Class	Index Number	Name

新加坡海星中学

MARIS STELLA HIGH SCHOOL
MID-YEAR EXAMINATION
SECONDARY FOUR

ADDITIONAL MATHEMATICS **4047/01**
Paper 1
10 May 2019
2 hours

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use
80

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 A triangle has an area of $\left(58+8\sqrt{5}\right)$ cm^2 and a height of $\left(7+3\sqrt{5}\right)$ cm. Without using a calculator, find the exact length of its base, expressing in the form $a+b\sqrt{5}$, where a and b are integers. [4]

2 **(i)** On the same diagram, sketch the curves $y = 9x^{-\frac{1}{2}}$ and $y^2 = 4x$. [2]

(ii) Find the coordinates of the point(s) of intersection of the two curves. [2]

3 The equation of a curve is $y = 2xe^{x-k}$, where k is a constant. The curve passes through the point $(5,10)$.

(i) Find the value of k. [2]

(ii) For what values of x is y an increasing function of x? [3]

4 Express $\dfrac{16x^2-9x+18}{x^3+3x^2}$ in partial fractions. [5]

5 The function f is given by $f(x) = -3\sin\frac{x}{2} + 2$.

(i) State the amplitude and period of f. [2]

(ii) Sketch the graph of $y = f(x)$ for $0 \le x \le 4\pi$. By drawing a suitable straight line on the same axes, state the number of solutions to the equation $4\pi - x - 6\pi\sin\frac{x}{2} = 0$ for $0 \le x \le 4\pi$. [5]

6 **(i)** Given that $\cos(A+B)=3\cos(A-B)$ and $\tan A=-\frac{5}{2}$, find the value of $\cot B$. [3]

(ii) Prove that $\frac{1+\tan^2 x}{1-\tan^2 x}=\sec 2x$. [3]

7 The roots of the quadratic equation $2x^2 + x + 6 = 0$ are α and β.

(i) Express $\alpha^2 - \alpha\beta + \beta^2$ in terms of $(\alpha + \beta)$ and $\alpha\beta$. [1]

(ii) Form a quadratic equation whose roots are α^3 and β^3. [5]

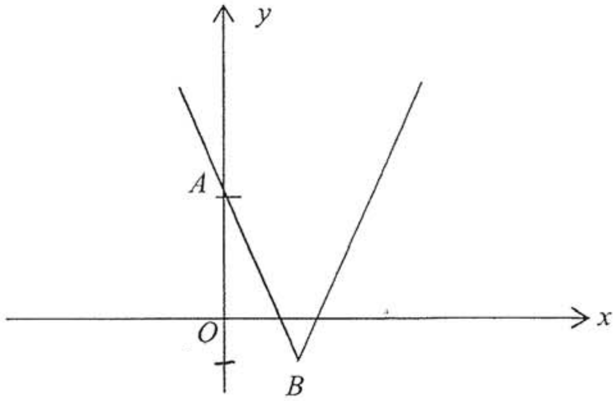
8 An antique grandfather clock manufactured using the finest wood in 1850 had an initial value \$2000. The clock appreciated in its value such that its value \$$V$ can be modelled by the equation $V = 20000 - Ae^{kt}$, where t is the number of years after its manufacture date.

(i) Find the value of A. [2]

(ii) In the year 1880, the clock reached five times its initial value. Show that $k = -0.01959$ correct to 4 significant figures. [3]

(iii) Explain why the value of the clock will not exceed \$20000. [2]

9 The diagram shows the graph of $y = |6 - 2x| - 1$.



(i) Find the coordinates of A and of B. [2]

(ii) By solving the equation $|6 - 2x| = 3x + 1$, find the x-coordinate of the point(s) of intersection between the graphs $y = |6 - 2x| - 1$ and $y = 3x$. [3]

(iii) State the range of values of m for the equation $|6 - 2x| = mx + 1$ to have no solution. [2]

10 A circle passes through the points $P(0,8)$ and $Q(8,12)$. The y-axis is a tangent to the circle at P.

(i) Find the equation of the circle. [5]

The tangent to the circle at Q intersects the x-axis and y-axis at A and B respectively.

(ii) Find the ratio of $AQ:QB$. [3]

11 **(i)** Expand $(1-2x)^9$ in ascending powers of x up to the term in x^3. [2]

(ii) Find the value of k, given that the coefficient of x in the expansion of $\left(3x+\frac{1}{kx^2}\right)(1-2x)^9$ is -53. [3]

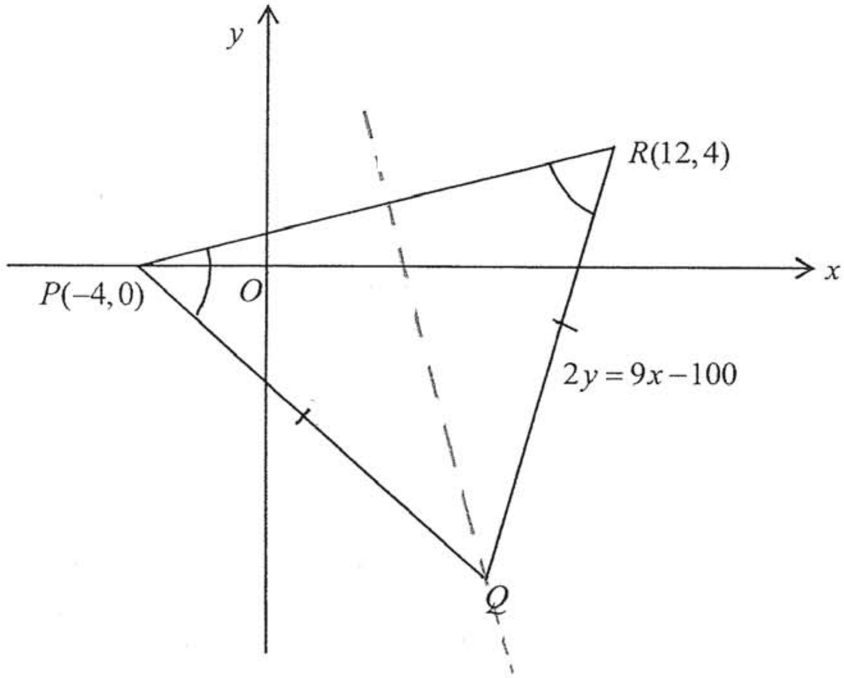
12 The equation of a curve is given by $y = \ln\sqrt{\dfrac{5x}{9x+4}}$.

(i) Find $\dfrac{dy}{dx}$, expressing it as a single fraction. [3]

(ii) Find the rate at which x is changing when the graph crosses the x-axis, given that y is increasing at a rate of 0.3 units per second. [4]

Solutions to this question by accurate drawing will not be accepted.

13



The diagram, which is not drawn to scale, shows a triangle PQR in which $PQ = QR$. The coordinates of the points P and R are $(-4, 0)$ and $(12, 4)$ respectively.

(i) Find the equation of the perpendicular bisector of PR. [3]

The equation of the line QR is $2y = 9x - 100$.

(ii) Find the coordinates of Q. [2]

(iii) Find the coordinates of S if $PQRS$ forms a rhombus.
Hence, or otherwise, find the area of the rhombus $PQRS$. [4]

End of paper

Class	Index Number	Name

新加坡海星中学

MARIS STELLA HIGH SCHOOL

MID-YEAR EXAMINATION

SECONDARY FOUR

ADDITIONAL MATHEMATICS **4047/02**

Paper 2

15 May 2019

2 hours

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use
80

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2} bc \sin A$$

1 The polynomial $f(x) = 2x^3 + ax^2 + bx + 8$, where a and b are constants, has a factor $(x+2)$ and leaves a remainder of 10 when divided by $(2x-1)$.

(i) Find the value of a and of b. [4]

(ii) Using the values of a and b found in part **(i)**, explain why the equation $f(x)=0$ has only one real root. Find this root. [4]

(iii) Hence, solve $x^3+3x^2+4x+32=0$. [2]

2 (a) Find the range of values of k for which $(k-3)x^2 + 4x + k$ is always positive for all real values of x. [4]

(b) Show that the roots of the equation $6x^2 + 4(m-1) = 2(x+m)$ are real if $m \leq 2\frac{1}{12}$. [3]

Page 6 missing - to copy questions from answers

(c) Solve the equation $\log_3(2x-1)-\frac{1}{2}\log_3(x^2+2)=\log_{25}5$. [5]

4 In a Science experiment, a container of liquid was heated to a temperature of K °C. It was then left to cool in a chiller such that its temperature, T °C, t minutes after removing the heat, is given by $T = Ke^{-qt}$, where q is a constant.

Measured values of t and T are given in the following table.

t (minutes)	2	4	7	10	12
T °C	71.1	57.0	40.8	29.3	23.4

(i) Using a scale of 1 cm to 1 unit on the t-axis and 4 cm to 1 unit on the ln T- axis, plot ln T against t and draw a straight line graph. [2]

(ii) Use the graph to estimate the value of K and of q. [4]

(iii) Estimate the temperature of the liquid 8 minutes after it was left to cool. [2]

5 **(a)** **(i)** Prove that $\dfrac{\sin x}{\sec x+1}+\dfrac{\sin x}{\sec x-1}=2\cot x$. [4]

(ii) Hence find, for $0 \leq x \leq 4$, the exact solutions of the equation

[3]

(b) Given that θ is obtuse and that $\sin\theta = \frac{1}{\sqrt{3}}$, express, **without the use of a calculator**,

$\frac{1}{\sin\theta - \cos\theta}$ in the form $\sqrt{a} - \sqrt{b}$ where a and b are integers. [4]

6 The equation of a curve is $y = \dfrac{a}{x} + bx + 1$, where a and b are constants. The normal to the curve at the point $Q(1, -1)$ is parallel to the line $4y - x = 20$. This normal meets the curve again at point P.

(i) Find the value of a and of b. [5]

(ii) Find the coordinates of point P. [3]

7 The equation of a curve is $y = \dfrac{x^2}{x-1}$, where $x \neq 1$.

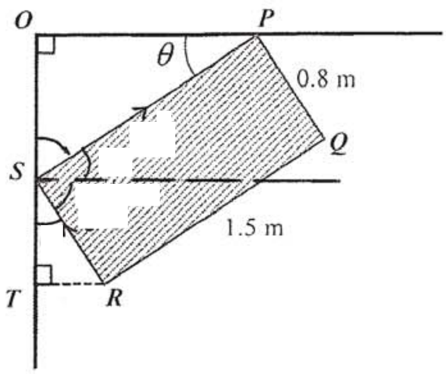
(i) Obtain an expression for $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. [4]

(ii) Find the coordinates of the stationary points of the curve and determine their nature. [4]

8 **(a)** Differentiate $\cot^4\left(\frac{\pi}{2}-2x\right)$ with respect to x. [3]

(b) Given that a curve has the equation $y=3\sin 2x-\cos x$, find the gradient of the curve when $x=\frac{\pi}{3}$, leaving your answer in exact form. [3]

9



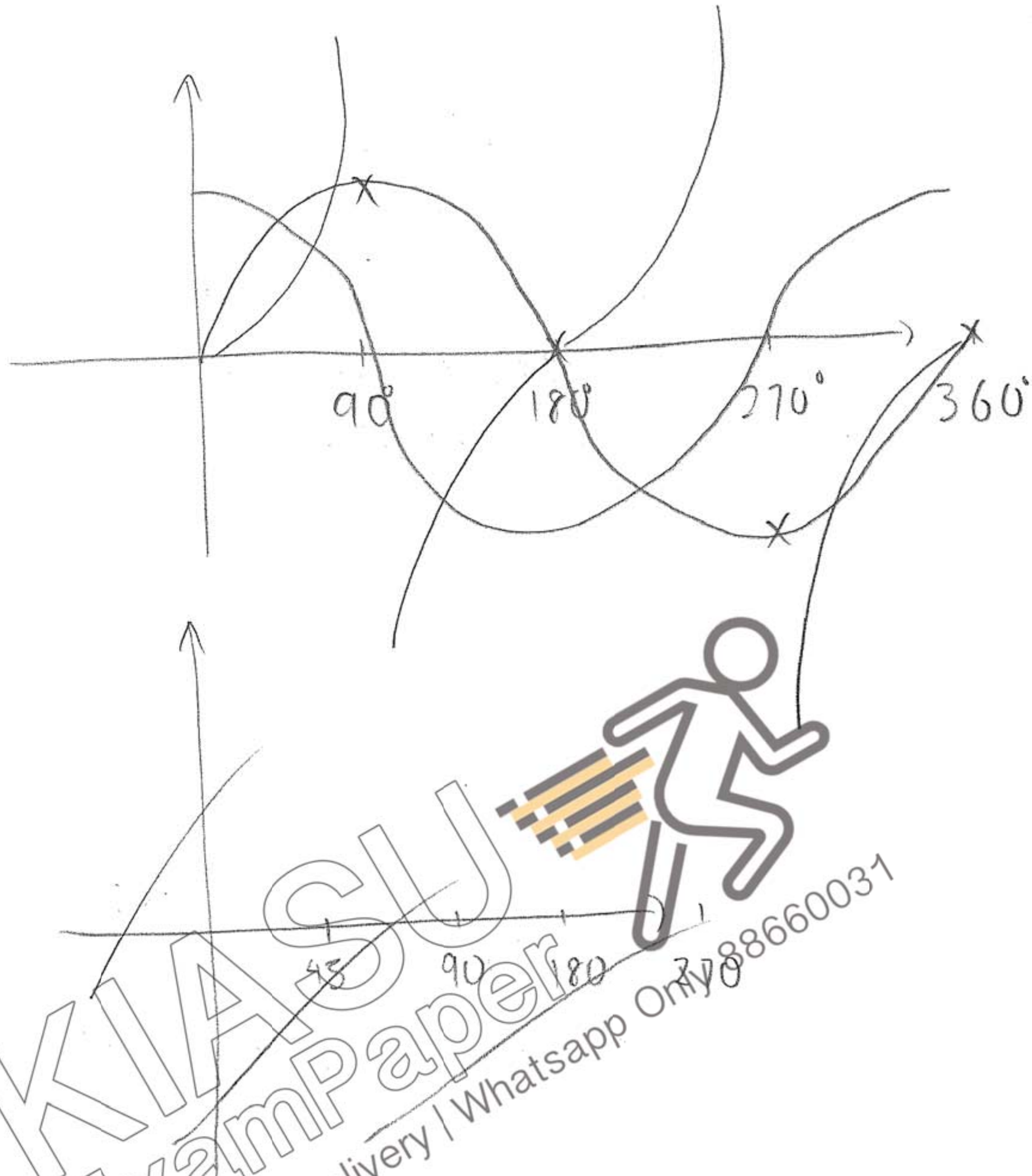
The diagram shows the top view of a rectangular desk, *PQRS*, in a corner of a room. The desk has a length of 1.5 m and width 0.8 m, $\angle POS = \angle STR = 90°$ and $\angle OPS = \theta$.

(i) Show that $OT = (1.5\sin\theta + 0.8\cos\theta)$ m. [3]

(ii) Express OT in the form $R\sin(\theta + \alpha)$, where $R > 0$ and α is acute. [3]

(iii) Given that θ can vary, find the maximum value of OT and the corresponding value of θ. [3]

End of paper



90°
180°
270°
360°
45
90
180
270

2019 Additional Mathematics Paper 1 Sec 4 MYE (Solutions)

1 A triangle has an area of $(58+8\sqrt{5})$ cm^2 and a height of $(7+3\sqrt{5})$ cm. Without using a calculator, find the exact length of its base, expressing in the form $+b\sqrt{5}$, where a and b are integers. [4]

1 Length of the base

$$=\frac{2(58+8\sqrt{5})}{7+3\sqrt{5}}$$

$$=\frac{116+16\sqrt{5}}{7+3\sqrt{5}}\times\frac{7-3\sqrt{5}}{7-3\sqrt{5}}$$

$$=\frac{812-348\sqrt{5}+112\sqrt{5}-240}{49-45}$$

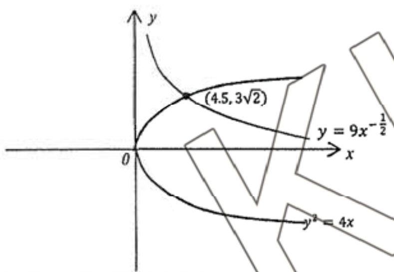
$$=\frac{572-236\sqrt{5}}{4}$$

$$=143-59\sqrt{5}\ \text{cm}$$

2 (i) On the same diagram, sketch the curves $y=9x^{-\frac{1}{2}}$ and $y^2=4x$. [2]

(ii) Find the coordinates of the point(s) of intersection of the two curves. [2]

2(i)



(ii)

$y=9x^{-\frac{1}{2}}$ -----(1)

$y^2=4x$ -----(2)

Sub (1) into (2): $(9x^{-\frac{1}{2}})^2=4x$

$81x^{-1}=4x$

$x^2=\frac{81}{4}$

$x=\frac{9}{2}$ or $-\frac{9}{2}$ (reject)

When $x=\frac{9}{2}$, $y=9(\frac{9}{2})^{-\frac{1}{2}}$

$=9\div\frac{\sqrt{9}}{\sqrt{2}}$

$=3\sqrt{2}$

The coordinates of the point of intersection is $(4\frac{1}{2}, 3\sqrt{2})$.

3 The equation of a curve is $y=2xe^{x-k}$, where k is a constant. The curve passes through the point $(5,10)$.

(i) Find the value of k. [2]

(i) For what values of x is y an increasing function of x. [3]

3(i) $y=2xe^{x-k}$

When $x=5, y=10$,

$10=2(5)e^{5-k}$

$1=e^{5-k}$

$e^0=e^{5-k}$

$5-k=0$

$k=5$

(ii) $y=2xe^{x-5}$

$\frac{dy}{dx}=2xe^{x-5}+2e^{x-5}$

$=2e^{x-5}(x+1)$

For y to be an increasing function of x,

$\frac{dy}{dx}>0$

Since $2e^{x-5}>0$, $x+1>0$

$x>-1$

4 Express $\frac{16x^2-9x+18}{x^3+3x^2}$ in partial fractions. [5]

4

$$\frac{16x^2-9x+18}{x^3+3x^2}=\frac{16x^2-9x+18}{x^2(x+3)}$$

Let $\frac{16x^2-9x+18}{x^3+3x^2}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+3}$

$16x^2-9x+18=Ax(x+3)+B(x+3)+Cx^2$

Let $x=-3$, $16(-3)^2-9(-3)+18=9C$

$9C=189$

$C=21$

Let $x=0$, $18=3B$

$B=6$

Comparing x^2 term, $16x^2=Ax^2+Cx^2$

$A+C=16$

$A+21=16$

$A=-5$

$$\frac{16x^2-9x+18}{x^3+3x^2}=\frac{-5}{x}+\frac{6}{x^2}+\frac{21}{x+3}$$

5 The function f is given by $f(x) = -3\sin\frac{x}{2} + 2$.

(i) State the amplitude and period of f. [2]

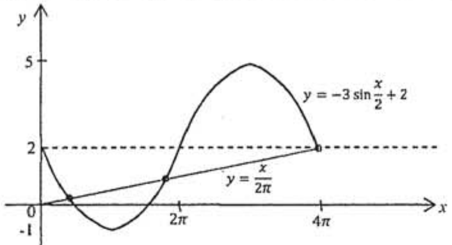
(ii) Sketch the graph of $y = f(x)$ for $0 \le x \le 4\pi$. By drawing a suitable straight line on the same axes, state the number of solutions to the equation $4\pi - x - 6\pi\sin\frac{x}{2} = 0$ for $0 \le x \le 4\pi$. [5]

5(i) Amplitude = 3

Period $= 2\pi \div \frac{1}{2}$

$= 4\pi$

(ii)



$4\pi - x - 6\pi\sin\frac{x}{2} = 0$

$\frac{4\pi - x - 6\pi\sin\frac{x}{2}}{2\pi} = \frac{0}{2\pi}$

$2 - \frac{x}{2\pi} - 3\sin\frac{x}{2} = 0$

$-3\sin\frac{x}{2} + 2 = \frac{x}{2\pi}$

Since there are 3 points of intersection between the graphs $y = -3\sin\frac{x}{2} + 2$ and $y = \frac{x}{2\pi}$, there are 3 solutions.

6 (i) Given that $\cos(A+B) = 3\cos(A-B)$ and $\tan A = -\frac{5}{2}$, find the value of $\cot B$. [3]

(ii) Prove that $\frac{1+\tan^2 x}{1-\tan^2 x} = \sec 2x$. [3]

6(i) $\cos(A+B) = 3\cos(A-B)$

$\cos A\cos B - \sin A\sin B = 3(\cos A\cos B + \sin A\sin B)$

$\cos A\cos B - \sin A\sin B = 3\cos A\cos B + 3\sin A\sin B$

$-4\sin A\sin B = 2\cos A\cos B$

$\frac{-4\sin A\sin B}{\cos A\cos B} = \frac{2\cos A\cos B}{\cos A\cos B}$

$-4\tan A\tan B = 2$

Sub $\tan A = -\frac{5}{2}$,

$-4\left(-\frac{5}{2}\right)\tan B = 2$

$10\tan B = 2$

$\tan B = \frac{1}{5}$

$\cot B = 5$

(ii) LHS $= \frac{1+\tan^2 x}{1-\tan^2 x} = \frac{1+\frac{\sin^2 x}{\cos^2 x}}{1-\frac{\sin^2 x}{\cos^2 x}}$

$= \frac{1+\frac{\sin^2 x}{\cos^2 x}}{1-\frac{\sin^2 x}{\cos^2 x}} \times \frac{\cos^2 x}{\cos^2 x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x - \sin^2 x}$

$= \frac{1}{\cos 2x}$

$= \sec 2x$

$=$ RHS (Proven)

7 The roots of the quadratic equation $2x^2 + x + 6 = 0$ are α and β.

(i) Express $\alpha^2 - \alpha\beta + \beta^2$ in terms of $(\alpha+\beta)$ and $\alpha\beta$. [1]

(ii) Form a quadratic equation whose roots are α^3 and β^3. [5]

7(i) $\alpha^2 - \alpha\beta + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta - \alpha\beta$

$= (\alpha+\beta)^2 - 3\alpha\beta$

(ii) Sum of roots: $\alpha + \beta = -\frac{1}{2}$

Product of roots: $\alpha\beta = \frac{6}{2} = 3$

For an equation whose roots are α^3 and β^3,

Sum of roots: $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$

$= (\alpha+\beta)[(\alpha+\beta)^2 - 3\alpha\beta]$

$= \left(-\frac{1}{2}\right)\left[\left(-\frac{1}{2}\right)^2 - 3(3)\right]$

$= \frac{35}{8}$

Product of roots: $\alpha^3\beta^3 = (3)^3$

$= 27$

The equation is $x^2 - \left(\frac{35}{8}\right)x + 27 = 0$

or $8x^2 - 35x + 216 = 0$

8 An antique grandfather clock manufactured using the finest wood in 1850 was valued at \$2000. The clock appreciated in its value such that its value \$$V$ can be modelled by the equation $V = 20000 - Ae^{kt}$, where t was the number of years after its manufacture date.

(i) Find the value of A. [2]

(ii) In the year 1880, the clock reached five times its initial value. Show that $k = -0.01959$ correct to 4 significant figures. [3]

(iii) Explain why the value of the clock will not exceed \$20000. [2]

8(i) When $t = 0$, $V = 2000$

$2000 = 20000 - Ae^{k(0)}$

$A = 20000 - 2000$

$= 18000$

(ii) $V = 20000 - 18000e^{kt}$

In the year 1880, $t = 30$, $V = 5(2000)$

$20000 - 18000e^{30k} = 10000$

$-18000e^{30k} = -10000$

$e^{30k} = \frac{5}{9}$

$$\ln e^{30k} = \ln \tfrac{5}{9}$$

$$30k = \ln \tfrac{5}{9}$$

$$k = \frac{\ln\frac{5}{9}}{30}$$

$$= -0.019592\ldots \text{ or } 3\ldots.$$

$$= -0.01959 \ (4\ sf)(shown)$$

(iii) For all values of $t \geq 0$, $e^{-0.01959t} > 0$

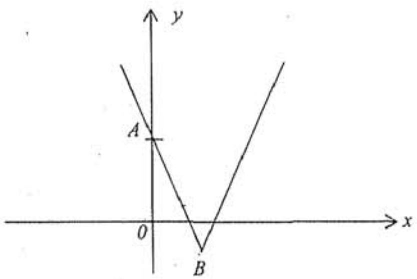
$$-18000e^{-0.01959t} < 0$$

$$20000 - 18000e^{-0.01959t} < 20000$$

$$V < 20000$$

Hence the value of the clock will not exceed $20000.

9 The diagram shows the graph of $y = |6 - 2x| - 1$.



(i) Find the coordinates of A and of B. [2]

(ii) By solving the equation $|6 - 2x| = 3x + 1$, find the x-coordinate of the point(s) of intersection between the graphs $y = |6 - 2x| - 1$ and $y = 3x$. [3]

(iii) State the range of values of m for the equation $|6 - 2x| = mx + 1$ to have no solution. [2]

9(i) When $x = 0$, $y = |6 - 2(0)| - 1$

$$= 5$$

$A(0, 5)$

At B, y is minimum when $6 - 2x = 0$

$$x = 3$$

$$y = -1$$

$B(3, -1)$

(ii) $|6 - 2x| = 3x + 1$

$6 - 2x = 3x + 1$ *or* $6 - 2x = -(3x + 1)$

$-5x = -5$ $\quad$ $x = -7$ (reject)

$x = 1$

(iii) For $|6 - 2x| = mx + 1$ to have no solutions

$$-2 \leq m < -\frac{1}{3}$$

10 A circle passes through the points $P(0, 8)$ and $Q(8, 12)$. The y-axisis a tangent to the circle at P.

(i) Find the equation of the circle. [5]

The tangent to the circle at Q intersects the x-axis and y-axis at A and B respectively.

(ii) Find the ratio of $AQ:QB$. [3]

10(i) y − coordinate of centre of circle $= 8$

Midpoint of $PQ = \left(\frac{0+8}{2}, \frac{8+12}{2}\right)$

$$= (4, 10)$$

Gradient of $PQ = \frac{12-8}{8-0}$

$$= \frac{1}{2}$$

Gradient of perpendicular bisector of $PQ = -2$

Equation of perpendicular bisector of PQ:

$$y - 10 = -2(x - 4)$$

$$y = -2x + 18$$

Sub $y = 8$, $\quad 8 = -2x + 18$

$$x = 5$$

Centre of the circle is $(5, 8)$.

$Radius^2 = (5 - 0)^2$

$$= 25$$

The equation of the circle is

$$(x - 5)^2 + (y - 8)^2 = 25$$

(ii) Gradient of line from Q to centre of circle$= \frac{12-8}{8-5}$

$$= \frac{4}{3}$$

Equation of tangent at $Q(8, 12)$:

$$y - 12 = -\frac{3}{4}(x - 8)$$

$$y = -\frac{3}{4}x + 18$$

When $y = 0$, $x = 24$

$A(24,0)$

When $x = 0$, $y = 18$.

$B(0,18)$

For the points $A(24,0)$, $Q(8, 12)$ and $B(0, 18)$,

$AQ:QB = 24 - 8:\ 8 - 0$ (Comparing difference in x or y-coordinates)

$= 2 : 1$

11 (i) Expand $(1 - 2x)^9$ in ascending powers of x up to the term in x^3. [2]

(ii) Find the value of k, given that the coefficient of x in the expansion of $\left(3x + \frac{1}{kx^2}\right)(1 - 2x)^9$ is -53. [3]

11(i)

$$(1 - 2x)^9 = \binom{9}{0}(-2x)^0 + \binom{9}{1}(-2x)^1 + \binom{9}{2}(-2x)^2 + \binom{9}{3}(-2x)^3 + \cdots$$

$$= 1 - 18x + 144x^2 - 672x^3 + \cdots$$

(ii)

$\left(3x+\frac{1}{kx^2}\right)(1-2x)^9$

$=\left(3x+\frac{1}{kx^2}\right)(1-18x+144x^2-672x^3+\cdots)$

Term in $x = 3x(1)+\frac{1}{kx^2}(-672x^3)$

coefficient of $x=-53$

$3-\frac{672}{k}=-53$

$k=12$

12 The equation of a curve is given by $y=\ln\sqrt{\frac{5x}{9x+4}}$.

(i) Find $\frac{dy}{dx}$, expressing it as a single fraction. [3]

(ii) Find the rate at which x is changing when the graph crosses the x-axis, given that y is increasing at a rate of 0.3 units per second. [4]

(i)

$y=\ln\sqrt{\frac{5x}{9x+4}}$

$=\frac{1}{2}[\ln 5x-\ln(9x+4)]$

$\frac{dy}{dx}=\frac{1}{2}\left[\frac{5}{5x}-\frac{9}{9x+4}\right]$

$=\frac{1}{2}\left[\frac{9x+4}{x(9x+4)}-\frac{9x}{x(9x+4)}\right]$

$=\frac{1}{2}\left[\frac{4}{x(9x+4)}\right]$

$=\frac{2}{x(9x+4)}$

(ii)

Let $y=0$, $\quad \ln\sqrt{\frac{5x}{9x+4}}=0$

$\frac{1}{2}[\ln 5x-\ln(9x+4)]=0$

$\ln 5x-\ln(9x+4)=0$

$\ln 5x=\ln(9x+4)$

$5x=9x+4$

$x=-1$

$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$

$0.3=\frac{2}{x(9x+4)}\times\frac{dx}{dt}$

When $x=-1$, $\frac{dx}{dt}=0.3\div\frac{2}{(-1)(-9+4)}$

$=\frac{3}{4}$

x is increasing at a rate of $\frac{3}{4}$ units per second.

13 **Solutions to this question by accurate drawing will not be accepted.**

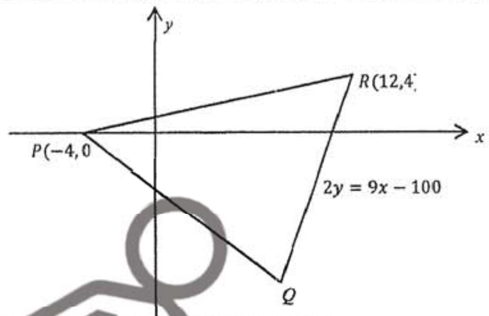
The diagram, which is not drawn to scale, shows a triangle PQR in which $PQ=QR$. The coordinates of the points P and R are $(-4,0)$ and $(12,4)$ respectively.

(i) Find the equation of the perpendicular bisector of PR. [3]

The equation of the line QR is $2y=9x-100$.

(ii) Find the coordinates of Q. [2]

(iii) Find the coordinates of S if $PQRS$ forms a rhombus. Hence, or otherwise, find the area of rhombus $PQRS$. [4]



(i)

Midpoint of $PR=\left(\frac{-4+12}{2},\frac{0+4}{2}\right)$

$=(4,2)$

Gradient of $PR=\frac{4-0}{12-(-4)}$

$=\frac{1}{4}$

Gradient of perpendicular bisector of $PR=-4$

Equation of perpendicular bisector of PR:

$y-2=-4(x-4)$

$y=-4x+18$

(ii)

$y=-4x+18$ ------(1)

$2y=9x-100$-----(2)

Sub (1) into (2): $2(-4x+18)=9x-100$

$-8x+36=9x-100$

$x=8$

$y=-14$

The coordinates of Q are $(8,-14)$.

(iii)

Let the coordinates of S be (x_S,y_S). If $PQRS$ forms a rhombus, then

Midpoint of QS = Midpoint of PR

$\left(\frac{8+x_S}{2},\frac{-14+y_S}{2}\right)=(4,2)$

$\frac{8+x_S}{2}=4$

$x_S=0$

$\frac{-14+y_S}{2}=2$

$y_S=18$

The coordinates of S are $(0,18)$.

Area of the rhombus $PQRS$

$=\frac{1}{2}\begin{vmatrix}-4 & 8 & 12 & 0 & -4\\ 0 & -14 & 4 & 18 & 0\end{vmatrix}$

$=\frac{1}{2}|(56+32+216+0)-(0-168+0-72)|$

$=\frac{1}{2}|544|$

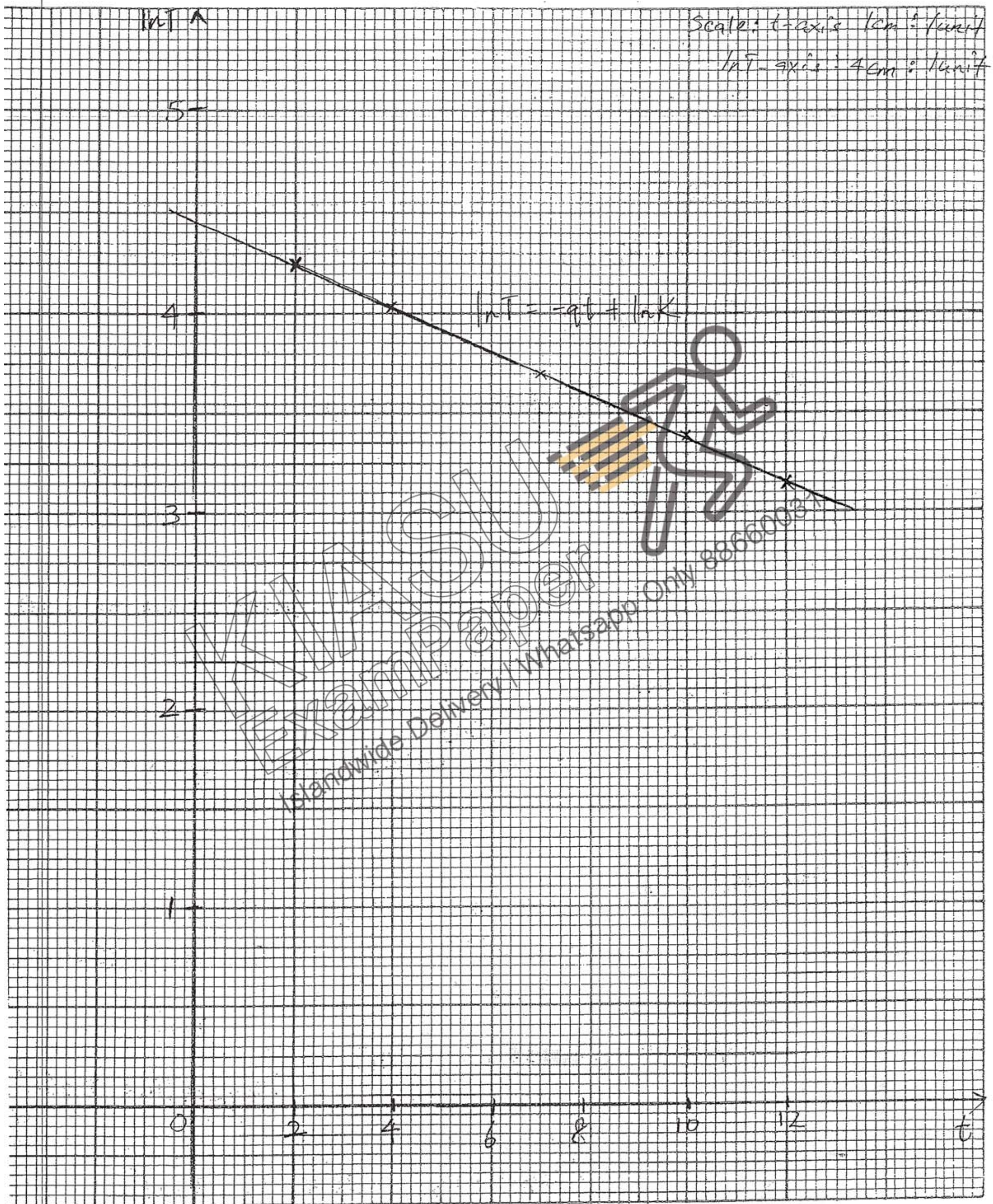
$=272$ unit sq

海 星 中 学 Maris Stella High School

(Q4) 姓 名 / Name: ____________ (   )

科目／Subject: ____________

班级／Class: ____________



1 The polynomial $f(x) = 2x^3 + ax^2 + bx + 8$, where a and b are constants, has a factor $(x+2)$ and leaves a remainder of 10 when divided by $(2x-1)$.

(i) Find the value of a and of b. [4]

(ii) Using the values of a and of b found in part (i), explain why the equation $f(x) = 0$ has only one real root. Find this root. [4]

(iii) Hence, solve $x^3 + 3x^2 + 4x + 32 = 0$. [2]

1	$f(x) = 2x^3 + ax^2 + bx + 8$
(i)	$f(-2) = 2(-2)^3 + a(-2)^2 + b(-2) + 8 = 0$ $4a - 2b = 8$ -------- Eqn (1)
	$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 + b\left(\frac{1}{2}\right) + 8 = 10$ $a + 2b = 7$ ----------- Eqn (2)
	Solving the equations, $b = 2$, $a = 7 - 2(2) = 3$
(ii	$f(x) = 2x^3 + 3x^2 + 2x + 8$ $= (x+2)(2x^2 + bx + 4)$ Term in x^2: $3x^2 = bx^2 + 4x^2$, $b = -1$ $f(x) = 2x^3 + 3x^2 + 2x + 8$ $= (x+2)(2x^2 - x + 4)$ [Getting Quadratic factor by long division also allowed]
	For the factor $2x^2 - x + 4$, Discriminant $= 1 - 4(2)(4)$ $= -31 < 0$ Hence, the equation $2x^2 - x + 4 = 0$ has no real roots. Therefore $f(x) = 0$ has only 1 real root. The root is $x = -2$
(iii	$2x^3 + 3x^2 + 2x + 32 = 0$ $2\left(\frac{x}{2}\right)^3 + 3\left(\frac{x}{2}\right)^2 + 2\left(\frac{x}{2}\right) + 8 = 0$ $\left(\frac{x}{2} + 2\right)\left(2\left(\frac{x}{2}\right)^2 - \left(\frac{x}{2}\right) + 4\right) = 0$ $x = -4$

2 (a) Find the range of values of k for which $(k-3)x^2 + 4x + k$ is always positive for all real values of x. [4]

(b) Show that the roots of the equation $6x^2 + 4(m-1) = 2(x+m)$ are real if $m \le 2\frac{1}{12}$. [3]

2(a)	$(k-3)x^2 + 4x + k > 0$ for all values of x Discriminant < 0 $16 - 4k(k-3) < 0$ $4k^2 - 12k - 16 > 0$ $k^2 - 3k - 4 > 0$ $(k-4)(k+1) > 0$ $k > 4$ or $k < -1$ Since $k - 3 > 0$, $k > 3$ $\therefore k > 4$
(b)	$6x^2 + 4(m-1) = 2(x+m)$ $6x^2 - 2x + 2m - 4 = 0$ Discriminant $= 100 - 48m$ Since $m \le 2\frac{1}{12}$ $25 - 12m \ge 0$ $100 - 48m \ge 0$ Since discriminant ≥ 0, $6x^2 + 4(m-1) = 2(x+m)$ has real roots if $m \le 2\frac{1}{12}$

3 (a) Simplify $\frac{9^{x+1} + 18(3^{2x})}{3^{2-x} \times 27^{x+1}}$ without the use of a calculator. [4]

(b) Solve the equation $4^{x+1} = 18(2^x) - 8$. [4]

(c) Solve the equation $\log_3(2x-1) - \frac{1}{2}\log_3(x^2+2) = \log_{25} 5$. [5]

3 (a)	$\dfrac{9^{x+1}+18(3^{2x})}{3^{2-x}\times 27^{x+1}}$
	$=\dfrac{3^{2(x+1)}+18(3^{2x})}{3^{2-x}\times 3^{3(x+1)}}$ $=\dfrac{3^{2x}(3^2+18)}{3^{2x+5}}$ $=\dfrac{3^{2x}(3^3)}{3^{2x}(3^5)}$ $=\dfrac{1}{3^2}$ $=\dfrac{1}{9}$
(b)	$4^{x+1}=18(2^x)-8$
	$4(2^{2x})=18(2^x)-8$ $4(2^x)^2-18(2^x)+8=0$ Let $2^x=A$, $4A^2-18A+8=0$ $2A^2-9A+4=0$ $(2A-1)(A-4)=0$ $A=\dfrac{1}{2}$ or $A=4$ $2^x=\dfrac{1}{2}$ $2^x=4$ $x=-1$ or $x=2$
(c)	$\log_3(2x-1)-\dfrac{1}{2}\log_3(x^2+2)=\log_{25}5$
	$2\log_3(2x-1)-\log_3(x^2+2)=1$ $\log_3\left(\dfrac{(2x-1)^2}{x^2+2}\right)=1$ $(2x-1)^2=3(x^2+2)$ $x^2-4x-5=0$ $(x-5)(x+1)=0$ $x=5$ or $x=-1$ (rej)

4 In a Science experiment, a container of liquid was heated to a temperature of K °C.

It was then left to cool in a chiller such that its temperature, T °C, t minutes after removing the heat, is given by $T=Ke^{-qt}$, where q is a constant.

Measured values of t and T are given in the following table.

t (minutes)	2	4	7	10	12
T °C	71.1	57.0	40.8	29.3	23.4

(i) Using a scale of 1 cm to 1 unit on the t-axis and 4 cm to 1 unit on the $\ln T$- axis, plot $\ln T$ against t and draw a straight line graph. [2]

$\ln T$	4.26	4.04	3.71	3.38	3.15
t	2	4	7	10	12

(ii) Use the graph to estimate the value of K and of q. [4]

(iii) Estimate the temperature of the liquid 8 minutes after it was left to cool. [2]

4(i)	Plot a straight line passing all the points with correct scale etc
(ii)	$T=Ke^{-qt}$ $\ln T=-qt+\ln K$
	Gradient $=\dfrac{3.15-4.45}{12-0}$ $=-\dfrac{13}{120}$ $-q=-\dfrac{13}{120}$ $q\approx\dfrac{13}{120}$
	$\ln K=4.475$ $K=e^{4.475}$ ≈ 87.8
(iii)	$T=87.8e^{-\frac{13}{120}(8)}$ ≈ 36.9 Temperature is 36.9°. Alternatively from graph, $t=8, \ln T=3.6$ $T=e^{3.6}\approx 36.6$ Temperature is 36.6°.

5 (a) (i) Prove that $\dfrac{\sin x}{\sec x+1}+\dfrac{\sin x}{\sec x-1}=2\cot x$. [4]

(ii) Hence find, for $0\le x\le 4$, the exact solution of the equation $\dfrac{\sin x}{\sec x+1}+\dfrac{\sin x}{\sec x-1}=\dfrac{2\tan x}{3}$. [3]

(b) Given that θ is obtuse and that $\sin\theta=\dfrac{1}{\sqrt{3}}$, express, **without the use of a calculator**, $\dfrac{1}{\sin\theta-\cos\theta}$ in the form $\sqrt{a}-\sqrt{b}$ where a and b are integers. [4]

5(a)(i)	$LHS=\dfrac{\sin x}{\sec x+1}+\dfrac{\sin x}{\sec x-1}$ $=\dfrac{\sin x(\sec x-1)+\sin x(\sec x+1)}{\sec^2 x-1}$ $=\dfrac{\tan x-\sin x+\tan x+\sin x}{\sec^2 x-1}$ $=\dfrac{2\tan x}{\sec^2 x-1}$ $=\dfrac{2\tan x}{\tan^2 x}$ $=2\cot x$ $=RHS$ (proved)
(ii)	$\dfrac{\sin x}{\sec x+1}+\dfrac{\sin x}{\sec x-1}=\dfrac{2\tan x}{3}$ $2\cot x=\dfrac{2\tan x}{3}$ $\tan^2 x=3$ $\tan x=\pm\sqrt{3}$ Basic angle $=\dfrac{\pi}{3}$ For $0\le x\le 4$, $x=\dfrac{\pi}{3},\dfrac{2\pi}{3}$

5b.	$1^2+x^2=\left(\sqrt{3}\right)^2$
	$x=\sqrt{2}$
	$\cos\theta=-\dfrac{\sqrt{2}}{\sqrt{3}}$
	$\dfrac{1}{\sin\theta-\cos\theta}=\dfrac{1}{\frac{1}{\sqrt{3}}+\frac{\sqrt{2}}{\sqrt{3}}}$ $=\dfrac{\sqrt{3}}{1+\sqrt{2}}\times\dfrac{1-\sqrt{2}}{1-\sqrt{2}}$ $=\dfrac{\sqrt{3}-\sqrt{6}}{-1}$ $=\sqrt{6}-\sqrt{3}$

6 The equation of a curve is $y=\dfrac{a}{x}+bx-1$, where a and b are constants. The normal to the curve at the point $Q(1,-1)$ is parallel to the line $4y-x=20$. This normal meets the curve again at point P.

(i) Find the value of a and of b. [5]

(ii) Find the coordinates of point P. [3]

6 (i)	Equation of line : $y=\dfrac{1}{4}x+5$ At $x=1$, gradient of normal $=\dfrac{1}{4}$ Gradient of tangent $=-4$ $\dfrac{dy}{dx}=-\dfrac{a}{x^2}+b$ $-a+b=-4$ ------ Eqn (1) sub $(1,-1)$ into $y=\dfrac{a}{x}+bx-1$ $a+b=0$ --------- Eqn (2) Solving : $a=2, b=-2$

(ii)	$y=\frac{2}{x}-2x-1$ Equation of normal is : $y+1=\frac{1}{4}(x-1)$ $y=\frac{1}{4}x-\frac{5}{4}$ $\frac{2}{x}-2x-1=\frac{1}{4}x-\frac{5}{4}$ $8-8x^2-4x=x^2-5x$ $9x^2-x-8=0$ $(9x+8)(x-1)=0$ $x=-\frac{8}{9}$ or $x=1$ $y=2\left(-\frac{9}{8}\right)-2\left(-\frac{8}{9}\right)-1$ $=-\frac{53}{36}$ Coordinates of P is $\left(-\frac{8}{9},-\frac{53}{36}\right)$

7 The equation of a curve is $y=\frac{x^2}{x-1}$, where $x\neq 1$.

(i) Obtain an expression for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(ii) Find the coordinates of the stationary points of the curve and determine their nature. [4]

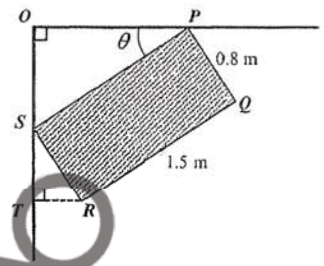
7(i)	$y=\frac{x^2}{x-1}$ $\frac{dy}{dx}=\frac{(x-1)(2x)-x^2(1)}{(x-1)^2}$ $=\frac{x^2-2x}{(x-1)^2}$ $\frac{d^2y}{dx^2}=\frac{(x-1)^2(2x-2)-2(x-1)(x^2-2x)}{(x-1)^4}$ $=\frac{(x-1)(2x^2-4x+2-2x^2+4x)}{(x-1)^4}$ $=\frac{2}{(x-1)^3}$
(ii)	For stationary point, $\frac{dy}{dx}=0$ $\frac{x^2-2x}{(x-1)^2}=0$ $x^2-2x=0$ $x(x-2)=0$ $x=0$ or 2 when $x=0, y=0$ $\frac{d^2y}{dx^2}=\frac{2}{(0-1)^3}$ $=-2<0$ $(0,0)$ is a maximum point. when $x=2, y=4$ $\frac{d^2y}{dx^2}=\frac{2}{(2-1)^3}$ $=2>0$ $(2,4)$ is a minimum point.

8 (a) Differentiate $\cot^4\left(\frac{\pi}{2}-2x\right)$ with respect to x. [3]

(b) Given that the curve has the equation $y=3\sin 2x-\cos x$, find the gradient of the curve when $x=\frac{\pi}{3}$, leaving your answer in exact form. [3]

8 (a)	Let $y=\cot^4\left(\frac{\pi}{2}-2x\right)$
	$y=\frac{1}{\tan^4\left(\frac{\pi}{2}-2x\right)}$ $=\frac{1}{\cot^4(2x)}$ $=\tan^4 2x$ $\frac{dy}{dx}=4\tan^3(2x)\left[2\sec^2(2x)\right]$ $=8\tan^3(2x)\sec^2(2x)$
	OR $y=\frac{1}{\tan^4\left(\frac{\pi}{2}-2x\right)}$ $=\tan^{-4}\left(\frac{\pi}{2}-2x\right)$ $\frac{dy}{dx}=-4\tan^{-5}\left(\frac{\pi}{2}-2x\right)\left[-2\sec^2\left(\frac{\pi}{2}-2x\right)\right]$ $=8\tan^{-5}\left(\frac{\pi}{2}-2x\right)\sec^2\left(\frac{\pi}{2}-2x\right)$
8(b)	$y=3\sin 2x-\cos x$,
	$\frac{dy}{dx}=6\cos 2x+\sin x$ At $x=\frac{\pi}{3}$, Gradient $=6\cos\frac{2\pi}{3}+\sin\frac{\pi}{3}$ $=-3+\frac{\sqrt{3}}{2}$

9



The diagram shows the top view of a rectangular desk, $PQRS$, in a corner of a room. The desk has a length of 1.5 m and width 0.8 m, $\angle POS=\angle STR=90^\circ$ and $\angle OPS=\theta$.

(i) Show that $OT=(1.5\sin\theta+0.8\cos\theta)$ m. [3]

(ii) Express OT in the form $R\sin(\theta+\alpha)$, where $R>0$ and α is acute. [3]

(iii) Given that θ can vary, find the maximum value of OT and the corresponding value of θ. [3]

9(i)	$\angle TSR=\theta$, $\cos\theta=\frac{ST}{0.8}$ $ST=0.8\cos\theta$ $\sin\theta=\frac{OS}{1.5}$ $OS=1.5\sin\theta$ $OT=OS+ST$ $=(1.5\sin\theta+0.8\cos\theta)$ m (Shown)
(ii)	$OT=1.5\sin\theta+0.8\cos\theta=R\sin(\theta+\alpha)$ where $R=\sqrt{1.5^2+0.8^2}=1.7$ $\tan\alpha=\frac{0.8}{1.5},\Rightarrow\alpha=28.072^\circ$ $\therefore\ OT=1.7\sin(\theta+28.1^\circ)$ (correct to 1 d.p.)
(iii)	Maximum value of $OT=1.7m$ when $\sin(\theta+28.072^\circ)=1$ $\theta+28.072^\circ=90^\circ$ $\theta=61.9^\circ$ (1 dp)