ST. MARGARET'S SECONDARY SCHOOL

Mid-Year Examinations 2019

Founded 1842

CANDIDATE NAME

CLASS

REGISTER NUMBER

ADDITIONAL MATHEMATICS

4047/01

Paper 1

7 May 2019

Secondary 4 Express

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **17** printed pages and a blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$, $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where *n* is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

[Turn over

1 (i) Given that $\frac{8}{(27^x)} = 2^{3x}$, find the exact value of 6^x. [2]

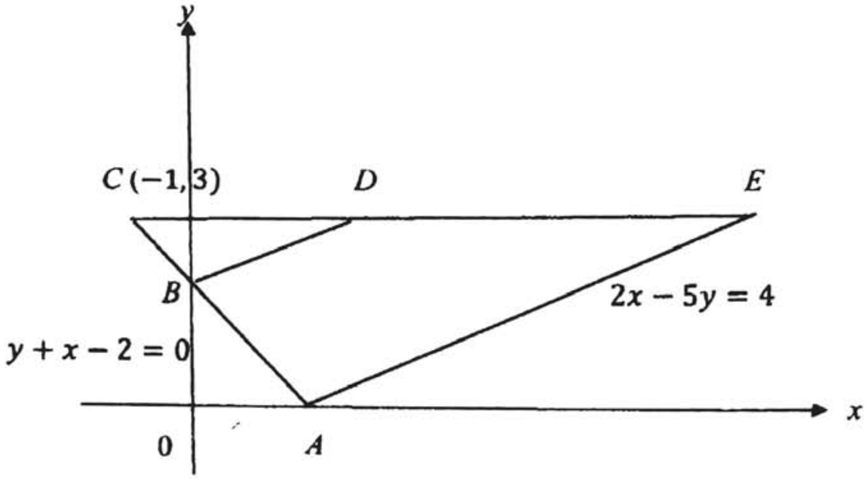
(ii) Solve the equation $(25^x)^x = 125^{5x-6}$. [3]

2 **(i)** A prism with a trapezium base has a volume of $24 + 17\sqrt{2}$ cm^3. The trapezium has a height of $\sqrt{2}$ cm and its parallel sides are $3\sqrt{2} + 2$ cm and 2 cm respectively. Find the height of the prism, leaving your answer in the form $a\sqrt{2} + b$ cm, where a and b are integers. [3]

(ii) Simplify $\frac{5}{\sqrt{2}} + 2\sqrt{50} - \frac{2}{\sqrt{8}}$. [2]

3 Variables x and y are related by the equation $y = \frac{11x-1}{9-x}$. Given that x and y are functions of t and that y increases from an initial value of 2.9 at a constant rate of 0.005 units/s, find the corresponding rate of change of x after 20 seconds. [5]

4 The diagram shows a trapezium $BDEA$ in which BD is parallel to AE. The side ED is parallel to the x-axis. It is extended to meet at point C which has coordinates $(-1, 3)$. The equation of AE is $2x - 5y = 4$ and the equation of AC is $y + x - 2 = 0$.



Find

(i) the coordinates of A, E and D. [4]

(ii) the ratio of area of triangle BCD to area of trapezium $BDEA$. [1]

5 **(i)** Differentiate $x^2 \ln x$ with respect to x. [2]

(ii) Hence find $\int x\ln x \ dx$. [3]

6 Given that $\sin A = \frac{24}{25}$ where A is acute, $\tan B = \frac{3}{4}$ and that A and B are in different quadrants, find, without evaluating A or B, the value of

(i) $\sin(A + B)$, [3]

(ii) $\cos\left(\frac{B}{2}\right)$. [2]

7 It is given that a curve has equation $= \mathrm{f}(x)$, where $\mathrm{f}(x) = (2x+3)(x-2)^2$.

(i) Find the coordinates of the stationary points of the curve. [4]

(ii) Hence, determine the nature of these stationary points. [3]

(iii) Sketch the graph of $f'(x)$ against x. [2]

8 (i) The quadratic function is defined by $y = 2x^2 - 8x - 15$, where x is real.

(a) Find the set of values of x for which $y \leq 3x^2$. [3]

(b) Find the set of values of k for which the equation $y = kx - 23$ has no real roots. [3]

(ii) Show that the line $y = \frac{x}{p} + \frac{p}{2}$ is a tangent to the curve $y^2 = 2x$ for all real values of p. [3]

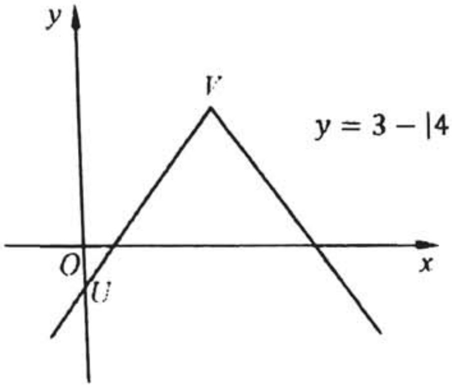
9 (i) Sketch the graph of $= 3\sqrt{x}$. [1]

(ii) On the same axes, sketch the graph of $y = \frac{12}{\sqrt{x^3}}$, $x > 0$. [1]

(iii) Calculate the x co-ordinate of the point of intersection of your graphs in exact form. [2]

(iv) Determine, with explanation, whether the tangents to the graphs at the point of intersection are perpendicular. [4]

10



The diagram shows part of the graph $y = 3 - |4 - x|$ intersecting the y-axis at U.

V is the highest point on the graph.

(i) Find the coordinates of U and V. [2]

The equation of a line is $y = mx + 3$, where m is a constant.

(ii) In the case where $m = -2$, find the coordinates of any point of intersection of the line and the graph of $y = 3 - |4 - x|$. [3]

(iii) Determine the range of values of m for which the line intersects the graph of $y = 3 - |4 - x|$ in two points. [2]

11 The roots of the quadratic equation $2x^2 + 5x - 4 = 0$ are α and β.

Find

(i) the value of $\alpha^3 + \beta^3$, [4]

(ii) a quadratic equation with roots $\frac{1}{\alpha^3}$ and $\frac{1}{\beta^3}$. [4]

12 (i) Show that $\frac{4\cos 2x}{1+\cos 2x}$ can be written as $+b sec^2 x$, where a and b are integers. [4]

(ii) Solve, for $0^o < x < 180^o$, the equation $\frac{4\cos 2x}{1+\cos 2x} = 4\tan x - 5$. [4]

(iii) State the number of solutions of the equation $\frac{4\cos 2x}{1+\cos 2x} = 4\tan x - 5$ in the range $-360^o < x < 360^o$. [1]

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ST. MARGARET'S SECONDARY SCHOOL

Mid-Year Examinations 2019

CANDIDATE NAME			
CLASS		REGISTER NUMBER	

ADDITIONAL MATHEMATICS **4047/02**

Paper 2 **10 May 2019**

Secondary 4 Express **2 hours 30 minutes**

Candidates answer on the Question Paper

Additional Materials: Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
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At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

This document consists of **18** printed pages.

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$$\Delta = \frac{1}{2}ab\sin C$$

1 A curve has the equation $y = x^2 e^{2-3x}$.

Find the range of values of x for which y is an increasing function of x. [3]

2 Find all the exact values of x which satisfies the equation

$8 \cos x - 2 \sin x \cos x + 4 - \sin x = 0$ for $0 \le x \le 9$. [3]

3 Differentiate xe^{5x} with respect to x. Hence find $\int xe^{5x}\ \mathrm{d}x$. [4]

4 Evaluate the following definite integrals, giving your answer in exact form.

(a) $\int_0^1 \frac{3e^{5x}-7}{e^{2x}}\ \mathrm{d}x$, [3]

(b) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{4-\cos^2 2x}{1-\sin^2 2x}\ \mathrm{d}x$. [3]

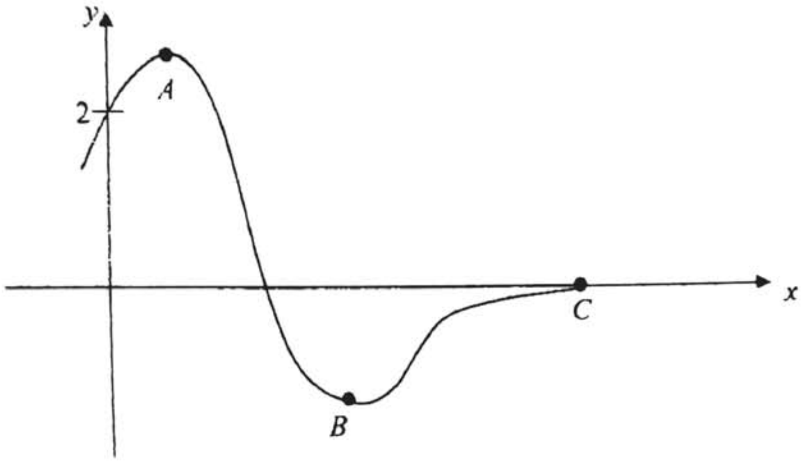
5 Express $\dfrac{x^3}{x^2+3x+2}$ in the form $Ax + B + \dfrac{C}{x+2} + \dfrac{D}{x+1}$, where A, B, C and D are constants. Hence evaluate $\displaystyle\int \frac{x^3}{x^2+3x+2}\,\mathrm{d}x$. [6]

6 (i) Find the number of real roots of the equation $3x^3 + 2x^2 + 10 = 3x$. [4]

(ii) Hence solve the equation $3 + 2y - 3y^2 + 10y^3 = 0$. [2]

7 The diagram shows part of a graph whose gradient function is given by

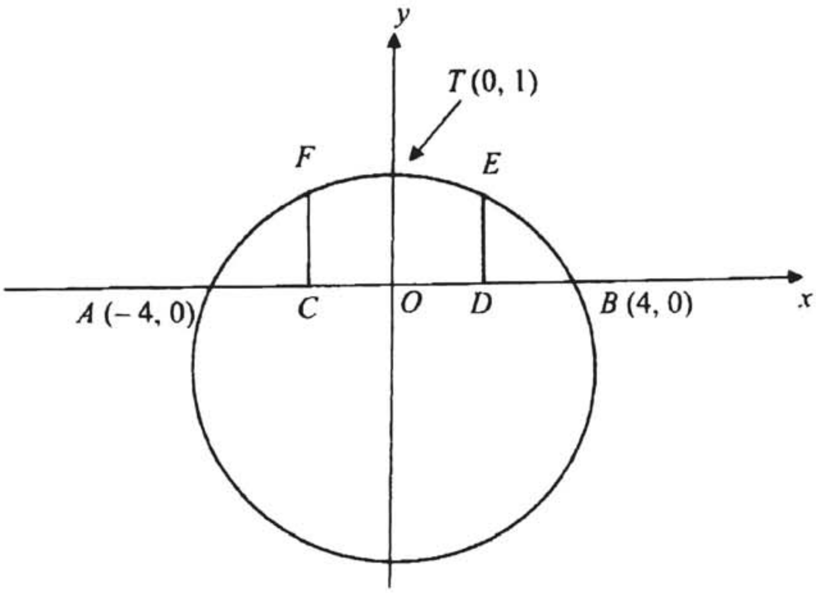
$\frac{dy}{dx} = 2\cos 2x - 2\sin x$. A, B and C are stationary points on the graph.



(i) Show that C is a point of inflexion. [4]

(ii) Find the equation of the curve. [2]

8 The diagram shows the arch *AFTEB* of a stone bridge. The bridge forms an arc of a circle and the length *AB* forms a chord of the circle. *AB* is 8 m and the top of the bridge *T* is 1 m vertically above *AB*. *C* and *D* are the midpoints of *OA* and *OB*. *CF* and *DE* are two vertical pillars supporting the arch.



(i) Show that the equation of the circle is $x^2 + y^2 + 15y - 16 = 0$. [4]

(ii) Find the height of the pillar *CF*. [2]

9 (i) Given that $y = \cos^3 x$, show that $\frac{dy}{dx} = 3\sin^3 x - 3\sin x$. [2]

(ii) Hence evaluate $\int_0^{\frac{\pi}{3}} \sin^3 x \, dx$. [5]

10 (a) Solve the equation $\ln(2x + e) = 1 + \frac{1}{\log_x e}$, leaving your answer in the exact form. [3]

10 (b) Without using a calculator, find the exact value of y if $(5y)^{\ln 5} = (2y)^{\ln 2}$. [5]

11 **Answer the whole question on a sheet of graph paper.**

The table below shows the experimental values of the variables x and y.

x	0.5	1.5	3	4.5	5.5	6
y	4.43	6.24	7.44	11.4	15.7	18.7

It is known that x and y are related by an equation of the form $y = e + ab^x$.

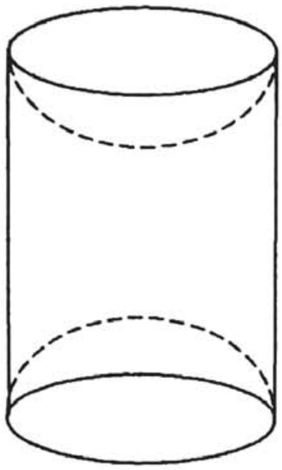
One of the y values is incorrect.

(i) Plot a straight line graph of $\ln(y - e)$ against x. [4]

(ii) Use your graph to identify the abnormal reading and estimate its correct value. [2]

(iii) Use the graph to estimate the value of a and of b. [2]

12 A solid right circular cylinder with base radius r cm and height h cm has a hemisphere hollowed out from each end as shown in the diagram.



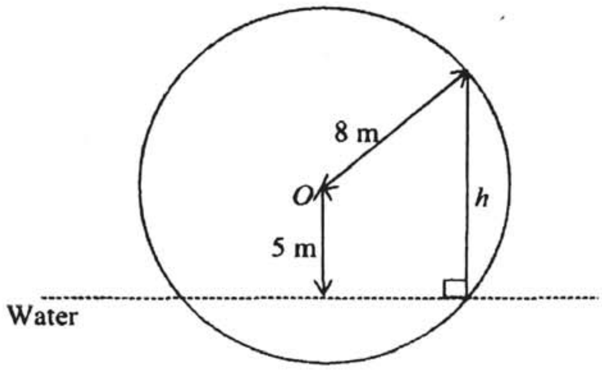
Given that the surface area is 128π cm^2,

(i) show that the volume of the solid, V cm^3, is given by $V = \frac{2\pi r}{3}\left(96 - 5r^2\right)$. [3]

(ii) find the value of r for which V is stationary, [2]

(iii) find the corresponding value of V and determine whether it is a maximum or a minimum value. [3]

13 A waterwheel rotates 5 revolutions anticlockwise in 1 minute. A bucket B is attached to the waterwheel. Tammy starts a stopwatch when the bucket B is at its highest height above water level. The radius of the waterwheel is 8 m and its centre is 5 m above the water level.



The height of the bucket B above water level is given by $h = a \cos bt + c$, where t is the time, in seconds, since Tammy started the stopwatch.

(i) Determine the value of each of the constant a, b, and c. [5]

(ii) For how long in each revolution is $h < 0$? [3]

(iii) Explain what does the answer in (ii) mean. [1]

14 (a) In the expansion of $\left(x^9 - \frac{1}{3x}\right)^{10}$, determine if there is a x^9 term. [3]

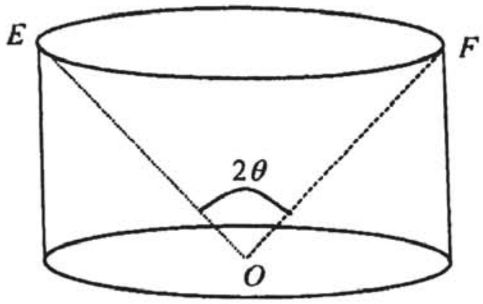
(b)(i) Find the first three terms in the expansion of $\left(2 - \frac{1}{x}\right)^8$ in descending powers of x. [2]

(ii) Hence find the values of a, b and c given that the first three terms in the expansion of $(a + bx)\left(2 - \frac{1}{x}\right)^8$ are $128x$, -256 and $\frac{c}{x}$ respectively. [5]

15 An open cylindrical tank with O as the centre of the base is shown in the diagram.

It is given that $\angle EOF = 2\theta$ where $0° < \theta < 90°$ and $OF = 2$ cm.

The external total surface area of the cylindrical tank is S cm^2.



(i) Show that $S = 2\pi\,(2\sin 2\theta - \cos 2\theta + 1)$. [4]

(ii) Express $S = 2\pi\,(2 \sin 2\theta - \cos 2\theta + 1)$ in the form $2\pi\,[R \sin (2\theta - \alpha) + 1]$

where $R > 0$ and $0° < \theta < 90°$. [3]

(iii) Find the maximum possible value of S and the corresponding value of θ. [3]

Anwer

1 $0 < x < \frac{2}{3}$

2 $\frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}$

3 $\frac{1}{5}\left(xe^{5x} - \frac{1}{5}e^{5x}\right) + C_2$

4(a) $e^3 + \frac{7}{2e^2} - \frac{9}{2}$ (b) $2\sqrt{3} - \frac{\pi}{6}$

5 $\frac{x^2}{2} - 3x + 8\ln(x+2) - \ln(x+1) + C$

6(i) 1 real root (ii) -0.5

7(ii) $y = \sin 2x + 2\cos x$

8(ii) 0.761 m, – 15.8 (reject)

9(ii) $\frac{5}{24}$

10(a) $x = \frac{e}{e-2}$ (b) 0.1

11(ii) abnormal reading $y = 6.24$, correct $y = 5.30$

(iii) $a = 1.42$, $b = 1.50$

12(ii) $r = 2.53$ (iii) $V = 339$ cm^3, V is maximum

13(i) $a = 8$, $c = 5$, $b = \frac{\pi}{6}$ (ii) 3.42 s

(iii) It is th duration of time that bucket is in the water.

14(a) no x^9 term (b)(i) $256 - \frac{1024}{x} + \frac{1792}{x^2} + \cdots$

(c) $a = 1$, $b = 0.5$, $c = -128$

15(ii) $S = 2\pi[\sqrt{5}sin(2\theta - 26.6°) + 1]$

(iii) max $s = 20.3$ cm^2 when $\theta = 58.3^0$