COMMONWEALTH SECONDARY SCHOOL
MID-YEAR EXAMINATION 2019

MATHEMATICS
PAPER 1

Name: _____ ( ) Class: _____

SECONDARY FOUR EXPRESS
SECONDARY FIVE NORMAL ACADEMIC

Friday 03 May 2019
10 30 – 12 30
2 h

4048/1

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

Name of setter: Mr Toinh Long Teng

For Examiner's Use	
Presentation	
Accuracy	
Total	80

Parent's Signature: _____

This paper consists of **17** printed pages including the cover page.

1

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta \text{, where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta \text{, where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 Write the following in order of size, in descending order.

$$\pi, \, 3.14\dot{2}, \, \frac{22}{7}, \, 3.14\dot{2}$$

Answer .   [2]

_____

2 Factorise completely $16p^4 - 81q^4$.

Answer .... [2]

_____

3 Given that $\dfrac{1}{27} = 9^k$, find k.

Answer $k = ...$ [2]

_____

4 The stem-and-leaf diagram shows the times, in minutes, taken by some students to complete a task.

$$
\begin{array}{c|ccccc}
3 & 2 & 4 & 5 & 6 \\
4 & 0 & 1 & 1 & 9 & 9
\end{array}
$$

Key: $3 \mid 4$ represents 34 minutes

For these times, find

(a) the lower quartile,

Answer .   minutes [1]

(b) the mean,

Answer   .minutes [1],

(c) the standard deviation,
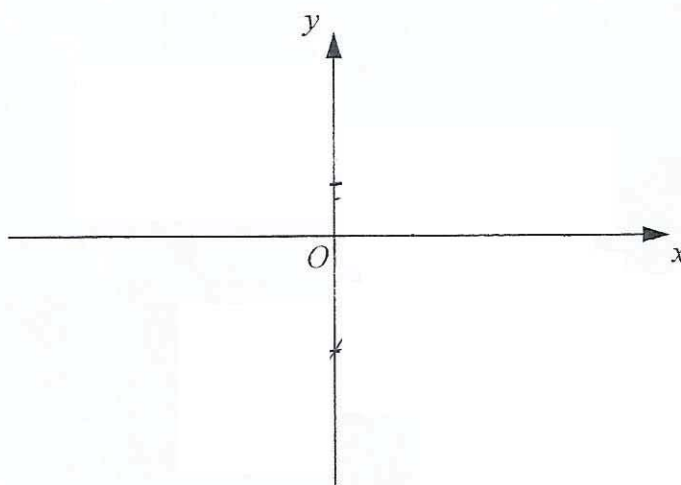
$$\sqrt{\frac{x^2}{n} - \bar{x}^2} \;=\; 5.81$$

Answer   minutes [1]
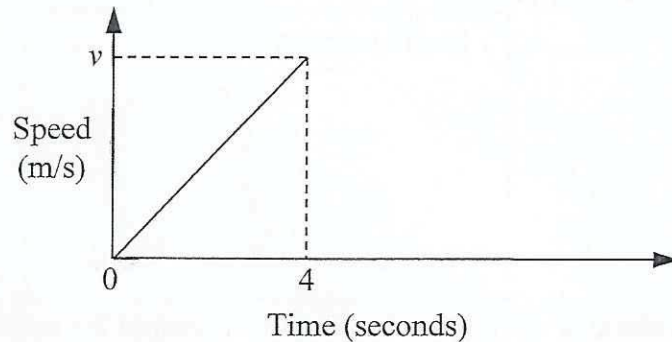
---

5 **Sketch** the graph of $y = -(2-x)^2 + 1$ on the axes below.

Indicate clearly the values where the graph crosses the x- and y-axes and the coordinates of any turning points.

[2]

---

6   The diagram shows the speed-time graph for part of a car's journey between two sets of traffic lights.

The distance travelled in the first 4 seconds is 20 metres.

(a)   Calculate the value of v.

Answer $v = ..$ [1]

The car then decelerated at twice the rate of the acceleration.

(b)   Calculate the average speed of the car for the entire journey.

Answer .   m/s [2]

7   A club is made up of members that are either a child, an adult or a senior citizen. There are 19 children and 14 senior citizens. The club wishes to maintain that at most three-fifths of the members are adults.
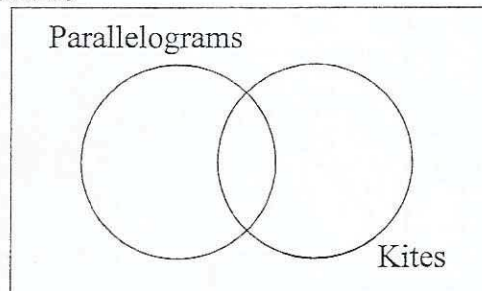
By forming an inequality, find the maximum number of adults possible.

Answer .   [3]

8 The Venn diagram illustrates the relationship between two different types of quadrilaterals.

Quadrilaterals

Parallelograms

Kites

(a) What special shape is represented by the intersection of the sets representing Parallelograms and Kites?

Answer .... [1]

(b) Using an appropriate symbol, complete the statement:
{Parallelograms}.....{Trapeziums}

Answer .... [1]

---

9 A sum of money was divided between *A*, *B* and *C* in the ratio 2 : 3 : 4.
If the money had been divided equally between them, *A* would have received an extra $40.
What was the total sum of money?

Answer .... [2]

---

10 The probability that a phone will ring in a thirty-minute interval is $\frac{5}{6}$. What is the probability that the phone will not ring in a one-hour interval?
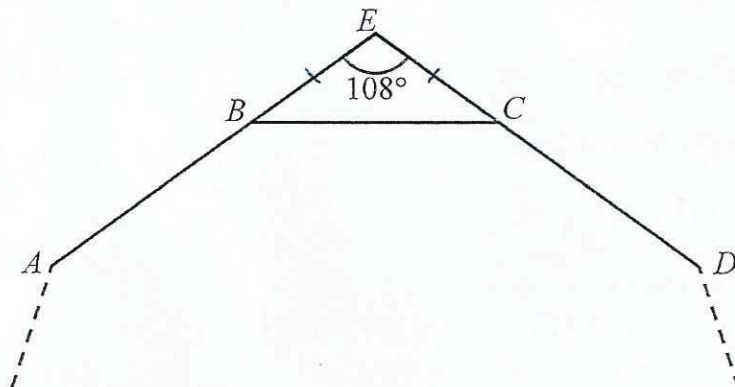
Answer .... [2]

---

11 Adam can lay 70 bricks in 30 minutes.
Charlie can lay 80 bricks in 45 minutes.
Adam and Charlie work together to lay a total of 1000 bricks.
If they continue to lay bricks at the same rate, how long will it take them to lay the 1000
bricks? Give your answer in hours and minutes, to the nearest minute.

Answer .... [3]

12 The diagram shows three of the sides, *AB*, *BC* and *CD* of a regular polygon. *AB* produced and
DC produced meets at *E*. Angle *BEC* = 108°

Find the number of sides of the regular polygon.

Answer .... [3]

13 Solve $\dfrac{5x}{14} - \dfrac{2x-3}{21} = 1$.

Answer $x =$.[2]

14 Solve $x^2 + 4x - 11 = 0$ by using completing the square.
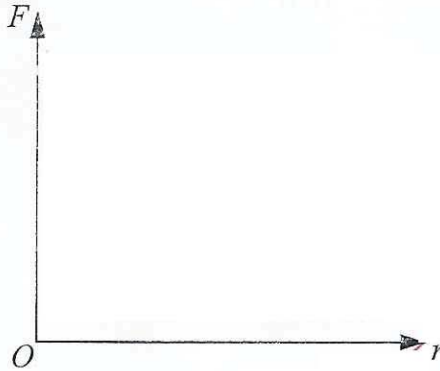
Answer $x =$ or $x =$ [3]

15 Given that the value of $\dfrac{1}{x^2} + \dfrac{1}{y^2} = 3$ and $xy = 4$, find the value of $(x+y)^2$.

Answer ..   [3]

16 The attractive force, F newtons, between two stars, is inversely proportional to the square of the distance between the centres of the two stars, r km.

(a) Sketch the graph of F against r.

[1]

(b) The distance between the centres of star A and star B is 150% larger than the distance between the centres of star C and star D. Find the ratio of the attractive force between star A and star B to the attractive force between star C and star D.
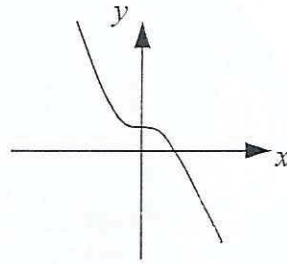
Answer .... [3]

17 (a) Write down a possible equation for each of the sketch graphs below.
In each case select one of the equations from the box below.

$y = x^2 - 3$	$y = -x^2 + 3$	$y = 3^x$
$y = -x^3 + 3$	$y = 3^{-x}$	$y = x^3 + 3$

(i)

Answer .... **[1]**

(ii)
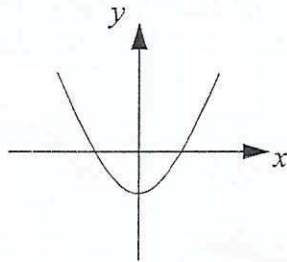
Answer .... **[1]**

(iii)
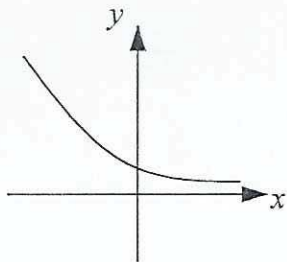
Answer .... **[1]**
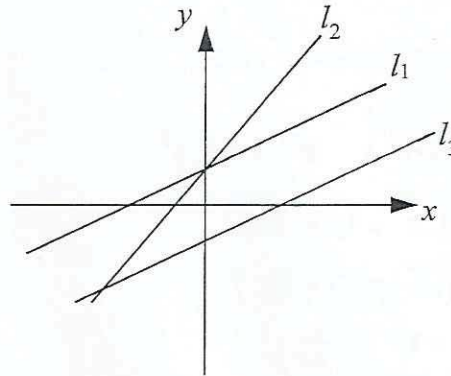
(b) The diagram shows three lines l_1, l_2 and l_3.

The equation of l_1 is $y = 2x + 1$.

(i) l_1 and l_2 have the same y-intercept. State a possible value of the gradient of l_2.

Answer .   [1]

(ii) l_1 and l_3 have the same gradient. State a possible equation of l_3.

Answer ..   [1]

---

18 A solid cylinder has radius r cm and height h cm. A solid hemisphere has radius r cm. The total surface area of the cylinder and hemisphere are equal.

Work out, in terms of r, the volume of the cylinder.

Answer   . cm^3 [3]

19 Daniel invested a sum of money in an account paying compound interest at r% per year. After 3 years, the money had earned **total interest** of 20% of the principal.

Find the value of r, correct to one decimal place.

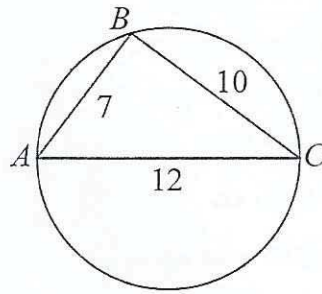*Answer* .... [3]

_____

20 Two glasses are geometrically similar. The height of the small glass is 9 cm and the height of the large glass is 15 cm.

If a big tank of fruit juice can fill up 125 small glasses, how many large glasses could be filled from the same big tank?

Answer .... [3]

_____

21

In the diagram, A, B and C are points on a circle. $AB = 7$ cm, $BC = 10$ cm and $AC = 12$ cm. Explain, with reasons, whether AC is a diameter of the circle.

Answer

[3]

22 It is given that $T = 2\pi \sqrt{\dfrac{L}{g}}$.

(a) Calculate the value of T when $L = 1.2$ and $g = 9.81$. Write your answer correct to three decimal places.

Answer .. [1]

(b) Rearrange the formula to make g the subject.

Answer $g =$. [3]

23 The Singapore River is 3.2 kilometres long. It is represented on a map with a distance of 2 cm.

   (a) Express the scale of the map in the form $1 : n$.

Answer   [2]

   (b) The length of the Ayer Rajah Expressway is 27 km, corrected to the nearest kilometre. Find the greatest possible distance of the expressway on the map.

Answer .   cm [1]

   (c) The Jurong Lake District is represented by an area of 1.4 square centimetres on the map. Find the actual area, in square kilometres, of the district.

Answer   km^2 [2]

24 (a) Express 84 as a product of its prime factors, leaving your answer in index notation.

Answer ..   [1]

(b) Find the highest common factor of 84 and $2 \times 3^3 \times 5$.

Answer   [1]

(c) The lowest common multiple of 84 and x is $2^3 \times 3 \times 7^2$. Find the smallest possible value of x.

Answer .   [2]

25 The *n*th term of a sequence is given by $n(n+3)$.

(a) Find the fifth term.

Answer .... [1]

(b) One term in the sequence is 154. Find the value of *n* for this term.

Answer n = ... [2]

(c) Explain why every term in the sequence is an even number.

Answer ....

.... [2]

26
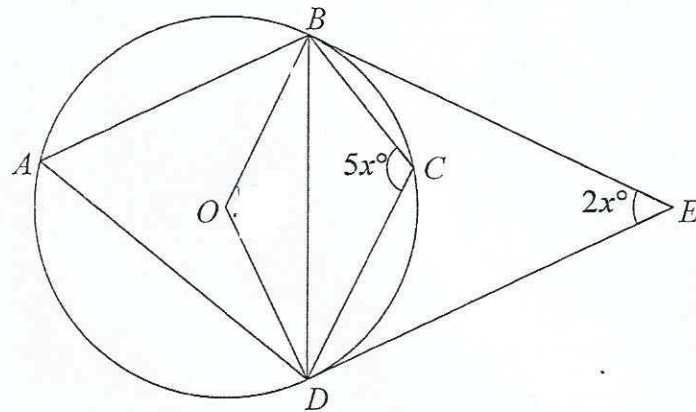
In the diagram, A, B, C and D are points on a circle, centre O. BE and DE are tangents to the circle at points B and D respectively.
Angle $BCD = 5x°$ and angle $BED = 2x°$.

Find x.
Showing your working clearly and give reasons.

Answer $x =$ [4]

END OF PAPER

COMMONWEALTH SECONDARY SCHOOL
MID YEAR EXAMINATION 2019

MATHEMATICS
PAPER 2

Name: _____ (   )   Class: _____

SECONDARY FOUR EXPRESS
SECONDARY FIVE NORMAL ACADEMIC
4048/2

Monday 6 May 2019
08 00 – 10 30
2h 30 min

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.

Write in dark blue or black pen.

You may use a pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.

If working is needed for any question it must be shown with the answer.

Omission of essential working will result in loss of marks.

The use of an approved scientific calculator is expected, where appropriate.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

Name of setter: Mrs Ang YM
　　　　　　　　　Mr Koh HY

For Examiner's Use	
Total	100

Parent's Signature: _____

This paper consists of **13** printed pages including the cover page.

[Turn over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2} ab \sin C$$

$$\text{Arc length} = r\theta \text{ , where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2} r^2 \theta \text{ , where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1. **(a)** Solve the inequality $\dfrac{3-x}{5} < 1 + \dfrac{2x+1}{4}$. [2]

 (b) Express as a single fraction in its simplest form $\dfrac{4y}{3-2y} - \dfrac{y}{(2y-3)^2}$. [2]

 (c) Simplify $\dfrac{(-3h)^2}{8h^3 j^5} \div \dfrac{27h^4 j^2}{4j^3}$. [2]

 (d) Simplify $\left(\dfrac{256p^{16}}{q^{20}r^{-4}}\right)^{-\frac{1}{4}}$. [2]

 (e) Solve the equation $\dfrac{10}{x^2-9} - \dfrac{3}{x+3} = 1$. [3]

---

2. One astronomical unit (1 au) is a unit of length defined as 149597870700 metres, which is roughly the average distance between the earth and the sun.

 (a) Express one astronomical unit in metres, correct to 3 significant figures in standard form. [1]

 (b) The speed of sound is 343 m/s. How long, in seconds, does sound take to travel a distance of 1 au? Give your answer in standard form, correct to 3 significant figures. [2]

 (c) The average distance between the earth and the moon is 384400 km. Express this distance as a percentage of 1 au. [2]

 (d) A rocket travels a distance of one metre in 8000 nanoseconds (ns) and $1\text{ns} = 10^{-9}$ s. How long, in seconds, does the rocket take to travel 1 au. Give your answer in standard form, correct to 3 significant figures. [2]

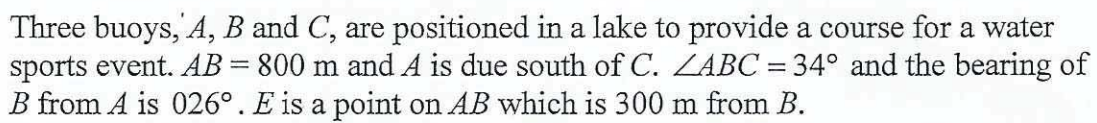**3.** **Answer the whole of this question on a sheet of graph paper.**

A stone was projected directly up a slope. Its distance, y metres, from the bottom of the slope, t seconds after it was projected, is given in the table below.

t	0	0.5	1	2	3	4	5	6
y	0	1.80	3.10	5.50	6.85	7.75	8.30	8.40

(a) Using a scale of 2 cm to represent 1 second, draw a horizontal t-axis for $0 \le t \le 6$. Using a scale of 2 cm to represent 1 metre, draw a vertical y-axis for $0 \le y \le 9$.
On your axes, draw the graph of y against t. [3]

(b) Use your graph to find the distance of the stone from the bottom of the slope when $t = 2.5$. [1]

(c) **(i)** By drawing a tangent, find the gradient of the curve at $t = 3$. [2]

(ii) State briefly what this gradient represents. [1]

(d) At the instance the stone was projected, an object was released down the slope from a point 8 metres from the bottom. This object moved directly down the slope at a constant speed of 2 m/s.

(i) On the same axes, draw the graph representing the motion of this object. [1]

(ii) Use your graphs to find when the stone and the object passed each other. [1]

---

4. A is the point $(-2, 7)$ and B is the point $(6, -4)$.

(a) Find the length of the line AB. [2]

(b) Find the equation of the line AB. [2]

(c) The equation of another line l is $8y + 11x = -16$.
Show how you can tell that the line l is parallel to the line AB. [2]

(d) The coordinates of C is $(2, k)$. Given that the points A, B and C are collinear, find the value of k. [2]

5.

Three buoys, *A*, *B* and *C*, are positioned in a lake to provide a course for a water sports event. *AB* = 800 m and *A* is due south of *C*. ∠*ABC* = 34° and the bearing of *B* from *A* is 026°. *E* is a point on *AB* which is 300 m from *B*.

(a) Calculate *AC*. [2]

(b) Find the area of triangle *ABC*. [2]

(c) Calculate the bearing of *A* from *B*. [2]

(d) A helicopter, *H*, is hovering at a point vertically above *E*.

    (i) The angle of elevation of the helicopter from *B* is 11°. Calculate the vertical height of the helicopter above *E*. [2]

    (ii) *F* is a point on *BC* which is nearest to the helicopter. Calculate the angle of depression of *F* from the helicopter. [3]

6. **(a)** Given that $\mathbf{P} = \begin{pmatrix} w & 1 \\ 0 & 2 \end{pmatrix}$ and $\mathbf{P^2} = \begin{pmatrix} 6w-9 & w+2 \\ 0 & 4 \end{pmatrix}$, where w is a constant.

Find the value(s) of w. [3]

(b) A waffle maker produces three different types of waffles: Red bean, Chocolate and Peanut, for distribution to its outlets at various locations.

The table below shows the quantity delivered to each location each time.

	Chocolate	Red bean	Peanut
Outlet 1	225	140	125
Outlet 2	265	115	245
Outlet 3	245	125	175

(i) Represent the data in the above table by a 3×3 matrix A. [1]

(ii) Hence, find, by matrix multiplication, the total number of waffles delivered to each outlet. [2]

(c) The following table shows the selling price and the cost price of 1 unit of each type of waffle.

|	Chocolate	Red Bean	Peanut
Selling Price ($)	1.20	0.80	1.00
Cost Price ($)	0.60	0.50	0.40

(i) Represent the data in the above table by a matrix C such that AC gives the total selling price and total cost price of each outlet. Hence, evaluate AC. [3]

**(ii)** Find the profit earned by outlet 2. [1]

7. A rectangular wall is 208 cm by 169 cm. The border around the wall is to be covered with tiles. The tiles measure x cm by $(x-5)$ cm. Each tile is placed so that its longer side is vertical. Some of the tiles are shown in the diagram below.
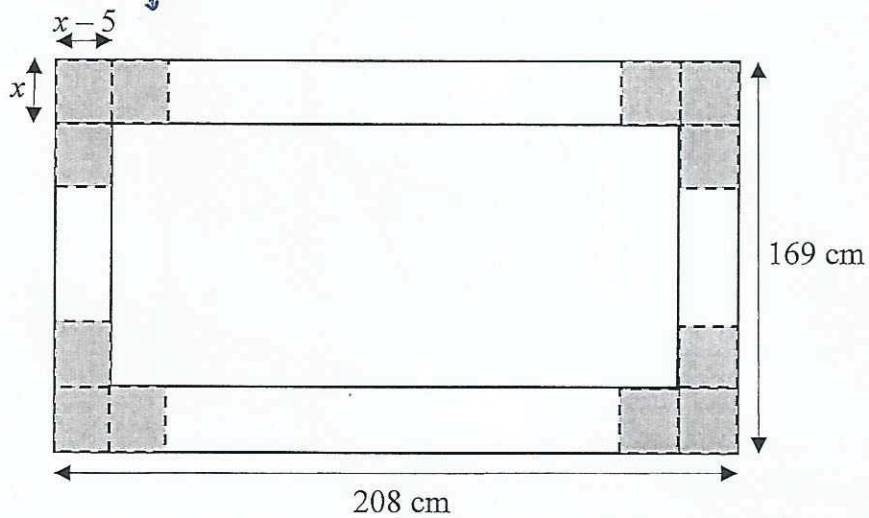
(a) Express, in terms of x, the number of tiles that will fit
    (i)   across the top row of the border,   [1]
    (ii) along one vertical side of the border.   [1]

(b) Given that 74 tiles are required to fill the whole border around the wall, form an equation and show that it reduces to $3x^2 - 44x + 65 = 0$.   [3]

(c) Solve the equation $3x^2 - 44x + 65 = 0$.   [2]

(d) Explain why one of the solutions in part (c) must be rejected as the length of the tile.   [1]

(e) Find the area of the wall that will be filled with tiles.   [2]

8. The diagram shows the cross-section design of the medal for a sports event.
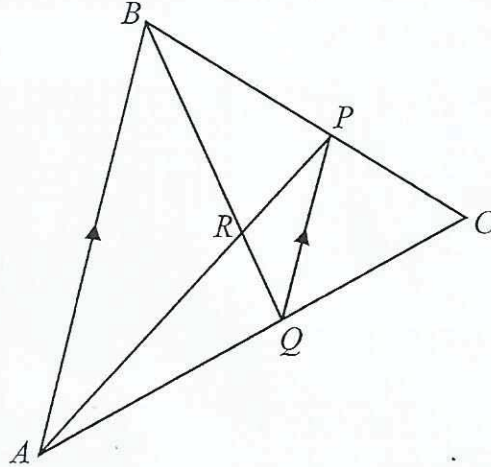   It is made up of two identical shapes.
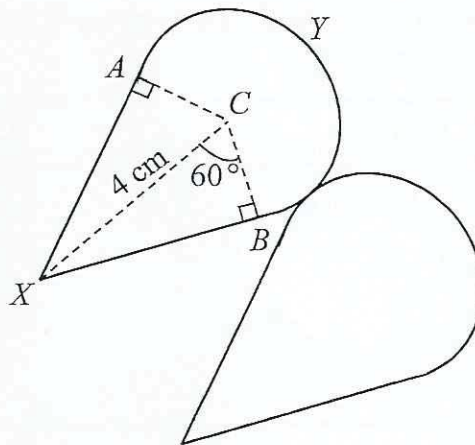
AYB is an arc of a circle with centre *C*.
*XA* and *XB* are tangents to the circle.
XC = 4 cm and ∠*XCB* = 60°.

(a) Calculate the length of *BC*. [2]

(b) Find the total cross-sectional area of the medal. [3]

(c) If the medal has a thickness of 0.7 cm, find the volume of
     this piece of medal. [2]

9. In the diagram, *AB* is parallel to *QP*.

(a) Prove that triangle *PQR* is similar to triangle *ABR*. [3]

(b) State another pair of similar triangles. [1]

(c) Given that *PQ* = 5 cm, *PR* = 4 cm and *AB* = 12 cm, calculate the length of *AR* using similar triangles. [2]

(d) Find the numerical value of the ratio of $\dfrac{\text{Area of triangle } PQR}{\text{Area of triangle } ABR}$. [1]

(e) Given that the area of triangle *CPQ* is 15 cm², find the area of *ABPQ*. [2]

10. **(a)** In the afternoon, on a certain stretch of road, the speeds of 100 vehicles were recorded. The cumulative frequency curve shows the distribution of the speeds of the vehicles.
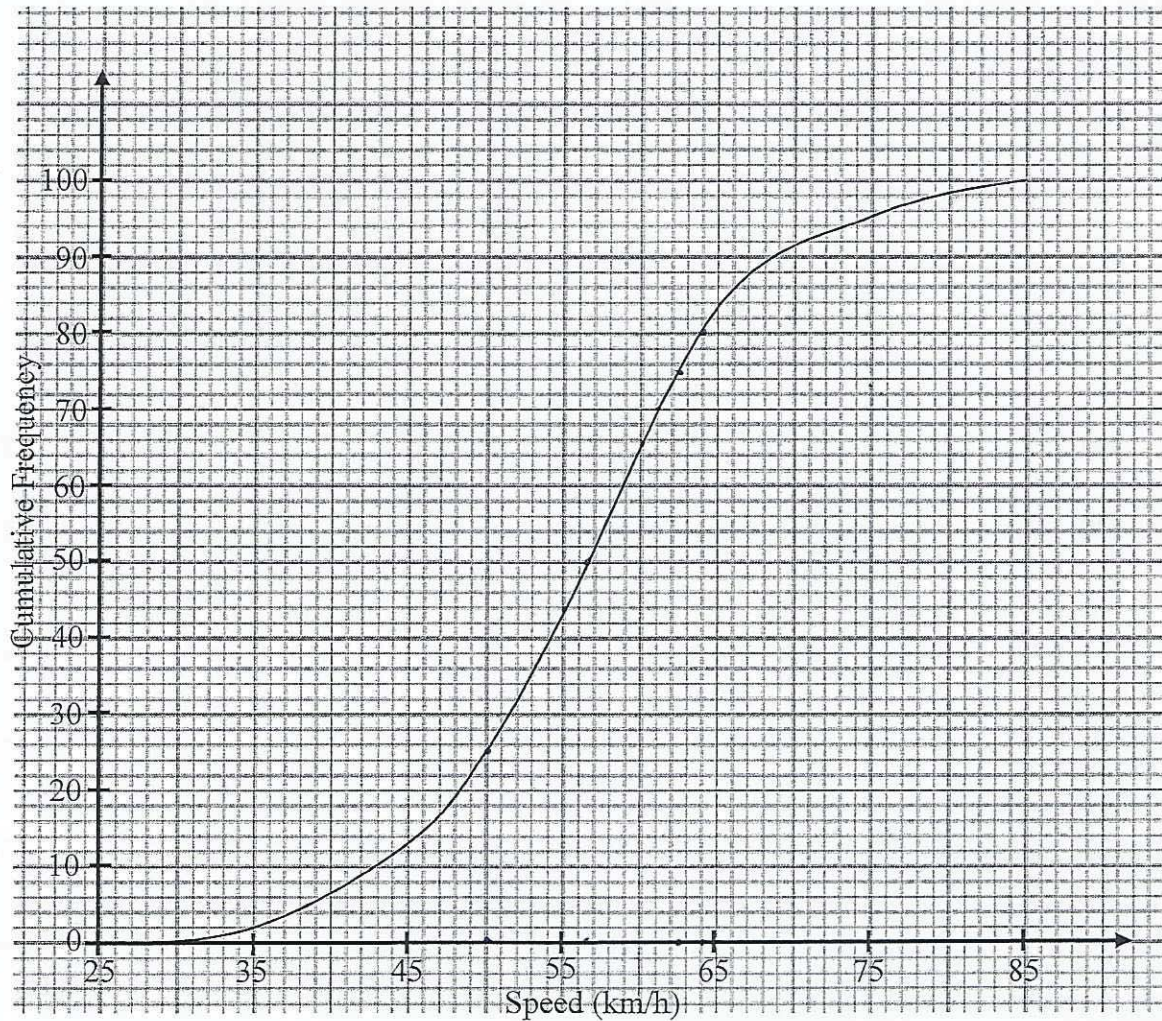
(i) Use the curve to estimate

    (a) the median speed, [1]

    (b) the interquartile range of the speeds, [1]

    (c) the 80th percentile, [1]

    (d) the percentage of vehicles that travels within the speed limit if the speed limit on the road is 60 km/h. [1]

(ii) From the curve, find the probability that at least one of the two vehicles chosen at random travels at a speed more than 55 km/h. [2]

(b) The speeds of 100 vehicles passing the same stretch of road at night were also recorded. The box-and-whisker plot shows the distribution of the speeds.


```
25      35      45      55      65      75      85
```
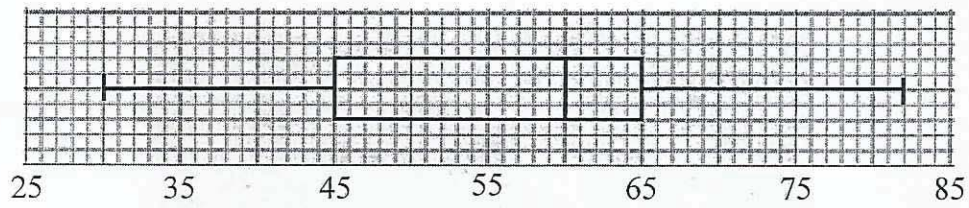
(i) Make two comments comparing the speeds of the vehicles in the afternoon and at night. [2]

(ii) Find the range of the speeds of the vehicles at night. [1]

11. Jewel Changi Airport is set to become an iconic landmark of Singapore and a key tourist attraction. The building occupies an area of about 134 000 m² on the ground.

(a) A map has a scale of 1 : 7500. Calculate the area of the building on the map. [2]

(b) The diagram below shows the view of the building from the top. A Skytrain Bridge runs through the building at about 18 m above the ground of Forest Valley.

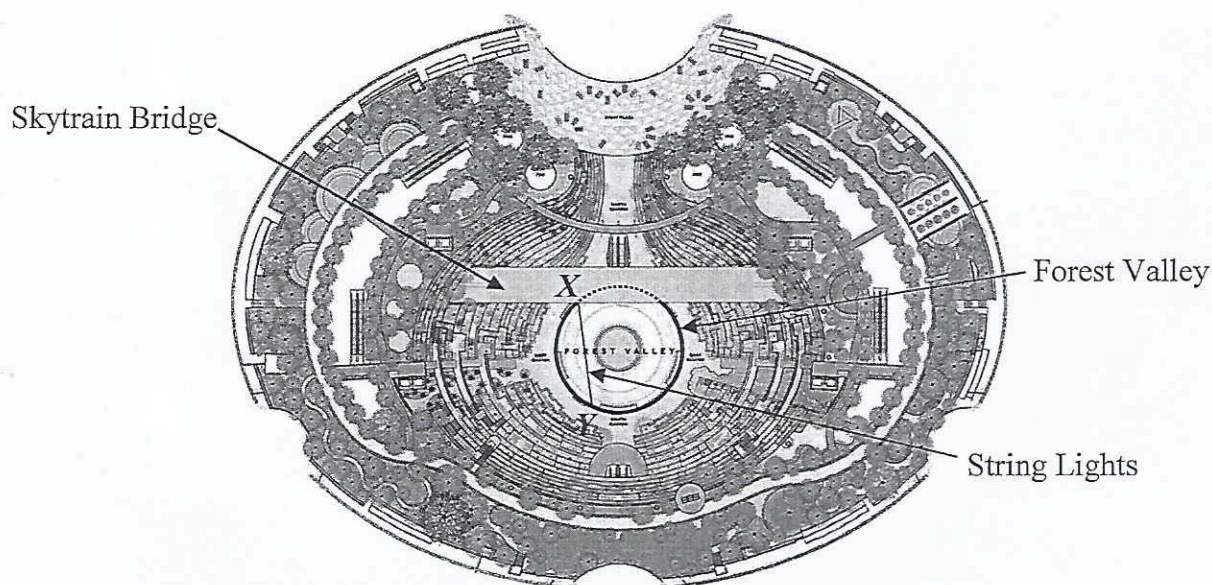

Diagram 1 [Source: Jewel Changi Development & Straits Times Graphic]

(i) The actual circumference of the whole circular Forest Valley is about 105 m. Find the actual radius of the whole circular Forest Valley in terms of π, in metres. [1]
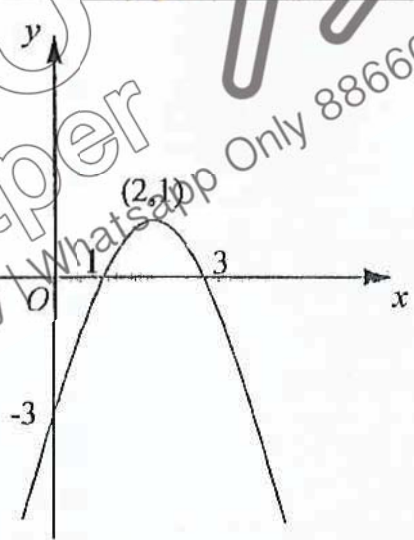
(ii) Mr Yap, a lighting designer, plans to hang string lights from point X on the skytrain bridge, down to point Y on the ground, which is on the circumference of the Forest Valley as illustrated in diagram 1.

Assuming XY is taut, he claims that he will need at least 34 m of string lights.
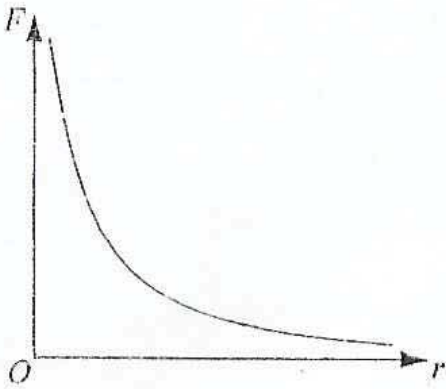
Given that Z is another point on the ground, which is also on the circumference of the Forest Valley, vertically below point X, and the minor arc length ZY is 35 m, verify if his claim is true or false, by showing your working clearly. [6]

END OF PAPER

1	$\dfrac{22}{7}$, $3.14\dot{2}$, $3.\dot{1}4\dot{2}$, π
2	$16p^4 - 81q^4$ $= \left(4p^2\right)^2 - \left(9q^2\right)^2$ $= \left(4p^2 - 9q^2\right)\left(4p^2 + 9q^2\right)$ $= \left(2p - 3q\right)\left(2p + 3q\right)\left(4p^2 + 9q^2\right)$
3	$\dfrac{1}{27} = 9^k$ $3^{2k} = 3^{-3}$ $k = -1.5$
4a	$\dfrac{34 + 35}{2} = 34.5$
4b	$39\dfrac{2}{3}$ or 39.7
4c	$\sqrt{\dfrac{14465}{9} - \left(39\dfrac{2}{3}\right)^2} = 5.81$ (3 s.f.)
5	
6a	$\dfrac{1}{2}(4)(v) = 20 \Rightarrow v = 10$
6b	$t = 6$ total distance $= \dfrac{1}{2}(6)(10) = 30$ avg speed $= \dfrac{30}{6} = 5$ m/s

7	Let x be the number of adults.
	$$\frac{x}{x+19+14} \leq \frac{3}{5}$$
	$$5x \leq 3x+99$$
	$$x \leq 49.5$$
	Therefore, the maximum number of adults is 49.
8a	Rhombus
8b	$\subset$ or $\subseteq$
9	1 unit - $40
	Total sum of money $= 40 \times 9 = \$360$
10	$$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$
11	Adam's rate $= \dfrac{70}{30}$ Charlie's rate $= \dfrac{80}{45}$ $\Big\}$ Total rate $= \dfrac{70}{30} + \dfrac{80}{45} = \dfrac{37}{9}$
	time $= \dfrac{1000}{\frac{37}{9}} = 243.24$ min (5 s.f.) $= 4$ hours 3.24 min
	Time taken is 4 hours 4 minutes.
12	$\angle EBC = \dfrac{180° - 108°}{2} = 36° \Rightarrow$ no. of sides $= \dfrac{360°}{36} = 10$
13	$$\frac{5x}{14} - \frac{2x-3}{21} = 1$$
	$$15x - 2(2x-3) = 42$$
	$$15x - 4x + 6 = 42$$
	$$11x = 36 \Rightarrow x = \frac{36}{11} = 3\frac{3}{11}$$
14	$$(x+2)^2 - 4 - 11 = 0$$
	$$(x+2)^2 = 15$$
	$$x+2 = \pm\sqrt{15}$$
	$$x = -2 \pm \sqrt{15} \Rightarrow x = 1.87 \text{ or } -5.87 \text{ (3 s.f.)}$$
15	$$\frac{1}{x^2} + \frac{1}{y^2} = 3$$
	$$x^2 + y^2 = 3x^2y^2$$
	$$(x+y)^2 = 3x^2y^2 + 2xy$$
	$$= 3(4)^2 + 2(4)$$
	$$= 56$$

16a	
16b	$r_{AB} = \dfrac{5}{2} r_{CD}$ $r_{AB}^2 = \dfrac{25}{4} r_{CD}^2$ $\dfrac{F_{AB}}{F_{CD}} = \dfrac{r_{CD}^2}{r_{AB}^2}$ $= \dfrac{4}{25}$ Answer: 4 : 25
17ai	$y = -x^3 + 3$
aii	$y = x^2 - 3$
aiii	$y = 3^{-x}$
bi	Any value greater than 2
bii	$y = 2x + c$, where $c < 0$
18	$2\pi r^2 + 2\pi rh = 2\pi r^2 + \pi r^2$ $h = \dfrac{r}{2}$ vol. of cylinder $= \pi r^2 h$ $= \dfrac{1}{2}\pi r^3$
19	$P\left(1 + \dfrac{r}{100}\right)^3 = 1.2P$ $1 + \dfrac{r}{100} = \sqrt[3]{1.2}$ $r = 6.3$ (1 d.p.)

20	$\dfrac{l_1}{l_2} = \dfrac{9}{15} = \dfrac{3}{5}$ $\dfrac{V_1}{V_2} = \left(\dfrac{3}{5}\right)^3 = \dfrac{27}{125}$ $V_1 = \dfrac{27}{125}V_2$ $125V_1 = 27V_2$ Therefore, 27 large glasses could be filled.
21	$7^2 + 10^2 \neq 12^2$ $\therefore AB^2 + BC^2 \neq AC^2$ By Pythagoras Theorem, $\angle ABC \neq 90°$ By Right-Angle in Semicircle, AC is not a diameter.
22a	2.198
22b	$\dfrac{T}{2\pi} = \sqrt{\dfrac{L}{g}} \Rightarrow \dfrac{T^2}{4\pi^2} = \dfrac{L}{g} \Rightarrow g = \dfrac{4\pi^2 L}{T^2}$
23a	$2 : 3.2 \times 1000 \times 100$ $1 : 160000$
23b	$\dfrac{27.5}{3.2} \times 2 = 17.1875\,\text{cm}$
23c	$4\,\text{cm}^2$ represents $10.24\,\text{km}^2$ $1.4\,\text{cm}^2$ represents $3.584\,\text{km}^2$
24a	$2^2 \times 3 \times 7$
24b	$2 \times 3 = 6$
24c	$2^3 \times 1 \times 7^2 = 392$
25a	$5(5+3) = 40$
25b	$n(n+3) = 154$ $n^2 + 3n - 154 = 0$ $(n-11)(n+14) = 0$ $n = 11$ or $n = -14$ (rejected)
25c	When n is odd, $n+3$ is even. When n is even, $n+3$ is odd. Product of odd and even is always even.
26	$\angle BAD = 180° - 5x°$ (angles in opposite segments) $\angle BOD = 2 \times \angle BAD$ (angle at centre $= 2 \times$ angle at circumference) $= 360° - 10x°$ $360° - 10x° + 2x° + 90° \times 2 = 360°$ (tangent $\perp$ radius, total interior angle of quadrilateral) $x = 22.5$

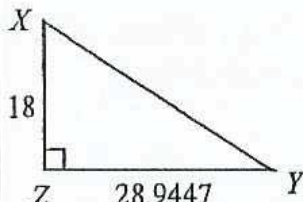| Qn No	Solutions
1a	$\dfrac{3-x}{5} < 1 + \dfrac{2x+1}{4}$
	$\dfrac{3-x}{5} < \dfrac{4+2x+1}{4}$
	$4(3-x) < 5(5+2x)$
	$12 - 4x < 25 + 10x$
	$-14x < 13$
	$x > -\dfrac{13}{14}$
1b	$\dfrac{4y}{3-2y} - \dfrac{y}{(2y-3)^2}$
	$= \dfrac{-4y}{2y-3} - \dfrac{y}{(2y-3)^2}$ or $= \dfrac{4y}{3-2y} - \dfrac{y}{(3-2y)^2}$
	$= \dfrac{-4y(2y-3) - y}{(2y-3)^2}$ $= \dfrac{4y(3-2y) - y}{(3-2y)^2}$
	$= \dfrac{11y - 8y^2}{(2y-3)^2}$ $= \dfrac{11y - 8y^2}{(3-2y)^2}$
	$= \dfrac{y(11-8y)}{(2y-3)^2}$ $= \dfrac{y(11-8y)}{(3-2y)^2}$
1c	$\dfrac{(-3h)^2}{8h^3 j^5} \div \dfrac{27h^4 j^2}{4j^3}$
	$= \dfrac{9h^2}{8h^3 j^5} \times \dfrac{4j^3}{27h^4 j^2}$
	$= \dfrac{1}{6h^5 j^4}$
1d	$\left(\dfrac{256p^{16}}{q^{20} r^{-4}} \right)^{-\frac{1}{4}}$
	$= \left(\dfrac{q^{20} r^{-4}}{256 p^{16}} \right)^{\frac{1}{4}}$
	$= \dfrac{q^5 r^{-1}}{4p^4}$
	$= \dfrac{q^5}{4p^4 r}$ or $\dfrac{q^5 r^{-1}}{4p^4}$ or $\dfrac{1}{4} p^{-4} q^5 r^{-1}$

1e	$\dfrac{10}{x^2-9}-\dfrac{3}{x+3}=1$
	$10-3(x-3)=x^2-9$
	$10-3x+9=x^2-9$
	$x^2+3x-28=0$
	$(x+7)(x-4)=0$
	$x=-7 \quad$ or $\quad x=4$
2a	1.50×10^{11} m (3 sig. fig)
2b	Time taken for sound to travel 1 au $=\dfrac{1.4959\times10^{11}}{343}$
	$\quad\quad\quad\quad = 4.36\times10^8$ s (3 sig. fig)
2c	$\dfrac{384400\times10^3}{1.4959\times10^{11}}\times100$
	$=0.257\%$ (3 sig. fig)
2d	Time taken $=1.4959\times10^{11}\times8000\times10^{-9}$
	$\quad\quad\quad = 1.20\times10^6$ s (3 sig.fig)
3	Answers on the last page
4a	$AB=\sqrt{(-2-6)^2+(7-(-4))^2}$
	$\quad=\sqrt{64+121}$
	$\quad=\sqrt{185}=13.6$ units
4b	Gradient of $AB=\dfrac{7-(-4)}{-2-6}=-\dfrac{11}{8}$
	Equation of AB :
	$y-7=-\dfrac{11}{8}(x+2) \quad$ or $\quad 7=-\dfrac{11}{8}(-2)+c$
	$\quad\quad\quad\quad\quad\quad c=\dfrac{17}{4}$
	$y=-\dfrac{11}{8}x+\dfrac{17}{4}\quad\quad\quad\quad y=-\dfrac{11}{8}x+\dfrac{17}{4}$
	$8y=-11x+34 \quad\quad\quad\quad 8y=-11x+34$
4c	$8y+11x=-16$
	$8y=-11x-16$
	$y=-\dfrac{11}{8}x-2$
	Since they have the same gradient, they are parallel.

4d	$8k = -11(2) + 34$ or $\dfrac{7-k}{-2-2} = -\dfrac{11}{8}$
	$k = 1\dfrac{1}{2}$ $\qquad\qquad\qquad k = 1\dfrac{1}{2}$

5a	Using Sine Rule,
	$\dfrac{\sin 120°}{800} = \dfrac{\sin 34°}{AC}$
	$AC = 516.560$
	$= 517\,\text{m}$ (3 sig. fig)

5b	Area of triangle $ABC = \dfrac{1}{2}(800)(516.560)\sin 26°$
	$= 90577.9$
	$= 90600\,\text{m}^2$ (3 sig. fig)

5c	Bearing of A from $B = 360° - 120° - 34°$ or $180° + 26°$
	$= 206°$

5d	(i) Let the vertical height of the helicopter above E be h
	$\tan 11° = \dfrac{h}{300}$
	$h = 300\tan 11°$
	$h = 58.314$
	$h = 58.3\,\text{m}$ (3 sig. fig)
	(ii) $\sin 34° = \dfrac{EF}{300}$
	$EF = 300\sin 34°$
	Let the angle of depression of F from the helicopter be θ
	$\tan \theta = \dfrac{58.314}{300\sin 34°}$
	$\theta = 19.167$
	$\theta = 19.2°$ (1 d.p.)

6a	$\begin{pmatrix} w & 1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} w & 1 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 6w-9 & w+2 \\ 0 & 4 \end{pmatrix}$
	$\begin{pmatrix} w^2 & w+2 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 6w-9 & w+2 \\ 0 & 4 \end{pmatrix}$
	$w^2 = 6w - 9$
	$w^2 - 6w + 9 = 0$
	$(w-3)^2 = 0$
	$w = 3$

6b	(i)	$A = \begin{pmatrix} 225 & 140 & 125 \\ 265 & 115 & 245 \\ 245 & 125 & 175 \end{pmatrix}$
	(ii)	$\begin{pmatrix} 225 & 140 & 125 \\ 265 & 115 & 245 \\ 245 & 125 & 175 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$
		$= \begin{pmatrix} 490 \\ 625 \\ 545 \end{pmatrix}$
		Total number of waffles delivered to outlet 1 is 490, outlet 2 is 625 and outlet 3 is 545.
6c	(i)	$C = \begin{pmatrix} 1.20 & 0.60 \\ 0.80 & 0.50 \\ 1.00 & 0.40 \end{pmatrix}$ or $C = \begin{pmatrix} 0.60 & 1.20 \\ 0.50 & 0.80 \\ 0.40 & 1.00 \end{pmatrix}$
		$AC = \begin{pmatrix} 225 & 140 & 125 \\ 265 & 115 & 245 \\ 245 & 125 & 175 \end{pmatrix} \begin{pmatrix} 1.20 & 0.60 \\ 0.80 & 0.50 \\ 1.00 & 0.40 \end{pmatrix}$ or
		$AC = \begin{pmatrix} 225 & 140 & 125 \\ 265 & 115 & 245 \\ 245 & 125 & 175 \end{pmatrix} \begin{pmatrix} 0.60 & 1.20 \\ 0.50 & 0.80 \\ 0.40 & 1.00 \end{pmatrix}$
		$= \begin{pmatrix} 507 & 255 \\ 655 & 314.50 \\ 569 & 279.50 \end{pmatrix}$ or $\begin{pmatrix} 255 & 507 \\ 314.50 & 655 \\ 279.50 & 569 \end{pmatrix}$
	(ii)	Profit earned by outlet 2 = \$ 340.50
7a	(i)	$\dfrac{208}{x-5}$ tiles
	(ii)	$\dfrac{169}{x}$ tiles
7b		$2\left(\dfrac{208}{x-5}\right) + 2\left(\dfrac{169}{x}\right) - 4 = 74$
		$\dfrac{416}{x-5} + \dfrac{338}{x} = 78$
		$416x + 338(x-5) = 78x(x-5)$
		$416x + 338x - 1690 = 78x^2 - 390x$
		$78x^2 - 1144x + 1690 = 0$
		$3x^2 - 44x + 65 = 0$ (shown)

7c	$(3x-5)(x-13)=0$
	$x=\dfrac{5}{3}=1\dfrac{2}{3}$ or $x=13$
7d	If $x=1\dfrac{2}{3}$, the <u>width</u> of the tile <u>will be negative</u>.
7e	Taking $x=13$,
	area of the wall that will be filled with tiles $=13(13-5)\times74$
	$=7696\,\text{cm}^2$
8a	$\cos60°=\dfrac{BC}{4}$
	$BC=2\,\text{cm}$
8b	Total cross-sectional area of medal
	$=2\left[\dfrac{1}{2}(2^2)\left(\dfrac{4\pi}{3}\right)+2\left(\dfrac{1}{2}\right)(4)(2)\sin60°\right]$
	$=30.6115$
	$=30.6\,\text{cm}^2\ (3\text{ sig. fig})$
8c	Volume of the medal $=30.6115\times0.7$
	$=21.428$
	$=21.4\,\text{cm}^3\ (3\text{ sig. fig})$
9a	$\angle BAR=\angle QPR$ (alternate angles)
	$\angle ABR=\angle PQR$ (alternate angles)
	$\angle ARB=\angle PRQ$ (vertically opposite angles)
	Since all corresponding angles are equal, triangle PQR is similar to triangle ABR.
9b	Triangle CPQ and Triangle CBA
9c	Using similar triangles,
	$\dfrac{AR}{PR}=\dfrac{AB}{PQ}$
	$\dfrac{AR}{4}=\dfrac{12}{5}$
	$PB=9.6\,\text{cm}$ or $9\dfrac{3}{5}\,\text{cm}$
9d	$\dfrac{\text{Area of triangle }PQR}{\text{Area of triangle }ABR}=\left(\dfrac{5}{12}\right)^2=\dfrac{25}{144}$
9e	$\left(\dfrac{5}{12}\right)^2=\dfrac{15}{\text{Area of triangle }CBA}$
	Area of triangle $CBA=86.4\,\text{cm}^2$

		Area of $ABPQ = 71.4$ cm^2
10a	(i)	(a) median $= 56.5$ km/h
		(b) Interquartile range $= 62.5 - 50$
		$= 12.5$ km/h
		(c) 80th percentile $= 64$ km/h
		(d) 65%
	(ii)	Probability $= 1 - \left(\dfrac{43}{100} \times \dfrac{42}{99} \right)$
		$= \dfrac{1349}{1650}$
10b	(i)	The median speed in the afternoon is 56.5 km/h while the median speed at night is 60 km/h. Vehicles at night travel faster. The interquartile range in the afternoon is 12.5 km/h while the interquartile range at night is 20 km/h. The speeds of the vehicles in the afternoon are less widespread (or more consistent) than the speeds at night due to smaller interquartile range in the afternoon.
	(ii)	Range $= 82 - 30 = 52$ km/h
11a		1 cm : 75 m
		1 cm^2 : 5625 m^2
		Area of building on the map $= \dfrac{134000}{5625}$
		$= 23.8$ cm^2 (3 sig. fig)
11b	(i)	actual circumference $= 105$ m
		$2\pi r = 105$
		Actual radius $= \dfrac{105}{2\pi}$ m
	(ii)	Let the centre of forest valley be O and angle ZOY be θ radian
		$\left(\dfrac{105}{2\pi} \right) \theta = 35$
		$\theta = \dfrac{2\pi}{3}$
		Using Cosine Rule,
		$YZ^2 = \left(\dfrac{105}{2\pi} \right)^2 + \left(\dfrac{105}{2\pi} \right)^2 - 2\left(\dfrac{105}{2\pi} \right)\left(\dfrac{105}{2\pi} \right)\cos \dfrac{2\pi}{3}$
		$YZ = 28.9447$ m

	By Pythagoras Theorem, $$XY^2 = 18^2 + YZ^2$$ $$XY = 34.1 \text{ m (3 sig. fig)}$$	
	His claim is false.	
3b	$y = 6.2$	
3c	(i) gradient $= \dfrac{8.5 - 5}{4.5 - 1.3} = 1.09375$	
	(ii) The gradient represents the speed of the stone at $t = 3$.	
3d	(ii) At $t = 1.65$	
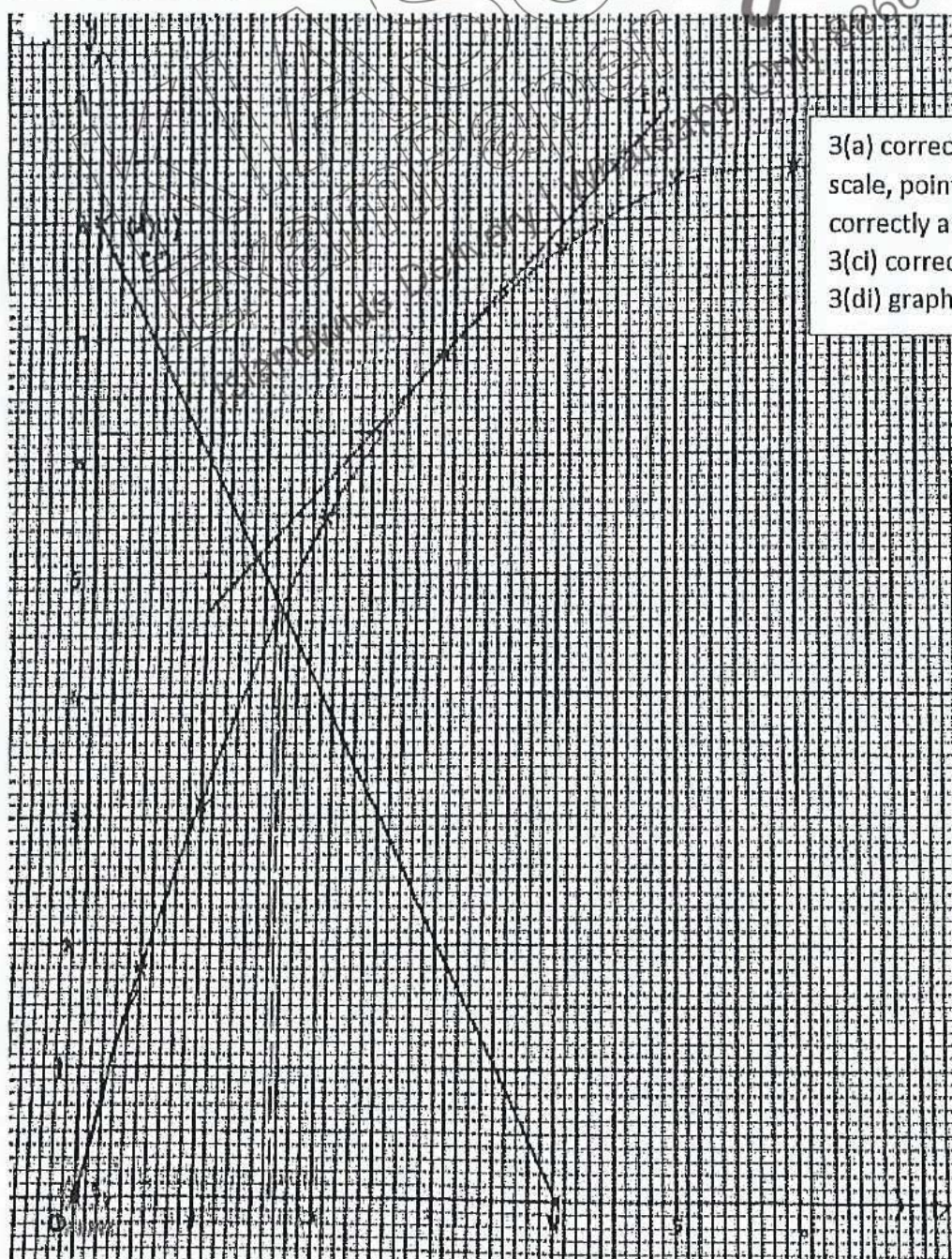
3(a) correct labelled axes &
scale, points plotted
correctly and smooth curve
3(ci) correct tangent
3(di) graph of object