Name: ____________________ ( )	Class: Sec ______

80

GREENDALE SECONDARY SCHOOL
Preliminary Examination 2019

Additional Mathematics **4047/01**

Paper 1 18 September 2019

Secondary 4 Express / 5 Normal Academic **2 hours**

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your index number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **80.**

Question	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12
Marks												

No of additional bookets/ writing paper used		**No of additional graph paper used**	

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

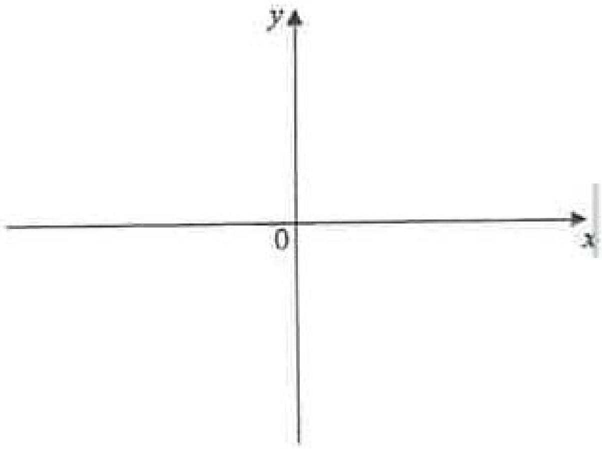
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\text{Area of } \Delta = \frac{1}{2}bc\sin A$$

Answer all the questions.

1 Express $\dfrac{10x^2-7x+10}{(3x-2)(x^2+2)}$ in partial fractions. [5]

2 **(i)** Sketch the graph of $y^2 = 3x$. **[1]**



(ii) Find the coordinates of the points of intersection of the curve $y^2 = 3x$ and the line $3y = 6x - 5$. **[4]**

3 The variables x and y are such that when the values of xy are plotted against $\sqrt{x}$, a straight line is obtained.

It is given that $y=\frac{1}{2}$ when $x=1$, and that $y=-\frac{1}{4}$ when $x=4$.

(i) Express y in terms of x. [4]

(ii) Find the value of y when $x=16$. [1]

4 (i) Show that $\frac{\cos 2x - \cos 4x}{2\sin^2 x} = 1 + 2\cos 2x$. [3]

(ii) Hence find, for $0^\circ < x < 360^\circ$, the values of x for which $\frac{\cos 2x - \cos 4x}{2\sin^2 x} = 2$. [3]

5 The roots of a quadratic equation $4x^2 - 37x + 9 = 0$ are α^2 and β^2, where $\alpha < 0 < \beta$ and $\beta < |\alpha|$.

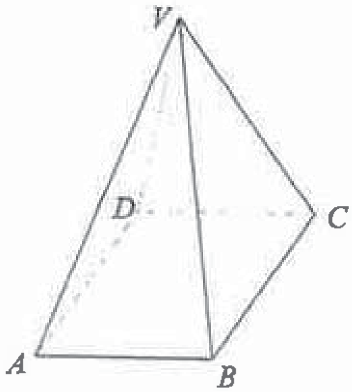
(i) Show that $\alpha\beta = -\frac{3}{2}$ and find the value of $\alpha + \beta$. [4]

(ii) Find a quadratic equation whose roots are $\frac{\alpha}{\alpha+\beta}$ and $\frac{\beta}{\alpha+\beta}$. [2]

6 A curve is such that $\frac{d^2y}{dx^2} = 1 - \frac{4}{(2x+5)^2}$ and has a stationary point at $P(-2, 5)$.

Find the equation of the curve. [5]

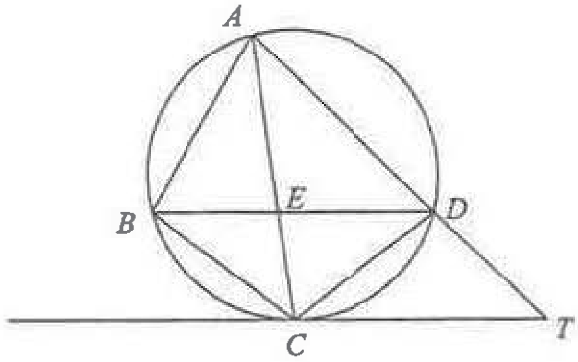
7 *VABCD* is a right pyramid with a square base *ABCD*, as shown in the diagram. The volume of the pyramid is $\left(6\sqrt{3}-8\right)$ cm^3 and the height is $\left(1+2\sqrt{3}\right)$ cm.



(i) Show that $AB^2 = 12 - 6\sqrt{3}$. **[3]**

(ii) Find the value of VA^2, giving your answer in the form $p + q\sqrt{3}$ where p and q are rational numbers. **[4]**

8 The diagram shown is not drawn to scale.

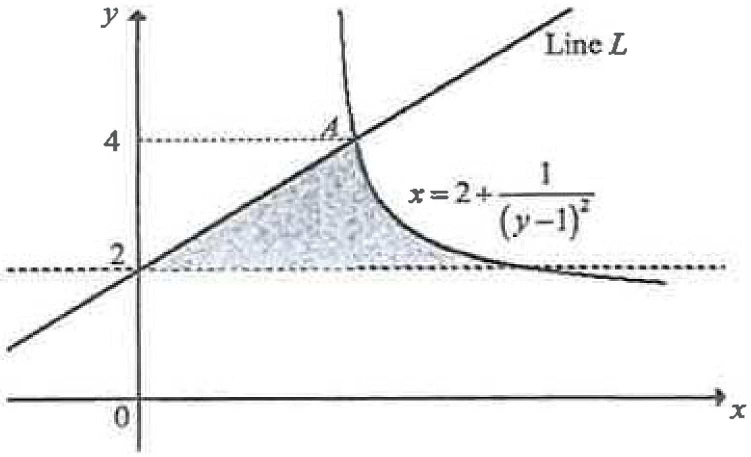


A, B, C and D are four points on the circle such that $CB = CD$. The chords AC and BD meet at E. The tangent to the circle at C meets AD extended at T.

(i) Prove that BD is parallel to CT. **[3]**

(ii) Show that $CT^2 = AT \times DT$ **[4]**

9 The diagram shows part of the curve $x = 2 + \frac{1}{(y-1)^2}$, $y \neq 1$. A line L intersects the curve at A, where $y = 4$, and cuts the y-axis at $y = 2$.



(i) Find the equation of line L. **[3]**

(ii) Find the area of the shaded region bounded by the line L, the line $y = 2$ and the curve $x = 2 + \frac{1}{(y-1)^2}$. **[4]**

10 An experiment to measure the growth of bacteria was conducted.

At 0900 on Monday, 1000 bacteria were introduced to the culture. At 1700 on the same day, the number of bacteria had grown to 1492. It is known that the number of bacteria, N, at t hours from the start of the experiment, is given by $N = pe^{kt}$, where p and k are constants.

(i) Find the value of p and of k. [3]

(ii) Calculate the number of bacteria at 0900 on Tuesday. [2]

(iii) Determine the earliest day and time (to the whole hour) at which there is at least 20 000 bacteria. [3]

11 The equation of a circle C_1 is $x^2+6x+y^2-16y+24=0$, and its centre is P.

(i) Find the coordinates of P and the radius of C_1. **[2]**

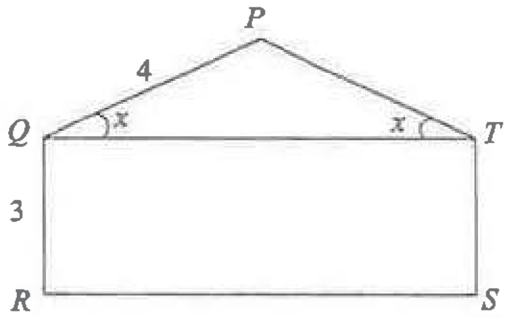
AB is a chord of C_1 and M is the midpoint of AB, where $M(-1,12)$.

(ii) Find the equation of the chord AB. **[3]**

A second circle C_2 with centre Q also passes through A and B.

(iii) Given that $PM:MQ=1:2$, show that one possible point for Q is $(3,20)$ and find the coordinates of another point. **[4]**

12 *PQRST* is a pentagon as shown in the diagram. *QRST* is a rectangle with $QR = 3$ cm. *PQT* is a triangle with $PQ = 4$ cm and $\angle PQT = \angle PTQ = x$ radians.



(i) Show that $QT = k\cos x$, where k is a positive integer to be found. [2]

(ii) Show that the area of the pentagon, A cm^2 is given by $A = 8\sin 2x + 24\cos x$. [2]

[Question 12 continues on next page]

[Question 12 continues]

(iii) Find the stationary value of A and determine whether it is a maximum or a minimum. **[6]**

END-OF-PAPER

Name: ______________________________ ( )	Class:S

	100

GREENDALE SECONDARY SCHOOL
Preliminary Examination 2019

Additional Mathematics Paper 2 **4047/2**

17 September 2019

Secondary 4 Express / 5 Normal Academic **2 hours 30 mins**

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Q	1	2	3	4	5	6	7	8	9	10	11
M											

No of additional booklets/ writing paper used		No of additional graph paper used	

Target Before : Target After :

This document consists of **16** printed pages including this cover page.

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where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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$$\Delta = \frac{1}{2} ab \sin C$$

1 A curve has the equation $y=(ax-3)\ln x$, where $x>0$, $x\neq\frac{3}{a}$ and a is a positive constant. The normal to the curve at the point where the curve crosses the x-axis is parallel to the line $x+5y-4=0$. Find the value of a. [7]

2 **(a)** Differentiate the following with respect to x.

(i) $\ln(\cos 2x)$ **[2]**

(ii) $\frac{x}{2}\tan 2x$ **[2]**

(b) Using your results from part **(a)**, find $\int 2x\sec^2 2x\, dx$. **[4]**

3 (i) Given that the constant term in the binomial expansion of $\left(\frac{2}{x}-\frac{x^2}{k}\right)^6$ is 60, find the value of the positive constant k. [4]

(ii) Using the value of k found in part (i), find the term independent of x in the expansion of $(1+x^3)\left(\frac{2}{x}-\frac{x^2}{k}\right)^6$. [4]

4 (a) A particle moves along the curve $y = 3x^2 - 2x + 5$. At the point P, the x-coordinate of the particle is increasing at a rate of 0.002 units/sec and the y-coordinate is increasing at 0.02 units/sec.
Find the coordinates of P. **[4]**

(b) The equation of a curve is $y = x^3 + 5x^2 - 8x + k$, where k a constant.
Find the set of values of x for which y is decreasing. **[4]**

5 **(i)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\ln 2x}{x^3}\right)=\frac{1}{x^4}-\frac{3\ln 2x}{x^4}$. [4]

(ii) Hence, integrate $\frac{\ln 2x}{x^4}$ with respect to x. [3]

(iii) Given that the curve $y=\mathrm{f}(x)$ passes through the point $\left(1,\frac{8}{9}\right)$ and is such that $\mathrm{f}'(x)=\frac{\ln 2x}{x^4}$, find $\mathrm{f}(x)$. [2]

6 Mr Tan drives his car along a straight road. As he passes a point A he applies the brake and his car slows down, coming to a rest at point B. For the journey from A to B, the distance, s meters, of the car from A, t seconds after passing A, is given by

$$s = 600\left(1 - e^{\ 6}\right) - 12t$$

(i) Find an expression, in terms of t, for the velocity of the car during the journey from A to B. [2]

(ii) Find the velocity of the car at A. [1]

(iii) Find the time taken for the journey from A to B. [3]

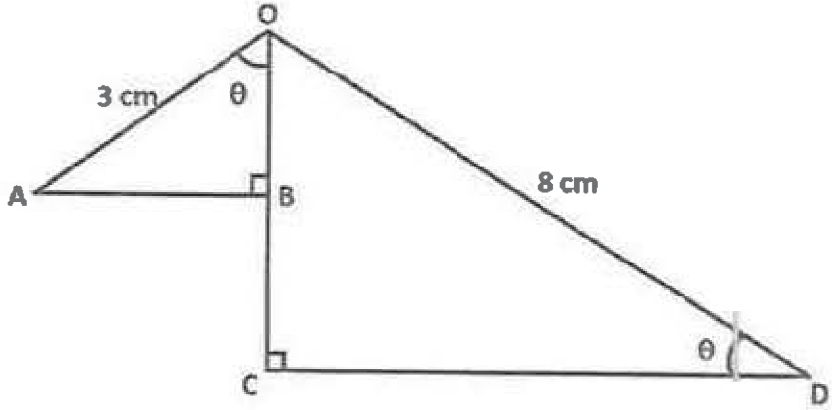
(iv) Find the average speed of the car for the journey from A to B. [3]

7 Solve each of the following equations.

(i) $e^{2\ln x} + \ln e^{2x} = 8$ [5]

(ii) $\log_5 50 + 4\log_{25} x - \log_5 (2x+4) = 2$ [5]

8 In the diagram, triangles OAB and OCD are right-angled triangles.
Angle AOB = angle $ODC = \theta$, $OA = 3$ cm and $OD = 8$ cm.



(i) Show that the length of $AB + CD = 3\sin\theta + 8\cos\theta$ **[1]**

(ii) Express $3\sin\theta + 8\cos\theta$ in the form $R\sin(\theta + \alpha)$ where $R > 0$ and α is acute. **[4]**

8 **(iii)** Find the maximum length of $AB + CD$ and the corresponding value of θ. **[3]**

(iv) Find the value of θ, if B is the midpoint of OC. **[2]**

9 The function f is defined by $f(x) = 4\cos 2x - 3$.

(i) State the amplitude of f. [1]

(ii) State the period of f in terms of π. [1]

The equation of a curve is $y = 4\cos 2x - 3$ for $0 \le x \le \pi$

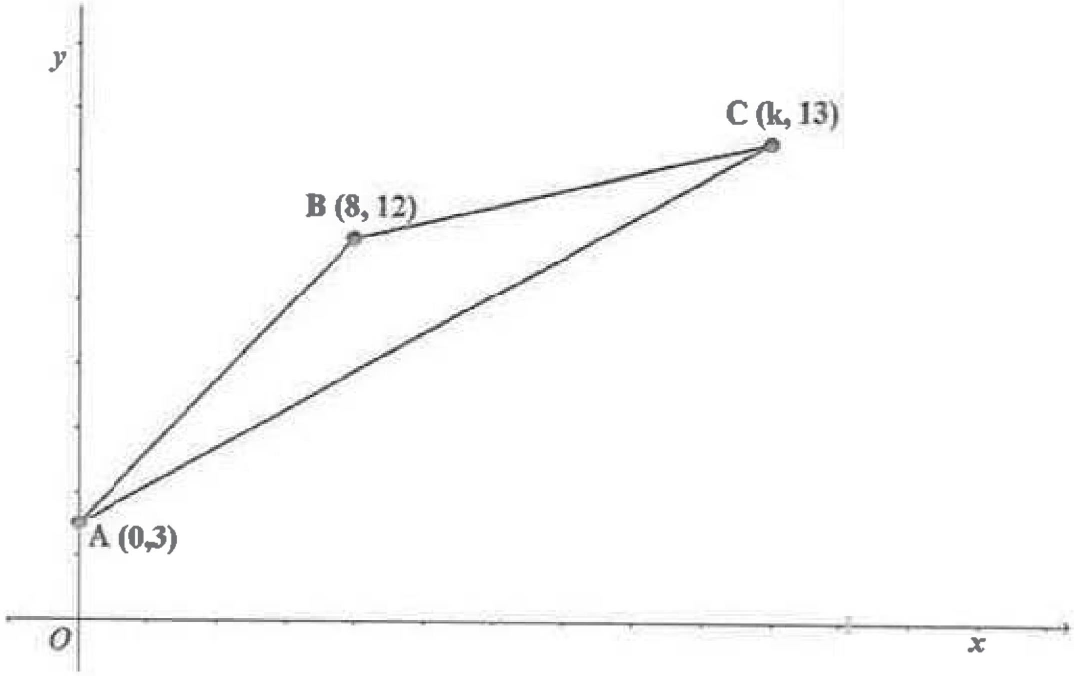
(iii) Find the minimum value of the curve. [1]

(iv) Find the x-coordinates of the points where the curve meets the x-axis. [3]

9 (v) Sketch the graph of $y = |4\cos 2x - 3|$ for $0 \le x \le \pi$. [3]

(vi) Hence, find the range of values of c, for which $|4\cos 2x - 3| = c$ has exactly two solutions only. [1]

10 The diagram shows a triangle ABC with vertices at $A(0, 3)$, $B(8, 12)$ and $C(k, 13)$.



Given that $AB = BC$,

(i) find the value of k. [4]

10 A line is drawn from B to meet the x-axis at D such that $AD = CD$.

(ii) Name the quadrilateral $ABCD$. **[1]**

(iii) Find the equation of BD and the coordinates of D. **[4]**

(iv) Find the area of the triangle ABC. **[2]**

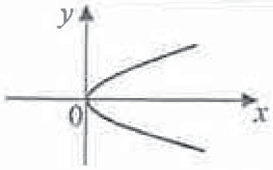
11 **(a)** **(i)** Find the range of values of x for which $x^2 - 8x +$ [illegible] 0 **[2]**

(ii) Hence, find the range of values of x for which $(x+2)^2 - 8x - 1 < 0$. **[3]**

(b) Show that $my = x^2 - 4(x-1)$ meets the curve $y = x^2 - 3x + 2$ at two distinct points for all real values of m, except $m = 0$ and $m = 1$. **[5]**

End of Paper

2019 PRELIMINARY EXAMINATION
SECONDARY 4E5N
AMATH PAPER 1 – MARK SCHEME

1		Let $\frac{10x^2-7x+10}{(3x-2)(x^2+2)}=\frac{A}{3x-2}+\frac{Bx+C}{x^2+2}$ $10x^2-7x+10=A(x^2+2)+(Bx+C)(3x-2)$ Sub $x=\frac{2}{3}$ to get $A=4$ Sub $x=0$ to get $C=-1$ Sub $x=1$ (or any other value) to get $B=2$ $$\frac{10x^2-7x+10}{(3x-2)(x^2+2)}=\frac{4}{3x-2}+\frac{2x-1}{x^2+2}$$
		TOTAL: 5m
2	(i)	
	(ii)	Equate both equations to reduce to one variable $2y^2-3y-5=0$ $(2y-5)(y+1)=0$ Solve for x and y The points of intersection are $\left(\frac{25}{12},\frac{5}{2}\right)$ and $\left(\frac{1}{3},-1\right)$
		TOTAL: 5m
3	(i)	Coordinates of two points on the straight line are $\left(1,\frac{1}{2}\right)$ and $(2,-1)$ Gradient of line = $\frac{\frac{1}{2}-(-1)}{1-2}=-\frac{3}{2}$ Equation of curve is $$\frac{xy-(-1)}{\sqrt{x}-2}=-\frac{3}{2}$$ $$y=\frac{4-3\sqrt{x}}{2x}$$
	(ii)	$y=-\frac{1}{4}$
		TOTAL: 5m

4	(i)	$\frac{\cos 2x-\cos 4x}{2\sin^2 x}=\frac{\cos 2x-(2\cos^2 2x-1)}{2\sin^2 x}$ $=\frac{(1+2\cos 2x)(1-\cos 2x)}{2\sin^2 x}$ $=\frac{(1+2\cos 2x)(1-1+2\sin^2 x)}{2\sin^2 x}$ $=1+2\cos 2x$ (shown)
	(ii)	$1+2\cos 2x=2$ $\cos 2x=\frac{1}{2}$ $2x=60^\circ, 300^\circ, 420^\circ, 660^\circ$ $x=30^\circ, 150^\circ, 210^\circ, 330^\circ$
		TOTAL: 6m
5	(i)	$\alpha^2\beta^2=\frac{9}{4}$ $\alpha<0<\beta \quad \Rightarrow \quad \alpha\beta<0$ Hence $\alpha\beta=-\sqrt{\frac{9}{4}}=-\frac{3}{2}$ (shown) $\alpha^2+\beta^2=\frac{37}{4} \Rightarrow (\alpha+\beta)^2=\alpha^2+2\alpha\beta+\beta^2=\frac{25}{4}$ $\beta<\lvert\alpha\rvert$ and $\alpha<0<\beta \Rightarrow \alpha+\beta<0$ Hence $\alpha+\beta=-\sqrt{\frac{25}{4}}=-\frac{5}{2}$
	(ii)	SOR: $\frac{\alpha}{\alpha+\beta}+\frac{\beta}{\alpha+\beta}=\frac{\alpha+\beta}{\alpha+\beta}=1$ POR: $\frac{\alpha}{\alpha+\beta}\times\frac{\beta}{\alpha+\beta}=\frac{\alpha\beta}{(\alpha+\beta)^2}=\frac{-\frac{3}{2}}{\left(-\frac{5}{2}\right)^2}=-\frac{6}{25}$ The equation is $x^2-x-\frac{6}{25}=0$ (or $25x^2-25x-6=0$)
		TOTAL: 6m

6		$\frac{dy}{dx}=\int\left[1-\frac{4}{(2x+5)^2}\right]dx=x+\frac{2}{2x+5}+c$ Sub $x=-2$, $\frac{dy}{dx}=0$ to get $c=0$, so $\frac{dy}{dx}=x+\frac{2}{2x+5}$ $y=\int\left[x+\frac{2}{2x+5}\right]dx=\frac{1}{2}x^2+\ln(2x+5)+k$ Sub $x=-2$, $y=5$ to get $k=3$ $y=\frac{1}{2}x^2+\ln(2x+5)+3$
		TOTAL: 5m
7	(i)	$\frac{1}{3}(AB)^2\left(1+2\sqrt{3}\right)=6\sqrt{3}-8$ $AB^2=\frac{18\sqrt{3}-24}{1+2\sqrt{3}}\times\frac{1-2\sqrt{3}}{1-2\sqrt{3}}$ $=\frac{18\sqrt{3}-108-24+48\sqrt{3}}{1-12}$ $=\frac{66\sqrt{3}-132}{-11}=12-6\sqrt{3}$ (shown)
	(ii)	Let M be midpoint of AC. By Pythagoras Theorem, $AC^2=AB^2+BC^2=2AB^2$, so $AM^2=\left(\frac{1}{2}AC\right)^2=\frac{1}{2}AB^2=6-3\sqrt{3}$ $VA^2=AM^2+VM^2$ $=\left(6-3\sqrt{3}\right)+\left(1+2\sqrt{3}\right)^2=19+\sqrt{3}$
		TOTAL: 7m
8	(i)	$\angle BDC=\angle CBD$ (base angles in isos $\triangle$) $=\angle TCD$ (alternate segment theorem) By the alternate-angle property, BD is parallel to CT.
	(ii)	$\angle TCD=\angle TAC$ (alternate segment theorem) $\angle CTD=\angle ATC$ (common angle) Hence ΔTCD is similar to ΔTAC (AA-test) $\frac{CT}{AT}=\frac{DT}{CT}\Rightarrow CT^2=AT\times DT$ (shown)
		TOTAL: 7m

9	(i)	When $y=4$, $x=2+\frac{1}{(4-1)^2}=\frac{19}{9}$ Gradient of line $L=\frac{4-2}{\frac{19}{9}-0}=\frac{18}{19}$ Equation of L is $y=\frac{18}{19}x+2$
	(ii)	Area $=\int_2^4\left[2+\frac{1}{(y-1)^2}\right]\mathrm{d}y-\frac{1}{2}(2)\left(\frac{19}{9}\right)$ $=\left[2y-\frac{1}{y-1}\right]_2^4-\frac{19}{9}$ $=\left[\left(8-\frac{1}{3}\right)-(4-1)\right]-\frac{19}{9}=\frac{23}{9}$ sq units
		TOTAL: 7m
10	(i)	$p=1000$ Sub $t=8$, $N=1492$ and $p=1000$ (found value) $1492=1000e^{8k}$ $8k=\ln\left(\frac{1492}{1000}\right)$ $k=0.05001\approx0.05$
	(ii)	Sub $t=24$ $N=1000e^{0.05(24)}=3320.1\approx3320$
	(iii)	$1000e^{0.05t}\geq20000$ $t\geq\frac{\ln\left(\frac{20000}{1000}\right)}{0.05}$ $t\geq59.9$ hours = 2 days 11.9 hours On Wednesday 2100 (or 9pm)
		TOTAL: 8m
11	(i)	$(x+3)^2+(y-8)^2=49$ Centre $P=(-3, 8)$, Radius = 7
	(ii)	Gradient of $PM=\frac{8-12}{-3-(-1)}=2$ Gradient of chord $AB=-\frac{1}{2}$ (AB perpendicular to PM) Equation of chord AB is

		$\dfrac{y-12}{x-(-1)}=-\dfrac{1}{2}$ $2y=-x+23$
	(iii)	Note that P, M and Q lie on a straight line. Case 1: M is between P and Q $x_Q=x_M+2(x_M-x_P)=-1+2(-1-(-3))=3$ $y_Q=y_M+2(y_M-y_P)=12+2(12-8)=20$ So coordinate of Q is $(3,20)$ (shown) Case 2: P is the midpoint of Q and M $x_P=\dfrac{x_Q+x_M}{2} \Rightarrow x_Q=2x_P-x_M=2(-3)-(-1)=-5$ $y_P=\dfrac{y_Q+y_M}{2} \Rightarrow y_Q=2y_P-y_M=2(8)-(12)=4$ So coordinate of Q is $(-5,4)$.
		TOTAL: 9m
12	(i)	Using cosine rule, $QT^2=4^2+4^2-2(4)(4)\cos(\pi-2x)$ $=32+32\cos 2x$ $=32+32(2\cos^2 x-1)$ $=64\cos^2 x$ $QT=\sqrt{64\cos^2 x}=8\cos x$
	(ii)	$A=\dfrac{1}{2}(4)(4)\sin(\pi-2x)+3(8\cos x)$ $=8\sin 2x+24\cos x$ (shown)
	(iii)	$\dfrac{dA}{dx}=16\cos 2x-24\sin x=0$ $16(1-2\sin^2 x)-24\sin x=0$ $4\sin^2 x+3\sin x-2=0$ $\sin x=0.4253$ or -1.175 (rejected) For stationary point, $x=0.4392$, $A=27.8799\approx 27.9\text{ cm}^2$ $\dfrac{d^2A}{dx^2}=-32\sin 2x-24\cos x$ When $x=0.4392$, $\dfrac{d^2A}{dx^2}=-46.35<0$, so $A=27.9\text{ cm}^2$ is a maximum area.
		TOTAL: 10m

Marking Scheme

1 A curve has the equation $y=(ax-3)\ln x$, where $x>0$, $x \neq \frac{3}{a}$ and a is a positive constant. The normal to the curve at the point where the curve crosses the x-axis is parallel to the line $x+5y-4=0$. Find the value of a. **[7]**

$(ax-3)\ln x=0$	M1
$\ln x=0$	
$x=1$	M1
$x+5y-4=0$	
$y=-\frac{1}{5}x+\frac{4}{5}$	
$m_{line}=-\frac{1}{5}$	M1
$m_{\perp}=5$	M1
$y=(ax-3)\ln x$	
$\frac{dy}{dx}=\frac{(ax-3)}{x}+a\ln x$	M1
$@x=1,\quad m_{tan}=\frac{a-3}{1}+a\ln 1$	M1
$=a-3$	
$\therefore\ a-3=5$	
$a=8$	A1

2a Differentiate the following with respect to x,
(i) $\ln(\cos 2x)$ **[2]**

$y = \ln(\cos 2x)$	
$\frac{dy}{dx} = \frac{1}{\cos 2x} \cdot -\sin 2x \cdot 2$	M1
$= -2\tan 2x$	A1

(ii) $\frac{x}{2}\tan 2x$ **[2]**

$y = \frac{x}{2}\tan 2x$	
$\frac{dy}{dx} = \frac{x}{2} \cdot \sec^2 2x \cdot 2 + \tan 2x \cdot \frac{1}{2}$	M1
$= x\sec^2 2x + \frac{1}{2}\tan 2x$	A1

b Using your results from part (a) find $\int 2x\sec^2 2x \, dx$. **[4]**

$\int x\sec^2 2x + \frac{1}{2}\tan 2x \, dx = \frac{x}{2}\tan 2x$	M1
$\int x\sec^2 2x \, dx = \frac{x}{2}\tan 2x - \frac{1}{2}\int \tan 2x \, dx$	M1
$2\int x\sec^2 2x \, dx = x\tan 2x - \int \tan 2x \, dx$	M1
$\int 2x\sec^2 2x \, dx = x\tan 2x + \frac{1}{2}\ln\cos 2x + c$	A1

3 (i) Given that the constant term in the binomial expansion of $\left(\frac{2}{x}-\frac{x^2}{k}\right)^6$ is 60, find the value of the positive constant k. [4]

$T_{r+1}=\left(\frac{2}{x}\right)^{6-r} C_r^6 \left(\frac{x^2}{k}\right)^r$	M1
$\to x^{-6+r}\times x^{2r}$	M1
$\therefore 3r-6=0$	
$r=2$	M1
$T_3=\left(\frac{2}{x}\right)^4 C_2^6\left(\frac{x^2}{k}\right)^2$	
$\frac{240}{k^2}=60$	
$k=2,\quad -2(NA)$	A1

(ii) Using the value of k found in part (i), find the term independent of x in the expression $\left(1+x^3\right)\left(\frac{2}{x}-\frac{x^2}{k}\right)^6$. [4]

$\left(1+x^3\right)\left(\frac{2}{x}-\frac{x^2}{k}\right)^6$	
$=\left(1+x^3\right)\left[\left(\frac{2}{x}\right)^6+\left(\frac{2}{x}\right)^5 C_1^6\left(-\frac{x^2}{2}\right)^1+\left(\frac{2}{x}\right)^4 C_2^6\left(-\frac{x^2}{2}\right)^2+\ldots\right]$	M2
$=\left(1+x^3\right)\left[\ldots-6\left(\frac{2^{5-1}}{x^{5-2}}\right)+15\left(\frac{2^{4-2}}{x^{4-4}}\right)+\ldots\right]$	M1
$=-96+60$	
$=-36$	A1

4a A particle moves along the curve $y = 3x^2 - 2x + 5$. At the point P, the x-coordinate of the particle is increasing at a rate of 0.002 units/sec and the y-coordinate is increasing at 0.02 units/sec. Find the coordinates of P. [4]

$y = 3x^2 - 2x + 5$	
$\frac{dy}{dx} = 6x - 2$	M1
$\frac{dx}{dt} = 0.002\ u/s \qquad \frac{dy}{dt} = 0.02\ u/s$	Both seen M1
$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	
$6x - 2 = \frac{0.02}{0.002}$	M1
$x = 2$	A1
$y = 3(2)^2 - 2(2) + 5$	
$= 13$	
$P(2, 13)$	

b The equation of a curve is $y = x^3 + 5x^2 - 8x + k$, where k is a constant. Find the set of values of x for which y is decreasing. [4]

$y = x^3 + 5x^2 - 8x + k$	
$\frac{dy}{dx} = 3x^2 + 10x - 8$	M1
For decreasing function, $\frac{dy}{dx} < 0$	
$3x^2 + 10x - 8 < 0$	M1
$(3x - 2)(x + 4) < 0$	M1
$-4 < x < \frac{2}{3}$	A1

5 (i) Show that $\frac{d}{dx}\left(\frac{\ln 2x}{x^3}\right)=\frac{1}{x^4}-\frac{3\ln 2x}{x^4}$ [4]

$\frac{d}{dx}\left(\frac{\ln 2x}{x^3}\right)=\frac{x^3\cdot\frac{1}{2x}\cdot 2-\ln 2x\cdot\left(3x^2\right)}{\left(x^3\right)^2}$	M1, M1
$=\frac{x^2}{x^6}-\frac{3x^2\ln 2x}{x^6}$	M1
$=\frac{1}{x^4}-\frac{3\ln 2x}{x^4}$	A1

(ii) Hence, integrate $\frac{\ln 2x}{x^4}$ with respect to x. [3]

$\int\frac{1}{x^4}-\frac{3\ln 2x}{x^4}\,dx=\frac{\ln 2x}{x^3}$	M1
$\int\frac{3\ln 2x}{x^4}\,dx=\int\frac{1}{x^4}\,dx-\frac{\ln 2x}{x^3}$	M1
$3\int\frac{\ln 2x}{x^4}\,dx=\frac{x^{-3}}{-3}-\frac{\ln 2x}{x^3}+c$	
$\int\frac{\ln 2x}{x^4}\,dx=-\frac{1}{9x^3}-\frac{\ln 2x}{3x^3}+c$	A1

(iii) Given that the curve $y=f(x)$ passes through the point $\left(1,\frac{8}{9}\right)$ and is such that $f'(x)=\frac{\ln 2x}{x^4}$, find f(x). [2]

$f(x)=\int\frac{\ln 2x}{x^4}\,dx$	
$y=-\frac{1}{9x^3}\left[1+3\ln 2x\right]+c$	M1
$\frac{8}{9}=-\frac{1}{9}\left[1+3\ln 1\right]+c$	
$c=1$	A1
$f(x)=-\frac{1}{9x^3}\left[1+3\ln 2x\right]+1$	

6 Mr Tan drives his car along a straight road. As he passes a point A he applies the brake and his car slows down, coming to a rest at point B.
For the journey from A to B, the distance, s meters, of the car from A, t seconds after passing A, is given by

$$s = 600\left(1 - e^{-\frac{t}{6}}\right) - 12t$$

(i) Find an expression, in terms of t, for the velocity of the car during the journey from A to B. **[2]**

$s = 600 - 600e^{-\frac{t}{6}} - 12t$	
$\frac{ds}{dt} = -600 \cdot e^{-\frac{t}{6}} \cdot \left(-\frac{1}{6}\right) - 12$	M1
$v = 100e^{-\frac{t}{6}} - 12$	A1

(ii) Find the velocity of the car at A. **[1]**

$v = 100e^{-\frac{t}{6}} - 12$	
$= 100 - 12$	
$= 88\ m/s$	B1

(iii) Find the time taken for the journey from A to B. **[3]**

$0 = 100e^{-\frac{t}{6}} - 12$	M1
$100e^{-\frac{t}{6}} = 12$	
$-\frac{t}{6} = \ln\left(\frac{12}{100}\right)$	M1
$t = 12.72\ s$	A1

(iv) Find the average speed of the car for the journey from A to B. **[3]**

$\text{Ave speed} = \frac{\text{tot dist}}{\text{tot time}}$	
$= \frac{600\left(1 - e^{-\frac{12.72}{6}}\right) - 12(12.72)}{12.72}$	M1 (num) M1 (den)
$= 29.5\ m/s$	A1

7 Solve each of the following equations.

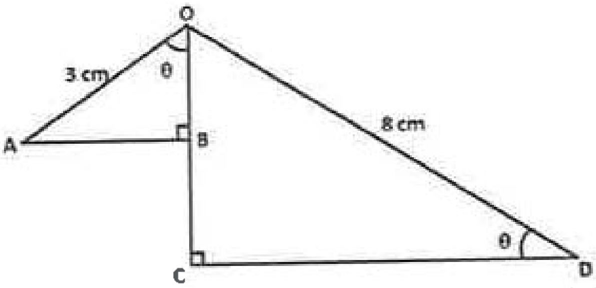
(i) $e^{2\ln x}+\ln e^{2x}=8$ **[5]**

$e^{2\ln x}+\ln e^{2x}=8$	
$e^{\ln x^2}+2x\ln e=8$	M1, M1
$x^2+2x-8=0$	M1
$(x+4)(x-2)=0$	
$x=2,\quad or\ x=-4\ (NA)$	A1, A1

(ii) $\log_5 50+4\log_{25}x-\log_5(2x+4)=2$ **[5]**

$\log_5 50+4\log_{25}x-\log_5(2x+4)=2$	
$\log_5 25\times 2+\frac{4\log_5 x}{\log_5 25}-\log_5(2x+4)=2$	M1, M1
$\log_5 5^2+\log_5 2+\frac{4\log_5 x}{\log_5 5^2}-\log_5 2(x+2)=2$	
$2+\log_5 2+\frac{4\log_5 x}{2}-[\log_5 2+\log_5(x+2)]=2$	M1
$2+\log_5 2+2\log_5 x-\log_5 2-\log_5(x+2)=2$	
$\log_5(x+2)=2\log_5 x$	
$x+2=x^2$	
$x^2-x-2=0$	M1
$(x-2)(x+1)=0$	
$x=2,\quad or\ x=-1\ (NA)$	A1

8 In the diagram, triangles OAB and OCD are right-angled triangles.



Angle AOB = angle $ODC = \theta$., $OA = 3$ cm and $OD = 8$ cm.

(i) Show that the length of $AB + CD = 3\sin\theta + 8\cos\theta$. [1]

$AB + CD = 3\sin\theta + 8\cos\theta$	B1

(ii) Express $3\sin\theta + 8\cos\theta$ in the form $R\sin(\theta+\alpha)$ where $R > 0$ and α is acute. [4]

$3\sin\theta + 8\cos\theta = R\sin(\theta+\alpha)$	
$= R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$	
$R\cos\alpha = 3$	M1
$R\sin\alpha = 8$	both
$\tan\alpha = \frac{8}{3}$	
$\alpha = 69.44^\circ$	M1
$R = \sqrt{73}$	M1
$3\sin\theta + 8\cos\theta = \sqrt{73}\sin(\theta + 69.44^\circ)$	A1

(iii) Find the maximum length of $AB + CD$ and the corresponding value of θ. [3]

Max = $\sqrt{73}$ *or* 8.544	B1
$\sin(\theta + 69.44^\circ) = 1$	M1
$\theta = 90^\circ - 69.44^\circ$	
$\theta = 20.56^\circ$	A1

(iv) Find the value of θ, if B is the midpoint of OC. [2]

$2OB = OC$	
$2(3\cos\theta) = 8\sin\theta$	M1
$\tan\theta = \frac{6}{8}$	
$\theta = 36.87^\circ$	A1

9 The function f is defined by $f(x) = 4\cos 2x - 3$.

(i) State the amplitude of f. **[1]**

Amplitude = 4	B1

(ii) State the period of f in terms of π. **[1]**

$\text{Period} = \frac{2\pi}{2}$ $= \pi$	B1

The equation of a curve is $y = 4\cos 2x - 3$ for $0 \le x \le \pi$.

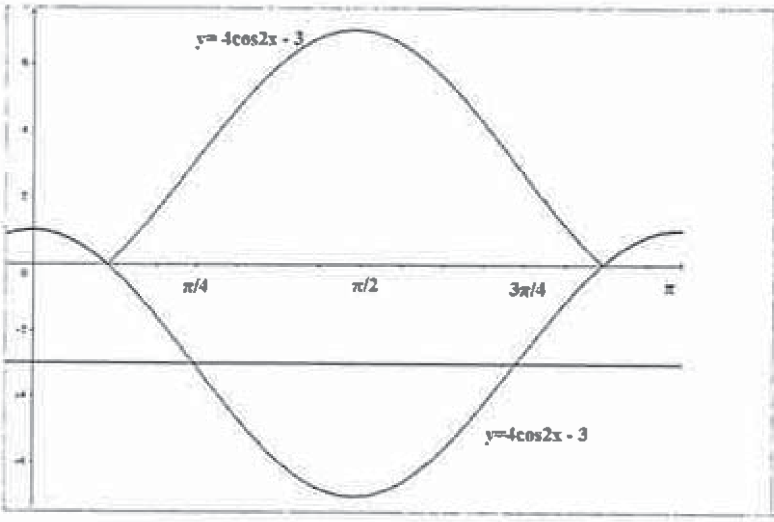
(iii) Find the minimum value of the curve. **[1]**

$\text{Min} = -4 - 3$ $= -7$	B1

(iv) Find the x-coordinates of the points where the curve meets the x-axis. **[3]**

$4\cos 2x - 3 = 0 \qquad 0 \le x \le \pi$	M1
$\cos 2x = \frac{3}{4} \qquad 0 \le 2x \le 2\pi$	M1
$2x = 0.7227, \quad 5.560$ $x = 0.3614, \quad 2.780$	A1

(v) Sketch the graph of $y = \lvert 3\cos 2x - 4 \rvert$ for $0 \le x \le \pi$. **[3]**



Shape B1
Correct keypoints:
x-axis B1
y-axis B1

(vi) Hence, find the range of values of c, for which $\lvert 3\cos 2x - 4 \rvert = c$ has exactly two solutions only. **[1]**

$1 < c < 7$	B1

10 The diagram shows a triangle ABC with vertices at $A(0, 3)$, $B(8, 12)$ and $C(k, 13)$.

(i) Given that $AB = BC$, find the value of k. [4]

$AB^2 = BC^2$	
$(k-8)^2 + (13-12)^2 = (8-0)^2 + (12-3)^2$	M1
$(k-8)^2 = 64 + 81 - 1$	
$(k-8)^2 - 144 = 0$	M1
$(k-8+12)(k-8-12) = 0$	
$(k+4)(k-20) = 0$	M1
$k = 20, \quad k = -4 (NA)$	A1

A line is drawn from B to meet the x-axis at D such that $AD = CD$.

(ii) Name the quadrilateral $ABCD$. [1]

Kite	B1

(iii) Find the equation of BD and the coordinates of D. [4]

Property of Kite ▵. Diagonals intersect at 90°	
$m_{AC} = \frac{13-3}{20-0}$	M1
$= \frac{1}{2}$	
$m_{BD} = -2$	M1
$12 = -2(8) + c$	
$c = 28$	
$y = -2x + 28$	A1
$0 = -2x + 28$	
$x = 14$	
$D(14, 0)$	A1

(iv) Find the area of the triangle ABC. [2]

$A = \frac{1}{2}\begin{vmatrix} 0 & 8 & 20 & 0 \\ 3 & 12 & 13 & 3 \end{vmatrix}$	M1
$= \frac{1}{2}[(264) - (104)]$	
$= 80 \text{ units}^2$	A1

11a (i) Find the range of values of x for which $x^2-8x+15\geq 0$ **[2]**

$x^2-8x+15\geq 0$ $(x-5)(x-3)\geq 0$ $x\leq 3 \quad or\ x\geq 5$	M1 A1

(ii) Hence, find the range of values of x for which $(x+2)^2-8x-1<0$ **[3]**

$(x+2)^2-8(x+2)+16-1<0$ $(x+2)^2-8(x+2)+15<0$ $[(x+2)-5][(x+2)-3]<0$ $(x-3)(x-1)<0$ $1<x<3$	M1 M1 A1

b Show that $my=x^2-4(x-1)$ meets the curve $y=x^2-3x+2$ at two distinct points for all real values of m, except $m=0$ and $m=1$. **[5]**

$my=x^2-4(x-1)$ $y=\frac{x^2-4x+4}{m}$ $y=x^2-3x+2$	M1
$\frac{x^2-4x+4}{m}=x^2-3x+2$ $x^2-4x+4=mx^2-3mx+2m$	M1
$(m-1)x^2+(4-3m)x+(2m-4)=0$	M1
$b^2-4ac=(4-3m)^2-4(m-1)\cdot 2(m-2)$ $=16-24m+9m^2-8(m^2-3m+2)$ $=16-24m+9m^2-8m^2+24m-16$ $=m^2$	M1
$m^2>0$ $\therefore b^2-4ac>0$ $\therefore$ 2 distinct roots	A1