NAME: _____ ( )   CLASS: _____

FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2019
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

MATHEMATICS   4048/01

Paper 1

Date: 27 August 2019   Duration: 2 hours

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact,
give the answer to 3 significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the
answer in terms of π.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part
question.
The total number of marks for this paper is 80.

For Examiner's Use	
Paper 1	/80
Paper 2	/100
Total	%

Setter: Miss Shamsiah Zainalabidin

This question paper consists of <u>19</u> printed pages including the cover page.

Name: _____ (   )   Class: _____

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Name: _____ ( )   Class: _____

Answer all the questions.

1   (a)   Express 3780 as a product of its prime factors.

Answer (a) .... [1]

    (b)   **Hence**, find the smallest integer by which 3780 must be multiplied to obtain a perfect square.

Answer (b) .... [1]

2   Given that $A = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$, $B = (-3 \quad 9)$ and $C = \begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix}$, find

    (a) $\dfrac{1}{2} BA$,

Answer (a) .... [1]

    (b) C^2.

Answer (b) .... [1]

3   The curve $y = (x-1)(x+k)$ has a minimum point Q as shown.

$Q(-4,-25)$
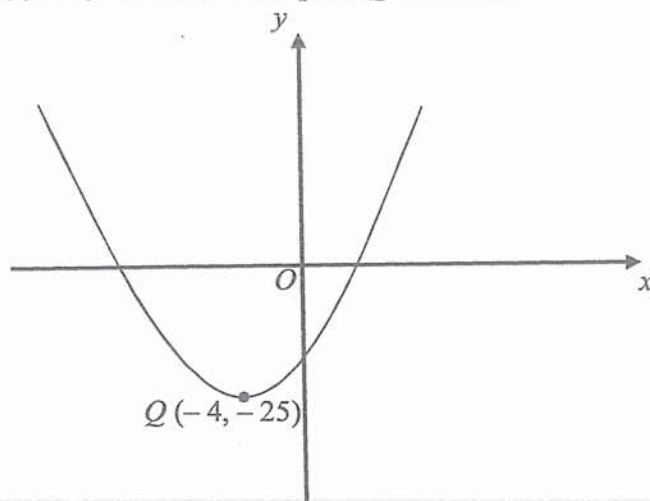
(a)   Write down the equation of the line of symmetry of this curve.

Answer (a) …............................. [1]

(b)   Write down the value of k.

Answer (b) k =…........................ [1]

4   The volume, V cm^3, of a cylinder is directly proportional to r^2h, where r is the radius and h is the height of the cylinder. Find the percentage change in the volume when the radius is increased by 50% and the height is decreased by 20%.

Answer… ……………....……………… % [2]

5   Charlene wanted to impress her parents by showing the rapid increase in her marks in 5 tests. Suggest, with reason, which graph she should use to impress her parents.

Graph 1

Graph 2
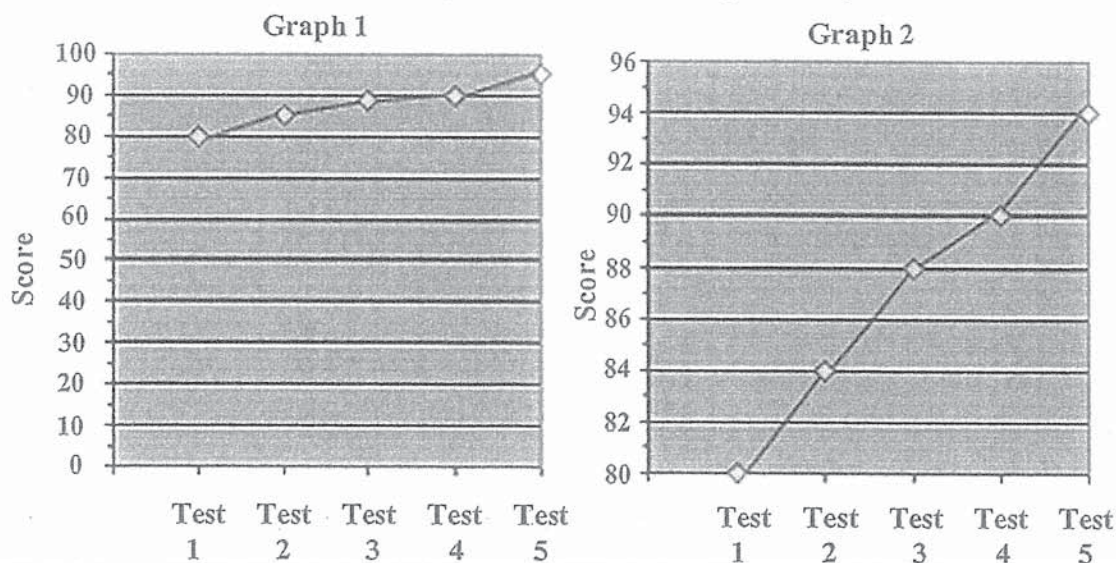
Answer..

....

....

.... [2]

_____

6   The estimated atomic mass of 12 billion nitrogen atoms is 2.80×10^{-9} grams.
   (a)   Express the mass of 1 nitrogen atom in picograms, leaving your answer in standard form.
         [1 pico $= 10^{-12}$]

Answer (a) .... picograms[1]

   (b)   The atomic mass of a helium atom is 6.684×10^{-24} g. Express the ratio of the mass of a helium atom to a nitrogen atom in the form of $n : 1$, leaving your answer in standard form.

Answer (b) .... :   1   [2]

7   The height of Mount Kiki is 4.2 km and the temperature at the foot of the mountain is 31°C. The temperature decreases constantly at a rate of 8 °C for every 700 m.

(a)   Calculate the temperature at the peak of the mountain.

Answer (a).........................°C [1]

(b)   Calculate the height from the foot of the mountain at which the temperature is 7 °C.

Answer (b)........................ m [1]

(c)   A man took 6 hours 43 minutes to climb from the foot of the mountain to the peak. Given that he reached the peak at 12 25, at what time did he begin his climb?
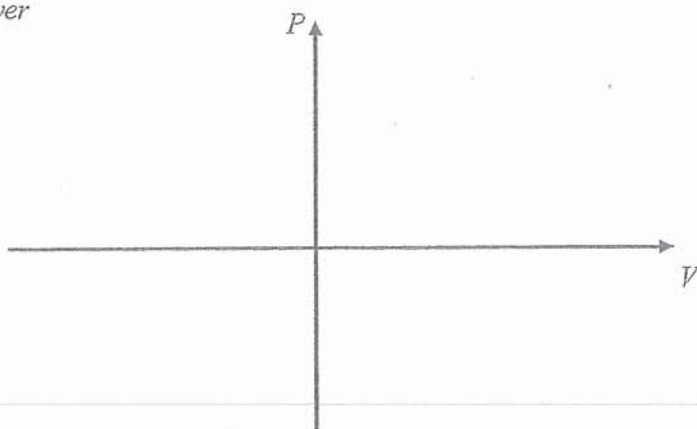
Answer (c)........................... [1]

8   The pressure, P, of a fixed mass of gas at a constant temperature is inversely proportional to the volume, V, of the gas.

(a)   Sketch the graph of P against V on the axis provided.

Answer　　　　　　　　　　　　　　　　　　　　　　　　　　　[1]

(b)   When the pressure of the gas is 4 Nm^{-2}, the volume is 8 m^3. Find P when $V = 12$ m^3.

Answer (b) .....…............. Nm^{-2} [1]

9   A map is drawn to a scale of 1 : 120 000.

(a)   Calculate the actual distance, in km, represented by 6.3 cm on the map.

Answer (a) .....…................. km [1]

(b)   A lake has an actual area of 3.9 km^2. Find the area of the lake on the map, in square centimetres.

Answer (b) .....…................. cm^2 [2]

10   Consider the number pattern:
$$1 + 3 = \ 4 = 2 \times 2$$
$$1 + 3 + 5 = \ 9 = 3 \times 3$$
$$1 + 3 + 5 + 7 = 16 = 4 \times 4$$
$$1 + 3 + 5 + 7 + 9 = 25 = 5 \times 5$$

(a)   Write down the sixth line in the pattern.

Answer (a) .... [1]

(b)   Using the above number pattern, find the sum $1 + 3 + 5 + 7 + 9 + \ldots + 81$

Answer (b) .... [1]

11   John deposited $10 000 in a bank paying an interest of 10% per annum, compounded half yearly. Calculate the amount of interest he would receive after 2 years.
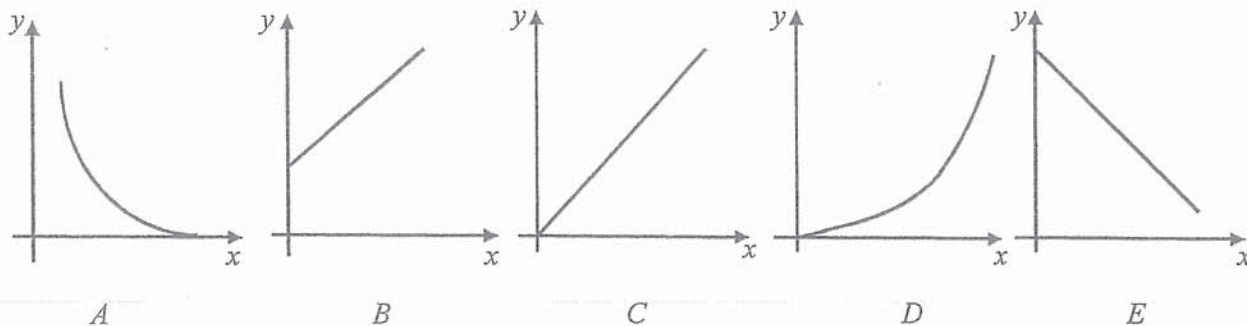
Answer $ .... [3]

12   Match the correct graphs A to E, found below to represent each of the following statements.

    (a)   The cost, y, of taxi fare which consists of a fixed charge plus an amount proportional to the distance travelled, x.

    (b)   The volume, y, of a sphere is proportional to the cube of the radius x.

    (c)   The distance travelled by an object, y, varies directly with the time taken, x.

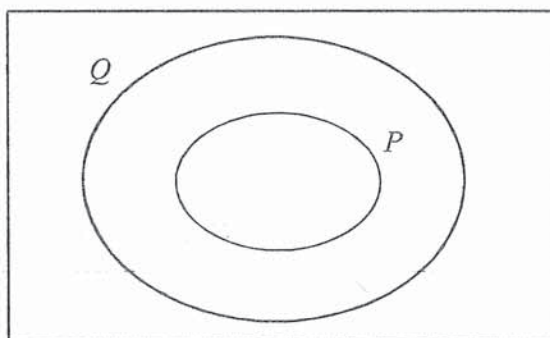   A   B   C   D   E

Answer (a) .... [1]

             (b) .... [1]

             (c) .... [1]

---

13   (a)   On the Venn diagram in the answer space, shade the region which represents $(P \cup Q')'$.

Answer

                                                             [1]

    (b) $\varepsilon = \{\, x : x$ is an integer between 0 and 21 $\}$

           $A = \{\, x : x$ is a multiple of 5 $\}$

           $B = \{\, x : x$ is not a prime number $\}$

           (i)   List the elements contained in the set $(A \cup B)'$.

Answer(b)(i).. [1]

           (ii)   Find $n(A \cap B)$.

Answer(b)(ii).. [1]

14 (a) Solve the inequality $4 - 3x \le \dfrac{2-x}{3} < \dfrac{4+x}{5}$.

Answer (a) .... [2]

(b) Hence, represent the solution on the number line below. [1]

Answer

x

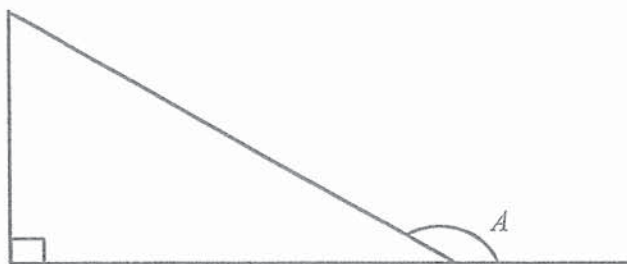15 (a) Express $x^2 - 9x + 45$ in the form $(x-p)^2 + q$.

Answer(a) .... [1]

(b) Hence, solve the equation $x^2 - 9x + 45 = 50$, giving your answers correct to two decimal places.

Answer(b) x = ... or .... [2]

16   In the diagram, $\sin A = \dfrac{5}{13}$ and angle A is an obtuse angle.

Leaving your answer, as a fraction, find the value of

(a) $\sin A - \cos A$,

Answer (a) ....…....…................ [2]

(b) $\cos (180° - A) + \tan (A - 90°)$.

Answer (b)............................... [2]

17   **(a)**   Factorise completely $4a^2 + 2ab - 14xb - 28ax$.

Answer(a) .... [2]

(b)   Solve the equation $\dfrac{3x+1}{7} = -\dfrac{3-x}{4}$.

Answer(b) $x =$ .... [2]

Name: _____ (   )   Class: _____

18   The graph below shows the speed-time graph of a moving object.
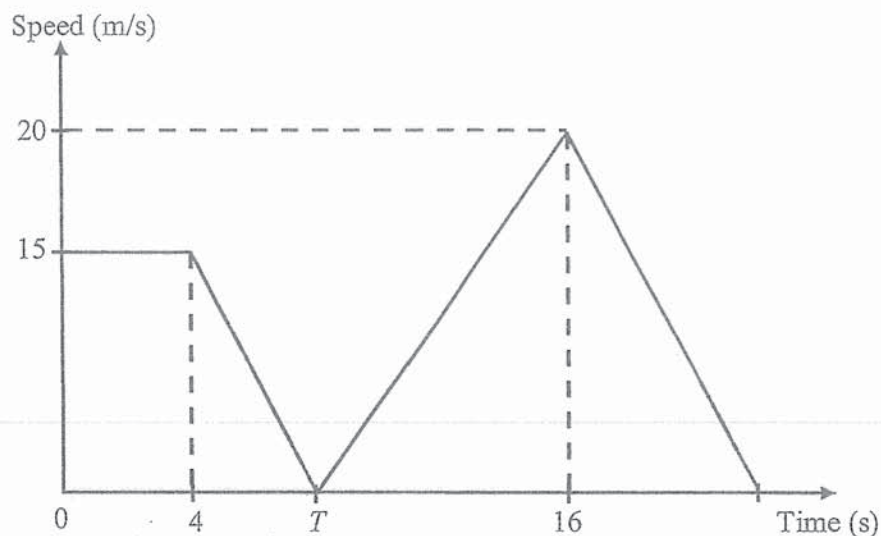
(a)   Describe the motion of the object for the first 4 seconds.

Answer(a)...

....[1]

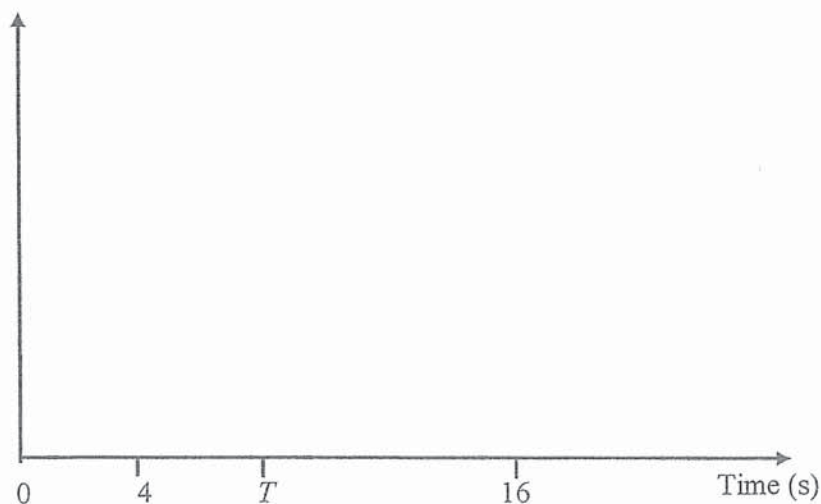(b)   Given that after 4 seconds, the object started to decelerate at a rate of 5 m/s^2, find the value of T.

Answer(b) ....[1]

(c)   Sketch the distance time graph of the object for the first 16 seconds.

Answer(c)   [2]

19   Construct a quadrilateral $PQRS$ such that $PQ = 10$ cm, $QS = QR = 9$ cm, $RS = 5$ cm
     and $\angle PQR = 120°$. PQ has already been drawn.   [2]
     (a) Construct the perpendicular bisector of PQ.   [1]
     (b) The perpendicular bisector meets PS at T. Hence, measure and write down the length RT.

Answer (a)

P _____ Q

Answer(b)RT = ... cm [1]

Name: _____ (   )   Class: _____ [2]

20   The table below shows the drug testing results of 36 athletes for 2018 Olympic Games.

		Drug test Results	
		Positive	Negative
Suspected of drugs consumption	Yes	11	1
	No	3	21

(a)   Present the results in the probability tree diagram below.

Answer(a)   [2]

Suspected of
drugs
consumption

Drug Test
Result

(   )   Positive

(   )   Yes

$\frac{1}{12}$ Negative

$\frac{3}{24}$ Positive

(   )   No
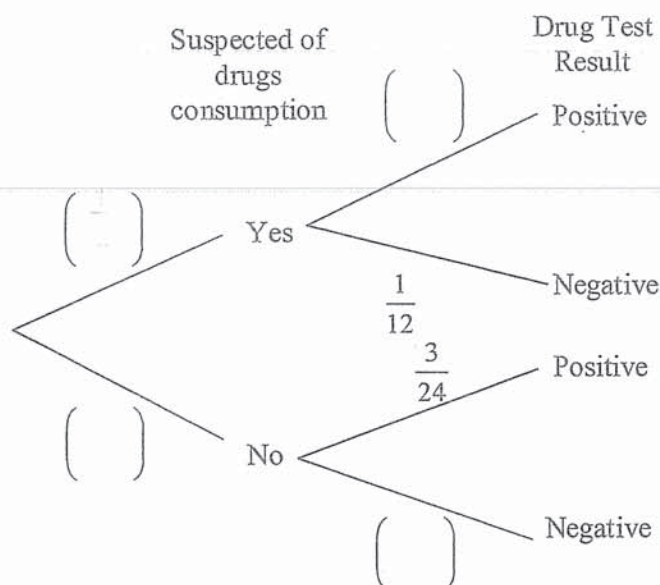
(   )   Negative

(b)   Find the probability that an athlete
       (i)   is suspected of taking drugs and tested positive,

Answer(b)(i)....………………… [1]

       (ii)   receives a negative test result.

Answer(b)(ii)....………………… [2]

Name: _____ (   )   Class: _____

21 (a) Simplify $\left(\dfrac{a^4 - a^3}{a^3}\right) - 3(a)^0$.

Answer (a) .... [2]

(b) Given that $\dfrac{1}{9^{1-3x}} = 243^{\frac{x}{2}-1}$, find the value of x.

Answer (b) x = ... [3]

Name: _____ (   )   Class: _____

22   The numbers 4, 6, 7, 9, 2, 5, 9, 12, 2, x and y have a mean of 7 and a mode of 9.
      Find
      (a)   the values of the two numbers x and y, given that $x < y$,

                                        Answer (a) x = ..., y = ... [2]

      (b)   the median,

                                        Answer (b) .... [1]

      (c)   the standard deviation of this set of eleven numbers.

                                        Answer (c) .... [2]

23   The figure below consists of a pentagon and five identical equilateral triangles.

(a)   Calculate angle *EGH*.

Answer(a).…..………..…………°[2]

(b)   Explain why *AI = AB*.

Answer.……..…….…..…………………………………………………................

…………..…….………………………………………………………………………

……….……..……..………………………………………………………………………

……………..……..………………………………………………..........................… [1]

(c)   Calculate angle *AEI*.

Answer(c).…..……….…………°[2]

Name: _____ (   )   Class: _____

24   (a)   Expand and simplify $y(y-2)+12y^2-6y$.

Answer(a) .... [2]

(b)   Express $\dfrac{6}{3y+7}-\dfrac{1}{49-9y^2}$ as a single fraction in its simplest form.

Answer(b) .... [3]

End of paper
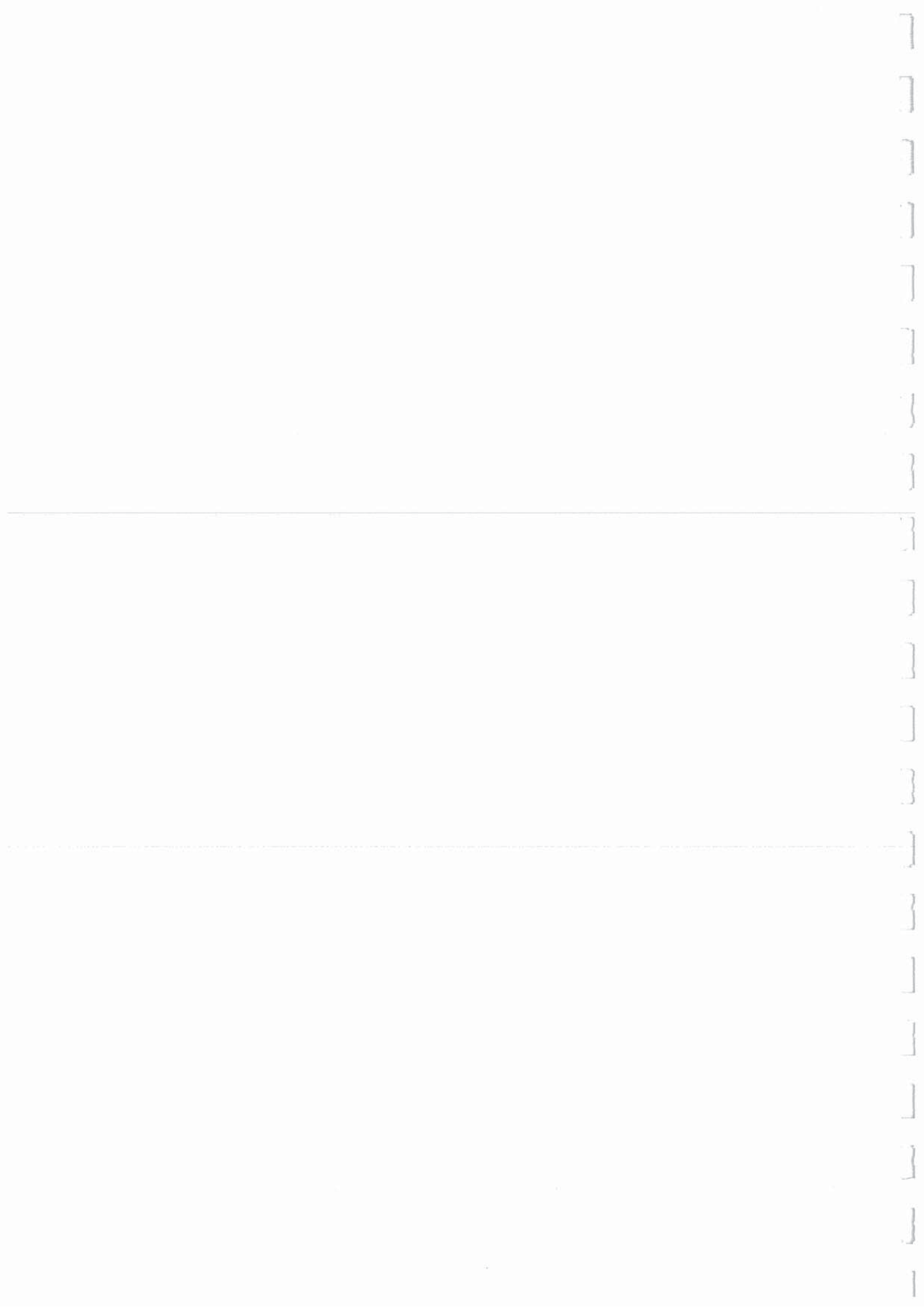
FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2019
SECONDARY 4 EXPRESS/ 5 NORMAL (ACADEMIC)

MATHEMATICS   4048/02

Paper 2

Date: 28 August 2019   Duration: 2 hours 30 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

_____

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place
in the case of angles in degrees, unless a different level of accuracy is specified in the
question.
The use of a scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part
question.

The total number of marks for this paper is 100.

For Examiner's Use	
Paper 2	/ 100

Setter: Miss Lee CP

This paper consists of 30 printed pages including 4 blank pages.

Name: _____ (   )   Class: _____

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi r l$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Area of a triangle $ABC = \frac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Name: _____ (   )   Class: _____

Answer all the questions.

1 (a) Express $\dfrac{3x}{x-3} + \dfrac{2}{x+4}$ as a single fraction.   [2]

(b) Using factorisation, simplify fully $(x^2 + 5)^2 - (x^2 - 3)^2$.   [2]

1   **(c)**   Solve $2x^3 - 13x^2 - 24x = 0$.   [3]

(d) n is an integer. Showing your working clearly, explain why the sum of $\frac{1}{2}n(n+1)$
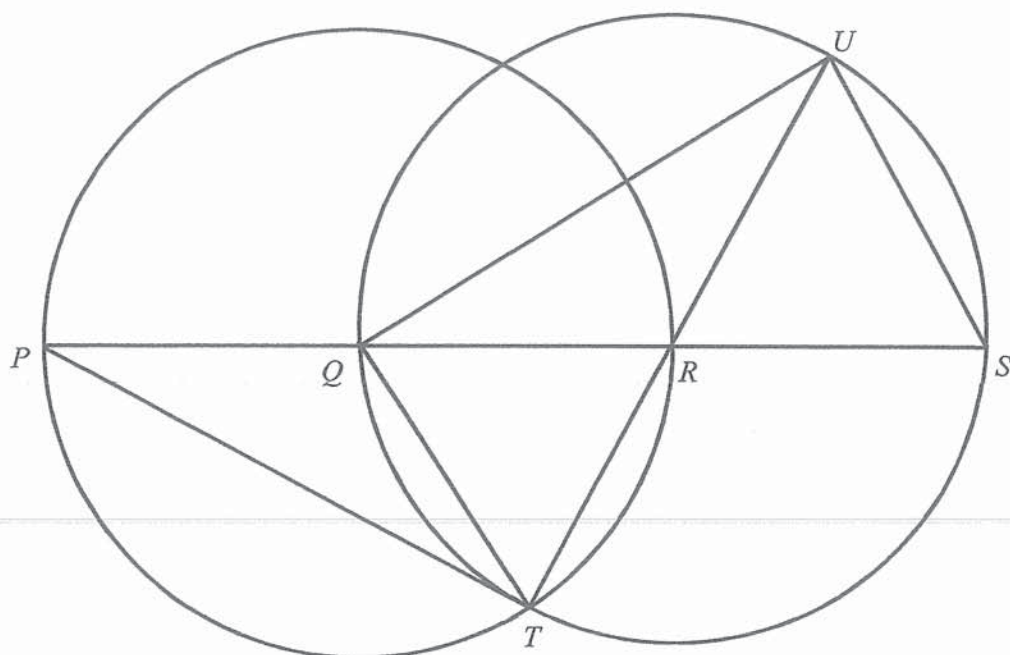
and $\frac{1}{2}(n+1)(n+2)$ is always a square number.   [2]

BLANK PAGE

2   The diagram shows two circles with equal radii.
       P, T and R are points on the circle with centre Q.
       Q, T, S and U are points on the circle with centre R.
       $PQRS$ is a straight line.

       (i)   Show that the triangles PTR and UQT are congruent.   [3]

2   (ii)   Name another triangle that is congruent to *PTR*.   [1]

     (iii)   Explain why *TQ* is parallel to *SU*.   [1]

     (iv)   Stating the reasons clearly, find the value of angle *UQR*.   [1]

3
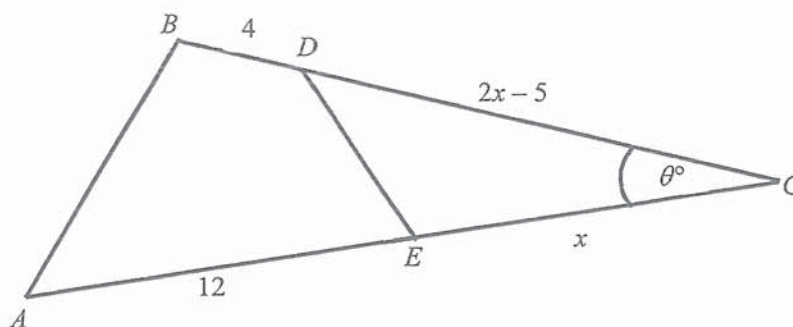
AEC and BDC are straight lines. $AE = 12$ cm and $BD = 4$ cm.

$CE = x$ cm and $CD = (2x - 5)$ cm. Angle $ACB = \theta°$.

(a)   Show that $\dfrac{Area\ of\ triangle\ CDE}{Area\ of\ triangle\ ABC} = \dfrac{CE \times CD}{AC \times BC}$.   [2]

(b)   It is given that $\dfrac{Area\ of\ triangle\ CDE}{Area\ of\ triangle\ ABC} = \dfrac{1}{3}$.

Using the result from part (a), form an equation in x and show that it simplifies to

$2x^2 - 19x + 6 = 0$.   [3]

3   (c)   (i)   Solve the equation $2x^2 - 19x + 6 = 0$, giving your answers correct to
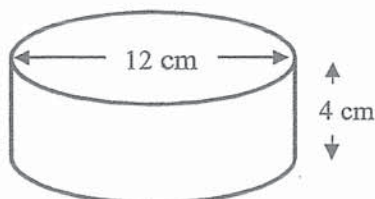
2 decimal places.   [3]

(ii)   State, with a reason, which of these solutions does not apply to triangle CDE.

[1]

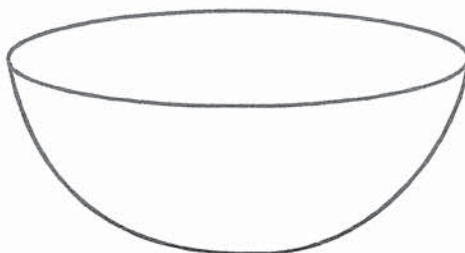(d)   Given that $\theta = 25$, calculate DE.   [3]

4   The diagram below shows an open cylindrical container with a diameter of 12 cm and height of 4 cm.

12 cm

4 cm

Container

(a)   Assuming the thickness of the container is negligible, calculate the area of material needed to make one container. Give your answer correct to the nearest square centimetre.   [3]

Pan

(b)   A hemispherical pan is completely filled with 13 litres of soup. As many containers as possible are completely filled with the soup from the pan.

(i)   Calculate the number of containers which are filled.   [3]

4   (b)   (ii)   Calculate the volume of soup which is left in the pan, giving your answer in
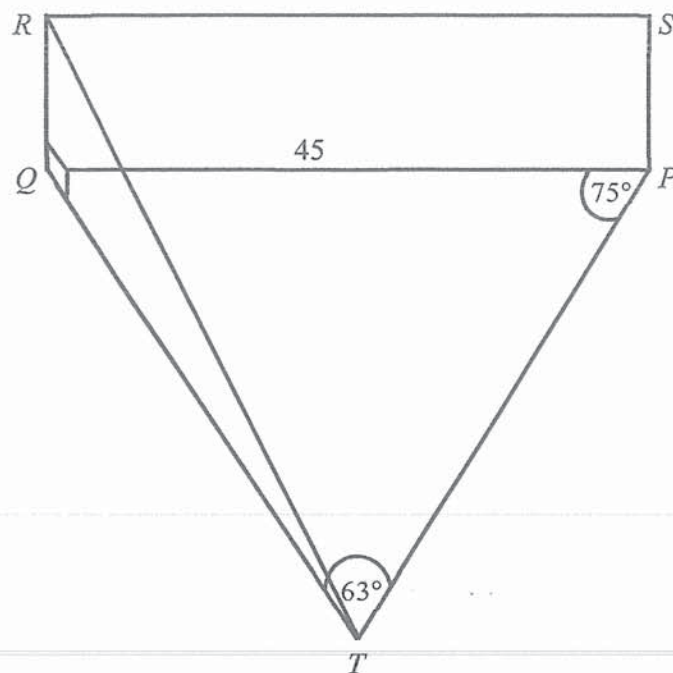   cubic centimetres.   [2]

(iii)   Calculate the radius of the hemispherical pan, giving your answer correct to
           the nearest millimetre.   [2]

(c)   Peter has two different containers, which are geometrically similar to each other. The
         heights of the containers are in the ratio of 2 : 3. Write down the ratio of the volumes
         of soup these containers hold when full.   [1]

5

In the diagram, the rectangle *PQRS* represents a vertical cliff face.

The foot of the cliff, *PQ*, runs from East to West, and is at sea level.

A ship is in the sea at *T*.

Angle $QPT = 75°$, angle $PTQ = 63°$ and $PQ = 45$ m.

(a)   Find the bearing of *T* from *Q*.   [2]

5   (b)   Show that $QT = 48.8$ m, correct to three significant figures.   [2]

(c)   Calculate the shortest distance from the ship to the cliff.   [2]

5   (d)   The angle of depression of the ship when viewed from R is $16°$.

(i)   Find the height of the cliff.   [2]

(ii)   Calculate the greatest possible value of the angle of elevation of the top of the
cliff when viewed from the ship.   [2]

Name: _____ ( ) Class: _____

BLANK PAGE

6   When x copies of a book are printed, the cost C of each copy is given by the formula
     $C = 10 + \dfrac{2400}{x}$.

(a)   The table gives some values of x and the corresponding values of C.
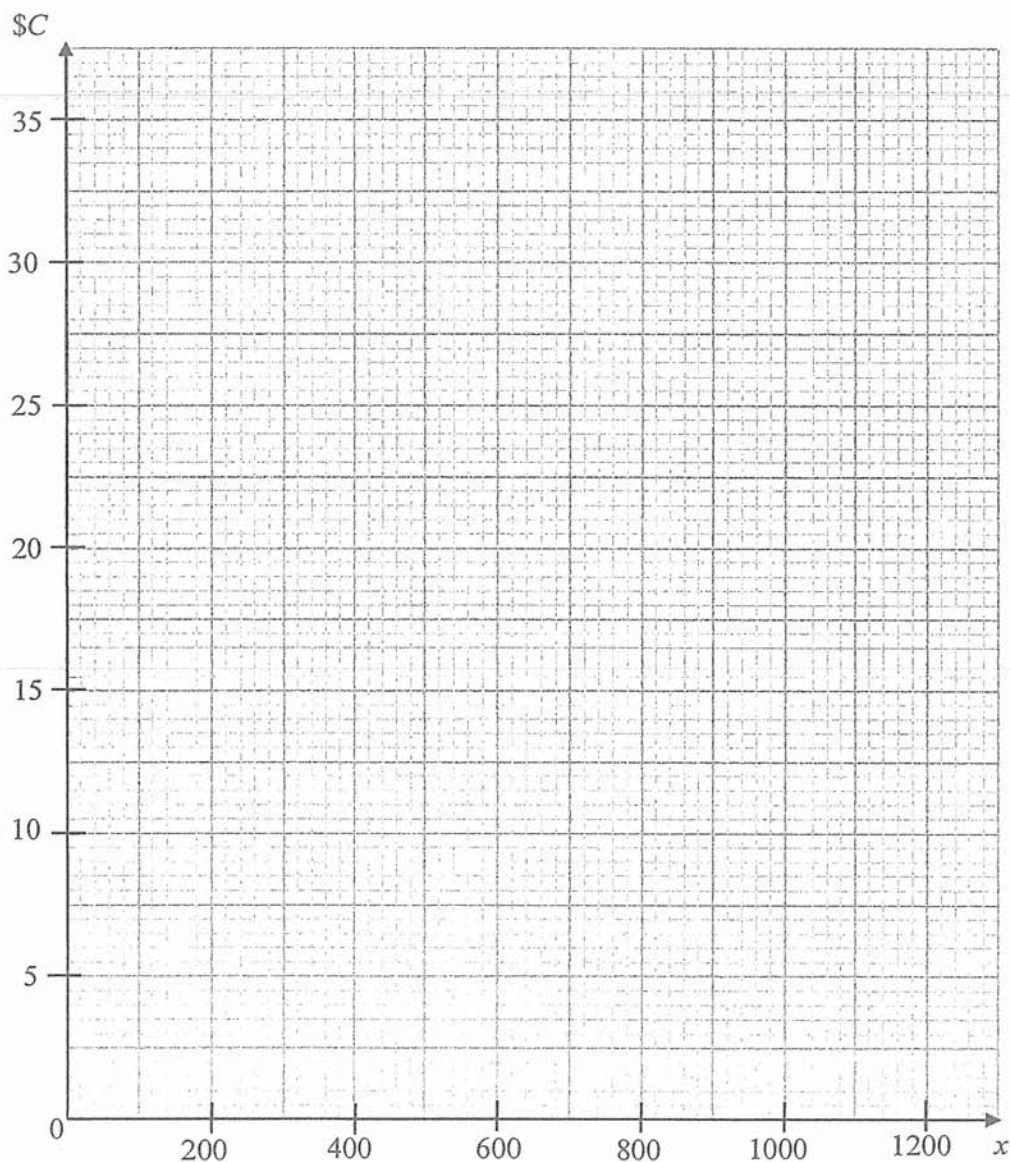
x	100	200	300	400	600	800	1200
C	34	22	18	16	14	13	p

(i)   Find the value of p.   [1]

(ii)   On the grid, plot the points given in the table and join them with a smooth curve.   [3]

6   (b)   Use your graph to estimate the number of books to be printed if the cost of
            printing each book is $15.   [1]

     (c)   (i)   By drawing a tangent, find the gradient of the curve at the point
                   where $x = 300$.   [2]

            (ii)   Describe briefly what this gradient represents.   [1]

     (d)   In order to sell x books, the selling price of each book must be $\$\left(25 - \dfrac{x}{60}\right)$.

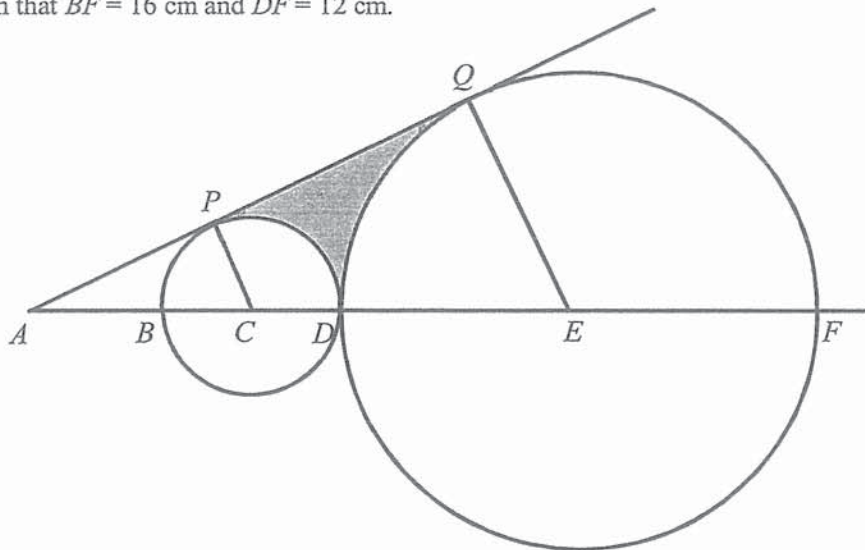   (i)   On the axes, used in part (a), draw the graph of $C = 25 - \dfrac{x}{60}$ for the values of
                   x from 0 to 1200.   [2]

            (ii)   Use your graphs to find the range of the number of books that should
                   be printed if no loss to be incurred.   [1]

7   The diagram shows a tangent APQ to two circles with centre C and E.
        The points A, B, C, D, E and F lie on the same straight line.
        It is given that $BF = 16$ cm and $DF = 12$ cm.

(a)   (i)   Show that the triangles APC and AQE are similar.   [2]

        (ii)   Hence, find the length of AB.   [2]

7   (b)   Show that angle EAQ is $\dfrac{\pi}{6}$.   [1]
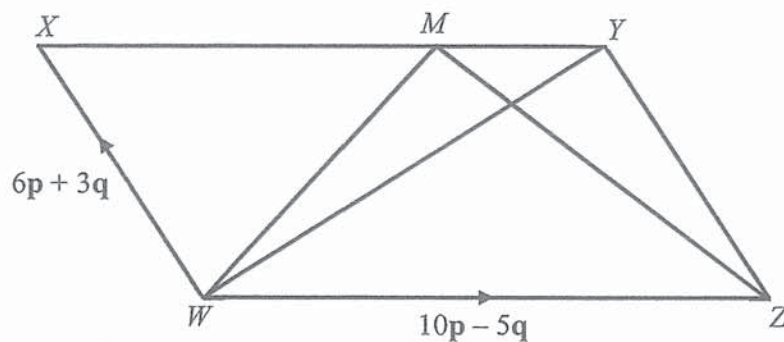
     (c)   Calculate the perimeter of the shaded region.   [4]

8   (a)   In the diagram, $WXYZ$ is a parallelogram.

M is a point on XY such that $XM : MY = 3 : 2$, $\overrightarrow{WX} = 6p + 3q$ and $\overrightarrow{WZ} = 10p - 5q$.

    (i)   Find, in terms of p and/or q,

       (a) $\overrightarrow{WM}$,   [1]

       (b) $\overrightarrow{ZM}$.   [1]

8   (a)   (ii)   (a)   Find area of triangle WMX : area of $WXYZ$.   [1]

(b)   The area of triangle WMX is 8 units^2.
Hence, calculate the area of $WXYZ$.   [1]

(iii)   Given that N is on WX produced such that ZMN is a straight line.
Express $\overrightarrow{WN}$ in terms of p and q.   [1]

8   (b)   Coordinates of A and B are (–3, 3) and (7, –13) respectively.

    (i)   Write $\overrightarrow{AB}$ as a column vector.   [1]

    (ii)   Find the acute angle formed by the line AB with the horizontal axis.   [2]

    (iii)   If the gradient of $AB = -\dfrac{2m}{n}$, express $\overrightarrow{AB}$ in terms of m and n.   [1]

Another vector $\overrightarrow{CD}$ is parallel to $\overrightarrow{AB}$ and has the magnitude thrice that of $\overrightarrow{AB}$.

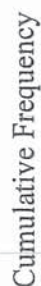   (iv)   Write down the possible vectors of $\overrightarrow{CD}$.   [2]

Name: _____ (   )   Class: _____

BLANK PAGE

Name: _____ ( ) Class: _____

9 The cumulative frequency curve below shows the travelling time of 120 working adults travelling to work daily by train.

(a) Use the graph to estimate
    (i) the median of travelling time, [1]

    (ii) the 20th percentile of travelling time, [1]

9 (a) (iii) the interquartile range of travelling time, [2]

(iv) the percentage of the total number of adults who spend more than
45 minutes travelling to work every day. [1]

(b) Another 120 working adults travelled to work by bus. The travelling time is illustrated
in the box and whisker diagram below.
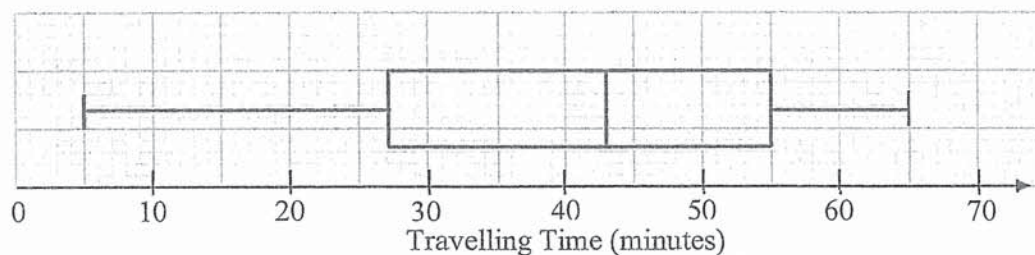
Find the median travelling time and the interquartile range. Hence, compare and

comment on the travelling times by train and bus in two different ways. [3]

9   (c)   One working adult is chosen at random. Assume that the travelling times between the train and bus are independent. The working adult makes the first trip on Monday by train and the second trip on Tuesday by bus.

Expressing each answer as a fraction in its lowest terms, calculate the probability that the working adult took

(i)   more than 55 minutes on both trips,   [1]

(ii)   more than 55 minutes on one trip, but not the other.   [2]

BLANK PAGE

10   The diagram below shows a race in the Olympic Games. For certain races, the athletes do not all start from the same part of the track. This is called "staggered start".
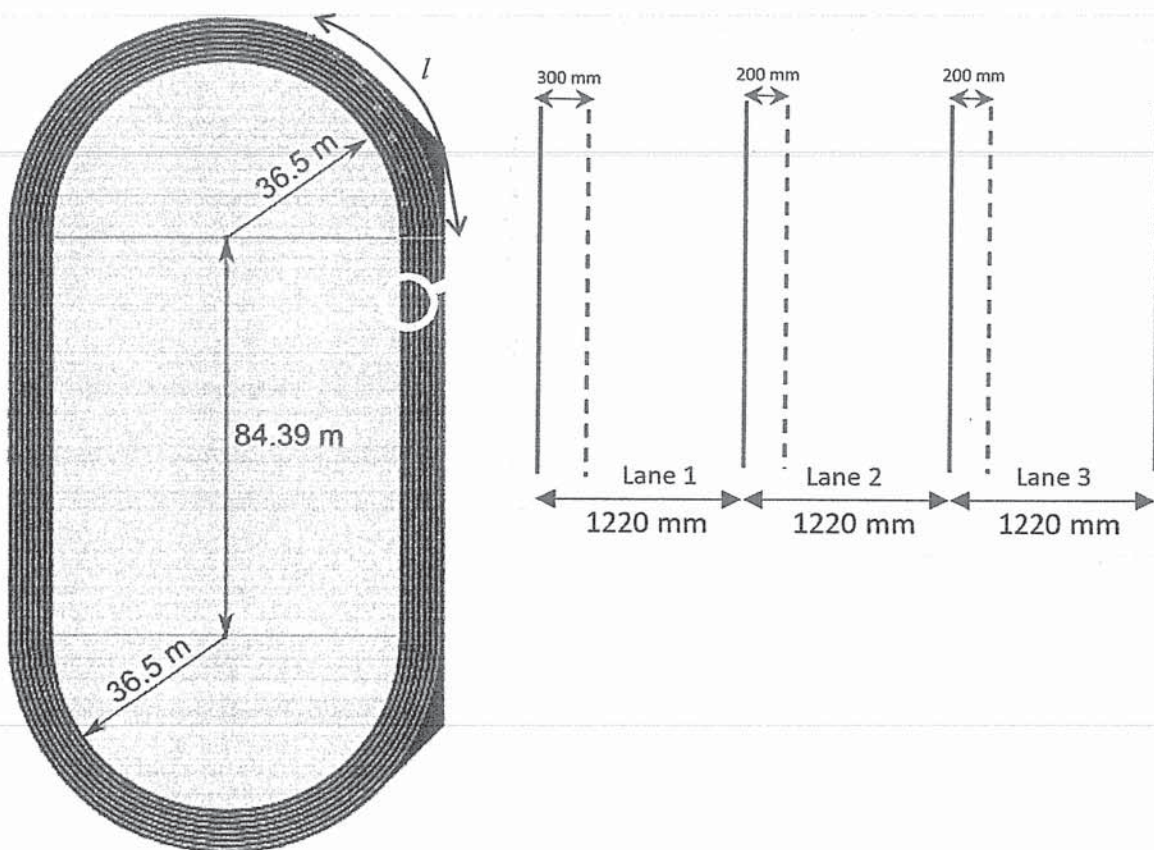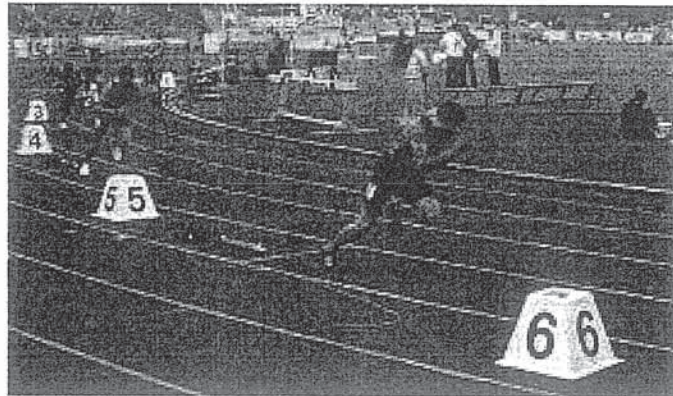

Figure 1

The grass field comprises two semi-circular ends of radius 36.5 m and two straight lengths of 84.39 m each. The field is surrounded by a running track of 8 lanes, each of width 1220 mm.

The route along which the running distance is measured for each lane is as below:

Lane 1
300 mm from inner edge of the lane
Lanes 2 to 8
200 mm from inner edge of the lane

10   (a)   (i)   Show that the total distance that an athlete in Lane 1 would have to run
                    to complete one lap of the track is 400 m.   [2]

       (ii)   Show that the staggered start line for Lane 8 is 53.03 m from the start line for
               Lane 1 (distance of l) as seen in Figure 1.   [3]

       (iii)   Explain why a "staggered start" is needed for each runner in Lane 1 to
               Lane 8 to complete one lap of the track.   [1]

10   (b)   An athlete wants to incorporate in his training a minimum of 150 minutes of brisk
           walking weekly, at the an average speed of 6.8 km/h. He claims that he needs to walk
           briskly around the track in **Lane 8** five rounds daily to hit his target.
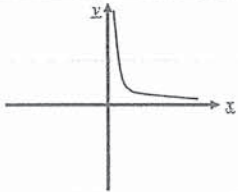   Justify whether his claims is true or false. Show your working clearly.   [4]
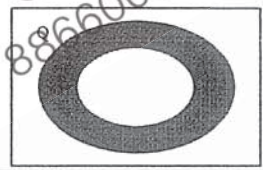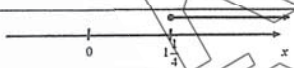
2019 Sec 4Exp/5NA Preliminary Examination
Mathematics Paper 1
Answer Key

1a	$2^2 \times 3^3 \times 5 \times 7$	1b	105	2a	(28.5)
2b	$\begin{pmatrix} 7 & 14 \\ -7 & -14 \end{pmatrix}$	3a	x = –4	3b	k = 0
4	80%	5a	Charlene should use **Graph 2** as the **scale for the vertical axis is bigger and does not start from zero**, making the **difference in marks** between each test looks **bigger**.	6a	2.325×10^{-7} picograms
6b	2.86×10^{-5} : 1	7a	$-17\,°C$	7b	2100 m
7c	05 42	8a		8b	$P = \dfrac{32}{12} = 2.67\,Nm^{-2}\,or\,2\dfrac{2}{3}\,Nm^{-2}$
9a	7.56 km	9b	2.71 cm² (3 s.f.)	10a	$1+3+5+7+9+11+13 = 49 = 7 \times 7$
10b	$1+3+5+7+9+...+81 = 41 \times 41 = 1681$	11	$2155.06 (2 d.p)	12a	B
12b	D	12c	C	13a	
13bi	{2,3,7,11,13,17,19}	13bii	3	14a	$x \geq 1\dfrac{1}{4}$
14b		15a	$(x-4.5)^2 - 24.75$	15b	25.28 or 15.23 (2 d.p.)
16a	$\dfrac{17}{13}$	16b	$\dfrac{216}{75}$	17a	$2(a-7x)(2a+b)$
17b	x = – 5	18a	The object is moving at **constant speed of 5 ms⁻¹ with zero acceleration** for the first 4 sec	18b	7 sec
19c	10.0 ± 0.1 cm	20bi	$\dfrac{11}{36}$	20bii	$\dfrac{7}{12}$
21a	a – 4	21b	$x = \dfrac{6}{7}$	22a	y = 12
22b	7	22c	3.55	23a	168°
23c	36°	24a	$13y^2 - 8y$	24b	$\dfrac{41-18y}{(7+3y)(7-3y)}$
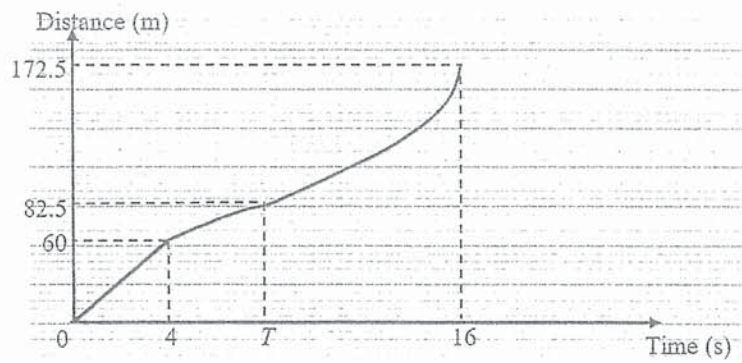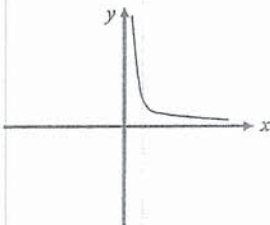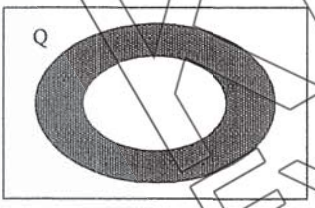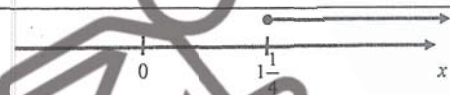
Marking Scheme for Sec 4 Exp/ 5NA Mathematics P1 2019

Qn	Solution	Marking Scheme
1a	$3780 = 2^2 \times 3^3 \times 5 \times 7$	B1
1b	$2^2 \times 3^3 \times 5 \times 7 \times (3 \times 5 \times 7) =$ perfect sq. Therefore, $3 \times 5 \times 7 = 105$	B1 B1
2a	$\dfrac{1}{2}BA$ $= \dfrac{1}{2}(-3 \quad 9)\begin{pmatrix} 2 \\ 7 \end{pmatrix}$ $= \dfrac{1}{2}(57)$ $= (28.5)$	 B1
2b	C^2 $= \begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ -1 & 4 \end{pmatrix}$ $= \begin{pmatrix} 7 & 14 \\ -7 & -14 \end{pmatrix}$	 B1
3a	$x = -4$	B1
3b	$\dfrac{1+(-k)}{2} = -4$ $1 - k = -8$ $-k = -8 - 1$ $k = 9$	 B1
4	$V = kr^2h$ new radius, $r_1 = 1.5r$, new height, $h_1 = 0.8h$ $V = kr^2h$ $V_{new} = k(1.5r)^2(0.8h)$ $V_{new} = \dfrac{V}{r^2h}(1.5r)^2(0.8h)$ $\therefore V_{new} = 1.8V$ % change in V $= \dfrac{1.8-1}{1} \times 100\%$ $= 80\%$	 M1 B1 or B2
5	Charlene should use **Graph 2** as the **scale** for the vertical axis is **bigger** and does not start from zero, making the **difference in marks** between each test looks **bigger**.	A1 B1

6a	Mass of 12 billion nitrogen atoms $= 2.80 \times 10^{-9}$ g $= 2.80 \times 10^3$ picograms Mass of 1 nitrogen atom $= \dfrac{2.80 \times 10^3}{12 \times 10^9}$ $= 0.2325 \times 10^{-6}$ $= 2.325 \times 10^{-7}$ picograms	 B1
6b	Helium : nitrogen 6.684×10^{-24} : $\dfrac{2.80 \times 10^{-9}}{12 \times 10^9}$ 6.684×10^{-24} : 0.2333×10^{-18} 28.6×10^{-6} : 1 2.86×10^{-5} : 1	M1 A1
7a	No. of times temp will decrease $= 4200 \div 700 = 6$ Amt. of decrease in temp. $= 8 \times 6 = 48°C$ Temp. at the peak of mountain $= 31 - 48 = -17 \; °C$	 B1
7b	Amt. of decrease in temp. $= 31 - 7 = 24 \; °C$ Height of mountain $= 24 \div 8 \times 700$ m $= 2100$ m	 B1
7c	Time he begins his climb $\begin{array}{rr} 12 & 25 \\ 6 & 43 \quad - \\ \hline 05 & 42 \end{array}$	 B1
8a		B1 (Negative region of the reciprocal graph should not be shown as its not applicable)
8b	$P = \dfrac{k}{V}$ $k = PV$ $= 4(8) = 32$ $\therefore P = \dfrac{32}{12} = 2.67 \, Nm^{-2} \, or \, 2\dfrac{2}{3} \, Nm^{-2}$	 B1

9a	1 cm : 120 000 cm 1 cm : $\dfrac{120000}{1000 \times 100} = 1.2$ km 6.3 cm : 6.3 × 1.2 km = 7.56 km	B1
9b	$(1 \text{ cm})^2$: $(1.2 \text{ km})^2$ 1 cm² : 1.44 km² $\dfrac{3.9}{1.44} \times 1$: 3.9 km² 2.71 cm² = (3 s.f.)	M1 A1
10a	1 + 3 + 5 + 7 + 9 + 11 + 13 = 49 = 7 × 7	B1
10b	1 + 3 + 5 + 7 + 9 + ... + 81 = 41 × 41 = 1681	B1
11	$T = P(1 + \dfrac{r}{100})^n$ $= 10000(1 + \dfrac{\frac{2}{2}}{100})^{2 \times 2}$ $= \$12155.0625$ Interest $= \$12155.0625 - \$10000$ $= \$2155.06 (2 d.p.)$	M1 M1 A1
12a	B	B1
12b	D	B1
12c	C	B1
13a	Answer ε 	B1
13bi	{2,3,7,11,13,17,19}	B1
13bii	3	B1

14a	$4 - 3x \le \dfrac{2-x}{3} < \dfrac{4+x}{5}$ $4 - 3x \le \dfrac{2-x}{3}$ or $\dfrac{2-x}{3} < \dfrac{4+x}{5}$ $-9x + x \le 2 - 12$ $-5x - 3x < 10 - 10$ $-8x \le -10$ $-8x < 2$ $x \ge 1\frac{1}{4}$ $x > -\frac{1}{4}$ $\therefore x \ge 1\frac{1}{4}$	M1 A1
14b		B1
15a	$x^2 - 9x + 45$ $= x^2 - 9x + \left(\dfrac{9}{2}\right)^2 - \left(\dfrac{9}{2}\right)^2 + 45$ $= (x - 4.5)^2 + 24.75$	B1
15b	$(x - 4.5)^2 = \pm\sqrt{50 - 24.75}$ $x - 20.25 = \pm 5.024938$ $x = 5.024938 + 20.25$ or $-5.024938 + 20.25$ $= 25.28$ (2 d.p.) or 15.23 (2 d.p)	M1 A1 for both correct answers
16a	Length of base of triangle $= \sqrt{13^2 - 5^2} = 12$ units $\sin \angle A - \cos \angle A$ $= \sin \angle A - (-\cos(180° - \angle A))$ $= \dfrac{5}{13} - \left(-\dfrac{12}{13}\right)$ $= \dfrac{17}{13}$	M1 A1
16b	$\cos(180° - \angle A) + \tan(\angle A - 90°)$ $= \dfrac{12}{13} + \tan[180° - (180° - \angle A) - 90°]$ $= \dfrac{12}{13} + \dfrac{12}{5}$ $= \dfrac{216}{75}$	M1 A1

17a	$4a^2 + 2ab - 14xb - 28ax$	
	$= 2a(2a+b) - 14x(b+2a)$	M1
	$= (2a - 14x)(2a+b)$	
	$= 2(a-7x)(2a+b)$	A1
17b	$\dfrac{3x+1}{7} = -\dfrac{3-x}{4}$	
	$4(3x+1) = -7(3-x)$	M1
	$12x + 4 = -21 + 7x$	
	$12x - 7x = -21 - 4$	
	$5x = -25$	
	$x = -5$	A1
18a	The object is moving at **constant speed of 5 ms^{-1} with zero acceleration** for the first 4 sec	B1
18b	Gradient = 5	
	$\dfrac{15}{Time} = 5$	
	$Time = \dfrac{15}{5}$	
	$\qquad = 3\,\text{sec}$	B1
	$T = 4 + 3 = 7\,\text{sec}$	
18c	Distance (m) graph with values 172.5, 82.5, 60 and Time (s) markings 0, 4, T, 16	All 3 parts of correct graph shape = B2. 1 wrong shape = deduct B1. 2 or more wrong shape = 0 marks
19a,b,	Refer to last page for the details	[2][1]
19c	10.0 ± 0.1 cm	B1

20a	Tree diagram: Suspected of drugs consumption. Yes branch $\frac{12}{36}$ or $\frac{1}{3}$ to Positive $\frac{11}{12}$ and Negative $\frac{1}{12}$. No branch $\frac{24}{36}$ or $\frac{2}{3}$ to Positive $\frac{3}{24}$ and Negative $\frac{21}{24}$ or $\frac{7}{8}$	Any 2 correct ans = B1 × 2 = B2
20bi	P(suspected of taking drugs and tested positive)	
	$= \dfrac{12}{36} \times \dfrac{11}{12}$	
	$= \dfrac{11}{36}$	B1
20bii	P(received negative results)	
	$= (\dfrac{12}{36} \times \dfrac{1}{12}) + (\dfrac{24}{36} \times \dfrac{21}{24})$	M1
	$= \dfrac{1}{36} + \dfrac{7}{12}$	
	$= \dfrac{21}{36}$	
	$= \dfrac{7}{12}$	A1
21a	$\left(\dfrac{a^4 - a^3}{a^3}\right) - 3(a)^0$	
	$= a^3\left(\dfrac{a-1}{a^3}\right) - 3$	M1
	$= (a-1) - 3$	
	$= a - 4$	A1

21b	$\dfrac{1}{(3^2)^{1-3x}} = (3^5)^{\frac{x}{2}-1}$ $(3^2)^{3x-1} = 3^{\frac{5x}{2}-5}$	M1
	$6x - 2 = \dfrac{5x}{2} - 5$	M1
	$6x - \dfrac{5x}{2} = -5 + 2$ $3.5x = -3$ $x = \dfrac{-3}{3.5}$ $x = \dfrac{6}{7}$	A1
22a	Let x = 9 as mode is 9	B1
	mean $= \dfrac{5+6+7+9+2+4+9+12+2+9+y}{11}$ $7 = \dfrac{65+y}{11}$ $77 = 65 + y$ $y = 77 - 65$ $y = 12$	B1
22b	Median position $= \dfrac{11+1}{2} = 6th$ Arranging the values in ascending order, 2, 2, 4, 5, 6, 7, 9, 9, 9, 12, 12 Hence the median at 6th position is 7	B1
22c	Standard deviation $= 3.55$	B2
23a	Int angle IGH $= \dfrac{180°}{3}$ (angle in equilateral triangle) $= 60°$ Angle in pentagon $= \dfrac{(5-2) \times 180°}{5}$ $= 108°$ Angle EDH = 60° + 108° $= 168°$	M1 A1
23b	Since the triangle is equilateral and all triangles are identical, AB = BC = AC = AI. Hence AI = AB	B1

23c	Angle GEI = Angle CEA $= \dfrac{180° - 108}{2}$ (base angles in isos.triangle) $= 36°$	M1
	$\therefore \angle AEI = 108° - (36° \times 2)$ $= 36°$	A1
24a	$y(y-2) + 12y^2 - 6y$ $= y^2 - 2y + 12y^2 - 6y$ $= 13y^2 - 8y$	M1 for correct expansion A1
24b	$\dfrac{6}{3y+7} - \dfrac{1}{49 - 9y^2}$ $= \dfrac{6}{3y+7} - \dfrac{1}{7^2 - (3y)^2}$ $= \dfrac{6}{3y+7} - \dfrac{1}{(7+3y)(7-3y)}$	M1
	$= \dfrac{6(7-3y) - 1}{(7+3y)(7-3y)}$	M1
	$= \dfrac{42 - 18y - 1}{(7+3y)(7-3y)}$ $= \dfrac{41 - 18y}{(7+3y)(7-3y)}$	A1

Fairfield Methodist School (Secondary)
2019 Sec 4Exp/5NA Preliminary Examination
Answer Key for Mathematics Paper 1

Answer Key			
1(a)	$\dfrac{3x^2 + 14x - 6}{(x-3)(x+4)}$	1(b)	$16(x^2 + 1)$ or $16x^2 + 16$
1(c)	$x = 0$ or $x = -1.5$ or $x = 8$	1(d)	Since $(n+1)^2 = (n+1)(n+1)$, therefore, it is a square number. Or $(n+1)^2$ has a repeated same factor, therefore it is a square number.
2(i)	Statement with reasons for SAA/RHS/SAS/SSS	2(ii)	Triangle QUS
2(iii)	Angle TQS = Angle SUT (Angle in same segment) = 60° (mentioned in (i) that triangle QTR is equilateral triangle) Angle QTR = 60° (triangle QTR is an equilateral triangle). Since angle QTR = Angle SUR, the form alternate angle, therefore TQ is parallel to SU. Or Angle TQU = Angle SUQ = 90° (Angle in a semi-circle) Since they are the same and they form interior angle, therefore, TQ is parallel to SU. Or by converse of property of interior angle, therefore, TQ is parallel to SU.		
2(iv)	30°	3(c)(i)	$x = 9.17$ or $x = 0.33$ (2 d.p.)
3(c)(ii)	When $x = 0.327$, $DC = 2x - 5 =$ 4.356 < 0. Therefore, x cannot be 0.33.	3(d)	6.35 (3 s.f.)
4(a)	264 cm² (nearest cm²)	4(b)(i)	28
4(b)(ii)	333 cm³ (nearest cm³)	4(b)(iii)	184 mm (nearest mm)
4(c)	8 : 27	5(a)	132°
5(b)	48.8 m (3 s.f.)	5(c)	h = 32.6 m (3 s.f.)
5(d)(i)	14.0 m (3 s.f.)	5(d)(ii)	23.2°
6(a)	$p = 12$	6(c)(i)	Accept answers: -0.023 to - 0.03
6(c)(ii)	The gradient represent the rate of decrease of the cost of printing each book when number of books is 300. Or The gradient represent the rate at which the cost of printing each book is decreasing.	6(d)(i)	Plot points (0, 25), (600, 15) and (1200, 5) Draw a line
6(d)(ii)	$210 \le x \le 700$ (accept 180 – 220; 680 – 730, with interval of 10)	7(a)(i)	Angle APC = 90° (tangent perpendicular radius) Angle AQE = 90° (tangent perpendicular radius) Therefore, angle APC = angle AQE Angle PAC = Angle QAE (common angle) Since there are two pairs of corresponding angles are equal, triangle APC and triangle AQE are similar.
7(a)(ii)	$AB = 2$	7(b)	$\angle CAP = \sin^{-1}\left(\dfrac{1}{2}\right) = 30° = \dfrac{\pi}{6}$
7(c)	17.4 cm (3 s.f.)	8(a)(i)(a)	$12p$

Answer Key			
8(a)(i)(b)	$2p + 5q$	8(a)(ii)(a)	$3 : 10$
8(a)(ii)(b)	$\frac{80}{3}$ units^2 or $26\frac{2}{3}$ or 26.7 (3 s.f)	8(a)(iii)	$\frac{15}{2}(2p + q)$
8(b)(i)	$\overrightarrow{AB} = \begin{pmatrix} 10 \\ -16 \end{pmatrix}$	8(b)(ii)	58.0° (1 d.p.)
8(b)(iii)	$\overrightarrow{AB} = k\begin{pmatrix} n \\ -2m \end{pmatrix}$ or $\overrightarrow{AB} = k\begin{pmatrix} -n \\ 2m \end{pmatrix}$	8(b)(iv)	$\begin{pmatrix} -30 \\ 48 \end{pmatrix}$
9(a)(i)	41 minutes	9(a)(ii)	25 minutes
9(a)(iii)	IQR = 48 − 29 = 19 mins	9(a)(iv)	$36\frac{2}{3}\%$ or $\frac{110}{3}\%$ or 36.7% (3 s.f.)
9(b)	The travelling time using train is much shorter (faster) than using a bus as the median time travelling with a train < median time travelling with a bus. The travelling time using train is less spread out (more consistent) than using travelling using a bus as the interquartile range of travelling time of a train < interquartile range of travelling time of a bus.	9(c)(i)	$\frac{7}{240}$
9(c)(ii)	$\frac{37}{120}$	10(a)(ii)	53.03 m (2 d.p.)
10(a)(iii)	Using staggered start, each runner runs exactly 400 meters or same distance.	10(b)	His claim in false.

No	Description	Marks	Total
1(a)	$\dfrac{3x}{x-3} + \dfrac{2}{x+4}$ $= \dfrac{3x(x+4) + 2(x-3)}{(x-3)(x+4)}$	M1	
	$= \dfrac{3x^2 + 12x + 2x - 6}{(x-3)(x+4)}$		
	$= \dfrac{3x^2 + 14x - 6}{(x-3)(x+4)}$	A1	2
1(b)	$(x^2 + 5)^2 - (x^2 - 3)^2$ $= [x^2 + 5 - (x^2 - 3)][x^2 + 5 + (x^2 - 3)]$	M1	
	$= (8)(2x^2 + 2)$ $= 16(x^2 + 1)$ or $16x^2 + 16$	A1	2
1(c)	$2x^3 - 13x^2 - 24x = 0$ $x(2x^2 - 13x - 24) = 0$		
	$x(2x+3)(x-8) = 0$	M1	
	$x = 0$ or $x = -1.5$ or $x = 8$	A1 (two correct), A1	3
1(d)	$\dfrac{1}{2}n(n+1) + \dfrac{1}{2}(n+1)(n+2)$		
	$= \dfrac{1}{2}(n+1)[n + n + 2]$		
	$= \dfrac{1}{2}(n+1)[2n+2]$		
	$= \dfrac{1}{2}(n+1)[2(n+1)]$		
	$= (n+1)^2$	B1	
	Since $(n+1)^2 = (n+1)(n+1)$, therefore, it is a square number. Or $(n+1)^2$ has a repeated same factor, therefore it is a square number.	B1	2
			8
2(i)	$QR = QT = RT$ (radius of circles centred at Q and R)	B1	
	Therefore, triangle QRT is an equilateral triangle. Angle Q = Angle R = Angle T = 60°	B1	
	$\angle QUT = 30°$ ($\angle$ at centre = $2 \times \angle$ at circumference) $\angle TPR = 30°$ ($\angle$ at centre = $2 \times \angle$ at circumference) Therefore, angle QUT = angle TPR Therefore, by SAA / AAS, triangle PTR and UQT are congruent.	B1	3

No	Description	Marks	Total
2(i)	Angle PTR = 90° (right angle in semi-circle) Angle UQT = 90° (right angle in semi-circle) Therefore, angle PTR = angle UQT	B1	
	$PR = UT$ (as both are diameter of the circle centred at Q and R respectively)	B1	
	$QR = QT = RT$ (radius of circles centred at Q and R) Therefore, by RHS congruency test, triangle PTR and UQT are congruent.	B1	
	$PR = UT$ (as both are diameter of the circle centred at Q and R respectively)	B1	
	$QR = QT = RT$ (radius of circles centred at Q and R) Therefore, triangle QRT is an equilateral triangle. Angle Q = Angle R = Angle T = 60°	B1	
	Therefore, by SAS congruency test, triangle PTR and UQT are congruent.	B1	
	Angle PTR = 90° (right angle in semi-circle) Angle UQT = 90° (right angle in semi-circle) Therefore, angle PTR = angle UQT	B1	
	$PR = UT$ (as both are diameter of the circle centred at Q and R respectively) $QR = QT = RT$ (radius of circles centred at Q and R) Therefore, triangle QRT is an equilateral triangle. Angle Q = Angle R = Angle T = 60°	B1	
	Angle PRT = 60° (int angle of equilateral triangle) Angle UTQ = 60° (int angle of equilateral triangle) Therefore, angle PRT = angle UTQ Therefore, by SAA / AAS congruency test, triangle PTR and UQT are congruent.	B1	
2(ii)	Triangle QUS	B1	1
2(iii)	Angle TQS = Angle SUT (Angle in same segment) = 60° (mentioned in (i) that triangle QTR is equilateral triangle) Angle QTR = 60° (triangle QTR is an equilateral triangle). Since angle QTR = Angle SUR, the form alternate angle, therefore TQ is parallel to SU. Since angle QTR = Angle SUR, by converse property of alternate angle, TQ is parallel to SU.	B1	
	Angle TQU = Angle SUQ = 90° (right angle in a semi-circle) Since they are the same and they form interior angle, therefore, TQ is parallel to SU. Since angle TQU = Angle SUQ, by converse property of interior angle, TQ is parallel to SU.	B1	1
2(iv)	Angle UQR = 30° (angle at circumference = $\dfrac{1}{2}$ angle at centre) Or Angle UQR = 30° (complementary angle and angle in semicircle)	B1	1
			6

No	Description	Marks	Total
3(a)	$\dfrac{Area\ of\ triangle\ CDE}{Area\ of\ triangle\ ABC}$ $=\dfrac{\frac{1}{2}\times CD\times CE\times\sin\theta}{\frac{1}{2}\times AC\times BC\times\sin\theta}$ $=\dfrac{CD\times CE}{AC\times BC}$	M1 AG1	2
3(b)	$\dfrac{CD\times CE}{AC\times BC}=\dfrac{1}{3}$ $\dfrac{(2x-5)\times x}{(12+x)\times(4+2x-5)}=\dfrac{1}{3}$ $3x(2x-5)=(12+x)(2x-1)$ $6x^2-15x=24x-12+2x^2-x$ $4x^2-15x-24x+x+12=0$ $4x^2-38x+12=0$ $2(2x^2-19x+6)=0$ $2x^2-19x+6=0$	M1 M1 (Expand and simplify) AG1	3
3(c)(i)	$2x^2-19x+6=0$ $x=\dfrac{19\pm\sqrt{(-19)^2-4(2)(6)}}{2(2)}$ $x=\dfrac{19\pm\sqrt{313}}{4}$ $x=9.1729$ or $x=0.3270484$ $x=9.17$ or $x=0.33$ (2 d.p.)	M1 A1, A1	3
3(c)(ii)	When $x=0.327$, $DC=2x-5=-4.356<0$. Therefore, x cannot be 0.33.	B1	1
3(d)	When $x=9.1729$, $2x-5=13.3458$ $DE=\sqrt{(13.3458)^2+9.1729^2-2(13.3458)(9.1729)\cos25°}$ $=\sqrt{40.3526}$ $=6.35237$ $=6.35$ (3 s.f.)	M2 A1	3
			12
4(a)	Total surface area of bowl $=\pi\times6^2+2\times\pi\times6\times4$ $=263.893$ $=264$ cm^2 (nearest cm^2)	M1, M1 A1	3
4(b)(i)	Number of bowls completely filled $=\dfrac{13l}{(\pi\times6^2\times4)cm^3}$ $=\dfrac{13\times1000cm^3}{(\pi\times6^2\times4)cm^3}$ $=28.7363$ $=28$	M1 M1 A1	3
4(b)(ii)	Volume of soup left $=13l-28\times(\pi\times6^2\times4)$ $=333.0984$	M1 A1	

No	Description	Marks	Total
	$=333$ cm^3 (nearest cm^3)		
Or 4(b)(ii)	Volume of soup left $=0.7363\times(\pi\times6^2\times4)$ $=333.094$ $=333$ cm^3 (nearest cm^3)	M1 A1	2
4(b)(iii)	Volume of hemisphere pan $=13$ litres $=13000$ cm^3 $\dfrac{2}{3}\pi r^3=13000$ $r=\sqrt[3]{\dfrac{13000\times3}{2\times\pi}}$ $=18.377$ cm $=184$ mm (nearest mm) or 18.4 cm (nearest mm)	M1 A1	2
4(c)	$\dfrac{V_s}{V_t}=\left(\dfrac{2}{3}\right)^3=\dfrac{8}{27}$ The ratio is $8:27$.	B1	
			11
5(a)	Angle $PQT=180°-63°-75°=42°$ Bearing of S from $Q=90°+42°=132°$	B1 B1	2
5(b)	$\dfrac{QT}{\sin75°}=\dfrac{45}{\sin63°}$ $QT=\dfrac{45}{\sin63°}\times\sin75°$ $=48.783775=48.8$ (3 s.f.)	M1 AG1	2
5(c)	Let shortest distance from ship to cliff be h. $\dfrac{1}{2}\times h\times45=\dfrac{1}{2}\times45\times48.783776\times\sin42°$ $h=48.783776\times\sin42°$ $h=32.642715$ $h=32.6$ m (3 s.f.)	M1 A1	2
Or 5(c)	$\sin42°=\dfrac{h}{48.8}$ $h=\sin42°\times48.8$ $=32.654$ $=32.7$ m (3 s.f.)	M1 A1	
5(d)(i)	Let the height of the cliff be H. $\tan16°=\dfrac{H}{48.783755}$ $H=\tan16°\times48.783755$ $=13.9883$ $=14.0$ m (3 s.f.)	Note: if $H=\tan16°\times48.8$ $=13.993=14.0$m M1 A1	2
5(d)(ii)	Angle of elevation $=\tan^{-1}\left(\dfrac{13.9883}{32.642715}\right)=23.196°$ $=23.2°$	M1 note can be FT1 A1	2
			12
6(a)(i)	$p=12$	B1	1
6(a)(ii)	Plot the points Draw a smooth curve	P2 C1	3
6(b)	490 (accept answer: $460-500$)	B1	1

No	Description	Marks	Total
6(c)(i)	Draw a tangent line Gradient of tangent $= -\dfrac{20.1}{800} = -0.025125$ $= -0.0251$ (3 s.f.) Accept answers: $(-0.023 \text{ to} - 0.03)$ Actual answer: $-0.02666 = -0.0267$	B1 B1 * answer must be a decimal number	2
6(c)(ii)	The gradient represent the <u>rate of decrease</u> of the <u>cost of printing each book</u> when number of books is 300. Or The gradient represent the <u>rate at which the cost of printing each book is decreasing.</u>	B1	1
6(d)(i)	Plot points (0, 25), (600, 15) and (1200, 5) Draw a line	P1 L1	2
6(d)(ii)	$210 \le x \le 700$ (accept $180 - 220$; $680 - 730$, with interval of 10)	B1	1
7(a)(i)	Angle $APC = 90°$ (tangent perpendicular radius) Angle $AQE = 90°$ (tangent perpendicular radius) Therefore, angle APC = angle AQE Angle PAC = Angle QAE (common angle) Since there are two pairs of corresponding angles are equal, triangle APC and triangle AQE are similar.	B1 B1	2
7(a)(ii)	$\dfrac{AC}{AE} = \dfrac{PC}{QE}$ $\dfrac{AB+2}{AB+4+6} = \dfrac{2}{6}$ $\dfrac{AB+2}{AB+10} = \dfrac{1}{3}$ $3(AB+2) = AB+10$ $3AB+6 = AB+10$ $2AB = 4$ $AB = 2$	M1 A1	2
7(b)	Angle EAQ = Angle CAP $\sin \angle CAP = \dfrac{PC}{AC} = \dfrac{2}{4} = \dfrac{1}{2}$ $\angle CAP = \sin^{-1}\left(\dfrac{1}{2}\right) = 30° = \dfrac{\pi}{6}$	A1 (for $\sin^{-1}\left(\dfrac{1}{2}\right)$)	1
7(c)	Perimeter of the shaded region $= \text{Arc } PD + PQ + \text{Arc } QD$ $= \dfrac{120}{360} \times 2 \times \pi \times 2 + \sqrt{12^2 - 6^2} - \sqrt{4^2 - 2^2} + \dfrac{60}{360} \times 2 \times \pi \times 6$ $= 4.18879 + 6.92820 + 6.28318$ $= 17.40017$ $= 17.4$ cm (3 s.f.) or 17.5 cm (3 s.f.) (if rounded off to 5 s.f. for working)	M1 (Arc $PD = \frac{4}{3}\pi$) M1 ($PQ = 4\sqrt{3}$) M1 (Arc $QD = 2\pi$) A1	4
8(a)(i)(a)	$\overrightarrow{WM} = \overrightarrow{WX} + \overrightarrow{XM} = 6p + 3q + \dfrac{3}{5}(10p - 5q)$ $\overrightarrow{WM} = 12p$	B1	1
8(a)(i)(b)	$\overrightarrow{ZM} = \overrightarrow{ZW} + \overrightarrow{WM}$ $\overrightarrow{ZM} = -10p + 5q + 12p$	Note: If notation is wrong, minus 1 marks for overall (a)(i)	

No	Description	Marks	Total
8(a)(i)(b) OR	$\overrightarrow{ZM} = 2p + 5q$ $\overrightarrow{ZM} = \overrightarrow{ZY} + \overrightarrow{YM} = 6p + 3q + \dfrac{2}{5}(-10p + 5q)$ $\overrightarrow{ZM} = 2p + 5q$	B1 B1	1
8(a)(ii)(a)	Area of triangle WMX : area of $WXYZ$ $= \dfrac{\text{Area of triangle } WMX}{\text{Area of triangle } WXY} \times \dfrac{\text{Area of triangle } WXY}{\text{Area of } WXYZ}$ $= \dfrac{3}{5} \times \dfrac{1}{2} = \dfrac{3}{10}$ Area of triangle WMX : area of $WXYZ$ $= 3 : 10$	B1	1
8(a)(ii)(b)	Area of $WXYZ = 8 \times \dfrac{10}{3} = \dfrac{80}{3}$ units² or $26\frac{2}{3}$ or 26.7 (3 s.f)	B1	1
8(a)(iii)	$\overrightarrow{WN} = \dfrac{5}{7}(6p + 3q) = \dfrac{15}{7}(2p + q)$	B1	1
8(b)(i)	$\overrightarrow{AB} = \begin{pmatrix} 3 \\ -3 \end{pmatrix} + \begin{pmatrix} 7 \\ -13 \end{pmatrix} = \begin{pmatrix} 10 \\ -16 \end{pmatrix}$	B1	1
8(b)(ii)	Acute angle $= \tan^{-1}\left(\dfrac{16}{10}\right) = 57.994 = 58.0°$ (1 d.p.)	M1 (tan ratio), A1	2
8(b)(iii)	$\overrightarrow{AB} = k\begin{pmatrix} n \\ -2m \end{pmatrix}$ or $\overrightarrow{AB} = k\begin{pmatrix} -n \\ 2m \end{pmatrix}$	B1	1
8(b)(iv)	$\overrightarrow{CD} = 3\overrightarrow{AB} = 3\begin{pmatrix} 10 \\ -16 \end{pmatrix} = \begin{pmatrix} 30 \\ -48 \end{pmatrix}$ $\overrightarrow{CD} = -3\overrightarrow{AB} = -3\begin{pmatrix} 10 \\ -16 \end{pmatrix} = \begin{pmatrix} -30 \\ 48 \end{pmatrix}$	B1 B1	2 9
9(a)(i)	41 minutes	B1	1
9(a)(ii)	25 minutes	B1	1
9(a)(iii)	$Q_1 = 29$ mins; $Q_3 = 48$ mins IQR $= 48 - 29 = 19$ mins	M1, A1 or B2	2
9(a)(iv)	Percentage of adults who spend more than 40 minutes travelling to work everyday $= \dfrac{120 - 76}{120} \times 100\% = \dfrac{44}{120} \times 100\%$ $= 36\frac{2}{3}\%$ or $\dfrac{110}{3}\%$ or 36.7% (3 s.f.)	B1	1
9(b)	Median for bus = 43 mins IQR for bus $= 55 - 27 = 28$ mins The travelling time using train is much shorter (faster) than using a bus as the median time travelling with a train < median time travelling with a bus. The travelling time using train is less spread out (more consistent) than using travelling using a bus as the interquartile range of travelling time of a train < interquartile range of travelling time of a bus.	B1 for both median and IQR of bus B1 B1	3
9(c)(i)	P(more than 55 minutes) $= \dfrac{120 - 106}{120} \times \dfrac{1}{4} = \dfrac{14}{120} \times \dfrac{1}{4} = \dfrac{7}{240}$	B1	1
9(c)(ii)	P(more than 55 minutes on one trip) $= \dfrac{120 - 106}{120} \times \dfrac{3}{4} + \dfrac{106}{120} \times \dfrac{1}{4}$ $= \dfrac{14}{120} \times \dfrac{3}{4} + \dfrac{106}{120} \times \dfrac{1}{4} = \dfrac{37}{120}$	M1 A1	2

No	Description	Marks	Total
			11
10(a)(i)	Total distance for Lane 1 $= 2 \times \pi \times (36.5 + 0.3) + (84.39 \times 2)$ $= 400.001$ m $= 400$ m (3 s.f.)	M1 for $36.5 + 0.3$ AG1	2
10(a)(ii)	Total distance for Lane 8 $= 2 \times \pi \times (36.5 + 0.2 + 1.22 \times 7) + (84.39 \times 2)$ $= 453.031$ Staggered start $= 453.031 - 400$ $= 53.03$ m (2 d.p.)	M1 for 1.22×7 M1 for the formula AG1	3
10(a)(ii)	Total distance for Lane 8 $=$		
10(a)(iii)	Using staggered start, each runner runs exactly 400 meters or run the same distance.	B1	1
10(b)	Speed $= 6.8$ km/h $= \dfrac{6.8 \times 1000}{1 \times 60} m/min$ $= 113\frac{1}{3} m/min$ Time taken to complete one round in Lane 8 $= \dfrac{453.031m}{6.8km/h} = \dfrac{453.031m}{113\frac{1}{3}m/min}$ $= 3.99733$ mins Time taken to complete five rounds in Lane 8 $= 3.99733 \times 5$ $= 19.9866$ mins Time taken to complete five rounds in Lane 8 in 1 week $= 19.98666 \times 7$ $= 138.906$ mins (< 150 minutes) His claim in false.	B1 (convert speeds) M1 (time for 1 round) M1 (time taken for 1 week) A1	4
Or 10b	Total distance for average speed of 6.8 km/h $= 6.8$ km/h $\times 150$ mins $= 6.8$ km/h $\times 2.5$ hours $= 17$ km $= 17000$ m Total distance covered for 5 round 1 week $= 453.03 \times 5 \times 7$ $= 15856.05$ m Since 15856.05 m < 17000 m, his claim is false.	B1 M1 for $\times 5$, M1 for $\times 7$ A1	
Or 10b	Total distance for average speed of 6.8 km/h $= 6.8$ km/h $\times 150$ mins $= 6.8$ km/h $\times 2.5$ hours $= 17$ km $= 17000$ m No. of rounds covered in 1 week $= \dfrac{17000}{453.03} = 37.525$ rounds No. of round covered in 1 week if he ran 5 rounds in one week $= 5 \times 7 = 35$ rounds. Since 35 rounds < 37.525 rounds, his claim is false.	B1 M1 M1 A1	
			10

$C = 10 + \dfrac{2400}{x}$

$C = 25 - \dfrac{x}{60}$

(c) (l) Tangent line