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ST. MARGARET'S SECONDARY SCHOOL

Preliminary Examinations 2019

CANDIDATE NAME

CLASS

REGISTER NUMBER

ADDITIONAL MATHEMATICS **4047/01**

Paper 1 **26 August 2019**

Secondary 4 Express / 5 Normal (Academic) **2 hours**

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **17** printed pages and a blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$, $\qquad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\dbinom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 The equation of a curve is $y = \frac{2x+5}{x+6}$, for $x \neq -6$.

Explain, with working, whether the curve has turning points. [3]

2 Find the values of the integers a and b for which $a + \sqrt{b}$ is a solution to the equation
$x\sqrt{27} - 2x\sqrt{2} = x\sqrt{75} - \sqrt{8}$. [4]

3 **(i)** Sketch the graph of $y^2 = \frac{1}{4}x$, for $x \leq 16$. [1]

(ii) Find the coordinates of the points of intersection of the curve $y^2 = \frac{1}{4}x$ and the line $6y - 4x + 10 = 0$. [4]

4 A particle is travelling in a straight line with a velocity of $v = 8t - \frac{t^2}{4}$ cm/s where t is the time in seconds after leaving a fixed point O.

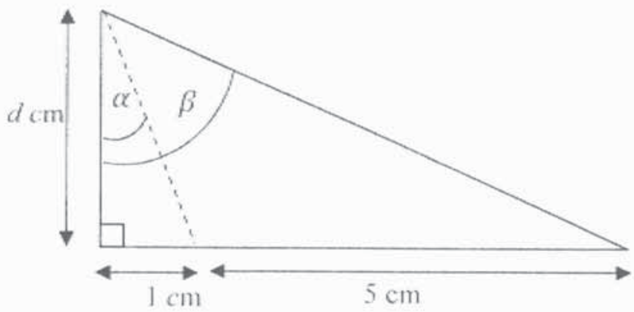
Calculate,

(i) its acceleration when the particle is at instantaneous rest. [3]

(ii) the value of t when the particle returns to O. [2]

5 The curve $y = f(x)$ has a gradient of -1 at the point $(2, 8)$. If $f''(x) = 6 - 6x$, find the equation of the curve. [4]

6



Find, in terms of d, an expression for

(i) $\tan\alpha$, [1]

(ii) $\tan\beta$, [1]

where α, β and d are shown in the diagram.

Hence obtain, in terms of d, an expression for

(iii) $\tan(\beta - \alpha)$. [2]

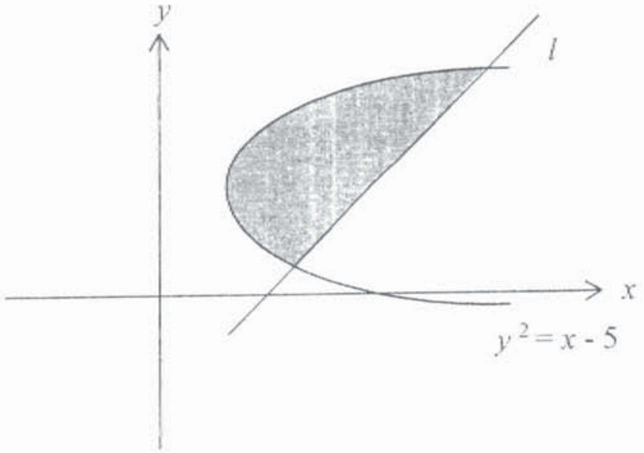
Given that $\beta - \alpha = 45°$, find the values of d. [2]

7 The quadratic equation $4x^2 - 44x + 1 = 0$ has roots $\alpha^2 - 1$ and $\beta^2 - 1$. Find the quadratic equation whose roots are α and β, where α and β are positive. [6]

8 **(i)** By using long division, divide $2x^3 - 11x^2 + 12x + 9$ by $2x + 1$. [2]

(ii) Express $\frac{13x^2-52x+32}{2x^3-11x^2+12x+9}$ in partial fractions. [5]

9 The diagram shows part of the curve $y^2 = x - 5$ and the line l. The equation of the line l is $4y + 2 = x$. Calculate the area of the shaded region. [6]

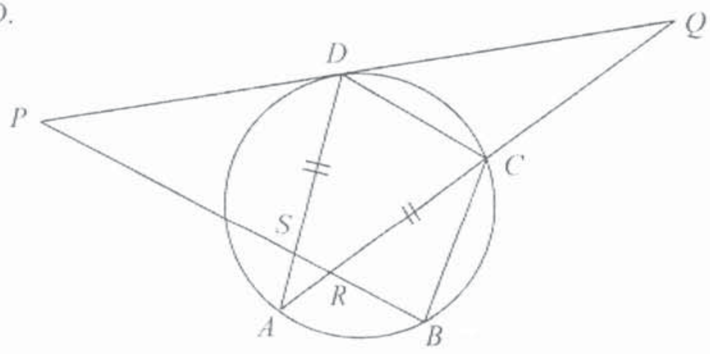


 [Turn over

10 **(i)** Prove $(\sin x + \cos x)\left(1 - \frac{1}{2}\sin 2x\right) = \sin^3 x + \cos^3 x$. [4]

(ii) Hence solve $(\sin x + \cos x)\left(1 - \frac{1}{2}\sin 2x\right) = 0$ for $-2\pi < x < 2\pi$. [3]

11 In the diagram below, A, B, C and D are points on the circle and QDP is a tangent to the circle at D.



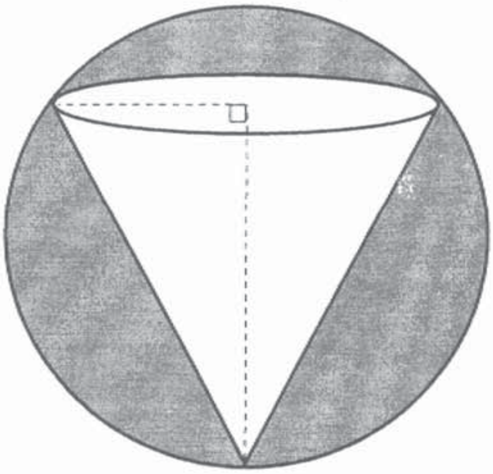
Given that $AD = AC$, prove that

(i) ΔQCD is similar to ΔQDA. [2]

(ii) $QD^2 - QC^2 = QC \times DA$. [4]

12 The diagram below shows a pendant in the shape of a sphere of radius 3 cm.

A right inverted circular cone of base radius r cm and height $(x + 3)$ cm is being removed from the solid sphere. [Volume of sphere $= \frac{4}{3}\pi r^3$; Volume of cone $= \frac{1}{3}\pi r^2 h$]



(i) Show that $r = (9 - x^2)^{\frac{1}{2}}$. [2]

(ii) Show that the volume of the cone is $V = \frac{1}{3}\pi\,(27 + 9x - 3x^2 - x^3)$. [2]

(iii) Given that x can vary, find the maximum value of V.

Hence, find the least amount of solid left in the pendant. [6]

13 The points $A(0, 2)$ and $B(8, 2)$ lie on the circumference of a circle C_1.

The line $x = -1$ is a tangent to C_1.

(i) Find the radius and the coordinates of the centre of C_1, given that the centre of the circle lies below the x-axis. [4]

(ii) Express the equation of C_1 in the form $x^2 + y^2 + 2px + 2qy + r = 0$, where p, q, r are integers. [2]

(iii) Find the equations of the tangents to C_1, which are parallel to x - axis. [2]

(iv) Another circle C_2 has its centre at B. Given that the area of C_2 is one-quarter that of C_1, find the equation of C_2. [3]

St. Margaret's Secondary School

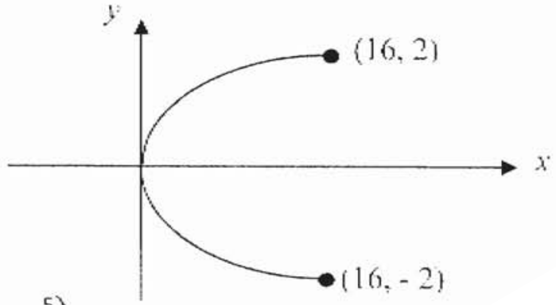
Sec 4E5N Preliminary Examinations

Additional Mathematics Paper 1

1. $\frac{dy}{dx} = \frac{7}{(x+2)^2}$, the curve has no turning point.

2. $x = -2 + \sqrt{6}$, $a = -2, b = 6$

3(i)



3(ii) (4, 1), $\left(\frac{25}{16}, -\frac{5}{8}\right)$

4(i) acceleration = -8 m/s^2 (ii) time = 48 s

5. $f(x) = 3x^2 - x^3 - x + 6$

6(i) $\frac{1}{d}$ (ii) $\frac{6}{d}$ (iii) $\frac{5d}{6+d^2}$; $d = 2$ or 3

7. $2x^2 - 4\sqrt{5}x + 7 = 0$

8(i)

$$\begin{array}{r|l} & x^2 - 6x + 9 \\ 2x+1 & 2x^3 - 11x^2 + 12x + 9 \\ & \vdots \end{array}$$

(ii) $\frac{5}{(2x+1)} + \frac{4}{(x-3)} - \frac{1}{(x-3)^2}$

9. $\frac{4}{3}$ units^2

10 (ii) $x = \frac{3}{4}\pi, \frac{7}{4}\pi, -\frac{\pi}{4}, -\frac{5}{4}\pi$

12(iii) $x = -3$ (rej) or 1; $x = 1$, $\frac{d^2y}{dx^2} < 0$, $\therefore$ max point;

Volume of cone = 33.5 cm^3; least volume of solid left = 79.6 cm^3

13(i) radius = 5 units; Centre(4, -1) (ii) $x^2 + y^2 - 8x + 2y - 8 = 0$

(iii) $y = 4$ & $y = -6$ (iv) $(x-8)^2 + (y-2)^2 = \frac{25}{4}$