NAN CHIAU HIGH SCHOOL

MID–YEAR EXAMINATION 2019
SECONDARY FOUR EXPRESS

For Marker's Use

MATHEMATICS	**4048/01**
Paper 1	**7 May 2019, Tuesday**
	2 hours
Candidates answer on the Question Paper.	

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a 2B pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 80.

This document consists of **19** printed pages including the cover page.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer all the questions.

1   Given that $64^k \times 16 = 1$, find the value of k.

Answer $k =$ .... [1]

---

2   The stem-and-leaf diagram shows the Science quiz marks scored by a group of 15 students.

```
0 | 7
1 | 2
2 | 4   X
3 | 0   4   5   6
4 | 3   5   6   6   6   7   9
```

Key: 0 | 7 represents 7 marks

Given that the median mark is twice the interquartile range, find the value of X.

Answer $X =$ .... [2]

---

3   The LCM and HCF of p and q are 60 and 6 respectively. Given that both p and q are between 6 and 60 and $p < q$, find the value of p and of q.

Answer $p =$ ...., $q =$ .... [2]

4 A, B and C are three non-empty sets satisfying the following conditions:

$$A \subset B, \quad A \cap C \neq \emptyset, \quad A \not\subset C \text{ and } C \not\subset B.$$

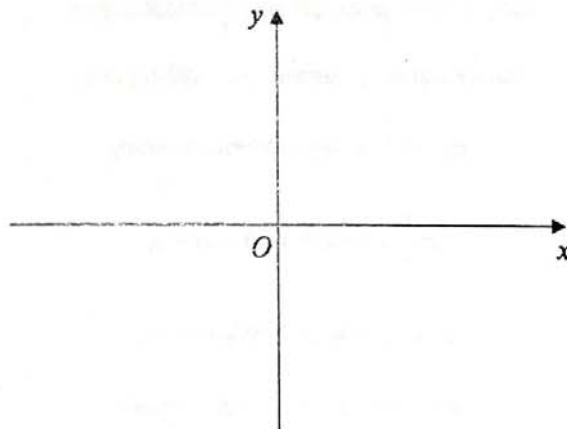Draw a clearly labelled Venn diagram to illustrate the above information.

Answer

[2]

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5 Sketch the graph of $y = (x + 2)(4 - x)$ on the axes below.

Indicate clearly the x-intercepts, y-intercept and the coordinates of the turning point.

Answer

[2]

6   Tide detergent is having its monthly promotion at MAMA Supermarket.

$1.79 NETT

TIDE Travel Size
Turbo Clean Liquid
Laundry - Free & Gentle
Volume: 10 oz

ANY 2 @ $17.90

TIDE
Turbo Clean Liquid
Laundry - Free & Gentle
Volume: 50 oz

OFFER $26.85

TIDE
Turbo Clean Liquid
Laundry - Free & Gentle
Volume: 150 oz

Show that the cost of the detergent is directly proportional to its volume.

Answer

[2]

---

7   The employees of a company are offered a wage increment which is calculated according to one of the following schemes:

Scheme A: An increment of 5% of their present wages.
Scheme B: An increment of 3% of their present wages plus additional $8 per week.

Employees earning $x per week at present will receive higher increment if they have chosen *Scheme A*. Find the range of values of x.

Answer .... [2]

8   Solve $\frac{1}{x} - \frac{3}{2x+1} = 1$.

Answer $x =$ .... [3]

9   Rearrange $y = \sqrt{\frac{2x+y}{3x-5}}$ to express x in terms of y.

Answer $x =$ .... [3]

10   Mr Lim invested $10 000 in a fund that pays a compound interest of 2.75% per annum, compounded quarterly. Find the interest earned, correct to the nearest dollar, at the end of 3 years.

Answer $.......................... [3]

11   The diagram shows a container in the shape of a prism with a triangular cross-section. Water is poured into the container until the depth of water is $\frac{3}{5}$ the height of the container.

If the volume of the container is 200 ml, find the volume of water in the container.

Answer .... ml   [3]

12 **(a)** Convert 80 km/h to m/s.

Answer .... m/s [1]

(b) In Marina Coastal Expressway (MCE), the speed limit of all vehicles is 80 km/h. A car is travelling at a constant speed of 30 m/s and passes a stationary traffic police motorcycle. The traffic police immediately gives chase, accelerating uniformly to reach a maximum speed of 40 m/s and continues with this speed until it overtakes the car.
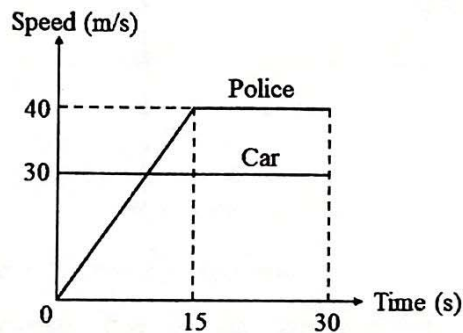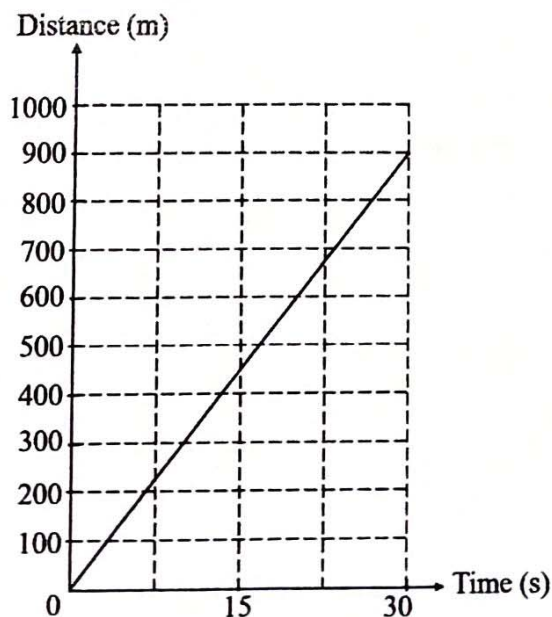

(i) Calculate the acceleration of the police motorcycle.

Answer .... m/s² [1]

(ii) In the axes below, draw the distance–time graph for the police motorcycle for the first 30 seconds. The distance–time graph for the car has been drawn for you.

Answer

[1]

13 *PQRS* is a parallelogram.

$$\overrightarrow{PQ} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}, \overrightarrow{PS} = \begin{pmatrix} 12 \\ 5 \end{pmatrix}.$$
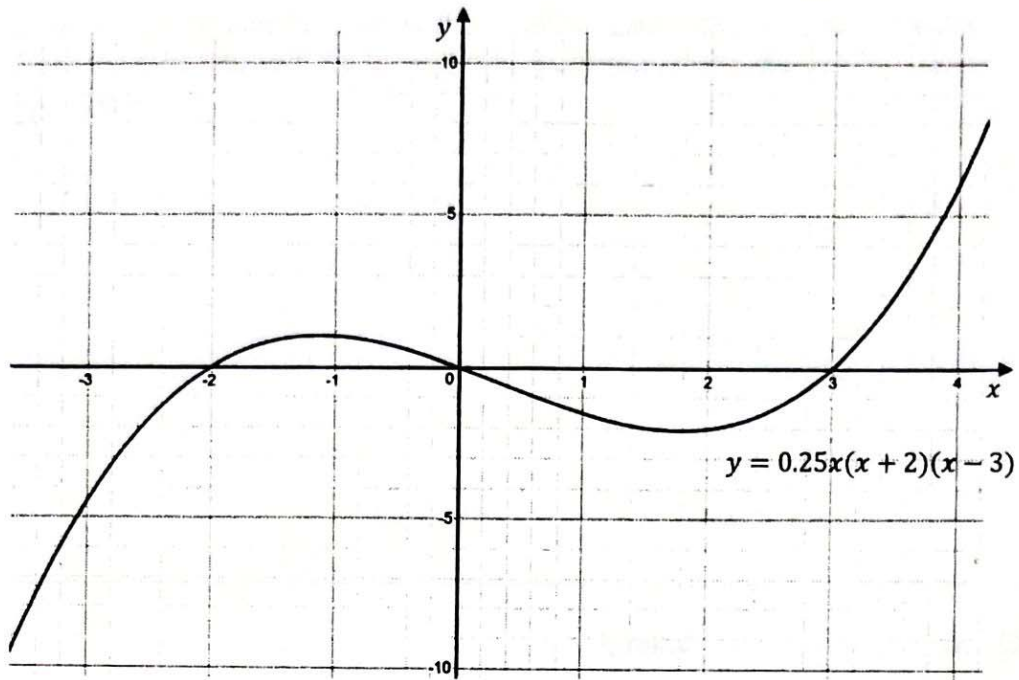
Calculate $|\overrightarrow{PR}|$.

Answer .... units   [3]

14 By observing the pattern in the last digit of 3^n and of 8^n, where $n > 0$, find the last digit in the subtraction $3^{31} - 8^{16}$.

Answer ....   [3]

15

$y = 0.25x(x+2)(x-3)$

The graph of $y = 0.25x(x+2)(x-3)$ was drawn.

By adding a suitable straight line on the graph above, solve the equation $x(x+2)(x-3) = 6x$.

Answer $x =$ .... [3]

16   The diagram below shows part of a site map of the newly launched Build-To-Order (BTO) flats at Fernvale Road.

Andy and his parents applied for this BTO project under the Multi-Generation Priority Scheme which allows married children to make a joint application with their parents for two units in the same project.

Their application has gotten them a good queue number in the selection of units. Andy has chosen the unit 580, marked as A.

State the unit which Andy's parents should choose if they wish to satisfy the following criteria:
   • equidistant from the multi storey carpark, C, and the LRT station, L, and
   • equidistant from the line LC and AC.

Source: http://esales.hdb.gov.sg/bp25/launch/19feb/bto/19FEBBTOSK_page_6280/$file/map-plan.html#site-plan
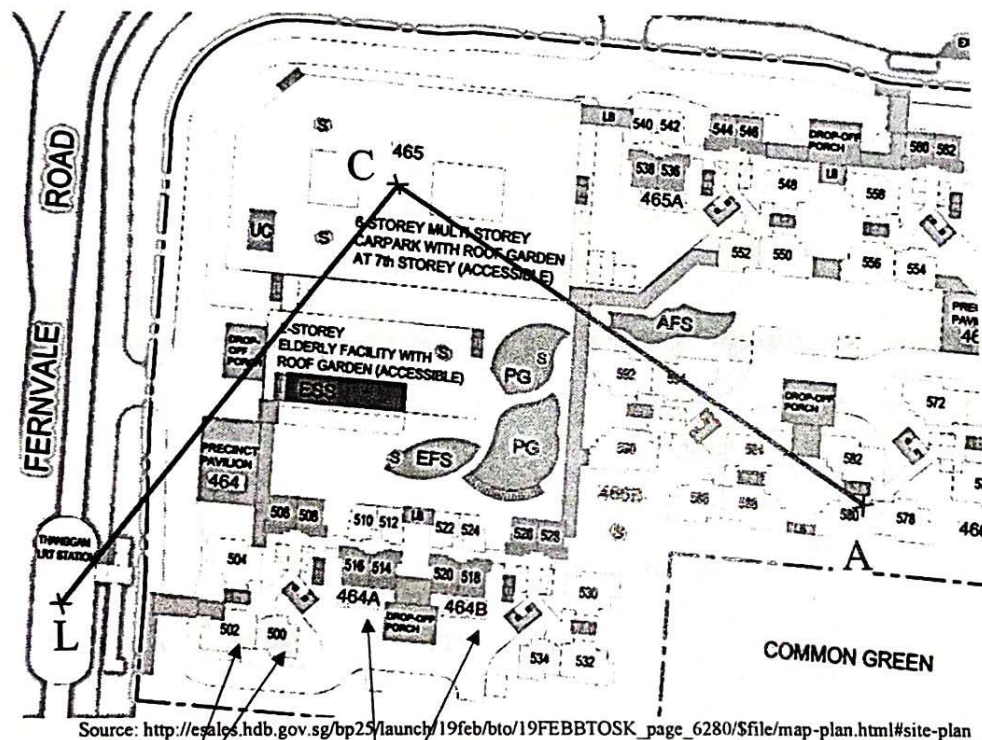
Unit numbers 502 and 500
i.e. #02–502,
#02–500

Block numbers 464A and 464B
i.e. Blk 464A,
Blk 464B

Answer .... [3]

17 In the diagram, O is the centre of the circle with radius 13 cm. PQ and RS are perpendicular equal chords of length 24 cm and intersecting at K.
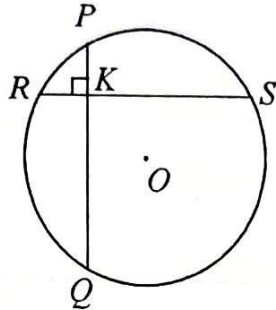
Calculate the length of OK.
Show your workings and give reasons.

Answer .... cm   [3]

18 For a n–sided polygon, the largest interior angle is 100° and the smallest interior angle is 20°. Find the value(s) of n.

Answer $n =$ .... [4]

19 (a) Factorise $(2x - 7)(x - 2) - 9$ completely.

Answer .... [2]

(b) Hence, solve $2(y - 1)^2 - 11y + 16 = 0$.

Answer .... [2]

20 When $x + 8$ is divided by y, the result is 4. When x is divided by y, its quotient is 2 and the remainder is 6. By forming two equations in terms of x and of y, solve for x and y.

Answer $x = $ ...., $y =$ .... [4]

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21 In the diagram, A, B, C and D lie on the circumference of the circle. ADP, ABQ, BCP and DCQ are straight lines. $\angle BAD = 54°$ and $\angle CPD = 27°$.
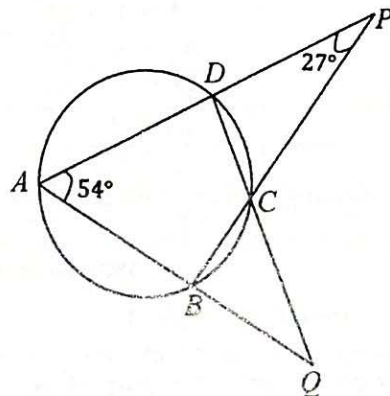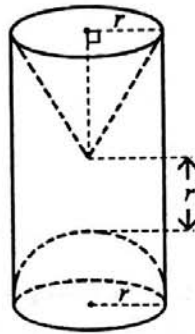

Find $\angle AQD$.
Show your working and give reasons.

Answer $\angle AQD = $ ....° [4]

22 A carpenter wants to create a wooden toy for his son.
He removed a right-angled cone and a hemisphere from a uniform cylindrical wood. The radius of the base of the cone and hemisphere are r cm. The distance between the top of the hemisphere and the vertex of the cone is also r cm.

Given that the curved surface area of the cone is equal to the curved surface area of the hemisphere, work out the volume of the toy, in terms of r.

Answer ....cm³ [4]

23 The histogram illustrates the length of stay (in days) in Australia for a group of Singaporean tourists last year.

Frequency

Length of stay (x days)
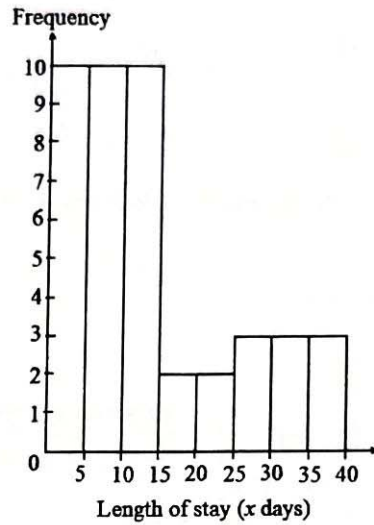
(a) Complete the following table.

Answer

Length of stay (days)	Numbers of tourists
$0 < x \le 5$	
$5 < x \le 10$	
$10 < x \le 15$	
$15 < x \le 20$	
$20 < x \le 25$	
$25 < x \le 30$	
$30 < x \le 35$	
$35 < x \le 40$	

[1]

(b) Calculate the fraction of the Singaporean tourists who stayed in Australia longer than 15 days.

Answer .... [1]

(c) Calculate an estimate for the mean and standard deviation of the length of stay.

Answer　　　　Mean = ...days

Standard deviation = ...days　[2]

24 **(a)** Explain why $(x-2)^4(x^2-4x+4)$ is both a perfect square and a perfect cube when x is an integer.
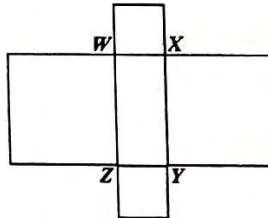
Answer

....

....

....

.... [2]

(b) The figure is made up of 2 big squares, 2 small squares and a rectangle *WXYZ*. The perimeter of rectangle *WXYZ* is 20 cm. The total area of all the four squares is 140 cm².

Find the area of *WXYZ*.

Answer .... cm² [4]

25 Mrs Tan sells three different types of cakes. The table below shows the number of cakes bought by Childcare Centre *A* and *B* for Children's Day Celebration.

	Sponge	Chocolate	Cheese
Childcare Centre *A*	32	27	20
Childcare Centre *B*	44	45	38

(a) Represent this information in a 2 × 3 matrix, **P**.

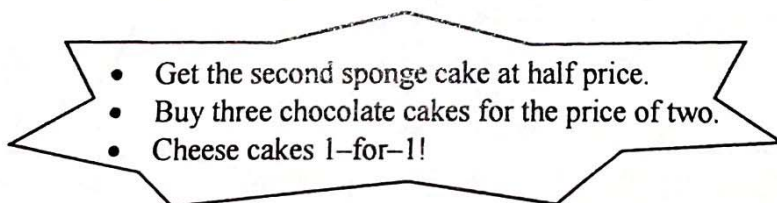*Answer* $\mathbf{P} = \begin{pmatrix} & & \\ & & \end{pmatrix}$ [1]

(b) The selling price for each sponge cake, chocolate cake and cheese cake is $3.20, $4.50 and $4.80 respectively.

The information can be represented as $\mathbf{Q} = \begin{pmatrix} 3.2 \\ 4.5 \\ 4.8 \end{pmatrix}$.

·Evaluate the matrix **PQ**.

Answer $\mathbf{PQ} = \begin{pmatrix} & \\ & \end{pmatrix}$ [1]

(c) Mrs Tan has a special promotion for cakes order for Children's Day celebration.

- Get the second sponge cake at half price.
- Buy three chocolate cakes for the price of two.
- Cheese cakes 1–for–1!

Write down a 3 × 3 matrix, **R**, such that **RQ** will give the discounted price for each type of cakes sold to Childcare Centre *A* and *B*.

Answer $\mathbf{R} = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$ [2]

25 (d) Evaluate matrix **S = PRQ**.

Answer **S =** $\begin{pmatrix} & \end{pmatrix}$ [2]

(e) Describe what the elements in matrix **S** represents.

Answer

....

....

.... [1]

– End –

NAN CHIAU HIGH SCHOOL
MID-YEAR EXAMINATION 2019
SECONDARY FOUR EXPRESS

MATHEMATICS   **4048/02**

Paper 2   **9 May 2019, Thursday**

Candidates answer on the Question Paper.   **2 hours 30 minutes**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a 2B pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total marks for this paper is 100.

This document consists of **25** printed pages including the cover page.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

1.   **(a)**   Simplify $\dfrac{4b^3}{3a} \div \dfrac{(-4ab^3)^2}{2a^{-4}b^2}$. Give your answer in positive indices.   [2]

     (b)   Factorise $14xb + 3ay - 2xy - 21ab$ completely.   [2]

     (c)   Solve the inequality $\dfrac{x}{5} < \dfrac{x+2}{3} \leq \dfrac{3-2x}{4}$.   [3]

(d) Simplify $\dfrac{2x(1-6x)}{4x^2-1} - \dfrac{2}{2x+1}$. [4]

(e) Express $2x^2 + 6x - 15$ in the form $a(x+b)^2 + c$, where a, b and c are constants.

Hence, solve the equation $2x^2 + 6x - 15 = 0$, giving your answers correct to four decimal places. [4]

2. *ABCDE* is a pentagon.
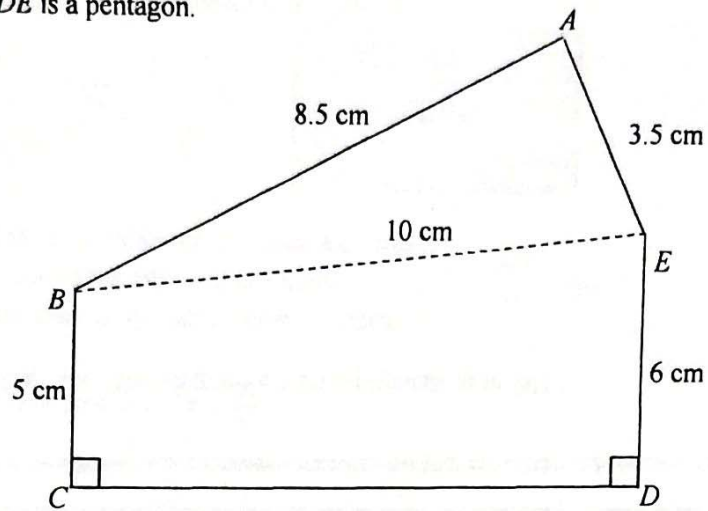
(a) Calculate the area of the pentagon *ABCDE*. [5]

(b) The pentagon *ABCDE* is a cross-section of a prism which is a model of a house as shown below.

(i) Calculate the total surface area of the model, including the base. [2]

(ii) Calculate the angle of depression of *H* from *E*. [3]

3. There was a newspaper article on Singapore's population published in 2018.

Singapore's population grows slightly to 5.638 million, with non-resident numbers stable.

Singapore's population rose 0.5 percent to 5.638 million from 2017 to 2018.

The slight increase over the past year was due to stable growth in citizen population, which rose 1 percent to 3.472 million citizens as of June 2018.

The 1 percent growth in citizens was due to citizen births and immigration.

Residents
3.994M

Citizens **3.472M**   PRs **0.522M**

+   **Non-Residents 1.644M**

Students 4%   Employment Pass Holders

Dependants of Citizens/ PRs/ Work Pass Holders 17%   11%   S Pass Holders 12%

Foreign Domestic Workers 15%   41%   Work Permit Holders
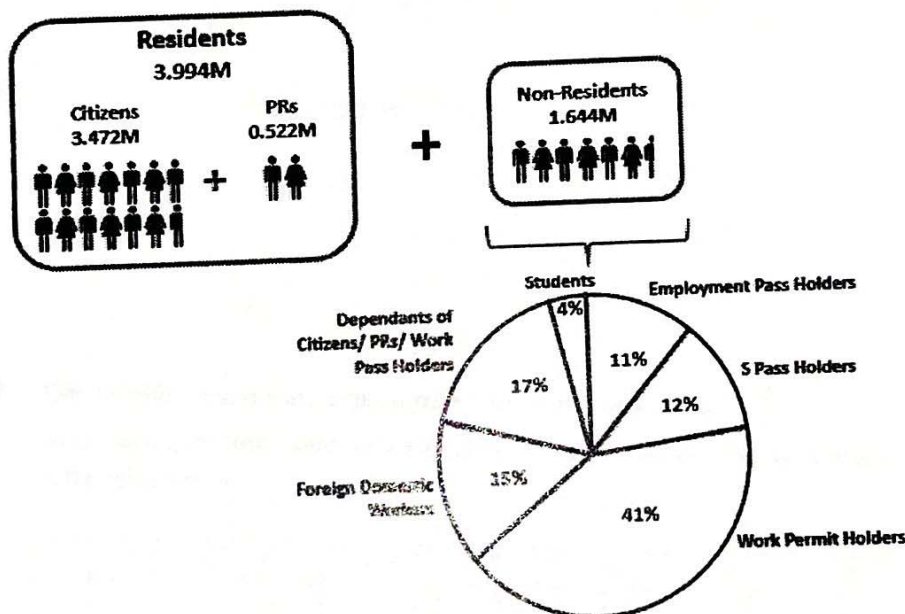
(a) Calculate the Singapore's total population in 2017.   [2]

(b) Express the number of Foreign Domestic Workers as a percentage of the Singapore's total population in 2018. [2]

(c) Based on the information shown in this article, do you agree that Singapore's total population will reach 6.3 million by 2030?
Support your answer with appropriate workings and state an assumption that you have made in your calculation. [3]

(d) State one aspect of the diagram that may be misleading and explain how this may lead to a misinterpretation of information. [2]

4. The diagram shows a rectangular table top.
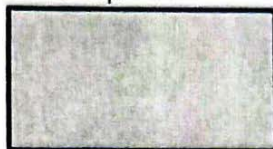
    x m

    The area of the table top is 5 square metres.
    The length of the table top is *x* metres.
    The perimeter of the table top is *y* metres.

    (a)   Show that $y = 2x + \frac{10}{x}$.   [2]

    (b)   The variables *x* and *y* are connected by the equation $y = 2x + \frac{10}{x}$.
             Some corresponding values of *x* and *y*, correct to one decimal place, are given
             in the table below.

x	0.5	1	1.5	2	4	6	8
y	*p*	12	*q*	9	10.5	13.7	17.3

    (i)   Find the value of *p* and of *q*.   [1]

    (ii) Using a scale of 1 cm to represent 1 m, draw a horizontal *x*-axis for
             $0 \le x \le 8$.
             Using a scale of 1 cm to represent 1 m, draw a vertical *y*-axis for
             $0 \le y \le 22$.
             On your axes, plot the points given in the table and join them with a
             smooth curve.   [3]
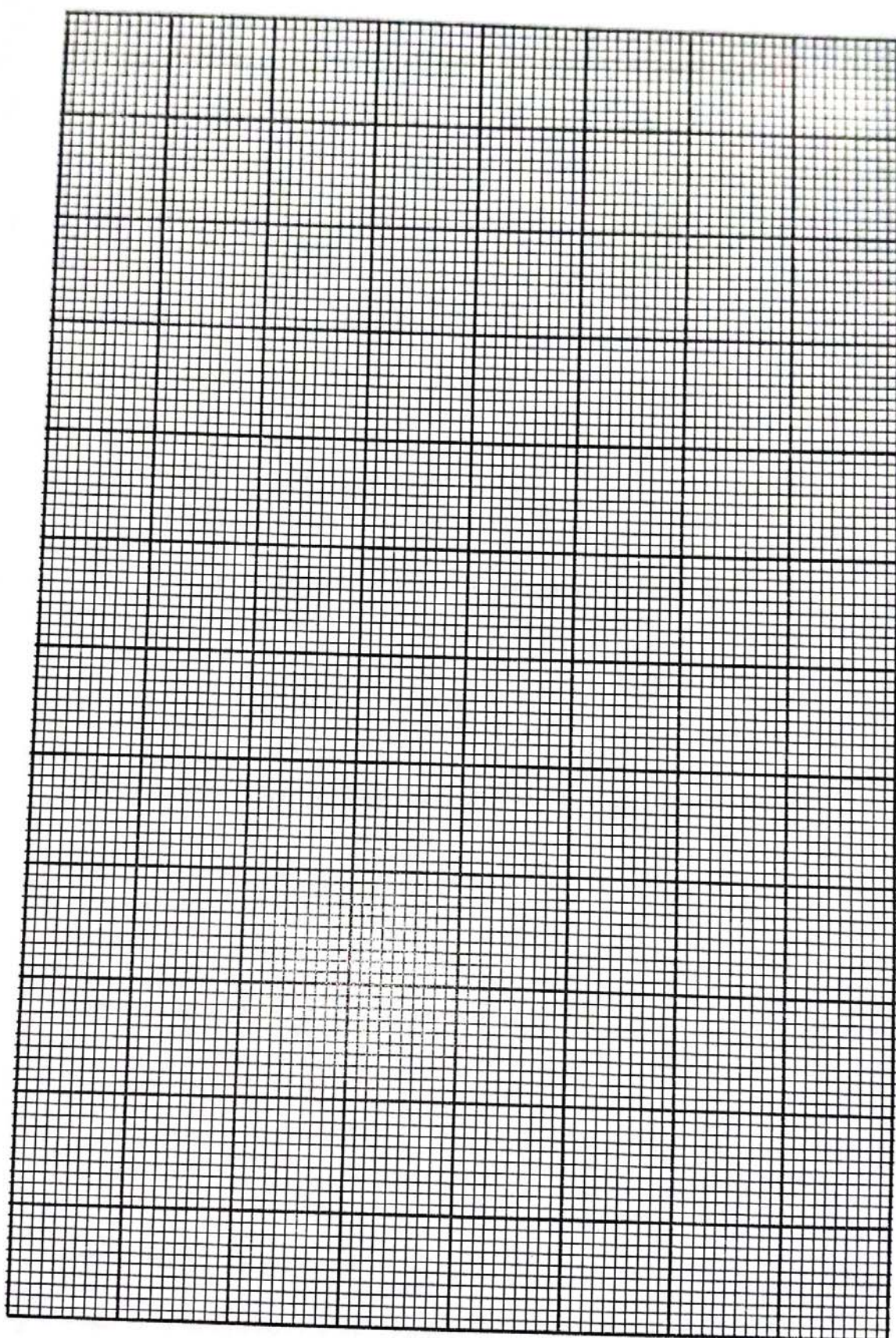
(iii) Use your graph to find the length of the table top if the perimeter of the table top is 16 m. [2]

(iv) By drawing a suitable straight line, find the *x* coordinate of the point at which the gradient of the curve is $\frac{1}{3}$. [2]

5. Given equation of a line L is $3x - 2y = 8$.

   (a) State the gradient of line L. [1]

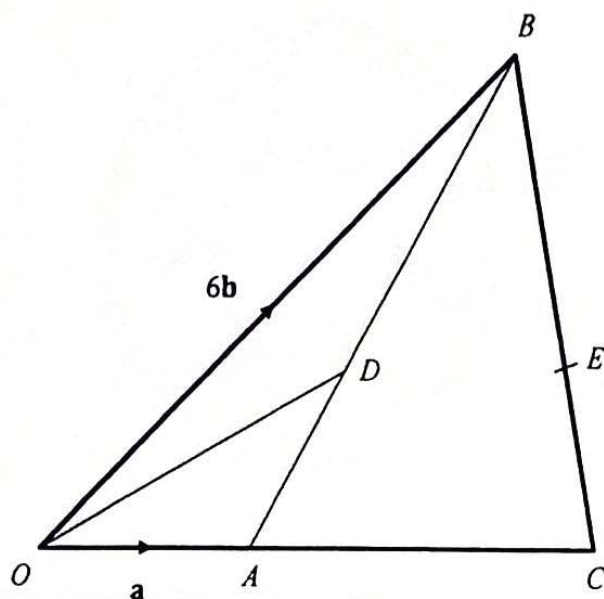   (b) If $P(k, -2)$ is a point on the line L, find the value of k. [1]

   (c) Find the equation of another line that is parallel to the x-axis and passes through P. [1]

   (d) Calculate the perpendicular distance from the origin, O, to the line L. [4]

6.

In the diagram, $\overrightarrow{OA} = \boldsymbol{a}$, $\overrightarrow{OB} = 6\boldsymbol{b}$ and $\overrightarrow{OA} = \frac{1}{3}\overrightarrow{OC}$.

D is a point on AB such that $3\overrightarrow{AD} = 2\overrightarrow{DB}$ and E is a point on BC such that $\overrightarrow{CE} : \overrightarrow{EB} = 4 : 5$.

(a) Express, as simply as possible, in terms of **a** and **b**,

    (i) $\overrightarrow{BA}$,   [1]

    (ii) $\overrightarrow{OD}$,   [1]

    (iii) $\overrightarrow{CB}$,   [1]

    (iv) $\overrightarrow{AE}$.   [2]

13

(b) Write down the relationship between *OD* and *AE*. Explain your answer.　　[2]

(c) Find the ratio of

   (i) area of triangle *CAE* : area of triangle *AOD* ,　　[2]

   (ii) area of triangle *CAE* : area of triangle *AOB* .　　[2]

7. **(a)** There are two boxes of sweets containing toffees and chocolates.

Box A contains 8 toffees and 4 chocolates, whereas box B contains 7 toffees and 3 chocolates.

Jolin loves chocolates.

One of the boxes is chosen at random and a sweet is taken out.

If she gets a chocolate, she will consume it.

If she did not get a chocolate from the first selection, she will place the sweet into the other box and she will select again from the other box.

Jolin limits herself to two selections.

Find, as a fraction in its simplest form, the probability that

(i) Jolin will have her favourite chocolate from the first selection, [3]

(ii) Jolin will have her favourite chocolate. [3]

(b) Your friend, Kenton gives you a chance to win $1000 by playing a game of "Guess the number". There are two options of the game that he allows you to choose.

Option A
He uses a random number generator to choose a number from 1 to 8.
If you guess it correctly, you win.

Option B
You flip a fair coin.
If the coin lands on head, Kenton will roll a fair 6-sided die. If you manage to guess what it rolled, you win.
If the coin lands on tail, Kenton will use a random number generator to choose a random number from 1 to 8. If you guess the number correctly, you win.

Which option will you choose? Explain your answer. [3]

8.
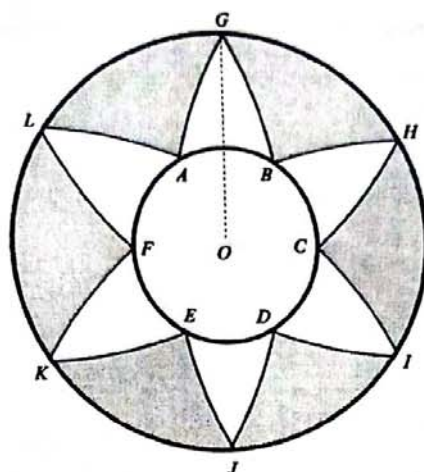
Figure 1

Figure 1 shows a dreamcatcher-inspired ornament which is made up of wire.
The ornament consists of an inner circle *ABCDEF* and an outer circle *GHIJKL*, both
with centre *O*, and 6 identical "petal" designs such as *AGB* and *BHC*.

Team Hō'ola will be making these ornaments to raise funds for their Voluntary
Welfare Organisation.

To estimate the amount of wire to be purchased, Head of Fundraising team, Janice
modelled the following:

- The arc *AG* is an arc of another circle with centre *X*, radius 18 cm and
  $\angle AXG = \frac{\pi}{6}$ radians.

This information is illustrated in Figure 2.
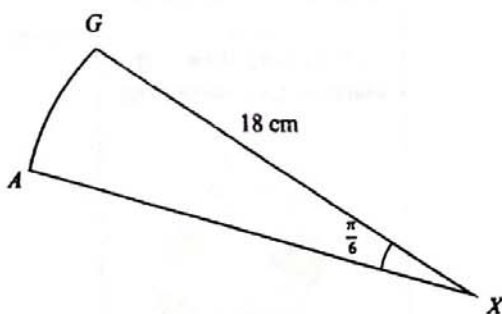
Figure 2

17

- A regular hexagon forms within the inner circle *ABCDEF* with *OB* = 5 cm.
  This information is illustrated in Figure 3.
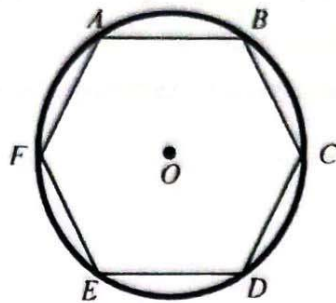
Figure 3

(a) Show that the radius of the outer circle is 13.31 cm, correct to 2 decimal places. [6]

(b) Team Hōʻola decided to make 50 dreamcatcher-inspired ornaments and Janice chanced upon the following promotion.

10 m silver craft bendable
DIY wire (2mm thick)
$9.28

What is the estimated cost to make these ornaments? [5]

(c) Find the area of the shaded region as shown in Figure 1. [5]

9. (a) The cumulative frequency curve below shows the time spent in minutes by a group of 80 teenagers on Instagram (a social media platform) on a particular day.
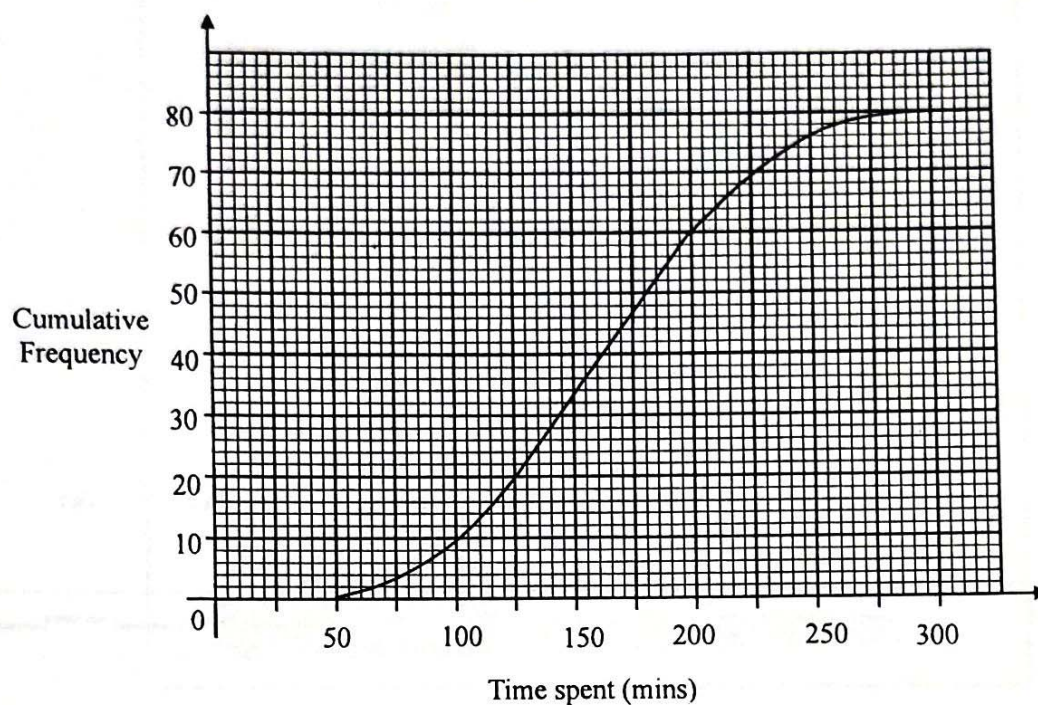
Time spent (mins)

Use the curve to estimate

(i)   the median,   [1]

(ii) the interquartile range.   [2]

(b) Through a market research, it was found out that the time spent on Facebook (another social media platform) is less popular and less consistent among the same group of 80 teenagers. A second cumulative frequency curve for the same group of 80 teenagers spending their time on Facebook is drawn.

Describe how the second cumulative frequency curve may differ from the curve for Instagram. [2]

(c) The box-and-whisker plot represents the distribution of the time spent for the same group of 80 teenagers on SnapChat (another social media platform).

Time Spent (mins)
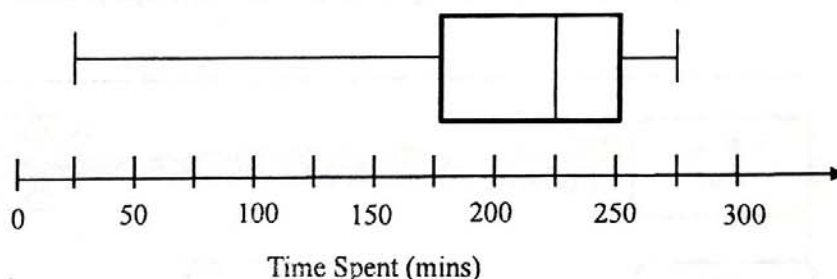
For this group of 80 teenagers, which of the social media platforms – Instagram, Facebook or Snapchat, is the most popular?

Support your answer with an appropriate statistical measure. [2]

10. Ms Tan got her new home recently and the layout of her house is shown in the diagram below. The layout is drawn to scale.
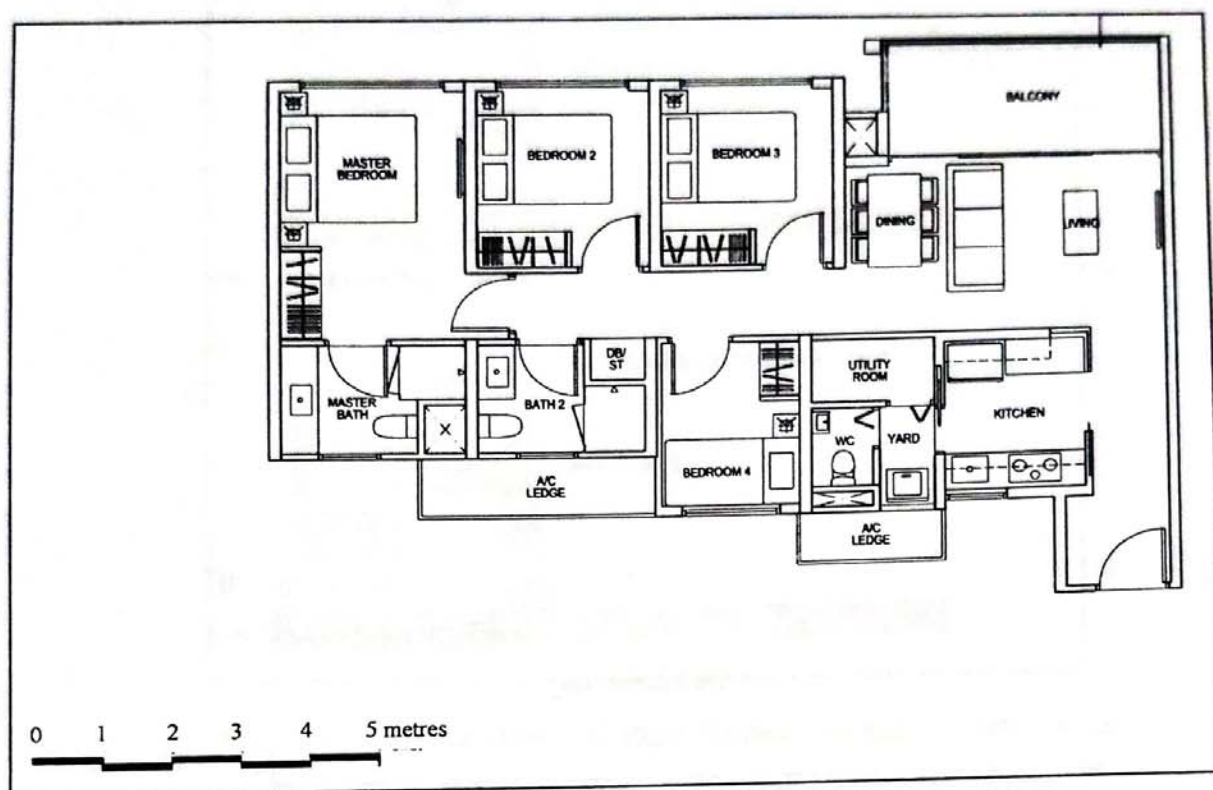
(a) Express the scale of the map in the form of $1:n$. [1]

(b) Ms Tan decides to get a television set for her living room.

She found the following infographic online.

Viewing distance guide: linking screen sizes to how far you sit from the TV

Viewing distance	Screen size
Less than 1.5m	Up to 32 inch
1.5 to 2m	32 to 39 inch
2 to 2.5m	40 to 45 inch
2.5 to 3m	46 to 56 inch
Over 3m	Over 56 inch
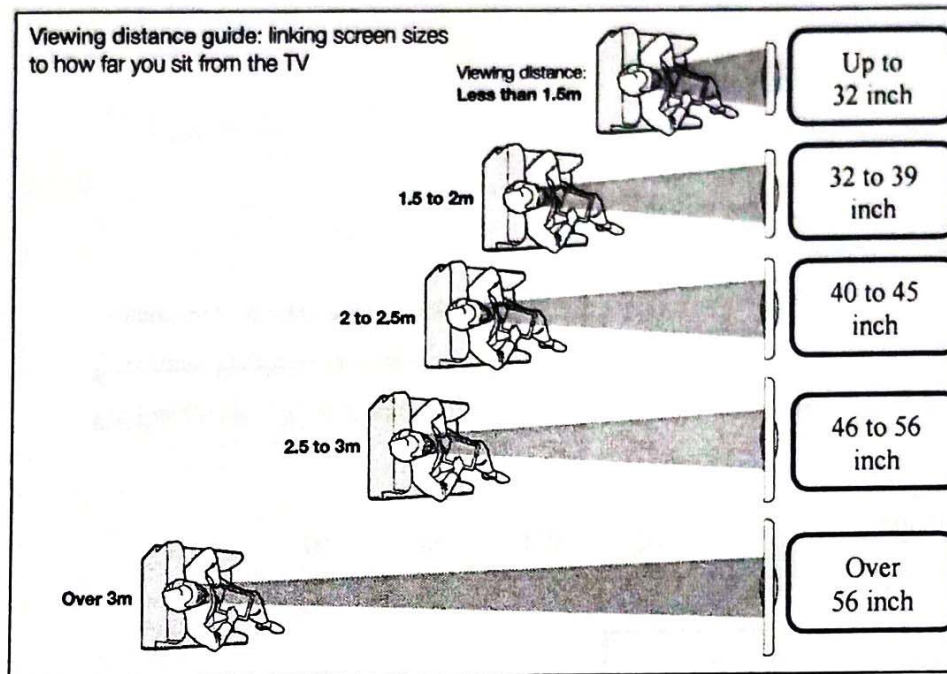
What is the range of television size which Ms Tan should get for her living room? [2]

(c) Ms Tan decided to shop for her television set online and she saw the following options:

LG OLED55B8STB 55" OLED 4K SMART TV

LG

SGD2,988.00

55-inch OLED 4K SMART TV

Quantity 1

Add to Cart

LG OLED65B8STB 65" OLED 4K SMART TV

LG

SGD4,988.00

65-inch OLED 4K SMART TV

Add To Cart
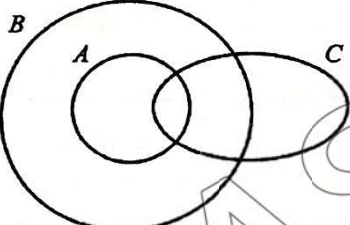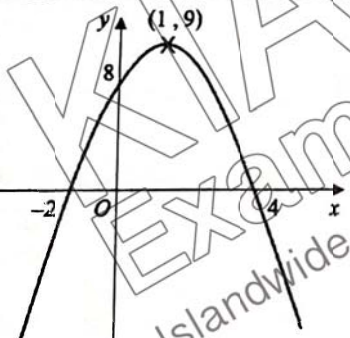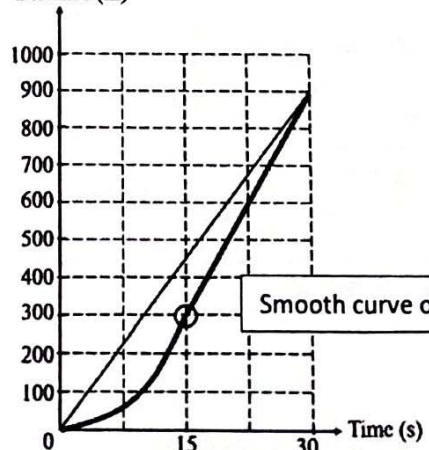
Ms Tan pays a downpayment of $2 000 for her television set as suggested in (b).
She pays the remaining amount over 3 months, with a simple interest rate of
7 % per annum.

Calculate her monthly instalment.   [3]

- End of Paper -

Answers

1	$4^{3k} \times 4^2 = 4^0$
	$3k + 2 = 0$
	$k = -\dfrac{2}{3}$ **exact only!**

2	Median = 36
	Interquartile Range = 18
	$Q_1 = 46 - 18$
	$= 28$
	$X = 8 \rightarrow$ digit in the ones place.

3	HCF $= 6 = 2 \times 3$
	LCM $= 60 = 2^2 \times 3 \times 5$
	$p = 2^2 \times 3 = 12$
	$q = 2 \times 3 \times 5 = 30$

4	

5	

6	$\dfrac{1.79}{10} = \dfrac{179}{1000}$,
	$\dfrac{17.9}{2 \times 50} = \dfrac{179}{1000}$,
	$\dfrac{26.85}{150} = \dfrac{179}{1000}$. **Must show for all three!**
	Since $\dfrac{cost}{volume} = $ **constant**, the cost is directly proportional to its volume.

7	$0.05x > 0.03x + 8$ or $1.05x > 1.03x + 8$
	$0.02x > 8$
	$x > 400$

8	$\dfrac{2x+1-3x}{x(2x+1)} = 1$ **Must show quad formula with values subbed in.**
	$1 - x = 2x^2 + x$
	$2x^2 + 2x - 1 = 0$
	$x = \dfrac{-2 \pm \sqrt{(2)^2 - 4(2)(-1)}}{2(2)} = 0.366$ or -1.37

9	$y^2 = \dfrac{2x+y}{3x-5}$
	$3xy^2 - 5y^2 = 2x + y$
	$3xy^2 - 2x = 5y^2 + y$
	$x(3y^2 - 2) = 5y^2 + y$
	$x = \dfrac{y(5y+1)}{3y^2-2}$ or $\dfrac{5y^2+y}{3y^2-2}$ or $\dfrac{5y^2+y}{2-3y^2}$

10	$10\,000 \left(1 + \dfrac{2.75 \div 4}{100}\right)^{3 \times 4} - 10\,000$
	$= 10\,856.92 - 10\,000$
	$= 856.92$
	$= \$857$ **Quarterly is 4 times!**

11	$\left(\dfrac{3}{5}\right)^2 = \dfrac{9}{25}$ **Not cube because one of the component is constant!!**
	$\dfrac{9}{25} \times 200 = 72$ ml

12a	$\dfrac{80 km}{1h} = \dfrac{80 \times 1000 \text{ m}}{3600 \text{ s}} = \dfrac{200}{9} = 22\dfrac{2}{9}$ **Mixed number or 3sf only!!**

12b	$\dfrac{40}{15} = 2\dfrac{2}{3}$

12c	**Smooth curve only!**

13	$\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} + \begin{pmatrix} 12 \\ 5 \end{pmatrix} = \begin{pmatrix} 9 \\ 9 \end{pmatrix}$ $\|\overrightarrow{PR}\| = \sqrt{9^2 + 9^2} = \sqrt{162} = 12.7$ units	PQ is not perpendicular to PS! Length PR ≠ length PQ + length QR!

14	$3^1 = 3 \qquad\qquad 8^1 = 8$ $3^2 = 9 \qquad\qquad 8^2 = 64$ $3^3 = 27 \qquad\quad\; 8^3 = 512$ $3^4 = 81 \qquad\quad\; 8^4 = 4096$ $3^5 = 243 \qquad\; 8^5 = 32768$ Last digit for 3^n is 3, 9, 7, 1 Last digit for 8^n is 8, 4, 2, 6 $\frac{31}{4} = 7\frac{3}{4} \rightarrow$ Remainder = 3 $\rightarrow$ Last digit = 7 $\frac{16}{4} = 4 \rightarrow$ Remainder = 0 $\rightarrow$ Last digit = 6 Last digit of $3^{31} - 8^{16} = 7 - 6 = 1$	We must see that you had chosen the digit "7" and "6".

15	$\frac{1}{4}x(x+2)(x-3) = \frac{6}{4}x$ Plot $y = \frac{3}{2}x$	Equation of line must be seen in this page. Or else, we will take it as you use calculator mode 3, 4.

$y = 1.5x$

G1

$y = 0.25x(x+2)(x-3)$

$x = -3$ or 0 or 4 $\rightarrow$ only accept **exact** answers.

16	Draw perpendicular bisector of *LC*. Draw angle bisector of *AC*. Unit 518.
17	Let *M* be the midpoint of *RS*. $OM = \sqrt{13^2 - 12^2} = 5$ (Perpendicular bisector of chord) $OK = \sqrt{5^2 + 5^2} = 7.07$ (Equal chords) Leave answer in 3sf as you are finding length!
18	$20 \leq \dfrac{(n-2)\times 180 - 100 - 20}{n-2} \leq 100$ $20(n-2) \leq (n-2)\times 180 - 120 \leq 100(n-2)$ $20n - 40 \leq 180n - 360 - 120 \leq 100n - 200$ $20n - 40 \leq 180n - 480$ or $180n - 480 \leq 100n - 200$ $440 \leq 160n$ or $80n \leq 280$ $2.75 \leq n$ or $n \leq 3.5$ $2.75 \leq n \leq 3.5$ $n = 3$ Guess & check allowed for this qn.
19a	$2x^2 - 4x - 7x + 14 - 9 = 2x^2 - 11x + 5 = (2x - 1)(x - 5)$ Factorise, not completing the square! Read carefully.
19b	$2(y-1)^2 - 11(y-1) + 5 = 0$ $x = y - 1$ → must be seen! Read question. It says "Hence". $y - 1 = \dfrac{1}{2}$ or 5 $y = 1.5$ or 6
20	$\dfrac{x+8}{y} = 4$ $\quad\quad$ $\dfrac{x}{y} = 2 + \dfrac{6}{y}$ $x + 8 = 4y$ $\quad\quad$ $x = 2y + 6$ $2y + 6 + 8 = 4y$ $2y = 14$ $y = 7$ $x = 20$ Do not write mixed number for algebra!! Recall: dividend = (divisor)(quotient) + remainder
21	$\angle BCD = 180 - 54 = 126$ (∠s in opp segment) $\angle DCP = 180 - 126 = 54$ (adj ∠ on a straight line) $\angle ADC = 27 + 54 = 81$ (ext ∠ of Δ) $\angle AQD = 180 - 54 - 81 = 45$ (∠ sum of Δ) Easier method: $\angle ABP = 180 - 54 - 27 = 99$ (∠ sum of Δ) $\angle ADQ = 180 - 99 = 81$ (∠s in opposite segment) $\angle AQD = 180 - 54 - 81 = 45$ (∠ sum of Δ)
22	$\pi r l = 2\pi r^2$ $l = 2r$ $h = \sqrt{(2r)^2 - r^2} = \sqrt{3}r$ Vertical height and slanted length are different! Volume $= \pi r^2(\sqrt{3}r + 2r) - \dfrac{1}{3}\pi r^2(\sqrt{3}r) - \dfrac{2}{3}\pi r^3$ $= \pi r^3\left(\sqrt{3} + 2 - \dfrac{\sqrt{3}}{3} - \dfrac{2}{3}\right)$ $= 7.82 r^3$ Refer to cover page if you had left your answers in π.

23a	Length of stay (days)	Numbers of tourists
	$0 < x \le 5$	10
	$5 < x \le 10$	10
	$10 < x \le 15$	10
	$15 < x \le 20$	2
	$20 < x \le 25$	2
	$25 < x \le 30$	3
	$30 < x \le 35$	3
	$35 < x \le 40$	3

23b	$\dfrac{13}{43}$

23c	Mean = $13\dfrac{77}{86}$ or 13.9 [3sf only! Do not round to nearest integer!] SD = 11.0

24a	$(x-2)^4(x^2-4x+4) = (x-2)^4(x-2)^2 = (x-2)^6$ → **must be seen!** Since the power 6 is both a **multiple of 2 and of 3**, it is both a perfect cube and a perfect square.

24b	$2x + 2y = 20$ $2x^2 + 2y^2 = 140$ $x + y = 10$ $x^2 + y^2 = 70$ $(x+y)^2 = x^2 + y^2 + 2xy$ $2xy = 100 - 70$ [Alternative method is to solve of x and y to get 8.16 or 1.84 and find xy = 15.0 cm².] $xy = 15$

25a	$P = \begin{pmatrix} 32 & 27 & 20 \\ 44 & 45 & 38 \end{pmatrix}$

25b	$PQ = \begin{pmatrix} 319.9 \\ 525.7 \end{pmatrix}$

25c	$R = \begin{pmatrix} \frac{3}{4} & 0 & 0 \\ 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$ [Work with exact! This is percentage, not money hence, it should not be 2dp! Only accept exact or 3sf.]

25d	$S = \begin{pmatrix} 32 & 27 & 20 \\ 44 & 45 & 38 \end{pmatrix}\begin{pmatrix} \frac{3}{4} & 0 & 0 \\ 0 & \frac{2}{3} & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 3.2 \\ 4.5 \\ 4.8 \end{pmatrix} = \begin{pmatrix} 24 & 18 & 10 \\ 33 & 30 & 19 \end{pmatrix}\begin{pmatrix} 3.2 \\ 4.5 \\ 4.8 \end{pmatrix} = \begin{pmatrix} 205.8 \\ 331.8 \end{pmatrix}$

25e	They represent the **total money collected from Childcare Centre A and B respectively** for the sales of cakes **after discount**. [Money collected from sales is not money earned! The word "earned/earnings" relates to profit.]

The End

For Marker's Use

NAN CHIAU HIGH SCHOOL
MID-YEAR EXAMINATION 2019
SECONDARY FOUR EXPRESS

MARKING SCHEME

MATHEMATICS	**4048/02**
Paper 2	**9 May 2019, Thursday**
Candidates answer on the Question Paper.	**2 hours 30 minutes**

1. (a) Simplify $\frac{4b^2}{3a} \div \frac{(-4ab^3)^2}{2a^{-4}b^2}$. Give your answer in positive indices. [2]

$\frac{4b^3}{3a} \div \frac{(-4ab^3)^2}{2a^{-4}b^2}$

$= \frac{4b^3}{3a} \times \frac{2a^{-4}b^2}{16a^2b^6}$ -------- [M1] convert expression to multiplication & use of power

$= \frac{b^3}{3a} \times \frac{b^2}{2a^6b^6}$

$= \frac{1}{6a^7b}$ -------- [A1]

(b) Factorise $14xb + 3ay - 2xy - 21ab$ completely. [2]

$14xb + 3ay - 2xy - 21ab$

$= 14xb - 2xy - 21ab + 3ay$

$= 2x(7b - y) - 3a(7b - y)$ -------- [M1] factorise common terms

$= (7b - y)(2x - 3a)$ -------- [A1]

(c) Solve the inequality $\frac{x}{5} < \frac{x+2}{3} \le \frac{3-2x}{4}$. [3]

$\frac{3-2x}{4} \ge \frac{x+2}{3}$ and $\frac{x+2}{3} > \frac{x}{5}$

$3(3 - 2x) \ge 4(x + 2)$ and $5(x + 2) > 3x$

$9 - 6x \ge 4x + 8$ and $5x + 10 > 3x$

$1 \ge 10x$ and $2x > -10$

$x \le \frac{1}{10}$ -------- [M1] and $x > -5$ -------- [M1]

$-5 < x \le \frac{1}{10}$ -------- [A1]

(d) Simplify $\dfrac{2x(1-6x)}{4x^2-1}-\dfrac{2}{2x+1}$. [3]

$\dfrac{2x(1-6x)}{4x^2-1}-\dfrac{2}{2x+1}$

$=\dfrac{2x(1-6x)}{(2x+1)(2x-1)}-\dfrac{2(2x-1)}{(2x+1)(2x-1)}$ -------- [M1] common denominator

$=\dfrac{2x-12x^2-4x+2}{(2x+1)(2x-1)}$

$=\dfrac{-12x^2-2x+2}{(2x+1)(2x-1)}$ -------- [M1] simplify numerator

$=\dfrac{-2(6x^2+x-1)}{(2x+1)(2x-1)}$

$=\dfrac{-2(3x-1)(2x+1)}{(2x+1)(2x-1)}$ -------- [M1] factorise numerator

$=\dfrac{-2(3x-1)}{2x-1}$ -------- [A1]

(e) Express $2x^2+6x-15$ in the form $a(x+b)^2+c$, where a, b and c are constants.
Hence, solve the equation $2x^2+6x-15=0$, giving your answers correct to four decimal places. [4]

$2x^2+6x-15$

$=2(x^2+3x-\dfrac{15}{2})$

$=2[(x+\dfrac{3}{2})^2-\dfrac{9}{4}-\dfrac{15}{2}]$ -------- [M1] ability to do completing the square

$=2[(x+\dfrac{3}{2})^2-\dfrac{39}{4}]$

$=2(x+\dfrac{3}{2})^2-\dfrac{39}{2}$ -------- [A1]

$2x^2+6x-15=0$

$2(x+\dfrac{3}{2})^2-\dfrac{39}{2}=0$

$(x+\dfrac{3}{2})^2=\dfrac{39}{4}$ -------- [M1] solving using completing the square

$x+\dfrac{3}{2}=\pm\sqrt{\dfrac{39}{4}}$

$x=1.6225$ or $x=-4.6225$ -------- [A1]

3

2. *ABCDE* is a pentagon.
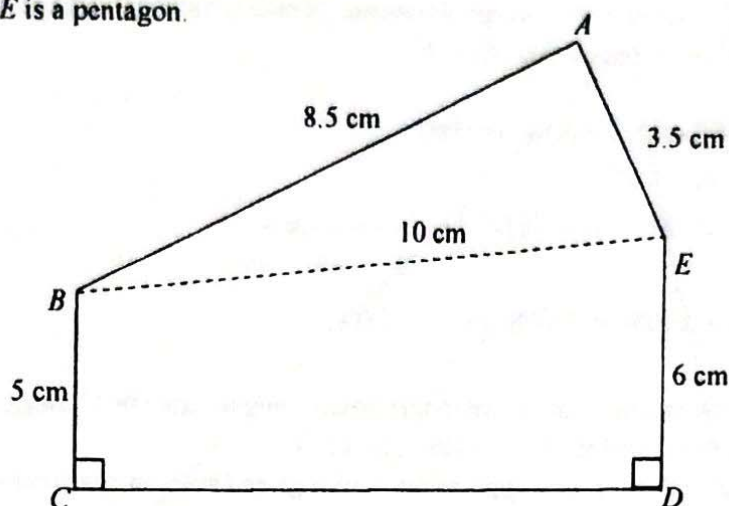
(a) Calculate the area of the pentagon *ABCDE*. [5]

$$10^2 = 8.5^2 + 3.5^2 - 2(8.5)(3.5)\cos B\hat{A}E \text{ -------- [M1] use of cosine rule}$$

$$\cos B\hat{A}E = \frac{8.5^2 + 3.5^2 - 10^2}{2(8.5)(3.5)}$$

$$\cos B\hat{A}E = -0.260504$$

$$\alpha = 74.90°$$

$$B\hat{A}E = 105.1° \text{ or } 285.1° \text{ (rej) ------- [M1] ability to find angle } B\hat{A}E$$

Or

$$A\hat{B}E = 19.75° \text{ or } 340.25° \text{ (rej)}$$

$$B\hat{E}A = 55.15° \text{ or } 304.83° \text{ (rej)}$$

$$\text{area of } \triangle ABE = \frac{1}{2}(8.5)(3.5)\sin 105.1° \text{ -------- [M1] finding area of triangle}$$

$$= 14.36 \text{ cm}^2$$

$$BE = \sqrt{10^2 - 1^2}$$

$$= \sqrt{99} \text{ cm}$$

$$\text{Area of } BCDE = \frac{1}{2}(5 + 6)(\sqrt{99}) \text{ -------- [M1] finding area of trapezium}$$

$$= 54.72 \text{ cm}^2$$

$$\text{Area of } ABCDE = 14.36 + 54.72$$

$$= 69.1 \text{ cm}^2 \text{ -------- [A1]}$$

(b) The pentagon *ABCDE* is a cross-section of a prism which is a model of a house as shown below.
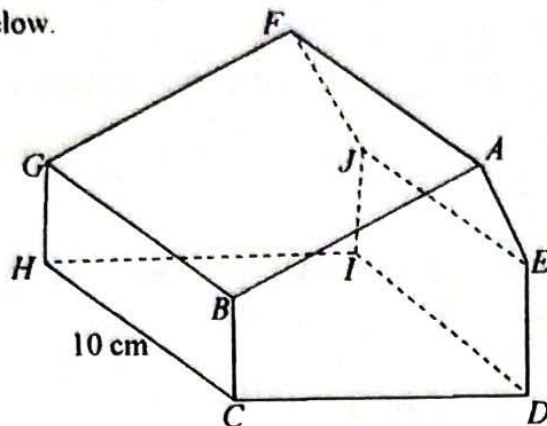
(i) Calculate the total surface area of the model, including the base. [2]

total surface area

$$= 2(69.08) + \underbrace{10(5 + 8.5 + 3.5 + 6 + \sqrt{99})}_{}$$

[M1] area of all surface area (excluding cross-sectional area)

$$= 468 \text{ cm}^2 \quad \text{-------- [A1]}$$

(ii) Calculate the angle of depression of *H* from *E*. [3]
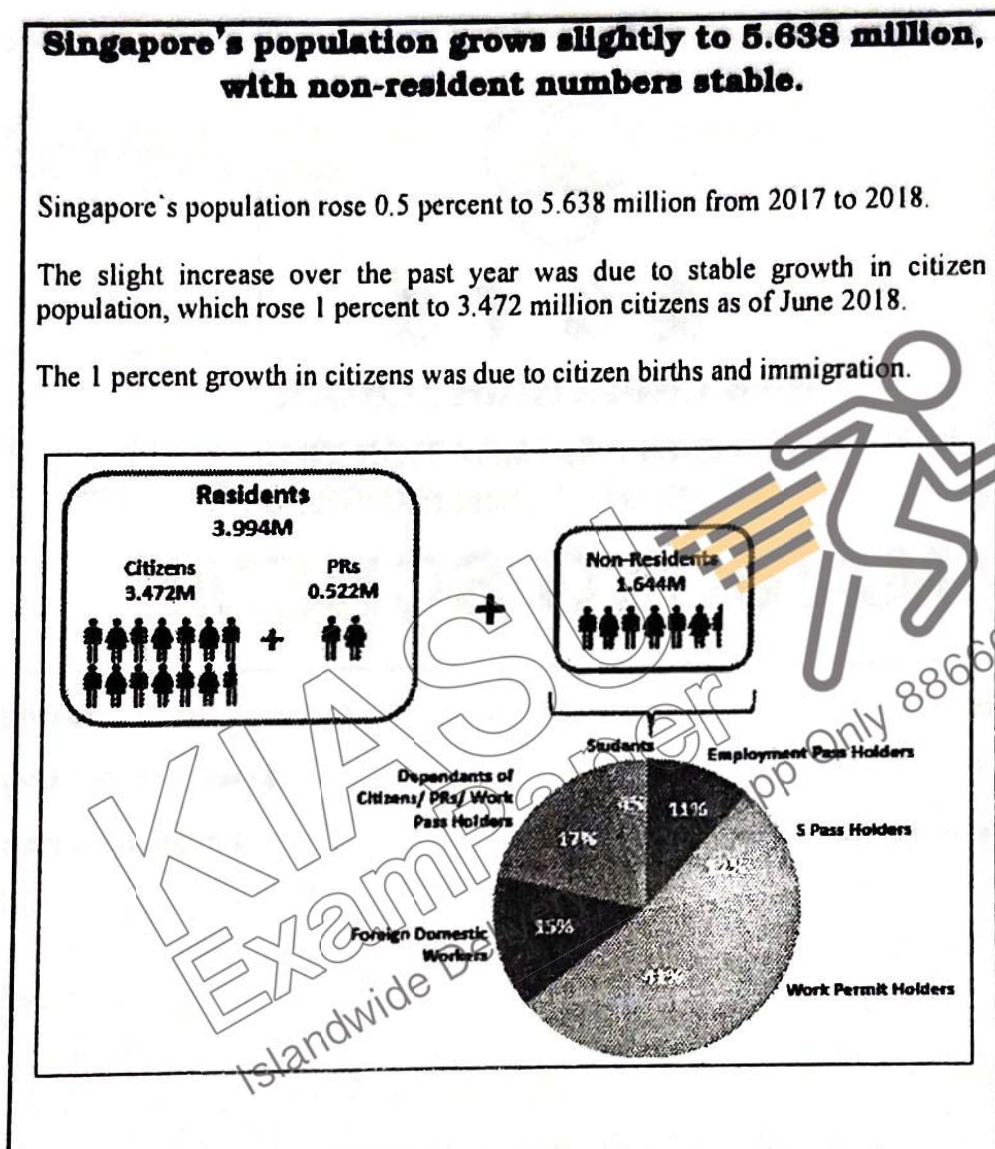
Let the angle of depression be θ

Student use
(i) angle of depression = angle of elevation or
(ii) angle of depression = $90° - 57.5°$

$$HD = \sqrt{10^2 + 99} \quad \text{-------- [M1] finding length HD}$$

$$= \sqrt{199} \text{ cm}$$

$$\tan\theta = \frac{6}{\sqrt{199}} \quad \text{-------- [M1] finding relevant angle (either angle EHD or HED)}$$

$$\theta = 23.0° \quad \text{-------- [A1]}$$

3. There was a newspaper article on Singapore's population published in 2018.

Singapore's population grows slightly to 5.638 million, with non-resident numbers stable.

Singapore's population rose 0.5 percent to 5.638 million from 2017 to 2018.

The slight increase over the past year was due to stable growth in citizen population, which rose 1 percent to 3.472 million citizens as of June 2018.

The 1 percent growth in citizens was due to citizen births and immigration.

Source:
https://www.channelnewsasia.com/news/singapore/singapore-s-population-grows-slightly-to-5-64m-with-non-resident-10763132

(a) Calculate the Singapore's total population in 2017.   [2]

100.5% → 5.638 million

100% → $\frac{100}{100.5}$ × 5.638 million -------- [M1] showing $\frac{100}{100.5}$ × 5.638

= 5.61 × 10^6 -------- [A1] or 5.61 million

(b) Express the number of Foreign Domestic Workers as a percentage of the Singapore's total population in 2018. [2]

Number of foreign domestic workers

$= \frac{15}{100} \times 1.644 \times 10^6$

$= 0.2466 \times 10^6$ -------- [M1] must in 10^6 or million

$\frac{0.2466 \times 10^6}{5.638 \times 10^6} \times 100\% = 4.37\%$ -------- [A1]

(c) Based on the information shown in this article, do you agree that "Singapore's total population will reach 6.3 million by 2030?"
Support your answer with appropriate workings and state an assumption that you have made in your calculation. [3]

$2030 - 2018 = 12$ years

Population of citizen by 2030

$= (1.01)^{12} \times 3.472 \times 10^6$ -------- [M1] finding the increase of citizen only

$= 3.912 \times 10^6$

total population by 2030

$= (3.912 + 0.522 + 1.644) \times 10^6$

$= 6.08 \times 10^6$ -------- [A1]

Assumption:
There is no increase for PRs and Non-Residents or
There is a constant growth of citizens at 1% per year -------- [M1]

(d) State one aspect of the diagram that may be misleading and explain how this may lead to a misinterpretation. [2]

Accept any logical answer

(i) Pictogram used alternate of a male and female figure --- [B1]
    [Reason] misleading readers that there is an equal number of male and female population. --- [B1]

(ii) Inaccurate use of ½ pictogram "figure" for "non-resident" & ¼ in "citizen" --- [B1]
    [Reason]
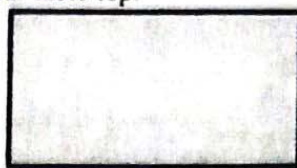   in PRs   : if   1 figure → 0.261 M
    in citizen :   3.472 M should be represented by 13.3 figures
                                        not 13 + ¾ figure --- [B1]

4. The diagram shows a rectangular table top

x m

The area of the table top is 5 square metres.
The length of the table top is x metres.
The perimeter of the table top is y metres.

(a) Show that $y = 2x + \dfrac{10}{x}$. [2]

Width of table top $= \dfrac{5}{x}$ -------- [M1]

Perimeter, $y = x + \dfrac{5}{x} + x + \dfrac{5}{x}$ -------- [M1]

$y = 2x + \dfrac{10}{x}$

(b) The variables x and y are connected by the equation $y = 2x + \dfrac{10}{x}$.

x	0.5	1	1.5	2	4	6	8
y	p	12	q	9	10.5	13.7	17.3

Find the value of p and of q. [1]

$p = 21$
$q = 9.7$ (accept also 9.67) -------- [B1] for both answers

(c) Using a scale of 1 cm to represent 1 unit, draw a horizontal x-axis for $0 \leq x \leq 8$.

Using a scale of 1 cm to represent 1 unit, draw a vertical y-axis for $0 \leq y \leq 22$.

On your axes, plot the points given in the table and join them with a smooth curve. [3]

8

$y = 2x + \frac{10}{x}$

$y = 14$

$m = \frac{1}{2}$
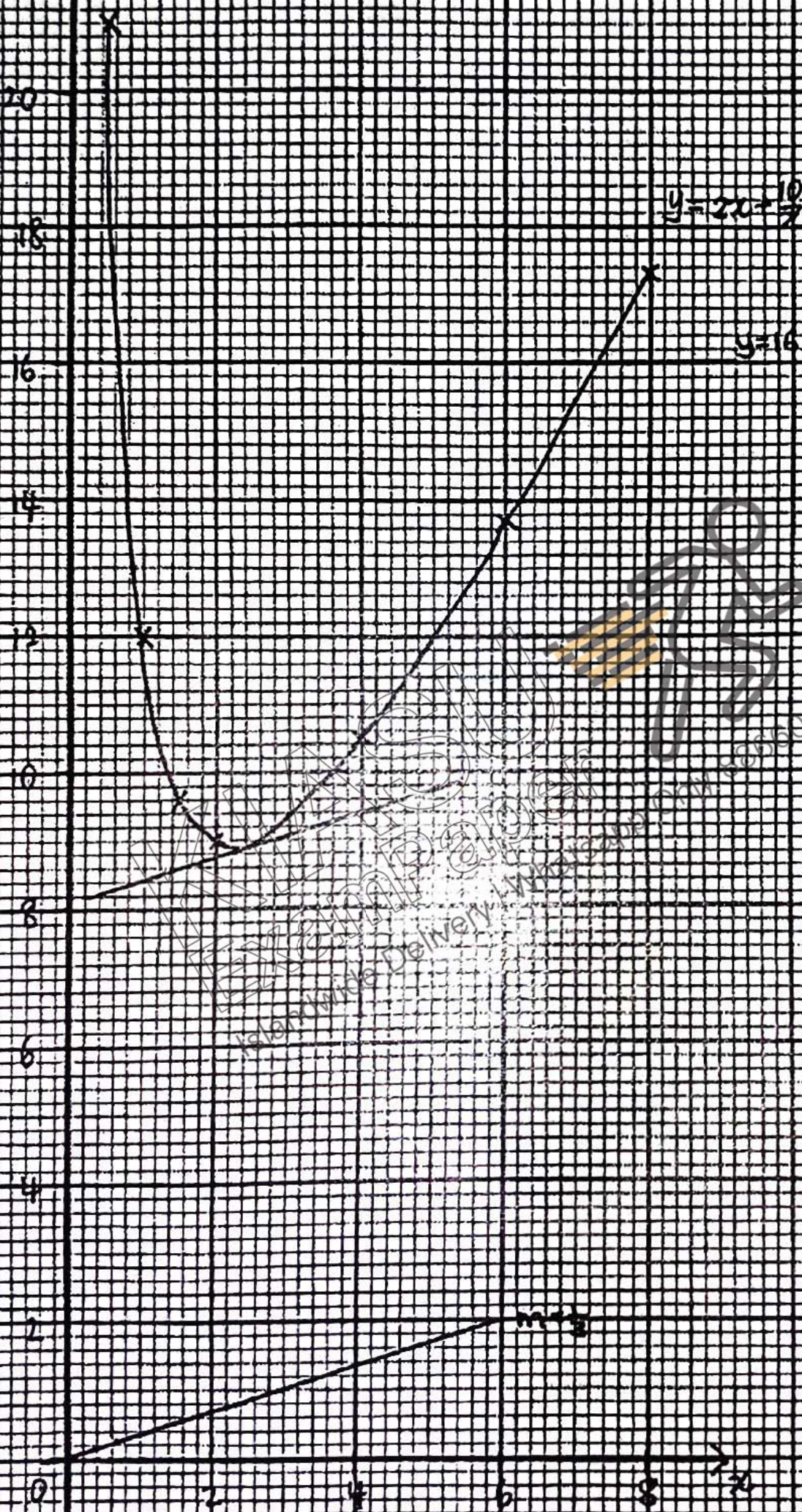
(d) Use your graph to find the possible width of the table top if the perimeter of the table top is 16 m. [2]

When $P = 16$,
$x = 0.7$ (± 0.1) ------ [B1] answer must be rejected as length is longer
or $x = 7.3$ (± 0.1) -------- [B1]

(e) By drawing a suitable straight line, find the x-coordinate of the point at which gradient of the curve is $\frac{1}{3}$. [2]

Draw a line with grad $= \frac{1}{3}$ on the graph -------- [M1]

x-coordinate $= 2.3$ (± 0.1) -------- [A1]

5. Given equation of a line L is $3x - 2y = 8$.

(a) State the gradient of line L. [1]

$2y = 3x - 8$

$y = \frac{3}{2}x - 4$

Gradient $= \frac{3}{2}$ -------- [B1]

(b) If $P(k, -2)$ is a point on the line L, find the value of k. [1]

$3k - 2(-2) = 8$

$k = \frac{4}{3}$ -------- [B1]

(c) Find the equation of another line that is parallel to the x-axis and passes through P. [1]

$y = -2$ -------- [B1]

(d) Calculate the perpendicular distance from the origin, O, to the line L. [4]

$3x - 2y = 8$

When $x = 0$, $y = -4$ $(0, -4)$

When $y = 0$, $x = \frac{8}{3}$ $(\frac{8}{3}, 0)$ -------- [M1] finding relevant coordinates

Distance between 2 points

$= \sqrt{\left(\frac{8}{3}\right)^2 + (4)^2}$ -------- [M1]
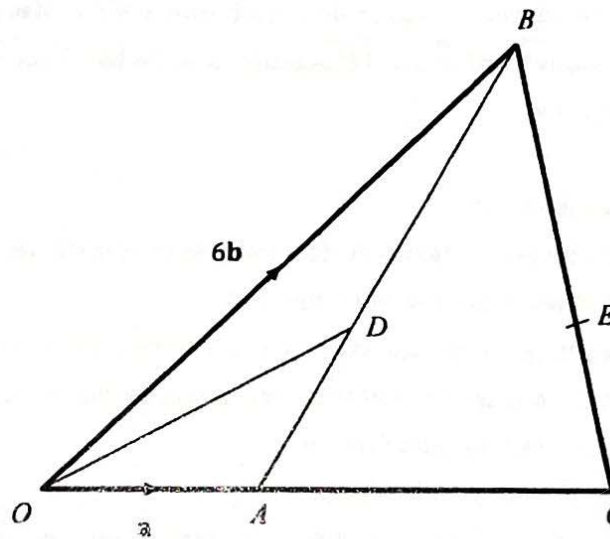
$= 4.8074$ units

Let the perpendicular distance be h.

$\frac{1}{2}(4)\left(\frac{8}{3}\right) = \frac{1}{2}(h)(4.8074)$ -------- [M1] suitable method to find h

$h = 2.22$ units -------- [A1] accept $h = 2.21$ units

11

6.

In the diagram, $\overrightarrow{OA} = a$, $\overrightarrow{OB} = 6b$ and $\overrightarrow{OA} = \frac{1}{3}\overrightarrow{OC}$.

D is a point on AB such that $3\overrightarrow{AD} = 2\overrightarrow{DB}$ and E is a point on BC such that

$\overrightarrow{CE} : \overrightarrow{EB} = 4 : 5$.

(a) Express, as simply as possible, in terms of **a** and **b**,

(i) $\overrightarrow{BA}$, [1]

$$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB}$$
$$= a - 6b \qquad \text{B1}$$

(ii) $\overrightarrow{OD}$, [1]

$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$$
$$= \overrightarrow{OA} - \frac{2}{5}\overrightarrow{BA}$$
$$= a - \frac{2}{5}(a - 6b)$$
$$= \frac{3}{5}(a + 4b) \qquad \text{B1}$$

12

(iii) $\overrightarrow{CB}$, [1]

$$\overrightarrow{CB} = \overrightarrow{OB} - \overrightarrow{OC}$$
$$= 6b - 3a$$
$$= 3(2b - a) \qquad \text{B1}$$

(iv) $\overrightarrow{AE}$. [2]

$$\overrightarrow{AE} = \overrightarrow{AC} + \overrightarrow{CE}$$
$$= 2a + \frac{4}{9}\overrightarrow{CB}$$
$$= 2a + \frac{4}{9}(6b - 3a)$$
$$= \frac{2}{3}(a + 4b) \qquad \text{B1}$$

(b) Write down the relationship between OD and AE. Explain your answer. [2]

$$\overrightarrow{OD} = \frac{3}{5}(a + 4b)$$
$$\overrightarrow{AE} = \frac{2}{3}(a + 4b)$$

Since

$$|\overrightarrow{OD}| = \frac{9}{10}|\overrightarrow{AE}|,$$ M1 – Relationship between OD and AE

OD and AE are parallel lines. A1 – Parallel lines

13

(c) Find the ratio of
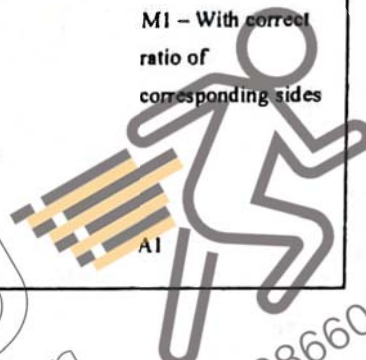
    (i)   area of triangle CAE : area of triangle AOD,   [2]

$$\frac{\text{area of triangle } CAE}{\text{area of triangle } AOD} = \frac{\frac{1}{2}(CA)(AE)\sin\theta}{\frac{1}{2}(OA)(OD)\sin\theta}$$

$$= \frac{AE}{OD} \times \frac{CA}{OA}$$

$$= \frac{10}{9} \times \frac{2}{1}$$

$$= \frac{20}{9}$$

M1 – With correct ratio of corresponding sides

20 : 9   A1

    (ii)   area of triangle CAE : area of triangle AOB [2]

$$\frac{\text{area of triangle } CAE}{\text{area of triangle } AOB}$$

$$= \frac{\text{area of triangle } CAE}{\text{area of triangle } AOD} \times \frac{\text{area of triangle } AOD}{\text{area of triangle } AOB}$$

$$= \frac{20}{9} \times \frac{\frac{1}{2}(h)(AD)}{\frac{1}{2}(h)(AB)}$$

$$= \frac{20}{9} \times \frac{2}{5}$$

$$= \frac{8}{9}$$

M1 – Common height relationship

8 : 9   A1

14

7. (a) There are two boxes of sweets containing toffees and chocolates.

Box A contains 8 toffees and 4 chocolates, whereas box B contains 7 toffees and 3 chocolates.

Jolin loves chocolates.

One of the boxes is chosen at random and a sweet is taken out.

If she gets a chocolate, she will consume it.

If she did not get a chocolate from the first selection, she will place the sweet into the other box and she will select again from the other box.

Jolin limits herself to two selections.

Find, as a fraction in its simplest form, the probability that

(i) Jolin will have her favourite chocolate from the first selection, [3]

$$P(\text{first selection}) = P(\text{Box A}) + P(\text{Box B})$$

$$= \left(\frac{1}{2}\right)\left(\frac{4}{12}\right) + \left(\frac{1}{2}\right)\left(\frac{3}{10}\right)$$

M1 – with ½

M1 – 4/12 + 3/10 (without ½)

$$= \frac{19}{60}$$

A1

(ii) Jolin will have her favourite chocolate. [3]

$$P(\text{favourite})$$

$$= P(\text{at first}) + P(\text{at second})$$

M1 – from (i)

$$= \frac{19}{60} + P(\text{no A, yes B}) + P(\text{yes A, no B})$$

$$= \frac{19}{60} + \left(\frac{1}{2}\right)\left(\frac{8}{12}\right)\left(\frac{3}{11}\right) + \left(\frac{1}{2}\right)\left(\frac{7}{10}\right)\left(\frac{4}{13}\right)$$

M1 – with ½ and 2 cases for P(at second) correct

$$= \frac{4421}{8580}$$

A1

15

(b) Your friend, Kenton gives you a chance to win $1000 by playing a game of "Guess the number". There are two options of the game that he allows you to choose.

Option A

He uses a random number generator to choose a number from 1 to 8.
If you guess it correctly, you win.

Option B

You flip a fair coin.
If the coin lands on head, Kenton will roll a fair 6-sided die. If you manage to guess what it rolled, you win.
If the coin lands on tail, Kenton will use a random number generator to choose a random number from 1 to 8. If you guess the number correctly, you win.

Which option will you choose? Explain your answer. [3]

Option A	
$P(\text{win}) = \dfrac{1}{8}$	B1
Option B	
$P(\text{win}) = P(H, \text{die}) + P(T, \text{No. Gen})$	
$= \left(\dfrac{1}{2}\right)\left(\dfrac{1}{6}\right) + \left(\dfrac{1}{2}\right)\left(\dfrac{1}{8}\right)$	M1
$= \dfrac{7}{48} \left(> \dfrac{1}{8} \right)$	
Since <u>Option B has a higher probability</u>, Option B should be chosen.	A1 – with both options' probability correctly calculated.

- A regular hexagon forms within the inner circle *ABCDEF* with *OB* = 5 cm.

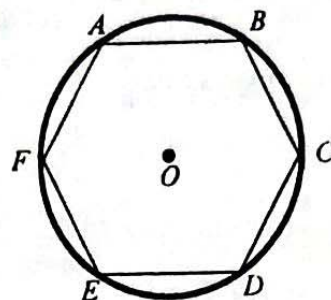 This information is illustrated in Figure 3.

Figure 3

(a) Show that the radius of the outer circle is 13.31 cm, correct to 2 decimal places. **[6]**

Let *M* be the midpoint of AB.

Consider $\triangle AGX$,

Using Cosine Rule,

$AG^2 = 18^2 + 18^2 - 2(18)(18)\cos\dfrac{\pi}{6}$ M1 – finding line

$AG = 9.31749$ cm   segment *AG*

Consider $\triangle AMO$,

$\angle AOM = 360° \div 12$ M1 – finding angle

$\qquad = 30°$ AOX or AOB = 60°

$AM = 5\sin 30°$ M1 – AX

$MO = 5\cos 30°$ M1 – XO

Consider $\triangle AGM$,

By Pythagoras theorem,

$GM = \sqrt{AG^2 - AM^2}$

$\qquad = \sqrt{9.31749^2 - (5\sin 30°)^2}$
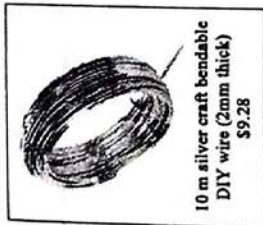
$\qquad = 8.9758$ cm

Radius $= GM + MO$

$\qquad = 8.9758 + 5\cos 30°$ M1 - GX + XO

= $111.36 (to 2dp)

A1

(b) Team Hō'ola decided to make 50 dreamcatcher-inspired ornaments and Janice chanced upon the following promotion.

10 m silver craft bendable DIY wire (2mm thick) $9.28

What is the estimated cost to make these ornaments?

[5]

arc $AG = (18)\left(\frac{\pi}{6}\right)$ M1

= 3π cm

Petals = $12 \times 3\pi$

= 36π cm

Circumference of small circle = $2\pi(5)$

= 10π

Circumference of big circle = $2\pi(13.30597)$ M1 – Both circumference found.

= 26.6119π

Amount of wire for 50 dreamcatchers   M1 – Total circumference multiply by 50

= $(26.6119\pi + 10\pi + 36\pi) \times 50$

= 3630.59π cm

= 36.3059π m   M1 – Conversion from cm to m and rounded up.

Rolls of wire needed

= $\dfrac{36.3059\pi}{10}$

= 11.4058

= 12 (rounded up)

Cost = 12 × $9.28

(c) Find the area of the shaded region as shown in Figure 1. [5]

area of AG segment

$$= \frac{1}{2}(18)^2\left(\frac{\pi}{6}\right) - \frac{1}{2}(18)(18)\sin\frac{\pi}{6} \qquad \text{M1}$$

$$= 3.823 \text{ cm}^2$$

$$\text{area of } \Delta AGX = \frac{1}{2}(8.9758)(5\sin 30°) \qquad \text{M1 – area of right}$$
$$\text{angle triangle AGX.}$$

$$= 11.21975 \text{ cm}^2$$

$$\text{area of } \Delta OXA = \frac{1}{2}(5\cos 30°)(5\sin 30°) \qquad \text{M1 – area of right}$$
$$\text{angle triangle OXA.}$$

$$= 5.41266 \text{ cm}^2$$

area of shaded region

$$= \pi(13.30597)^2 - 5.41266 \times 12 \qquad \text{M1 – big circle area}$$

$$\qquad - (3.823 + 11.21975) \times 12$$

$$= 310.7504 \text{ cm}^2$$

$$= 311 \text{ cm}^2 \qquad \text{A1}$$

21

9. (a) The cumulative frequency curve below shows the time spent in minutes by a group of 80 teenagers on Instagram (a social media platform) on a particular day.
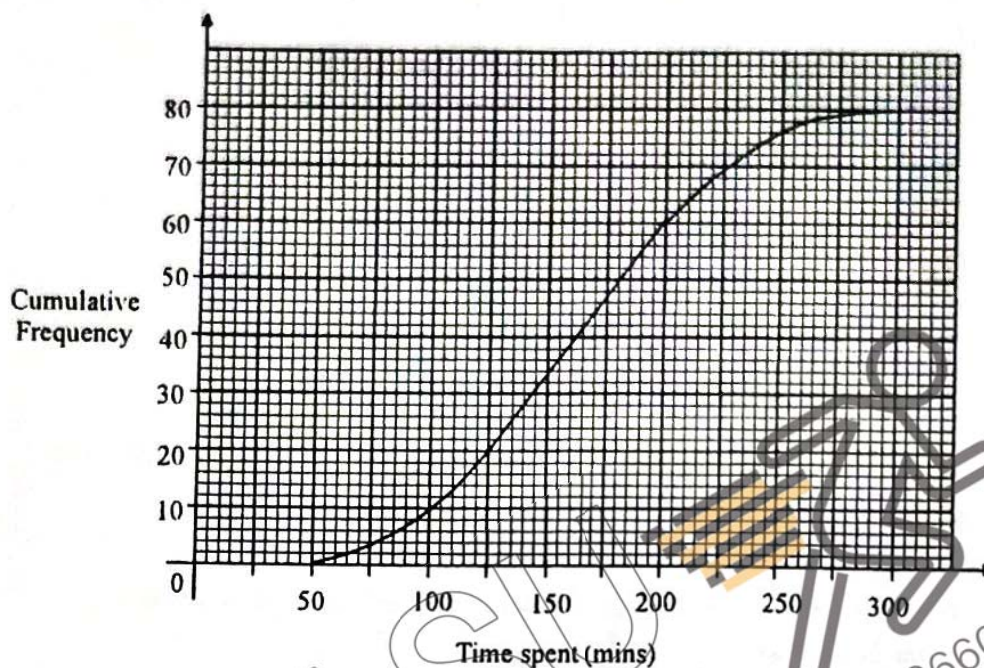
Use the curve to estimate

(i) the median, [1]

$$\frac{N}{2} = 40$$

median = 162.5 min   B1

(ii) the interquartile range. [2]

$$\frac{N}{4} = 20 \qquad \frac{3N}{4} = 60$$

Q1 = 125 min   Q3 = 200 min   B1 – both Q1 and Q3 correct

Interquartile Range = Q3 − Q1

= 200 − 125

= 75 min   A1

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(b) Through a market research, it was found out that the time spent on Facebook (another social media platform) is less popular and less consistent among the same group of 80 teenagers. A second cumulative frequency curve for the same group of 80 teenagers spending their time on Facebook is drawn.
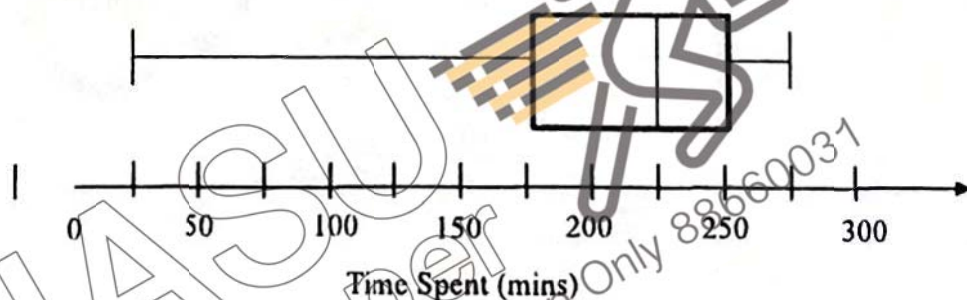
Describe how the second cumulative frequency curve may differ from the curve for Instagram. [2]

The curve will <u>shift to the left</u> and has a <u>gentler slope</u> as compared to the given curve.	B1 – shift to left B1 – Gentler slope

(c) The box-and-whisker plot represents the distribution of the time spent for the same group of 80 teenagers on SnapChat (another social media platform).

For this group of 80 teenagers, which of the social media platforms - Instagram, Facebook or Snapchat, is the most popular?
Support your answer with an appropriate statistical measure. [2]

|	
Median (Instagram) = 162.5 min Median (Facebook) < 162.5 min Median (SnapChat) = 225 min Since <u>Snapchat has the highest median</u>, it is the most popular.	M1 – Evidence for median especially for SnapChat A1 – Median as statistical comparison
*If students show no data evidence but mentioned about highest median, award 1 mark.	

10. Ms Tan got her new home recently and the layout of her house is shown in the diagram below. The layout is drawn to scale.
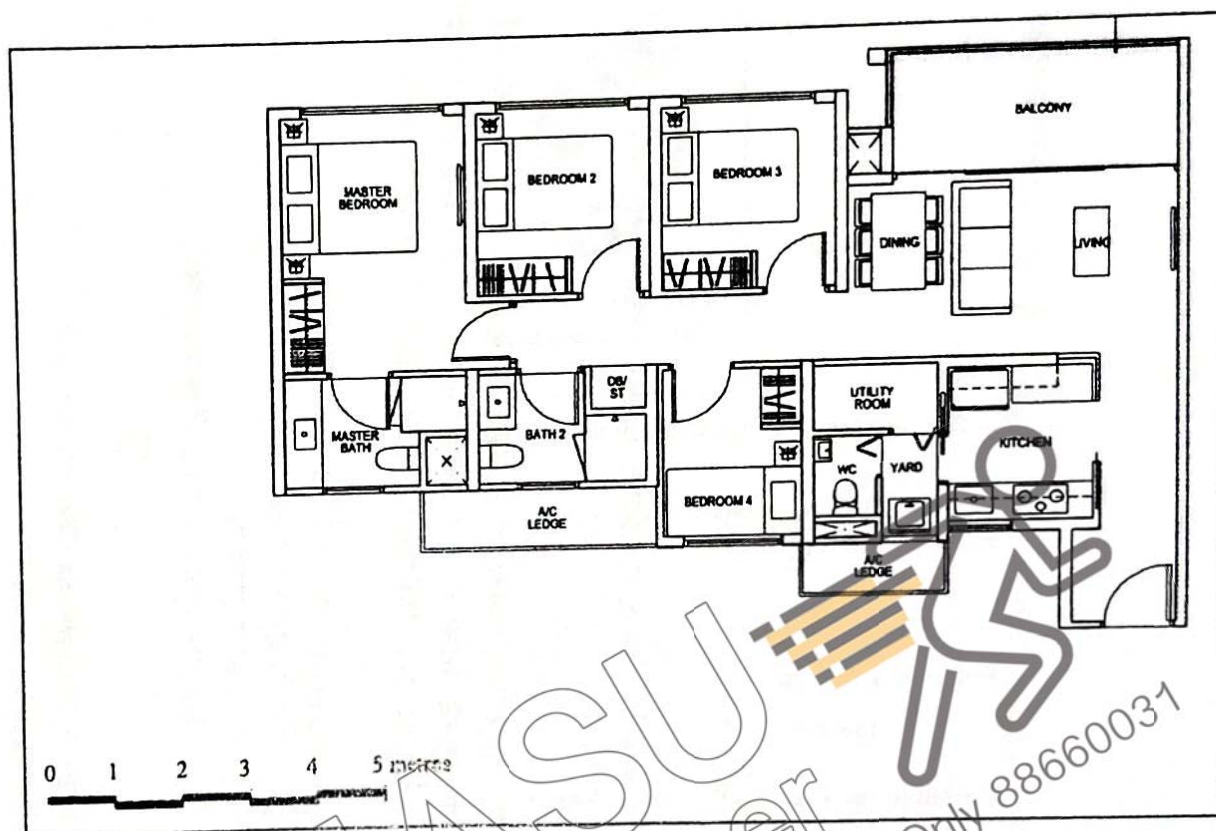
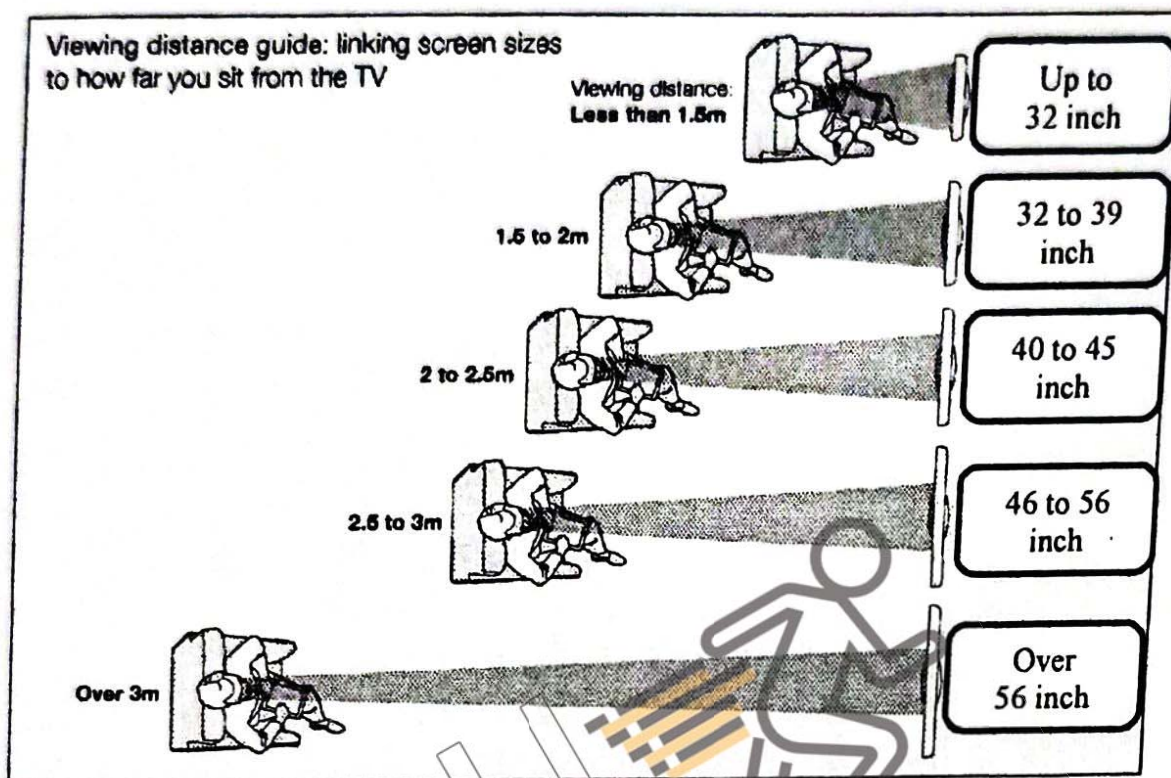(a) Express the scale of the map in the form of 1 : n [1]

1cm = 1m

1cm = 100 cm

1: 100   B1

(b) Ms Tan decides to get a television set for her living room.

She found the following infographic online.

Viewing distance guide: linking screen sizes to how far you sit from the TV

Viewing distance	Screen size
Less than 1.5m	Up to 32 inch
1.5 to 2m	32 to 39 inch
2 to 2.5m	40 to 45 inch
2.5 to 3m	46 to 56 inch
Over 3m	Over 56 inch

What is the range of television size which Ms Tan should get for her living room? [2]

Distance of Sofa from TV = 3 cm × 100 cm = 3 m	M1 - Measured distance can range from 2.3 cm to 3.1 cm
Range – 46 to 56 inch	A1

(c) Ms Tan decided to shop for her television set online and she saw the following options:

LG OLED55B8STB 55" OLED 4K SMART TV

55-inch OLED 4K SMART TV

SGD2,988.00

Quantity   1

LG OLED65B8STB 65" OLED 4K SMART TV

65-inch OLED 4K SMART TV

SGD4,988.00

Quantity   1

Ms Tan pays a downpayment of $2 000 for her television set as suggested in (b).
She pays the remaining amount over 3 months, with a simple interest rate of
7 % per annum.
Calculate her monthly instalment.   [3]

Ms Tan to purchase 55 inch TV.	
Remaining amount = $2988 − $2000	M1
= $988	
Interest = $\dfrac{988 \times 7 \times \frac{3}{12}}{100}$	M1 – with correct $988 and 3/12
= $17.29	
Monthly instalment = $\dfrac{\$988 + \$17.29}{3}$	
= $335.10 (to 2dp)	A1