PEICAI SECONDARY SCHOOL
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC
PRELIMINARY EXAMINATION 2019

CANDIDATE NAME

CLASS

REGISTER NUMBER

ADDITIONAL MATHEMATICS

4047/01
27 August 2019

2 hours

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your register number, class and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80**.

This document consists of **19** printed pages and **1** blank page.

Setter: Mrs Ariel Leong

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

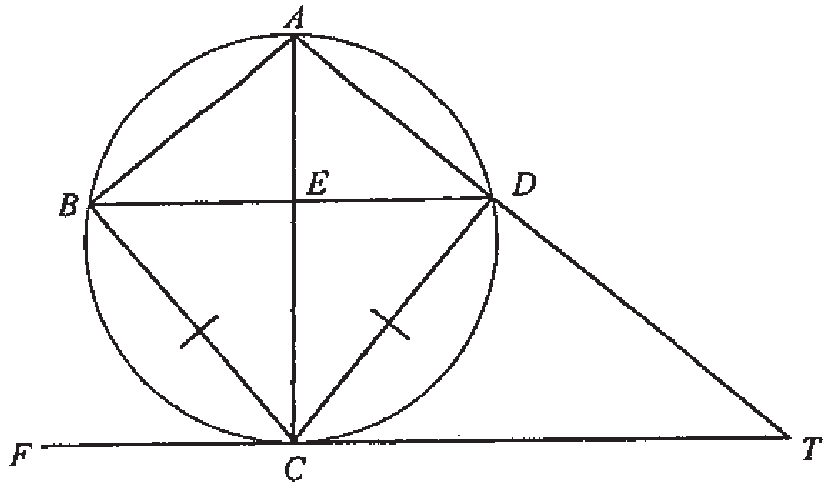
$$\Delta = \frac{1}{2} ab \sin C$$

1 Find the range of values of the constant p for which the line $y = px - 1$ intersects the curve $y = 2x^2 - 5x + 1$ at two points. [4]

2 Show that $\dfrac{36^{x+1}+15(6^{2x})}{3^{2x+1}}$ is divisible by 17, where x is a positive integer. [4]

3 Find the coordinates of the stationary points of the curve $y = 4x + \frac{25}{x}$, and determine the nature of these stationary points. [6]

4



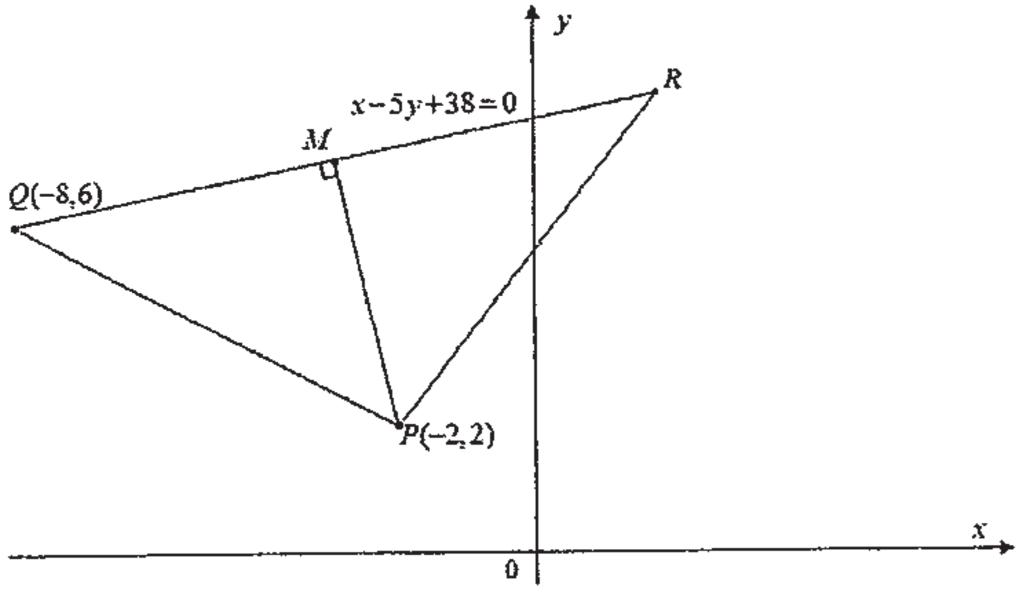
A, B, C and D are four points on the circle. CT is the tangent to the circle at point C. The diagonals of BD and AC intersect at point E and $BC = CD$.

(i) Prove that BD is parallel to FT. [3]

(ii) Show that $(CT)^2 = TD \times TA$. [3]

③

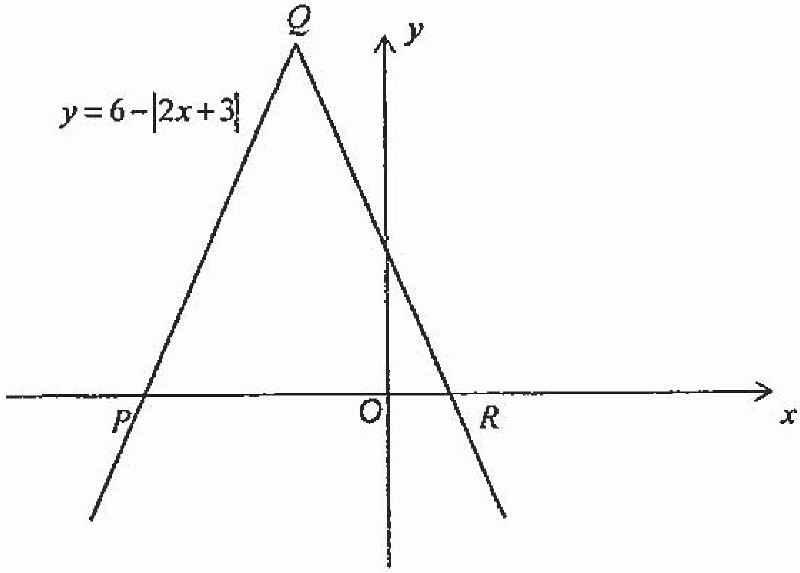
5



The diagram shows an isosceles triangle PQR with vertices $P(-2,2)$ and $Q(-8,6)$. $QP = PR$. The line QR with equation $x-5y+38=0$ passes through M, the mid-point of QR.

Find the area of triangle PQR. [6]

6



The diagram shows part of the graph $y=6-|2x+3|$.

(a) Find the coordinates of P, Q and R. [4]

(4)

(b) In each of the following cases determine the number solutions of the equation $6-|2x+3|=mx-1$. Justify your answer.

(i) $m=2$, [2]

(ii) $m=-\frac{1}{2}$. [2]

7 The roots of the quadratic equation $2x^2 - 4x + 3 = 0$ are α and β.

(i) Show that $\alpha^3 + \beta^3 = -1$. [3]

(ii) Find a quadratic equation whose roots are $\alpha^3 + 1$ and $\beta^3 + 1$. [4]

8 Solve the equation $\sin\theta\tan\theta + 2\sin\theta = 3\cos\theta$, where $\cos\theta \neq 0$, for $0° < \theta < 180°$. [6]

9 A particle moves in a straight line such that, at time t seconds after leaving a fixed point O, its velocity v m/s is given by $v = t^2\ (t - 4)$. Find

(i) the acceleration of the particle when $t = 3$, [2]

(ii) the value of t when the particle comes to an instantaneous rest, [2]

(iii) the time taken for the particle to return to the point O, [4]

(iv) the total distance travelled by the particle in the interval $t = 0$ to $t = 5$. [3]

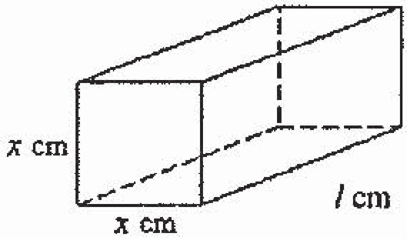
10 Variables x and y are connected by the equation $y = a^{x+b}$, where a and b are constants.
When a graph of $\lg y$ is plotted against x, a straight line passing through the points (3, 1) and (6, 4) is obtained. Find

(i) the value of a and of b, [4]

7

(ii) the coordinates of the point on the line at which $\lg y = 2x - 4$. [3]

11



The diagram shows a rectangular block of ice, x cm by x cm by l cm.
Volume of the ice is 1 litres.

(i) Show that the total surface area, A cm^2, is given by $A = 2x^2 + \frac{4000}{x}$. [2]

(ii) The block of ice is melting such that the total surface area is changing at a constant rate of 3 cm^2/s. Find the rate of decrease of x when $x = 5$. [4]

12 On a particular day, the water level at the beach of Tanjung Rima first reached a maximum of 2 m at 8 am. The lowest water level was forecast to be at 6 pm. The depth of water at the beach may be modelled by the equation

$$h = 0.8\cos(kt) + c$$

where h is the water level in metres and t is the number of hours after 8 am.

(i) Explain why this model suggests that the minimum water level will be 0.4 m.

[1]

(ii) Show that $c = 1.2$. [1]

(iii) Show that the value of k is $\frac{\pi}{10}$. [2]

9

The corals at the beach of Tanjung Rima are visible at low tide when the water level is less than 0.45 m.

(iv) Between what times in the daylight hours will corals be visible at Tanjung Rima? [5]

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SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC
PRELIMINARY EXAMINATION 2019

CANDIDATE NAME

CLASS

REGISTER NUMBER

ADDITIONAL MATHEMATICS
Paper 2

4047/02
28 August 2019

2 hours 30 minutes

Candidates answer on the Question Paper.
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The total number of marks for this paper is 100.

This document consists of **21** printed pages and **3** blank pages.

Setter: Mrs Ho Thuk Lan

Mathematical Formulae

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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1

1 (i) In the expansion of $(1+4x)^n$ the coefficient of x^2 is 8 times the coefficient of x. Show that $n=5$. [4]

(ii) Using your answer to part (i), find, in terms of p, the coefficient of x^2 in the expansion of $(1+px+3x^2)(1+4x)^n$. [1]

(iii) If the coefficient of x^2 in the expansion of $(1+px+3x^2)(1+4x)^n$ is 263, find the value of the constant p. [2]

2 **(i)** Differentiate $x\cos 3x$ with respect to x. [3]

(ii) Using your answer to part (i), find $\int x\sin 3x \, dx$. [3]

②

(iii) Hence show that $\int_0^{\frac{\pi}{3}} x\sin 3x \, dx = \frac{\pi}{9}$. [2]

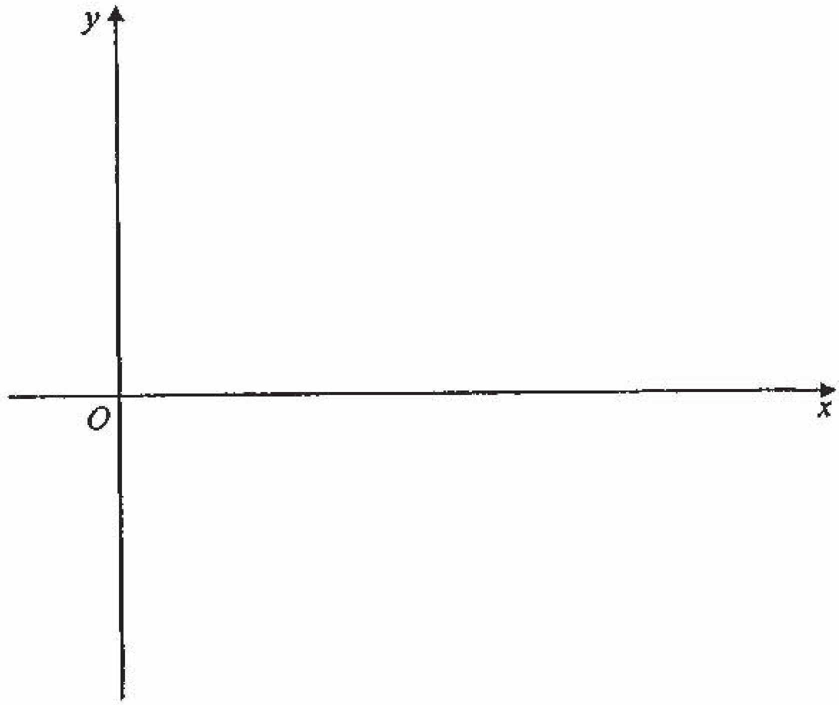
3 The equation of a curve is $\frac{x^2-1}{x^2+1}$.

(i) Obtain an expression for $f'(x)$. [2]

(ii) Find the equations of the tangent to the curve at the points where the curve meets the x – axis . [3]

(iii) Show that $f(x)$ is increasing when $x > 0$. [2]

4 (i) Sketch the graph of $y = \frac{1}{4}x^{\frac{2}{3}}$ for $x > 0$. [1]



(ii) On the same diagram sketch the graph of $y = -\frac{1}{4}x^{\frac{2}{3}}$ for $x > 0$. [1]

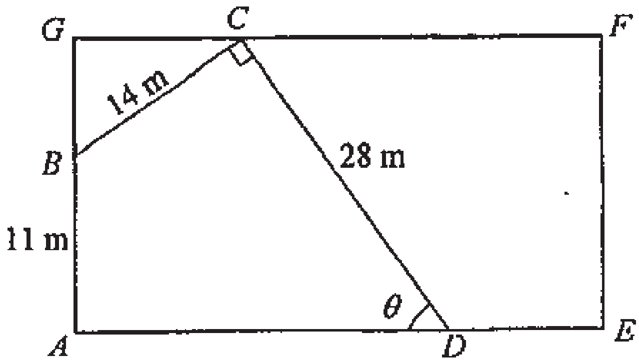
(iii) What is the relationship between the graphs of $y = \frac{1}{4}x^{\frac{2}{3}}$ and $y = -\frac{1}{4}x^{\frac{2}{3}}$? [1]

(iv) Calculate the coordinates of the point of intersection of $y = \frac{1}{4}x^{\frac{2}{3}}$ and $y = 4x^{-\frac{2}{3}}$ for $x > 0$. [2]

(v) On the same diagram sketch the graph of $y = 4x^{-\frac{2}{3}}$ for $x > 0$. [1]

(vi) Determine, with explanation, whether the tangents to the graphs of $y = \frac{1}{4}x^{\frac{2}{3}}$ and $y = 4x^{-\frac{2}{3}}$ for $x > 0$ at the point of intersection are perpendicular. [3]

5



The diagram shows a rectangular basketball court, *AEFG*.
From a point *A* on the court, players are to run along the straight paths *AB*, *BC*, *CD* and *DA*.
The lengths of *AB*, *BC* and *CD* are 11m, 14 metres and 28 metres respectively.
Angle *ADC* is θ, where $0^0 < \theta < 90°$.
The total distance covered by each player is *T* metres.

(i) Show that *T* can be expressed as $p + q\cos\theta + r\sin\theta$ where *p*, *q* and *r* are constants to be found. [3]

(ii) Express *T* in the form $p + R\cos(\theta - \alpha)$, where $R > 0$ and α is an acute angle. [4]

5

Given that the total distance is found to be 78 metres.

(iii) Find the value of θ. [2]

Given that the length of *DE* is $\left(5+2\sqrt{5}\right)$ m and the area of triangle *DEF* is $\left(45+42\sqrt{5}\right)$.

(iv) Find the length of *EF* in the form $a\sqrt{5}-b$. [2]

6 A circle, C_1, has centre $A(4,2)$ and radius $\sqrt{13}$.

(i) Write down the equation of circle, C_1. [1]

(ii) Determine whether the point $R(8, 3)$ lies inside or outside of circle, C_1. [2]

Circle, C_1 intersects the $x-$axis at points P and Q.

(iii) Find the mid-point of PQ. [4]

6

A second circle, C_2, with centre B and radius $\sqrt{18}$ also passes through P and Q.

(iv) *State the x–coordinate of B.* [1]

(v) Given that the y–coordinate of B is positive, find the centre of circle C_2. [3]

7 (a) Prove the identity $\frac{1}{1+\tan^2 x} = (1+\sin x)(1-\sin x)$. [3]

(b) Find all the angles between 0° and 360° that satisfy the equation $8\tan x = 3\cos x$. [4]

7

(c) Solve the equation $2\cos 2y - 5\cos y = 4$ for $0 \le y \le 2\pi$, giving your answers in radians. Correct your answers to 2 decimal places. [4]

8 A prism has a volume of $\left(6x^2-21x+25\right)$ cm^3 and a base area of $\left(2x^2-5x\right)$ cm^2.

(i) Find an expression for the height, h(x), of the prism. [1]

(ii) Using your answer to part **(i)**, express the height in partial fractions. [5]

(iii) Differentiate $\ln\left(2x-5\right)$ with respect to x. [1]

(8

(iv) Using your answers to part (ii) and (iii), find $\int_3^4 h(x)\,dx$ in the form $a+\ln\frac{b}{c}$ where a, b and c are integers. [5]

9 (a) Given that $\log_3 p = m$, $\log_{27} q = n$ and $\frac{p}{q} = 3^r$, express r in terms of m and n.

[3]

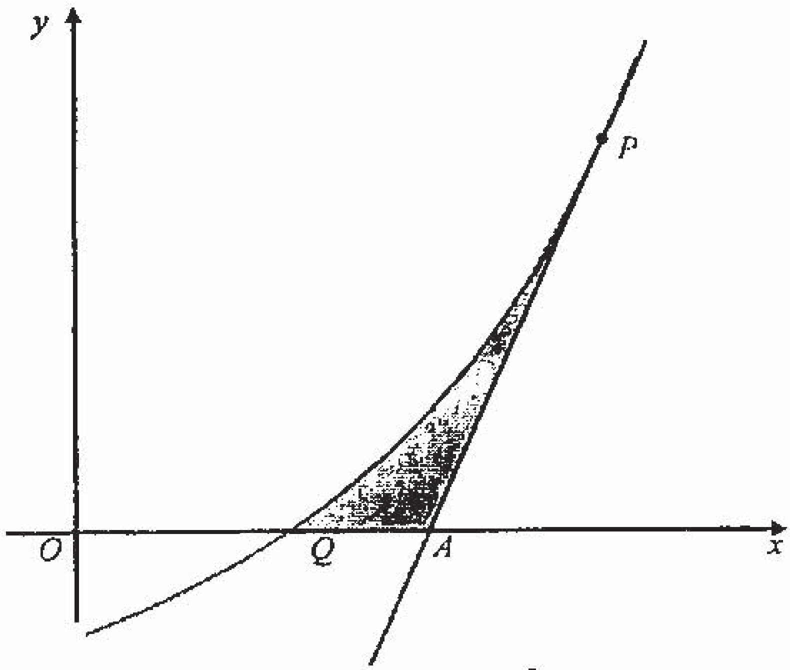
(b) Solve the equation

(i) $\log_3(3x-7) = \log_3(2x-3) - 2$, [4]

(9)

(ii) $3\log_5 y = 2 + \log_y 5$. [5]

10



The diagram shows part of the curve $y = \dfrac{9}{(7-x)^2} - 1$, cutting the x-axis at Q. The tangent at the point P on the curve cuts the x-axis at A. Given that the gradient of this tangent is $\dfrac{9}{4}$, calculate

(i) the coordinates of P, [5]

(ii) the area of the shaded region *PQA*. [7]

The End

Answer Key

1 $p < -9$, $p > -1$

2 Simplify to $17(4^x)$

Since 17 is a factor, the expression is divisible by 17

3 $\frac{d^2y}{dx^2} = 50x^{-2}$

(2.5, 20) is a min point, (–2.5, –20) is a max point

5 Area = 26 sq units

6a) P(–4.5, 0) Q(–1.5, 6) R(1.5, 0)

6b(i) One Solution. The line $y = 2x - 1$ is parallel to one part of the graph $y = 6 - |2x + 3|$ and the y-intercept of $y = 2x - 1$ is below the max point of $y = 6 - |2x + 3|$. Hence, the two graphs will only interest at one point.

6b(ii) Two solutions.

The line $y = -½ x - 1$ is not parallel to both parts of the graph $y = 6 - |2x + 3|$ and the y-intercept of y = = $-½ x - 1$ is below the max point of $y = 6 - |2x + 3|$. Hence, the two graphs will interest at two points.

7(i) –1 (ii) $x^2 - x + \frac{27}{8} = 0$

8 45°, 108.4°

9(i) $a = 3t^2 - 8t$; 3 m/s²

9(ii) at rest, v = 0, t = 4 (iii) $5\frac{1}{3}$ s

9(iv) Total distance = 32.25 m

10(i) a = 10, b = –2

10(iii) (2, 0)

11(i) $A = 2x^2 + 4x(1000/x^2)$

11(ii) 3/140 cm/s

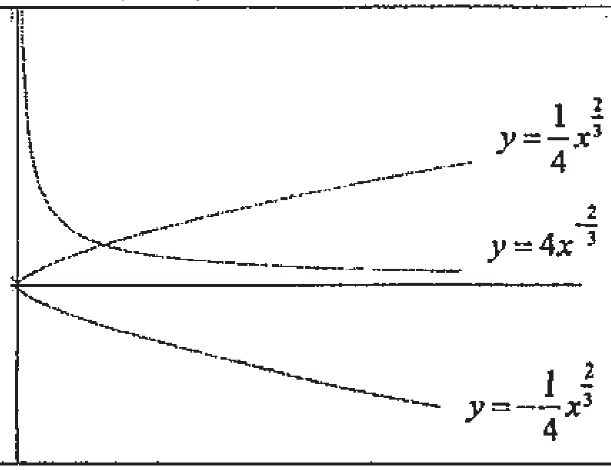
12(i) Min level = 2 – 2(0.8) = 0.4 m (ii) c = 2 – 0.8 or c = ½ (2+0.4)

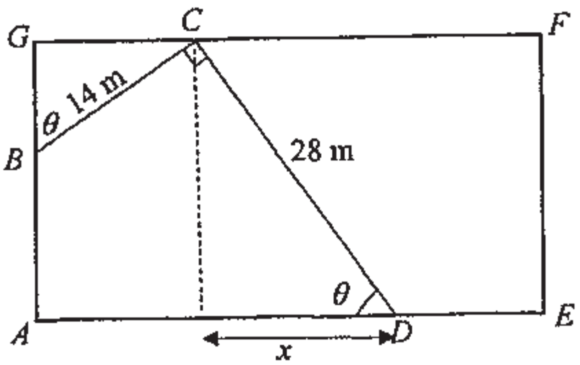
12(iii) k = π/10 (shown) (iv) $\frac{\pi}{10}t = \pi - 0.35542, \quad \pi + 0.35542$

$t = 8.869, \quad 11.131$

Between 16 53 to 19 08

Peicai Secondary School
Preliminary Exam 2019

1i	$4nx$
	$\frac{n(n-1)}{2}(16x^2)$
	$8(4n)=\frac{n(n-1)}{2}(16)$
	$n(n-5)=0$ $n=0$ (rejected)
1ii	$163+20p$
1iii	$163+20p=263$
	$p=5$
2i	$(\cos 3x)\left(\frac{d}{dx}(x)\right)+(x)\left(\frac{d}{dx}(\cos 3x)\right)$
	$\cos 3x = -3x\sin 3x$
2ii	$\frac{1}{3}\int(\cos 3x - x\cos 3x)dx$
	$\frac{\sin 3x}{3}$
	$\frac{1}{9}\sin 3x-\frac{1}{3}x\cos 3x+c$
2iii	$\left(\frac{1}{9}\sin\pi-\frac{1}{3}\left(\frac{\pi}{3}\right)\cos\pi\right)-\left(\frac{1}{9}\sin 0-\frac{1}{3}(0)\cos 0\right)=\left(0-\left(\frac{\pi}{9}\right)(-1)\right)-(0)=\frac{\pi}{9}$
3i	$\frac{(x^2+1)(2x)-(x^2-1)(2x)}{(x^2+1)^2}$ = $(x^2+1)(-1)(x^2+1)^{-1-1}(2x)+(x^2+1)^{-1}(2x)$
	$=\frac{4x}{(x^2+1)^2}$
3ii	$x=-1$ or $x=1$
	$y=x-1$
	$y=-x-1$
3iii	$4x>0,\ (x^2+1)^2>0$ → $f'(x)>0$ Hence f(x) is increasing
4i,ii,v	
4iii	Reflection in the x – axis

4iv	$\frac{1}{4}x^{\frac{2}{3}} = 4x^{-\frac{2}{3}}$ $x^{\frac{4}{3}} = 16$ (8, 1)
4vi	$\frac{1}{6}x^{-\frac{1}{3}} = -\frac{8}{3}x^{-\frac{5}{3}}$
	When $x = 8$, Product of the gradients $= \frac{1}{12} \times -\frac{1}{12} = -\frac{1}{144}$ Since $m_t m_n \neq -1$, Tangents are NOT perpendicular.
5	
5i	$x = 28\cos\theta$
	$GC = 14\sin\theta$
	$AD = 14\sin\theta + 28\cos\theta$
	$T = 53 + 14\sin\theta + 28\cos\theta$
5ii	$\sqrt{28^2 + 14^2} = \sqrt{980}$ or $14\sqrt{5}$
	$\tan^{-1}\frac{14}{28} = 26.6°$
	$T = 53 + 14\sqrt{5}\cos(\theta - 26.6°)$
5iii	$\cos(\theta - 26.565°) = \frac{25}{14\sqrt{5}}$, $\theta = 63.6°$ $\theta - 26.565° = 37.0037°$
5iv	$\frac{1}{2} \times EF \times (5 + 2\sqrt{5}) = 45 + 42\sqrt{5}$
	$EF = \frac{2(45 + 42\sqrt{5})}{(5 + 2\sqrt{5})} \times \frac{5 - 2\sqrt{5}}{5 - 2\sqrt{5}}$
	$EF = 48\sqrt{5} - 78$
6i	$(x-4)^2 + (y-2)^2 = 13$
6ii	$AR = \sqrt{17}$
	Since AR > $\sqrt{13}$, Point R lies OUTSIDE circle
6iii	$(x-4)^2 + (0-2)^2 = 13$
	$x = 7, x = 1$ $\left(\frac{7+1}{2}, 0\right)$ (4, 0)

[Turn over

6iv	4
6v	$(4-7)^2+(y-0)^2=18$ or $(4-1)^2+(y-0)^2=18$
	$y=3$ or $y=-3$(rejected) Centre = (4, 3)
7b	19.5°, 160.5°
7c	2.42, 3.86
8i	$h(x)=\dfrac{6x^2-21x+25}{2x^2-5x}$
8ii	By long division $6x^2-21x+25\div\left(2x^2-5x\right)=3$ Remainder $-6x+25$ or $a(2x^2-5x)+bx+c=6x^2-21x+25$
	$\dfrac{-6x+25}{2x^2-5x}=\dfrac{A}{x}+\dfrac{B}{2x-5}$ A= –5, B = 4
	$3-\dfrac{5}{x}+\dfrac{4}{2x-5}$
8iii	$\dfrac{2}{2x-5}$
8iv	$3+\ln\dfrac{2187}{1024}$
9a	$r=m-3n$
9bi	$x=2\dfrac{2}{5}$
9bii	$y=0.585$ or $y=5$
10i	$P=(5,\dfrac{5}{4})$
10ii	$\dfrac{9}{(7-x)^2}-1=0$
	$x=4$ or $x=10$ (rejected) $Q=(4,0)$
	Equation of AP is $y=\dfrac{9}{4}x-10$
	$x=\dfrac{40}{9}$ $A=(\dfrac{40}{9},0)$
	Area of triangle = $\dfrac{25}{72}$
	Area of shaded portion = $\int_4^5\left(\dfrac{9}{(x-x)^2}-1\right)dx-\dfrac{25}{72}$
	$\dfrac{11}{72}$

(12)