left(\frac{1}{x} \right) + \frac{3}{4} \left(\frac{1}{3x+1} \right) (3)$ $= \frac{3}{x} + \frac{9}{4(3x+1)}$ <p>M1 – differentiation of logarithms</p>

	$\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$ $= \left(\frac{3}{(15)} + \frac{9}{4(3(15)+1)} \right) \times (0.2)$ $= 0.0498 \text{ units}^2 / \text{s}$ <p>The area is increasing at 0.0498 units²/s.</p>	M1 A1
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10



$EFGH$ is a plot of land that comprises two smaller plots, triangle EFG and triangle EGH . EF and EH are perpendicular, angle $FEG = \theta$, $EH = 42$ m, $EG = 55$ m and $EF = 48$ m.

- (i) Show that the area, A m², of $EFGH$ can be expressed as

$$A = 1320 \sin \theta + 1155 \cos \theta. \quad [2]$$

- (ii) Express A in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

- (iii) Find the value of θ if the area is 1231 m². [2]

10(i)

$$\begin{aligned} \text{Area of triangle } EFG &= \frac{1}{2} \times EF \times EG \sin \angle FEG \\ &= \frac{1}{2} \times 48 \times 55 \sin \theta \\ &= 1320 \sin \theta \end{aligned}$$

M1 – area of triangle

$$\begin{aligned} \text{Area of triangle } EGH &= \frac{1}{2} \times EH \times EG \sin \angle FEG \\ &= \frac{1}{2} \times 42 \times 55 \sin(90^\circ - \theta) \\ &= 1155 (\sin 90^\circ \cos \theta - \cos 90^\circ \sin \theta) \\ &= 1155 \cos \theta \end{aligned}$$

M1 – use of trigonometry

$$\begin{aligned} A &= \text{Area of triangle } EFG + \text{Area of triangle } EGH \\ &= 1320 \sin \theta + 1155 \cos \theta \end{aligned}$$

(ii)

$$1320 \sin \theta + 1155 \cos \theta = R \sin(\theta + \alpha) = R \sin \theta \cos \alpha + R \cos \theta \sin \alpha$$

$$R = \sqrt{1320^2 + 1155^2} = \sqrt{3076425}$$

$$\tan \alpha = \frac{1155}{1320}, \quad \alpha = \tan^{-1}\left(\frac{1155}{1320}\right) = 41.1859^\circ$$

$$A = \sqrt{3076425} \sin(\theta + 41.1859^\circ)$$

Or

$$A \approx 165\sqrt{113} \sin(\theta + 41.2^\circ)$$

M1 – finding R and α
A1

A1

(iii)	$\sqrt{3076425} \sin(\theta + 41.1859^\circ) = 1231$ $\sin(\theta + 41.1859^\circ) = \frac{1231}{\sqrt{3076425}}$ $\text{Basic angle} = \sin^{-1}\left(\frac{1231}{\sqrt{3076425}}\right)$ $= 44.57449^\circ$ $\theta + 41.1859^\circ = 44.57449^\circ, 180 - 44.57449^\circ$ $\theta = 3.3885^\circ, 94.2397^\circ (\text{NA})$ $\theta = 3.4^\circ$	<p>M1</p> <p>A1</p>
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11	<p>(a) The variables x and y are related in such a way that when $\frac{x}{y}$ is plotted against $\frac{1}{x}$, a straight line is obtained. The line passes through (2, 9) and (5, 3). Find an expression for y in terms of x. [4]</p>		
11(i)	<table border="1"> <tr> <td data-bbox="268 421 1129 992"> <p>Let $Y = \frac{x}{y}, X = \frac{1}{x}$</p> <p>Gradient = $\frac{9-3}{2-5} = -2$</p> <p>$\therefore Y - 3 = -2(X - 5)$</p> <p>$Y = -2X + 13$</p> <p>$\frac{x}{y} = -\frac{2}{x} + 13$</p> <p>$\frac{x}{y} = \frac{13x-2}{x}$</p> <p>$y = \frac{x^2}{13x-2}$</p> </td><td data-bbox="1129 421 1422 992"> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> </td></tr> </table>	<p>Let $Y = \frac{x}{y}, X = \frac{1}{x}$</p> <p>Gradient = $\frac{9-3}{2-5} = -2$</p> <p>$\therefore Y - 3 = -2(X - 5)$</p> <p>$Y = -2X + 13$</p> <p>$\frac{x}{y} = -\frac{2}{x} + 13$</p> <p>$\frac{x}{y} = \frac{13x-2}{x}$</p> <p>$y = \frac{x^2}{13x-2}$</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p>
<p>Let $Y = \frac{x}{y}, X = \frac{1}{x}$</p> <p>Gradient = $\frac{9-3}{2-5} = -2$</p> <p>$\therefore Y - 3 = -2(X - 5)$</p> <p>$Y = -2X + 13$</p> <p>$\frac{x}{y} = -\frac{2}{x} + 13$</p> <p>$\frac{x}{y} = \frac{13x-2}{x}$</p> <p>$y = \frac{x^2}{13x-2}$</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p>		

(b) The table shows experimental values of two variables, x and y .

x	2	4	6	8
y	8.48	5.99	4.90	4.24

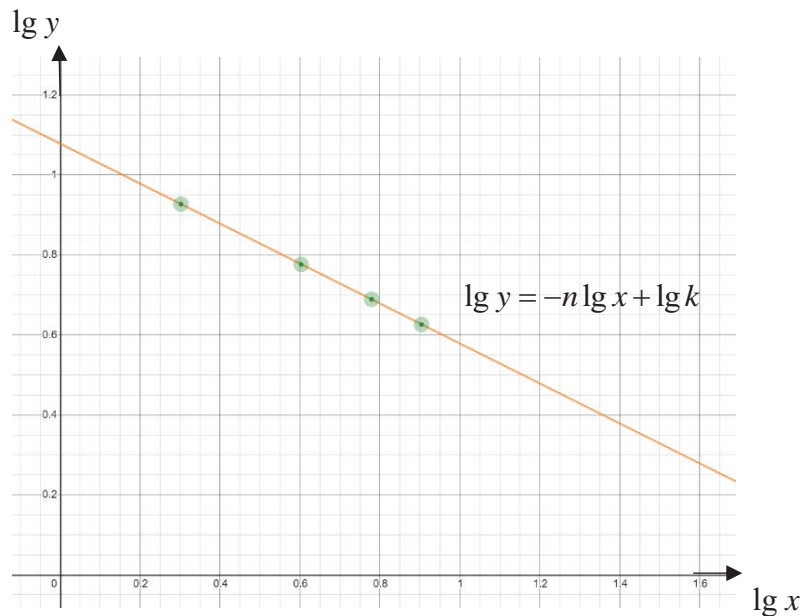
It is known that x and y are related by the equation $x^n y = k$, where n and k are constants. Draw a suitable straight line graph to represent the above data and use it to estimate the values of n and k . [6]

(ii)

$$x^n y = k$$

$$\lg y = -n \lg x + \lg k$$

x	2	4	6	8
y	8.48	5.99	4.90	4.24
$\lg x$	0.301	0.602	0.778	0.903
$\lg y$	0.928	0.777	0.690	0.627



$$\lg k = 1.08 \quad (1.06 - 1.10)$$

$$k = 12.0 \quad (11.5 - 12.6)$$

$$-n = \frac{1.00 - 0.60}{0.15 - 0.95}$$

$$= -0.500$$

$$\therefore n = 0.500 \quad (0.45 - 0.55)$$

M1

B1

G1 – correct points

G1 – y-intercept

Deduct 1 mark for labels

A1

A1

12	Solve the equation $3\operatorname{cosec}^2 x \sin x = 5(\cos x + \sin x)$, giving the principal values of x , in radians. [5]
	<div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> $3\operatorname{cosec}^2 x \sin x = 5(\cos x + \sin x)$ $3\operatorname{cosec}^2 x \sin x = 5\cos x + 5\sin x$ $3\operatorname{cosec}^2 x = \frac{5\cos x + 5\sin x}{\sin x}$ $= \frac{5\cos x}{\sin x} + \frac{5\sin x}{\sin x}$ $= 5\cot x + 5$ $3\operatorname{cosec}^2 x = 5\cot x + 5$ $3(\cot^2 x + 1) - 5\cot x - 5 = 0$ $3\cot^2 x + 3 - 5\cot x - 5 = 0$ $3\cot^2 x - 5\cot x - 2 = 0$ $(3\cot x + 1)(\cot x - 2) = 0$ $\cot x = -\frac{1}{3} \qquad \cot x = 2$ $\tan x = -3 \qquad \text{or} \qquad \tan x = \frac{1}{2}$ $x = -1.25 \qquad \qquad \qquad x = 0.464$ </div> <div style="width: 45%;"> <p>M1 – use of $\cot x = \frac{\cos x}{\sin x}$, $\operatorname{cosec} x = \frac{1}{\sin x}$, $\tan x = \frac{\sin x}{\cos x}$ etc</p> <p>M1 – use identity to change into an equation with single trigonometric term</p> <p>M1 – reach quadratic equation and factorize, or use formula</p> <p>A2 – deduct one mark if more than the principal values are given.</p> </div> </div>



ADDITIONAL MATHEMATICS

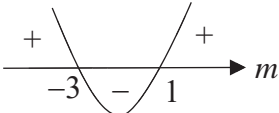
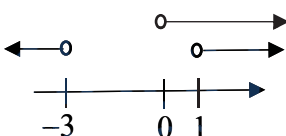
Paper 2

4047/02

28 August 2020 Friday

2 hours 30 minutes

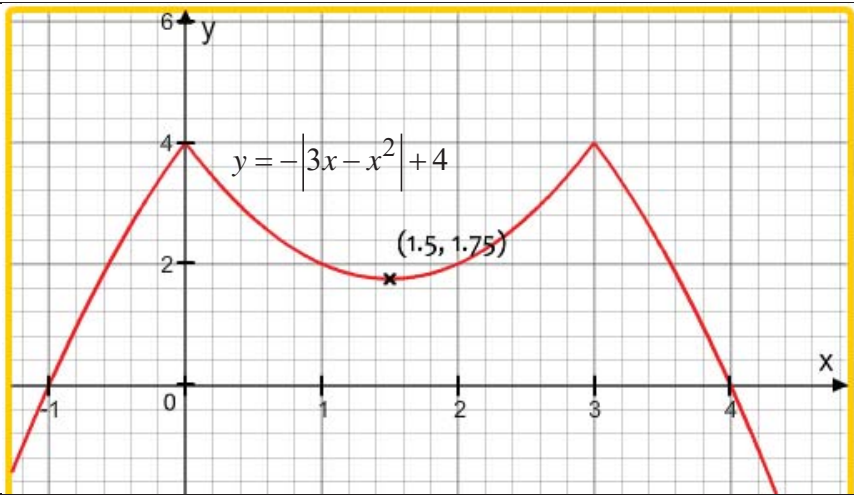
Marking Scheme

1a	<p>The equation of a curve is $y = mx^2 + (m-3)x + m$, where m is a constant. Find the range of values of m for which the curve lies completely above the x-axis.</p>	[5]
	<p>Since the curve is completely above the x-axis, the curve must be a quadratic curve with minimum point, $a > 0$ $\therefore m > 0$</p> <p>And there will be no point of intersection with the x-axis, $b^2 - 4ac < 0$ $(m-3)^2 - 4(m)(m) < 0$ $m^2 - 6m + 9 - 4m^2 < 0$ $-3m^2 - 6m + 9 < 0$ $m^2 + 2m - 3 > 0$ $(m+3)(m-1) > 0$ $m < -3$ or $m > 1$</p>   <p>Hence, $m > 1$</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p>
1b	<p>Given that $y = ax^2 - 4x + c$ is always negative, give an example of values of a and c which satisfy the condition.</p>	[2]
	<p>$b^2 - 4ac < 0$ $(-4)^2 - 4ac < 0$ $ac > 4$, $a < 0$</p> <p>$a = -1$ and $c = -5$ or $a = -2$ and $c = -2$ or $a = -5$ and $c = -1$ or any pairs of values that fulfill the above criteria</p>	<p>M1</p> <p>A1</p>

2a	Given that $2x^4 + 3x^3 + ax^2 - 9x + 9 = (x^2 - 1)(x - 2)Q(x) - 3x^2 + bx + c$ is an identity, state, with reason, the degree of $Q(x)$.	[1]
	Since degree of dividend = degree of quotient + degree of divisor, degree of $Q(x) = 1$ ALTERNATIVE: Must multiply with x^3 to give degree 4 in the polynomial on the left.	B1 for reason and answer
2b	Find the value of a , of b and of c .	[5]
	$2x^4 + 3x^3 + ax^2 - 9x + 9 = (x^2 - 1)(x - 2)Q(x) - 3x^2 + bx + c$ <p>When $x = 1$, $2 + 3 + a - 9 + 9 = -3 + b + c$ $a = -8 + b + c$ ----- (1)</p> <p>When $x = -1$, $2 - 3 + a + 9 + 9 = -3 - b + c$ $a = -20 - b + c$ ----- (2)</p> <p>(1) = (2): $-20 - b + c = -8 + b + c$ $-12 = 2b$ $b = -6$</p> <p>When $x = 2$, $32 + 24 + 4a - 18 + 9 = -12 - 12 + c$ $c = 47 + 4a + 24$ $c = 4a + 71$ ----- (3)</p> <p>Sub (3) and $b = -6$ into (2) $a = -20 + 6 + 4a + 71$ $a = 4a + 57$ $a = -19$</p> <p>Sub $a = -19$ into (2) $c = 4(-19) + 5$ $c = -5$</p> <p>$a = -19, \quad b = -6, \quad c = -5$</p>	<p>M1</p> <p>M1</p> <p>M1</p> <p>M1 for solving</p> <p>A1 for 3 answers</p>
2c	Hence, find the remainder when $2x^4 + 3x^3 + ax^2 - 9x + 9$ is divided by $(3x - 1)$.	[1]
	Let $f(x) = 2x^4 + 3x^3 - 19x^2 - 9x + 9$	

	$f\left(\frac{1}{3}\right) = 2\left(\frac{1}{3}\right)^4 + 3\left(\frac{1}{3}\right)^3 - 19\left(\frac{1}{3}\right)^2 - 9\left(\frac{1}{3}\right) + 9$ $f\left(\frac{1}{3}\right) = \frac{326}{81}$ <p>Therefore the remainder is $\frac{326}{81}$.</p>	B1
3a	Given that $p = 3^x$ and $q = 3^y$, express $\log_3 \frac{p^7 q}{243}$ in terms of x and y .	[4]
	$p = 3^x \Rightarrow x = \log_3 p \quad q = 3^y \Rightarrow y = \log_3 q$ $\log_3 \frac{p^7 q}{243} = 7 \log_3 p + \log_3 q - \log_3 3^5$ $\log_3 \frac{p^7 q}{243} = 7x + y - 5$	[M1 for change to log] M2 for using rules A1
3b	Given that $\log_2 x - \log_x x^2 = \frac{1}{3} - \log_8 2x$, find the value of x by leaving your answer in index form.	[4]
	$\log_2 x - \log_x x^2 = \frac{1}{3} - \log_8 2x$ $\log_2 x - 2 \log_x x = \frac{1}{3} - (\log_8 2 + \log_8 x)$ $\log_2 x - 2(1) = \frac{1}{3} - \frac{1}{3} \log_8 8 - \frac{\log_2 x}{\log_2 8}$ $\log_2 x - 2 = \frac{1}{3} - \frac{1}{3} - \frac{\log_2 x}{3}$ $3 \log_2 x - 6 = -\log_2 x$ $4 \log_2 x = 6$ $\log_2 x = \frac{3}{2}$ $x = 2^{\frac{3}{2}} \text{ or } x = 8^{\frac{1}{2}} \text{ or } x = 64^{\frac{1}{4}}$	M1 for power rule and product rule M1 for change of base rule M1 A1
4a	Without using a calculator, express $\frac{\sqrt{6} - \sqrt{5}}{\sqrt{15} + \sqrt{2}}$ in the form of $a\sqrt{10} + b\sqrt{3}$.	[4]
		M1 for rationalization

	$\frac{\sqrt{6}-\sqrt{5}}{\sqrt{15}+\sqrt{2}} = \frac{(\sqrt{6}-\sqrt{5})(\sqrt{15}-\sqrt{2})}{15-2}$ $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{15}+\sqrt{2}} = \frac{\sqrt{90}-\sqrt{12}-\sqrt{75}+\sqrt{10}}{13}$ $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{15}+\sqrt{2}} = \frac{3\sqrt{10}-2\sqrt{3}-5\sqrt{3}+\sqrt{10}}{13}$ $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{15}+\sqrt{2}} = \frac{4\sqrt{10}-7\sqrt{3}}{13}$ $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{15}+\sqrt{2}} = \frac{4}{13}\sqrt{10} - \frac{7}{13}\sqrt{3}$	<p>M1 for expansion</p> <p>M1 for simplifying</p> <p>A1</p>
4b	Without the use of a calculator, solve the equation $\sqrt[3]{27^x} - 81^{x+1} = 0$.	[3]
	$\sqrt[3]{27^x} - 81^{x+1} = 0$ $27^{\frac{x}{3}} - 81^{x+1} = 0$ $(3^3)^{\frac{x}{3}} - (3^4)^{x+1} = 0$ $3^x - 3^{4x+4} = 0$ $3^x = 3^{4x+4}$ <p>By comparing powers,</p> $x = 4x + 4$ $x = -\frac{4}{3}$	<p>M1 for converting all terms to base 3</p> <p>M1 for using if $a^m = a^n$ then $m = n$</p> <p>[A1]</p>
5ai	Given the curve $y = - 3x - x^2 + 4$, find the x -coordinates of the points where the curve meets the x -axis.	[2]
	<p>When $y = 0$,</p> $0 = - 3x - x^2 + 4$ $3x - x^2 = 4 \quad \text{or} \quad 3x - x^2 = -4$ $3x - x^2 - 4 = 0 \quad \text{or} \quad 3x - x^2 + 4 = 0$ $x^2 - 3x + 4 = 0 \quad \text{or} \quad x^2 - 3x - 4 = 0$ $b^2 - 4ac = (-3)^2 - 4(1)(4) \quad \text{or} \quad (x-4)(x+1) = 0$ $b^2 - 4ac = -7 \quad \text{or} \quad x = 4 \quad \text{or} \quad x = -1$	<p>M1 for considering positive and negative</p> <p>A1 for answers</p>
5aia	Sketch the curve $y = - 3x - x^2 + 4$, giving the coordinates of the maximum point and of the points where the curve meets the axes..	[3]

		<p>B1 for shape</p> <p>B1 for coordinates of max pt and x-intercepts.</p> <p>B1 for labelling.</p>
5b	Explain why there are only two solutions to the equation $- 3x - x^2 = k - 4$ for $k < 1.75$.	[2]
	$- 3x - x^2 = k - 4$ $- 3x - x^2 + 4 = k$ $y = k$ <p>$y = k$ is a horizontal line and when k is lesser than 2.25, it will be below the turning point and so it will only intersect the curve at the two outer arms thereby giving two solutions only.</p>	<p>B1</p> <p>B1</p>
5c	Determine the maximum value of m for which the line $y = mx + 1$ intersects the graph of $y = - 3x - x^2 + 4$ in three points.	[1]
	$m = \frac{4 - 1}{3 - 0}$ $m = 1$	B1
6(i)	Express $\frac{7x+11}{(x-1)(x+2)^2}$ in partial fractions.	[4]
	<p>Let $\frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2} = \frac{7x+11}{(x-1)(x+2)^2}$</p> $A(x+2)^2 + B(x-1)(x+2) + C(x-1) = 7x+11$ <p>when $x = 1$, $9A = 7 + 11$ $A = 2$</p> <p>when $x = -2$ $-3C = -14 + 11$ $C = 1$</p> <p>when $x = 0$ $4A - 2B - C = 11$ $4(2) - 2B - 1 = 11$</p>	<p>[M1 combining fractions together and equating the numerator]</p> <p>[M1 for substitution or any other method]</p>

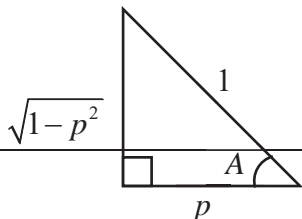
	$-2B = 4$ $B = -2$ $\therefore \frac{7x+11}{(x-1)(x+2)^2} = \frac{2}{x-1} - \frac{2}{x+2} + \frac{1}{(x+2)^2}$	[A2 for fully correct partial fractions. A1 only if at least two terms are correct]
6(ii)	Hence, find $\int \frac{7x+11}{2(x-1)(x+2)^2} dx$.	[3]
	$\int \frac{7x+11}{2(x-1)(x+2)^2} dx = \frac{1}{2} \int \frac{2}{x-1} - \frac{2}{x+2} + \frac{1}{(x+2)^2} dx$ $= \ln(x-1) - \ln(x+2) - \frac{1}{2(x+2)} + c$	<p>M1 for taking out coefficient</p> <p>A2 for all correct, A1 if at least two terms are correct</p>
7	A piece of wire which has a fixed length of k cm long is bent to form a rectangle. Show that the area of the rectangle is a maximum when it is a square.	[5]
	<p>Let the length of the rectangle be x.</p> <p>Breadth of rectangle = $\frac{k-2x}{2}$</p> <p>Let area of the rectangle be A.</p> $A = x \left(\frac{k-2x}{2} \right)$ $A = \frac{xk - 2x^2}{2}$ $A = \frac{1}{2} xk - x^2$ $\frac{dA}{dx} = \frac{1}{2} k - 2x$ <p>At the stationary value, $\frac{dA}{dx} = 0$</p> $\frac{1}{2} k - 2x = 0$ $2x = \frac{k}{2}$	<p>[M1]</p> <p>[M1]</p> <p>[M1]</p>

	$x = \frac{k}{4}$ $\frac{d^2 A}{dx^2} = -2 < 0$ Therefore, since the stationary value occurs when the sides of the rectangle are $\frac{k}{4}$ cm, and it is a maximum value, the maximum area of the rectangle occurs when it is a square.	[M1] [A1]
8	Given a circle with the equation $(2x+5)(x+2)+(2y+1)(y-5)=0$,	
8(i)	Express the equation of the circle in standard form.	[5]
	$(2x+5)(x+2)+(2y+1)(y-5)=0$ $2x^2+9x+10+2y^2-9y-5=0$ $2x^2+2y^2+9x-9y+5=0$ $x^2+y^2+\frac{9}{2}x-\frac{9}{2}y+\frac{5}{2}=0$ $x^2+\frac{9}{2}x+\left(\frac{9}{4}\right)^2+y^2-\frac{9}{2}y+\left(\frac{9}{4}\right)^2=-\frac{5}{2}+\left(\frac{9}{4}\right)^2+\left(\frac{9}{4}\right)^2$ $\left(x+\frac{9}{4}\right)^2+\left(y-\frac{9}{4}\right)^2=\frac{61}{8}$ Coordinates of centre = $\left(-\frac{9}{4}, \frac{9}{4}\right)$ Radius of circle = $\sqrt{\frac{61}{8}}$ units Equation of circle, $\left(x+\frac{9}{4}\right)^2+\left(y-\frac{9}{4}\right)^2=\frac{61}{8}$	M1 for expansion and simplification M1 – for getting the centre and radius [A1] [A1] [A1]
8(ii)	Find the length of the chord when the line $y = -2x$ cuts the circle.	[5]
	$y = -2x$ -(1) $\left(x+\frac{9}{4}\right)^2+\left(y-\frac{9}{4}\right)^2=\frac{61}{8}$ -(2) Sub (1) into (2) $\left(x+\frac{9}{4}\right)^2+\left(-2x-\frac{9}{4}\right)^2=\frac{61}{8}$	[M1 for substitution]

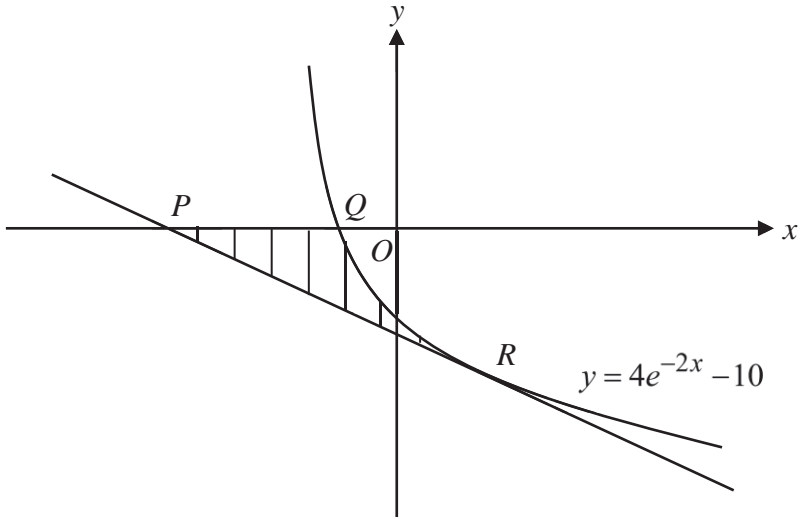
	$5x^2 + \frac{27}{2}x + \frac{5}{2} = 0$ $10x^2 + 27x + 5 = 0$ $(5x+1)(2x+5) = 0$ $x = -\frac{1}{5} \quad \text{or} \quad x = -\frac{5}{2}$ <p>when $x = -\frac{1}{5}, y = \frac{2}{5}$</p> <p>when $x = -\frac{5}{2}, y = 5$</p> <p>Thus the coordinates of the end points of the chord are $\left(-\frac{1}{5}, \frac{2}{5}\right)$ and $\left(-\frac{5}{2}, 5\right)$</p> <p>Length of chord = $\sqrt{\left(2\frac{3}{10}\right)^2 + \left(-4\frac{3}{5}\right)^2}$ $= 5.14$ units (to 3 s.f.)</p>	<p>[M1 for solving]</p> <p>[A1 for correct coordinates]</p> <p>[M1]</p> <p>[A1]</p>
9ai	Prove the identity $\sin x \cos x + \cot x \cos^2 x = \cot x$.	[4]
	<p>LHS = $\sin x \cos x + \cot x \cos^2 x$</p> <p>$= \cos x (\sin x + \cot x \cos x)$</p> <p>$= \cos x \left(\sin x + \frac{\cos x}{\sin x} \cos x \right)$</p> <p>$= \cos x \left(\sin x + \frac{\cos^2 x}{\sin x} \right)$</p> <p>$= \cos x \left(\frac{\sin^2 x + \cos^2 x}{\sin x} \right)$</p> <p>$= \cos x \left(\frac{1}{\sin x} \right)$</p> <p>$= \cot x = \text{RHS}$</p> <p>Or</p> <p>LHS = $\sin x \cos x + \cot x \cos^2 x$</p> <p>$= \sin x \cos x + \frac{\cos x}{\sin x} \cos^2 x$</p> <p>$= \frac{\sin^2 x \cos x}{\sin x} + \frac{\cos^3 x}{\sin x}$</p>	<p>[M1]</p> <p>[M1]</p> <p>[M1]</p> <p>[A1]</p> <p>[M1]</p>

	$= \frac{\sin^2 x \cos x + \cos^3 x}{\sin x}$ $= \frac{\cos x (\sin^2 x + \cos^2 x)}{\sin x}$ $= \frac{\cos x (1)}{\sin x}$ $= \cot x = \text{RH}$	 [M1] [M1] [A1]
9a ii	Hence, solve $\sin 3x \cos 3x + \cot 3x \cos^2 3x = 1$ for $0 \leq x \leq \pi$.	[3]
	<p>Since $\sin x \cos x + \cot x \cos^2 x = \cot x$</p> <p>Therefore, $\sin 3x \cos 3x + \cot 3x \cos^2 3x = \cot 3x$</p> <p>and</p> $\sin 3x \cos 3x + \cot 3x \cos^2 3x = 1 \Rightarrow \cot 3x = 1$ $0 \leq x \leq \pi$ $0 \leq 3x \leq 3\pi$ <p>Let the basic angle be α</p> $\tan \alpha = 1$ $\alpha = \frac{\pi}{4}$ $3x = \frac{\pi}{4}, \pi + \frac{\pi}{4}, 2\pi + \frac{\pi}{4}$ $3x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}$ $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4} \text{ rad}$	 [M1] [M1] [A1]
9bi	On the same axes, sketch the graphs of $y = 3 \sin x - 1$ and $y = \tan \frac{x}{2}$ for $0 \leq x \leq 2\pi$.	[5]
		<p>For tan curve, [B1] for shape, [B1] for asymptote</p> <p>For Sine curve, [B1] for shape [B1] for correct max and</p>

		<p>minimum values</p> <p>[B1] for correct period for both graphs</p> <p>[subtract 1 mark for wrong or no labels for axes or functions]</p>
9bii	Hence, state the number of solutions of $3 \sin x - 1 = \tan \frac{x}{2}$ for $0 \leq x \leq 2\pi$.	[1]
	From the sketch, the two functions intersect at three points. Hence there are three solutions for the equation for $0 \leq x \leq 2\pi$	[B1]
10a	<p>Two particles, A and B, leaves a point O at the same time and travel in the same direction along the same straight line.</p> <p>Particle A starts with a velocity of 9 m/s and moves with a constant acceleration of 2 m/s^2.</p> <p>Particle B starts from rest and moves with an acceleration of $a \text{ m/s}^2$, where $a = 1 + \frac{t}{3}$ and t seconds is the time since leaving O. Find an expression for the velocity of each particle in terms of t,</p>	[3]
	<p>For Particle A,</p> $v_A = \int 2 \, dt$ $v_A = 2t + c$ <p>When $t = 0$, $v_A = 9$</p> $c = 9$ $v_A = 2t + 9$ <p>For Particle B,</p> $v_B = \int \left(1 + \frac{t}{3}\right) dt$ $v_B = t + \frac{1}{6}t^2 + c$ <p>When $t = 0$, $v_B = 0$,</p> $c = 0$ $v_B = t + \frac{1}{6}t^2$	<p>B1</p> <p>M1</p> <p>A1</p>
10b	an expression for the displacement of each particle in terms of t ,	[3]

	<p>For Particle A,</p> $s_A = \int (2t + 9) dt$ $s_A = t^2 + 9t + c$ <p>When $t = 0$, $s_A = 0$</p> $c = 0$ $s_A = t^2 + 9t$ <p>For Particle B,</p> $s_B = \int \left(t + \frac{1}{6}t^2 \right) dt$ $s_B = \frac{1}{2}t^2 + \frac{1}{18}t^3 + c$ <p>When $t = 0$, $s_B = 0$,</p> $c = 0$ $s_B = \frac{1}{2}t^2 + \frac{1}{18}t^3$	<p>B1</p> <p>M1</p> <p>A1</p>
10c	the distance from O at which particle B collides with A ,	[3]
	<p>When particle B collides with particle A,</p> $s_A = s_B$ $t^2 + 9t = \frac{1}{2}t^2 + \frac{1}{18}t^3$ $18t^2 + 162t = 9t^2 + t^3$ $t^3 - 9t^2 - 162t = 0$ $t(t^2 - 9t - 162) = 0$ $t(t - 18)(t + 9) = 0$ $t = 0(\text{N.A.}) \quad \text{or} \quad t = 18 \quad \text{or} \quad t = -9(\text{N.A.})$ <p>Distance from $O = (18)^2 + 9(18) = 486 \text{ m}$</p>	<p>M1</p> <p>M1</p> <p>A1</p>
10d	the speed of each particle at the point of collision.	[2]
	Speed of particle $A = 2(18) + 9 = 45 \text{ m/s}$	B1
	Speed of particle $B = (18) + \frac{1}{6}(18)^2 = 72 \text{ m/s}$	B1
11	Given that $\cos A = p$ and that A is acute, express the following in terms of p .	
i	$\sin 2A$	[3]
		M1 for getting the length of opposite side

	$\sin 2A = 2 \sin A \cos A$ $= 2p\sqrt{1-p^2}$ <p>Or</p> $\cos^2 A + \sin^2 A = 1$ $\sin^2 A = 1 - \cos^2 A$ $\sin^2 A = 1 - p^2$ $\sin A = \sqrt{1-p^2} \text{ (reject negative as } A \text{ is acute)}$ $\sin 2A = 2 \sin A \cos A$ $= 2p\sqrt{1-p^2}$	M1 A1 M1 M1 A1
ii	$\cos \frac{A}{2}$	[3]
	$\cos A = 2 \cos^2 \frac{A}{2} - 1$ $\cos \frac{A}{2} = \pm \sqrt{\frac{\cos A + 1}{2}}$ $\cos \frac{A}{2} = \pm \sqrt{\frac{p+1}{2}}$ $\cos \frac{A}{2} = -\sqrt{\frac{p+1}{2}} \text{ (rejected) or } \cos \frac{A}{2} = \sqrt{\frac{p+1}{2}}$	M1 M1 for rejection A1
12	<p>The diagram shows the curve, $y = 4e^{-2x} - 10$. The curve crosses the x-axis at Q. The line PR is a tangent to the curve at R and intersects the x-axis at P. The x-coordinate of R is $\ln 2$.</p> <p>Find the area of the shaded region, PQR, which is the region enclosed by curve, the x-axis and the line x-axis and the line PR correct to 3 significant figures.</p>	[11]

		
	<p>At R, $x = \ln 2$ $y = 4e^{-2\ln 2} - 10 = -9$ $R(\ln 2, -9)$</p>	B1
	<p>$y = 4e^{-2x} - 10$ $\frac{dy}{dx} = 4(-2)e^{-2x} = -8e^{-2x}$ Gradient of line $PR = -8e^{-2\ln 2} = -2$ Equation of line PR $y - (-9) = -2(x - \ln 2)$ $y = -2x + 2\ln 2 - 9$ $y = -2x - 7.6137$</p>	M1 A1
	<p>At P, $y = 0$ $0 = -2x + 2\ln 2 - 9$ $x = \ln 2 - \frac{9}{2}$ $= -3.8069$ $P\left(\ln 2 - \frac{9}{2}, 0\right)$</p>	B1
	<p>At Q, $y = 0$ $0 = 4e^{-2x} - 10$ $4e^{-2x} = 10$ $e^{-2x} = \frac{5}{2}$ $-2x = \ln \frac{5}{2}$ $x = -\frac{1}{2} \ln \frac{5}{2} = -0.45815$ $Q\left(-\frac{1}{2} \ln \frac{5}{2}, 0\right)$</p>	M1 – change index form to log form A1
	<p>Area of the shaded region, PQR $= \text{Area of the triangle} - \text{Area between the curve and the } x\text{-axis}$</p>	M1

	$\text{Area} = \frac{1}{2} \times \left((\ln 2) - \left(\ln 2 - \frac{9}{2} \right) \right) \times (9) - \left \int_{-\frac{1}{2} \ln \frac{5}{2}}^{\ln 2} (4e^{-2x} - 10) dx \right $	M1
	$= \frac{81}{4} - \left \left[-2e^{-2x} - 10x \right]_{-\frac{1}{2} \ln \frac{5}{2}}^{\ln 2} \right $	M1
	$= \frac{81}{4} - \left \left(\left[-2e^{-2 \ln 2} - 10 \ln 2 \right] - \left[-2e^{-2 \left(-\frac{1}{2} \ln \frac{5}{2} \right)} - 10 \left(-\frac{1}{2} \ln \frac{5}{2} \right) \right] \right) \right $	M1
	$= \frac{81}{4} - -7.01295 $	
	$= 13.23705 = 13.2 \text{ units}^2$	A1

