YISHUN INNOVA JUNIOR COLLEGE
JC 2 PRELIMINARY EXAMINATION
Higher 2

CANDIDATE NAME			
CG		INDEX NO	

MATHEMATICS 9758/01

Paper 1

4 September 2019

3 hours

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your CG and name on the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiners' Use

Question	1	2	3	4	5	6	7
Marks							

Question	8	9	10	11
Marks				

Total marks	

This paper consists of 20 printed pages.

1 **(i)** Expand $\sin\left(\frac{\pi}{4}-2x\right)$ in ascending powers of x, up to and including the term in x^3. [3]

(ii) The first two non-zero terms found in part **(i)** are equal to the first two non-zero terms in the series expansion of $(a+bx)^{-1}$ in ascending powers of x. Find the exact values of the constants a and b. Hence find the third exact non-zero term of the series expansion of $(a+bx)^{-1}$ for these values of a and b. [3]

2 **(a)** Vectors **a** and **b** are such that $\mathbf{a} \neq 0$, $\mathbf{b} \neq 0$ and $|\mathbf{a}+\mathbf{b}| = |\mathbf{a}-\mathbf{b}|$. Show that **a** and **b** are perpendicular. [2]

(b) Referred to the origin O, points C and D have position vectors $\mathbf{c}$ and $\mathbf{d}$ respectively. Point P lies on OC produced such that $OC:CP=1:\lambda-1$, where $\lambda>1$. Point M lies on DP, between D and P, such that $DM:MP=2:3$. Write down the position vector of M in terms of λ, $\mathbf{c}$ and $\mathbf{d}$. Hence, find the area of triangle OPM in the form $k\lambda|\mathbf{c}\times\mathbf{d}|$, where k is a constant to be found. [4]

3 The function f is defined by

$$\mathrm{f}(x)=\begin{cases}(x-2a)^2 & \text{for } 0\le x<2a,\\ 2ax-4a^2 & \text{for } 2a\le x<4a,\end{cases}$$

where a is a positive real constant and that $\mathrm{f}(x+4a)=\mathrm{f}(x)$ for all real values of x.

(i) Sketch the graph of $y=\mathrm{f}(x)$ for $-3a\le x\le 8a$. [3]

(ii) Hence find the value of $\int_{-2a}^{8a}\mathrm{f}(|x|)\,\mathrm{d}x$ in terms of a. [3]

4 A curve C has parametric equations

$$x = t^2 \ , \ y = \frac{1}{\sqrt{t}} \ , t > 0 \ .$$

(i) The curve $y = \frac{8}{x}$ intersects C at point A. Without using a calculator, find the coordinates of A. [2]

(ii) The tangent at the point $P\left(p^2, \frac{1}{\sqrt{p}}\right)$ on C meets the x-axis at point D and the y-axis at point E. The point F is the midpoint of DE. Find a cartesian equation of the curve traced by F as p varies. [5]

5 The equation of a curve is $2xy + (1 + y)^2 = x$.

(i) Find the equations of the two tangents which are parallel to the y-axis. [4]

(ii) The normal to the curve at the point $A(1,\ 0)$ meets the y-axis at the point B. Find the area of the triangle OAB. [3]

6 The sum of the first n terms of a sequence is a cubic polynomial, denoted by S_n. The first term and the second term of the sequence are 2 and 4 respectively. It is known that $S_5 = 90$ and $S_{10} = 830$.

(i) Find S_n in terms of n. [4]

(ii) Find the 54th term of the sequence. [2]

(b) **(i)** Given that $\cos(2n-1)\alpha - \cos(2n+1)\alpha = 2\sin\alpha\sin 2n\alpha$ and α is not an integer multiple of π, show that

$$\sum_{n=1}^{N} \sin 2n\alpha = \frac{1}{2}\cot\alpha - \frac{1}{2}\operatorname{cosec}\alpha\cos(2N+1)\alpha .$$ [3]

(ii) Explain whether the series $\sum_{n=1}^{\infty} \sin\frac{2n\pi}{3}$ converges. [1]

7 **(a)(i)** Find $\int \cos(\ln x)\,\mathrm{d}x$. [3]

(ii) A curve C is defined by the equation $y = \cos(\ln x)$, for $\mathrm{e}^{-\frac{3}{2}\pi} \le x \le \mathrm{e}^{\frac{1}{2}\pi}$. The region R is bounded by C, the lines $x = \mathrm{e}^{-\frac{\pi}{2}}$, $x = \mathrm{e}^{\frac{\pi}{2}}$ and the x-axis. Find the exact area of R. [3]

(b) A curve is defined by the equation $y=\dfrac{\sqrt{e^{\cot x}}}{\sin x}$. The region bounded by this curve, the x-axis, the lines $x=\dfrac{\pi}{6}$ and $x=\dfrac{2\pi}{3}$, is rotated 2π radians about the x-axis to form a solid. Using the substitution $u=\cot x$, find the exact volume of the solid obtained. [4]

8 **(i)** Show that $y=\dfrac{x-x^2-1}{x-2}$ can be expressed as $y=\dfrac{A}{x-2}+B(x+1)$, where A and B are constants to be found. Hence, state a sequence of transformations that will transform the curve with equation $y=\dfrac{1}{3-x}-\dfrac{x}{3}$ onto the curve with equation $y=\dfrac{x-x^2-1}{x-2}$. [3]

(ii) On the same axes, sketch the curves with equations $y=\dfrac{x-x^2-1}{x-2}$ and $y=|2x+1|$, stating the equations of any asymptotes and the coordinates of the points where the curves cross the axes. [4]

Hence, find the exact range of values of x for which $\dfrac{x-x^2-1}{x-2} < |2x+1|$. [4]

9 **(a)** The function f is defined by $\mathrm{f}: x \mapsto 2+\dfrac{3}{x},\ x \in \mathbb{R},\ x>0$.

(i) Sketch the graph of $y=\mathrm{f}(x)$. Hence, show that f has an inverse. [2]

(ii) Find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [3]

(iii) On the same diagram as in part **(i),** sketch the graphs of $y = \mathrm{f}^{-1}(x)$ and $y = \mathrm{f}^{-1}\mathrm{f}(x)$. [2]

(iv) Explain why f^2 exists and find $\mathrm{f}^2(x)$. [2]

(b) The function h is defined by $\mathrm{h}: x \mapsto \dfrac{3-x}{x^2-1}$, $x \in \mathbb{R}$, $x \neq \pm 1$. Find algebraically the range of h, giving your answer in exact form. [4]

10 A tank initially contains 2000 litres of water and 20 kg of dissolved salt. Brine with C kg of salt per 1000 litres is entering the tank at 5 litres per minute and the solution drains out at the same rate of 5 litres per minute. The amount of salt in the tank at time t minutes is x kg. Assume that the solution is always uniformly mixed.

(i) Show that $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{400}(2C - x)$. Hence determine the value of C if the amount of salt in the tank remains constant at 120 kg after certain time has passed. [3]

(ii) Find x in terms of t. [5]

(iii) Sketch the graph of the particular solution, including the coordinates of the point(s) where the graph crosses the axes and the equations of any asymptotes. Find the time t when the amount of salt in the tank is 60 kg, giving your answer to the nearest minute. [3]

(iv) State one assumption for the above model to be valid. [1]

11 A factory produces power banks. The factory produces 1000 power banks in the first week. In each subsequent week, the number of power banks produced is 250 more than the previous week. The factory produces 7500 power banks in the Nth week.

(i) Find the value of N. [2]

(ii) After the Nth week, the factory produces 7500 power banks weekly. Find the total number of power banks that will be produced in the first 60 weeks. [3]

The sales manager predicts that the demand for power banks in this week is $a+bH$ if the demand for power banks in the preceding week is H, where a and b are constants and $b>1$. It is given that the demand for power banks in the first week is 50.

(iii) Show that the demand for power banks in the third week is $a+ba+50b^2$. [2]

(iv) Show that the demand for power banks in the nth week can be written as $a\left(\frac{b^{n-1}-1}{b-1}\right)+50b^{n-1}$. [1]

It is now given that $a = 300$ and $b = 1.05$.
In the first week, the number of power banks produced is still 1000. In each subsequent week, the number of power banks produced will be L more than the previous week.

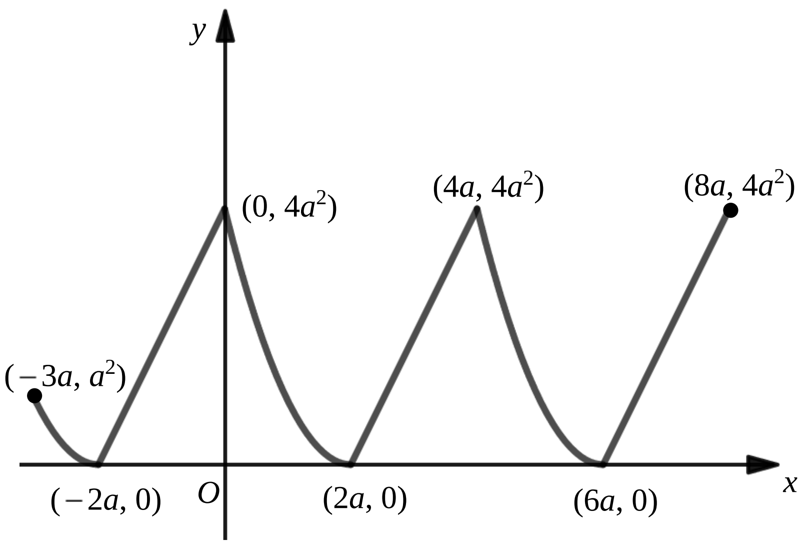
(v) The production manager decides to change the production plan so that the total production can meet the total demand in the first 60 weeks. Find the least value of L. [4]

- End of Paper -

Yishun Innova Junior College ♦ Mathematics Department
2019 JC 2 Mathematics H2 9758
Prelim Examination P1
Solutions

Qn	Solution	Remarks
1(i)	$\sin\left(\frac{\pi}{4}-2x\right)=\sin\frac{\pi}{4}\cos 2x-\cos\frac{\pi}{4}\sin 2x------*$ $=\frac{1}{\sqrt{2}}(\cos 2x-\sin 2x)$ $=\frac{1}{\sqrt{2}}\left[\left(1-\frac{(2x)^2}{2}+...\right)-\left(2x-\frac{(2x)^3}{3!}+...\right)\right]$ $=\frac{1}{\sqrt{2}}\left(1-2x-2x^2+\frac{4x^3}{3}+...\right)$	You cannot substitute $\frac{\pi}{4}-2x$ into the standard expansion formula directly. In general, we can apply the standard expansions when x is replaced by g(x), provided g(0) = 0. For instance, $g(x)=x+x^2$.
	<u>Alternative Method</u> Let $y=\sin\left(\frac{\pi}{4}-2x\right)$ $\frac{dy}{dx}=-2\cos\left(\frac{\pi}{4}-2x\right)$ $\frac{d^2y}{dx^2}=-4\sin\left(\frac{\pi}{4}-2x\right)$ $\frac{d^3y}{dx^3}=8\cos\left(\frac{\pi}{4}-2x\right)$ When x = 0, $y=\frac{1}{\sqrt{2}}$, $\frac{dy}{dx}=-\sqrt{2}$, $\frac{d^2y}{dx^2}=-2\sqrt{2}$, $\frac{d^3y}{dx^3}=4\sqrt{2}$ $\sin\left(\frac{\pi}{4}-2x\right)=\frac{1}{\sqrt{2}}-\sqrt{2}x-\frac{2\sqrt{2}}{2}x^2+\frac{4\sqrt{2}}{3!}x^3+\ldots$ $=\frac{1}{\sqrt{2}}-\sqrt{2}x-\sqrt{2}x^2+\frac{2\sqrt{2}}{3}x^3+\ldots$ $=\frac{1}{\sqrt{2}}\left(1-2x-2x^2+\frac{4x^3}{3}+...\right)$	Be careful with the signs when you doing the higher derivatives.

(ii)	$(a+bx)^{-1}=a^{-1}\left(1+\frac{b}{a}x\right)^{-1}$ $=a^{-1}\left[1+(-1)\left(\frac{b}{a}x\right)+\frac{(-1)(-2)}{2}\left(\frac{b}{a}x\right)^2+\ldots\right]$ $=\frac{1}{a}\left(1-\frac{b}{a}x+\left(\frac{b}{a}\right)^2x^2+\ldots\right)$ $\therefore a=\sqrt{2}$ and $\frac{b}{a}=2 \Rightarrow b=2\sqrt{2}$ Third non-zero term: $\frac{1}{a}\left(\frac{b}{a}\right)^2x^2=\frac{1}{\sqrt{2}}(2)^2x^2=\frac{4}{\sqrt{2}}x^2=2\sqrt{2}x^2$	Note that power is -1, not a positive integer so we need to use the series expansion of $(1+x)^n$ found in MF26 Don't forget to apply the power -1 to a after factorize a out. Third non-zero term and the coefficient of third non-zero term are different.
2(a)	$\lvert\mathbf{a}+\mathbf{b}\rvert=\lvert\mathbf{a}-\mathbf{b}\rvert$ $\lvert\mathbf{a}+\mathbf{b}\rvert^2=\lvert\mathbf{a}-\mathbf{b}\rvert^2$ $(\mathbf{a}+\mathbf{b})\bullet(\mathbf{a}+\mathbf{b})=(\mathbf{a}-\mathbf{b})\bullet(\mathbf{a}-\mathbf{b})$ $\lvert\mathbf{a}\rvert^2+2\mathbf{a}\bullet\mathbf{b}+\lvert\mathbf{b}\rvert^2=\lvert\mathbf{a}\rvert^2-2\mathbf{a}\bullet\mathbf{b}+\lvert\mathbf{b}\rvert^2$ $4\mathbf{a}\bullet\mathbf{b}=0$ $\mathbf{a}\bullet\mathbf{b}=0$ Hence **a** and **b** are perpendicular. <u>Alternative Method:</u> Since $\lvert\mathbf{a}+\mathbf{b}\rvert=\lvert\mathbf{a}-\mathbf{b}\rvert$, the diagonals of the parallelogram (with sides $\mathbf{a}$ and $\mathbf{b}$) are equal in length and thus the parallelogram must be a rectangle. Therefore, **a** and **b** are perpendicular.	There are two forms of vector product: dot (scalar) and cross (product). Hence expressions such as $\mathbf{a}^2$ will not make sense, as it will be ambiguous whether it means $\mathbf{a}\cdot\mathbf{a}$ or $\mathbf{a}\times\mathbf{a}$. However $\lvert\mathbf{a}\rvert^2$ is meaningful as $\lvert\mathbf{a}\rvert$ means the length of a vector, so it is a number. Next, as $\lvert\mathbf{a}+\mathbf{b}\rvert$ is the length of the vector $\mathbf{a}+\mathbf{b}$, so $\lvert\mathbf{a}+\mathbf{b}\rvert\neq\lvert\mathbf{a}\rvert+\lvert\mathbf{b}\rvert$, and therefore $\lvert\mathbf{a}+\mathbf{b}\rvert^2\neq\lvert\mathbf{a}\rvert^2+2\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert+\lvert\mathbf{b}\rvert^2$. It is wrong to say that $\lvert\mathbf{a}+\mathbf{b}\rvert=\lvert\mathbf{a}-\mathbf{b}\rvert$ implies $\mathbf{a}+\mathbf{b}=\mathbf{a}-\mathbf{b}$ or $\mathbf{a}+\mathbf{b}=-(\mathbf{a}-\mathbf{b})$. Take for example the vectors $\mathbf{i}$ and $\mathbf{j}$, they are obviously pointing in different directions, so neither $\mathbf{i}=\mathbf{j}$ nor $\mathbf{i}=-\mathbf{j}$, but $\lvert\mathbf{i}\rvert=\lvert\mathbf{j}\rvert=1$.

(b)	$\overrightarrow{OP} = \lambda\mathbf{c}$ $\overrightarrow{OM} = \dfrac{2\overrightarrow{OP}+3\overrightarrow{OD}}{5} = \dfrac{2\lambda\mathbf{c}+3\mathbf{d}}{5}$ **Area of triangle** $OPM = \dfrac{1}{2}\left\|\overrightarrow{OP}\times\overrightarrow{OM}\right\|$ $= \dfrac{1}{2}\left\|\lambda\mathbf{c}\times\left(\dfrac{2\lambda\mathbf{c}+3\mathbf{d}}{5}\right)\right\|$ $= \dfrac{1}{2}\left\|\lambda\mathbf{c}\times\dfrac{2\lambda\mathbf{c}}{5}+\lambda\mathbf{c}\times\dfrac{3\mathbf{d}}{5}\right\|$ $= \dfrac{1}{2}\left\|\lambda\mathbf{c}\times\dfrac{3\mathbf{d}}{5}\right\| = \dfrac{3}{10}\lambda\left\|\mathbf{c}\times\mathbf{d}\right\|$	Read carefully: *P* lies on *OC* produced. So *P* does not lie in between *O* and *C*. Ratio theorem is a very useful and quick way to get $\overrightarrow{OM}$. $\mathbf{c}\times\mathbf{c}$ is a vector product, so the result should be a vector. So $\mathbf{c}\times\mathbf{c}\neq 0$, it should be $\mathbf{0}$, the zero vector. Because of the definition of vector product, $\mathbf{c}\times\mathbf{c}\neq\left\|\mathbf{c}\right\|^2$, and $\mathbf{c}\times\mathbf{d}\neq\mathbf{d}\times\mathbf{c}$.
3(i)	 	-label all vertices and end points -Quadratic curve shape (part) for $0\le x\le 2a$ and straight line segment for $2a\le x\le 4a$
(ii)	$\int_{-2a}^{8a} \mathrm{f}\left(\|x\|\right)\,\mathrm{d}x = 3\int_0^{2a}(x-2a)^2\,\mathrm{d}x + 2\left(\dfrac{1}{2}(2a)(4a^2)\right)$ $= 3\left[\dfrac{1}{3}(x-2a)^3\right]_0^{2a} + 8a^3$ $= 8a^3+8a^3$ $= 16a^3$	To draw graph of f(|*x*|), keep RHS and reflect in the y-axis. Note that $(x-2a)^2$ is only defined for $0\le x\le 2a$ while $2ax-4a^2$ is only defined for $2a\le x\le 4a$. It is **wrong** to take for instance: $\int_{4a}^{6a}(x-2a)^2\,\mathrm{d}x$ as the area of the region from $4a\le x\le 6a$.

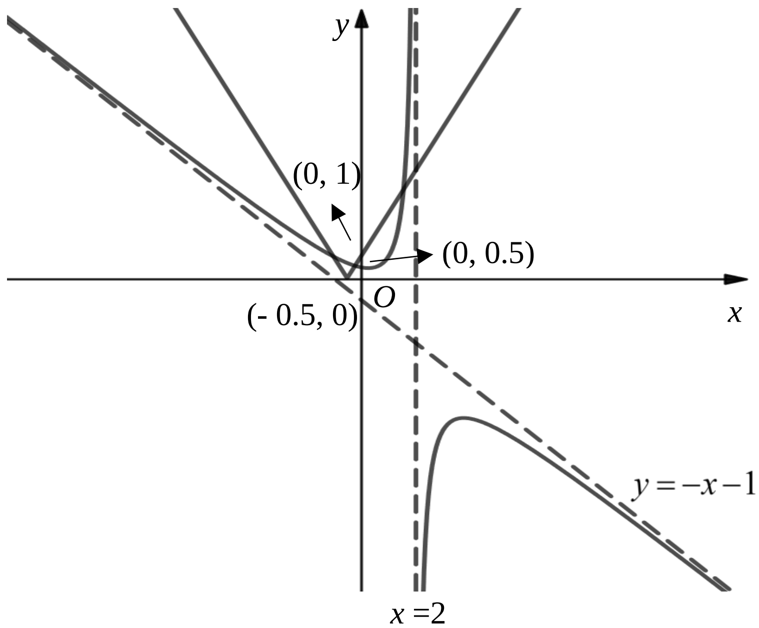
4(i)	Substitute $x=t^2$, $y=\frac{1}{\sqrt{t}}$ into $y=\frac{8}{x}$, $\frac{1}{\sqrt{t}}=\frac{8}{t^2}$ $t^{\frac{3}{2}}=8$ $t=4$ When $t=4$, $x=4^2=16$ $y=\frac{1}{\sqrt{4}}=\frac{1}{2}$ Coordinates of A is $\left(16,\frac{1}{2}\right)$.	Substitute the parametric equation of C into the Cartesian equation Avoid changing the parametric equation to Cartesian form
(ii)	$\frac{\mathrm{d}x}{\mathrm{d}t}=2t$, $\frac{\mathrm{d}y}{\mathrm{d}t}=-\frac{1}{2}t^{-\frac{3}{2}}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2}t^{-\frac{3}{2}}\times\frac{1}{2t}=-\frac{1}{4}t^{-\frac{5}{2}}$ Equation of tangent at point P on curve C, $y-\frac{1}{\sqrt{p}}=-\frac{1}{4}p^{-\frac{5}{2}}\left(x-p^2\right)$ At D , $y=0$ $\therefore 0-\frac{1}{\sqrt{p}}=-\frac{1}{4}p^{-\frac{5}{2}}\left(x-p^2\right)\Rightarrow x=5p^2$ At E , $x=0$ $\therefore y-\frac{1}{\sqrt{p}}=-\frac{1}{4}p^{-\frac{5}{2}}\left(0-p^2\right)\Rightarrow y=\frac{5}{4}p^{-\frac{1}{2}}$ Coordinates of F : $\left(\frac{5p^2}{2},\frac{\frac{5}{4}p^{-\frac{1}{2}}}{2}\right)\Rightarrow\left(\frac{5p^2}{2},\frac{5}{8\sqrt{p}}\right)$ $y=\frac{5}{8\sqrt{p}}$ $\Rightarrow\sqrt{p}=\frac{5}{8y}$, substitute into $x=\frac{5p^2}{2}$ $x=\frac{5}{2}\left(\frac{5}{8y}\right)^4=\frac{3125}{8192y^4}$ Cartesian equation of the curve traced by F is $x=\frac{3125}{8192y^4}$.	Take note that we are finding the gradient of tangent $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\div\frac{\mathrm{d}x}{\mathrm{d}t}$ Write down as header what you are trying to find, for example equation of tangent at point P. When $t=p$, Gradient of tangent at point P is $-\frac{1}{4}p^{-\frac{5}{2}}$ instead of $-\frac{1}{4}t^{-\frac{5}{2}}$ Find the midpoint F which follows the formula $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ From F, let $x=\frac{5p^2}{2}$, $y=\frac{5}{8\sqrt{p}}$ which is in the parametric form. Convert to Cartesian form so that we can trace how the path that point F moves as p varies.

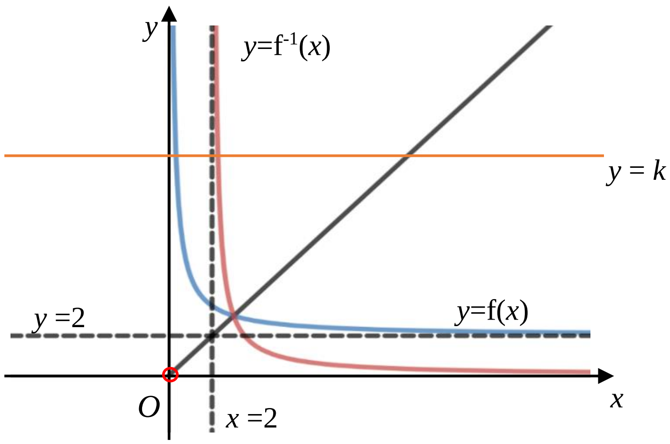
5(i)	$2xy+(1+y)^2=x$ Differentiating wrt x, $2y+2x\frac{\mathrm{d}y}{\mathrm{d}x}+2(1+y)\frac{\mathrm{d}y}{\mathrm{d}x}=1$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1-2y}{2(x+y+1)}$ When the tangent is parallel to the y-axis, $\frac{\mathrm{d}y}{\mathrm{d}x}$ is undefined. $\therefore 2(x+y+1)=0$ $y=-1-x$, substitute this into the equation of the curve, $2x(-1-x)+(1-1-x)^2=x$ $x^2+3x=0$ $x=0$ or $x=-3$	Do not confuse "tangent is parallel to y-axis", with x-axis: "parallel with y-axis": $\frac{\mathrm{d}y}{\mathrm{d}x}$ is undefined. "parallel with x-axis": $\frac{\mathrm{d}y}{\mathrm{d}x}=0$. Vertical lines in the Cartesian grid are of the form "x=…." instead of "y=…"
(ii)	Gradient of normal at $A=\frac{-1}{\frac{1-2(0)}{2(1+0+1)}}=-4$ Equation of normal at A: $y-0=-4(x-1)$ At B, $x=0\Rightarrow y=4$ => Coordinates of B are $(0,4)$ Area of triangle $OAB=\frac{1}{2}\times 4\times 1=2$ units^2	
6(a)(i)	$S_n=an^3+bn^2+cn+d$ $S_1=a(1)^3+b(1)^2+c(1)+d=2$ $\Rightarrow a+b+c+d=2$ ---- (1) $S_2=a(2)^3+b(2)^2+c(2)+d=6$ $\Rightarrow 8a+4b+2c+d=6$ ----(2) $S_5=a(5)^3+b(5)^2+c(5)+d=90$ $\Rightarrow 125a+25b+5c+d=90$ ----(3)	Don't assume that sequence is an AP/GP. It's neither. It's pointless to just write $S_5=T_1+T_2+...+T_5$ The sum is polynomial in n i.e. dependent on n and generally includes a constant term. Read question carefully. '4' is not S_2.

	$S_{10}=a(10)^3+b(10)^2+c(10)+d=830$ $\Rightarrow\quad 1000a+100b+10c+d=830\quad$ ----(4) Using GC: $a=1,\quad b=-2,\quad c=3,\quad d=0$ $S_n=n^3-2n^2+3n,\ \ n\geq 1,\ n\in\mathbb{Z}^+$	
(ii)	54^{th} term of the sequence $u_{54}=S_{54}-S_{53}$ $\quad=(54)^3-2(54)^2+3(54)-(53)^3+2(53)^2-3(53)$ $\quad=8376$	
(b)(i)	Given that $\cos(2n-1)\alpha-\cos(2n+1)\alpha=2\sin\alpha\sin 2n\alpha$ $\sin(2n\alpha)=\dfrac{\cos(2n-1)\alpha-\cos(2n+1)\alpha}{2\sin\alpha}$ $\therefore\displaystyle\sum_{n=1}^{N}\sin(2n\alpha)=\sum_{n=1}^{N}\frac{\cos(2n-1)\alpha-\cos(2n+1)\alpha}{2\sin\alpha}$ $=\dfrac{1}{2\sin\alpha}\begin{bmatrix}\cos\alpha-\cos 3\alpha\\ +\ \cos 3\alpha-\cos 5\alpha\\ +\ \cos 5\alpha-\cos 7\alpha\\ +\ \cdots\\ +\ \cos(2N-3)\alpha-\cos(2N-1)\alpha\\ +\ \cos(2N-1)\alpha-\cos(2N+1)\alpha\end{bmatrix}$ $=\dfrac{\cos\alpha-\cos(2N+1)\alpha}{2\sin\alpha}\qquad$ ------(*) $=\dfrac{\cos\alpha}{2\sin\alpha}-\dfrac{\cos(2N+1)\alpha}{2\sin\alpha}$ $=\dfrac{\cot\alpha}{2}-\dfrac{\operatorname{cosec}\alpha\cos(2N+1)\alpha}{2}$ (shown)	Use what was given. Don't waste time to derive it when it's already given. Note that $2\sin\alpha$ (independent of *n*) is a constant in this case Remember to cancel sufficient number of rows at the beginning and at the end.
(ii)	Let $\alpha=\dfrac{\pi}{3},\quad\displaystyle\sum_{n=1}^{N}\sin\left(\frac{2n\pi}{3}\right)$ $=\dfrac{\cot\frac{\pi}{3}}{2}-\dfrac{\cos ec\frac{\pi}{3}\cos(2N+1)\frac{\pi}{3}}{2}$ As $N\to\infty$, $\cos(2N+1)\dfrac{\pi}{3}$ takes values $\dfrac{1}{2}$ or -1. OR cannot converge to a constant number. $\therefore\displaystyle\sum_{n=1}^{\infty}\sin\frac{2n\pi}{3}$ does not converge.	Note that it's the letter *N* that tends to infinity and not the letter *n*.

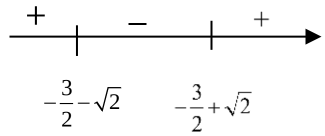
7(a)(i)	$\int \cos(\ln x)\ \mathrm{d}x = x\cos(\ln x) - \int x\left(-\frac{1}{x}\sin(\ln x)\right)\mathrm{d}x$ $= x\cos(\ln x) + \int \sin(\ln x)\ \mathrm{d}x$ $= x\cos(\ln x) + x\sin(\ln x) - \int x\left(\frac{1}{x}\cos(\ln x)\right)\ \mathrm{d}x$ $= x\cos(\ln x) + x\sin(\ln x) - \int \cos(\ln x)\ \mathrm{d}x$ $2\int \cos(\ln x))\ \mathrm{d}x = x\cos(\ln x) + x\sin(\ln x)$ $\int \cos(\ln x)\ \mathrm{d}x = \frac{1}{2}x\left(\cos(\ln x) + \sin(\ln x)\right) + C$	Use integration by parts directly. $\cos(\ln x)$ is a composite function. It is not a product of $(\cos x)(\ln x)$. Let u = $\cos(\ln x)$ and $\frac{\mathrm{d}v}{\mathrm{d}x} = 1$ apply integration by parts $uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\mathrm{d}x$ twice Bring $-\int \cos(\ln x)\ \mathrm{d}x$ to the LHS to stop the loop. Put + C in the final step
(ii)	Area = $\int_{\mathrm{e}^{-\frac{\pi}{2}}}^{\mathrm{e}^{\frac{\pi}{2}}} \cos(\ln x))\ \mathrm{d}x \quad = \frac{1}{2}\Big[\ x\cos(\ln x) + x\sin(\ln x)\ \Big]_{\mathrm{e}^{-\frac{\pi}{2}}}^{\mathrm{e}^{\frac{\pi}{2}}}$ $= \frac{1}{2}\mathrm{e}^{\frac{\pi}{2}}\left(\cos\left(\ln \mathrm{e}^{\frac{\pi}{2}}\right) + \sin\left(\ln \mathrm{e}^{\frac{\pi}{2}}\right)\right) -$ $\frac{1}{2}\mathrm{e}^{-\frac{\pi}{2}}\left(\cos\left(\ln \mathrm{e}^{-\frac{\pi}{2}}\right) + \sin\left(\ln \mathrm{e}^{-\frac{\pi}{2}}\right)\right)$ $= \frac{1}{2}\mathrm{e}^{\frac{\pi}{2}}\left(\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\right) - \frac{1}{2}\mathrm{e}^{-\frac{\pi}{2}}\left(\cos\left(-\frac{\pi}{2}\right) + \sin\left(-\frac{\pi}{2}\right)\right)$ $= \frac{1}{2}\left(\mathrm{e}^{\frac{\pi}{2}} + \mathrm{e}^{-\frac{\pi}{2}}\right)$	Read question carefully. Integrate from $e^{-\frac{\pi}{2}}$ to $e^{\frac{\pi}{2}}$. Use (i) answer. F(Upper limit)– F(lower limit) Note that $\ln \mathrm{e}^{\frac{\pi}{2}} = \frac{\pi}{2}$, $\sin\left(-\frac{\pi}{2}\right) = -1$
(b)	Using $u = \cot x$, $\frac{\mathrm{d}u}{\mathrm{d}x} = -\mathrm{cosec}^2 x = -\frac{1}{\sin^2 x}$ $\Rightarrow ...\mathrm{d}u = ... - \frac{1}{\sin^2 x}\ \mathrm{d}x$ When $x = \frac{\pi}{6} \Rightarrow u = \frac{1}{\tan x} = \sqrt{3}$ When $x = \frac{2\pi}{3} \Rightarrow u = -\frac{1}{\sqrt{3}}$	Differentiate the given substitution. Memorise the differentiation of the 6 trigo functions. Only differentiation of sec x and cosec x formula are in MF26 Find $\frac{\mathrm{d}u}{\mathrm{d}x}$ and hence dx = .. Need to change limit to u value. Write down the expression of the volume first. Then do substitution. $V = \int_{x_1}^{x_2} y^2\ \mathrm{d}x$

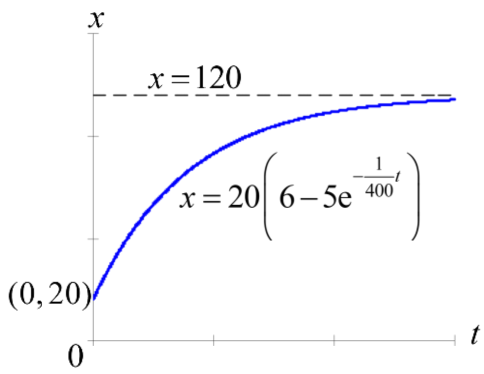
	Required volume is $=\pi\int_{\frac{\pi}{6}}^{\frac{2\pi}{3}}\left(\frac{e^{\frac{1}{2}\cot x}}{\sin x}\right)^2 \, dx=\pi\int_{\frac{\pi}{6}}^{\frac{2\pi}{3}}\frac{e^{\cot x}}{\sin^2 x} \, dx$ $=\pi\int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} e^{\cot x}\frac{1}{\sin^2 x} \, dx$ $=-\pi\int_{\sqrt{3}}^{-\frac{1}{\sqrt{3}}} e^u \, du$ or $\pi\int_{-\frac{1}{\sqrt{3}}}^{\sqrt{3}} e^u \, du$ $=\pi\left(e^{\sqrt{3}}-e^{-\frac{1}{\sqrt{3}}}\right)$	Change to $-\pi\int_{\sqrt{3}}^{-\frac{1}{\sqrt{3}}} e^u \, du$ Integrate e^u w.r.t u is e^u
8(i)	$y=\frac{x-x^2-1}{x-2}=\frac{-3}{x-2}-(x+1)$ $y=\frac{1}{3-x}-\frac{x}{3}$ $=\frac{-1}{x-3}-\frac{1}{3}x$ 1) A scaling parallel to the y-axis by a factor of 3. 2) A translation of 1 unit in the negative x-direction.	Be careful in your algebraic manipulation when trying to express $y=\frac{x-x^2-1}{x-2}$ in the form $y=\frac{A}{x-2}+B(x+1)$. Note that you are required to **describe** the sequence of transformations that will transform the curve with equation $y=\frac{1}{3-x}-\frac{x}{3}$ to $y=\frac{x-x^2-1}{x-2}=\frac{-3}{x-2}-(x+1)$, NOT the reverse. It is **INCORRECT to use** the word **“shift”** to describe translation and the word **“flip/rotate”** to describe reflection. One of the transformations involved is scaling parallel to the y-axis with a factor of 3, it is **<u>NOT with a factor of -3 or 3 units.</u>**

(ii)		Note that the curve $y=\frac{x-x^2-1}{x-2}$ has asymptotes $y=-x-1$ and $x=2$. You are expected to draw properly and clearly, including the behaviour of the curve $y=\frac{x-x^2-1}{x-2}$ near its asymptotes. Note that the oblique asymptote $y=-x-1$ meets the x-axis and y-axis at $(-1,0)$ and $(0,-1)$ respectively and the curve cuts the y-axis at the point $\left(0,\frac{1}{2}\right)$ which is NOT the minimum point. You are expected to get the <u>correct shape</u> of the graph from your GC. You are required to state the **coordinates of the points where the curves cross the axes.** For the modulus graph $y=\lvert 2x+1\rvert$, the coordinates are $\left(-\frac{1}{2},0\right)$ and $(0,1)$. It has a line of symmetry $x=-\frac{1}{2}$. You should have the sense of the relative positions for the points of intersection with the axes when sketching the two graphs, including the asymptotes on the same axes.
	$\frac{x-x^2-1}{x-2}=-2x-1$ From GC, the first point of intersection has x-coordinate -1 . $\frac{x-x^2-1}{x-2}=2x+1$ $x-x^2-1=(2x+1)(x-2)$ $3x^2-4x-1=0$ $x=\frac{4\pm\sqrt{16-4(3)(-1)}}{2(3)}$ $x=\frac{2\pm\sqrt{7}}{3}$	The question states that "**Hence**, find the **exact range** of …..", so you are required to use graphical method.

	Thus, the range of values of x is $x<-1,\ \ \frac{2-\sqrt{7}}{3}<x<\frac{2+\sqrt{7}}{3}$ or $x>2$	
9(a)(i)	 Since any horizontal line $y=k,\ k\in\mathbb{R}$ intersects the graph of f at most once, f is one-one and it has an inverse.	Must sketch the graph of f for the given domain only ($x>0$) with the asymptotes. Never use a particular line to explain one-one function and you must state the range of *k*. Note the 2 possible explanations. If you use any line $y=k,\ k\in\mathbb{R}$, then the line cuts the graph of f **at most once.** If you use any line $y=k,\ k\in R_f$, then the line cuts the graph of f **exactly once.** It is important to mention that f is one-one and not just f has an inverse.
(ii)	Let $y=2+\frac{3}{x}$ $\frac{3}{x}=y-2$ $x=\frac{3}{y-2}$ $\therefore f^{-1}(x)=\frac{3}{x-2}$ $D_{f^{-1}}=R_f=(2,\infty)$	To find the rule of f^{-1} : Let y = f (x) and then make x the subject
(iii)	See diagram in (i)	Must use the same scale for both axes when sketching the graphs of f and f^{-1} on the same diagram. The graph of f^{-1} is a reflection of the graph of f in the line $y=x$

		The graph of $f^{-1}f$ is not simply the line $y = x$. Must sketch for the correct domain and passing through (2, 2). The graphs of f, f^{-1} and $f^{-1}f$ must intersect at the same point.
(iv)	$D_f = (0, \infty)$ and $R_f = (2, \infty)$ Since $R_f \subseteq D_f$, f^2 exists. $f^2(x) = f(f(x))$ $= 2 + \dfrac{3}{2 + \frac{3}{x}}$ $= 2 + \dfrac{3x}{2x+3}$ $= \dfrac{2(2x+3)+3x}{2x+3}$ $= \dfrac{7x+6}{2x+3}$	It is not sufficient to state $R_f \subseteq D_f$. $D_f = (0, \infty)$, $R_f = (2, \infty)$ must be stated to justify the subset.
(b)	Given $h(x) = \dfrac{3-x}{x^2-1}$ To find the range of g, the graph must intersect the horizontal line $y = k$, therefore, $D \geq 0$ Let $k = \dfrac{3-x}{x^2-1}$ $kx^2 - k = 3 - x$ $kx^2 + x - (k+3) = 0$ $(1)^2 + 4k(k+3) \geq 0$ $1 + 12k + 4k^2 \geq 0$ $4k^2 + 12k + 1 \geq 0$ Consider $4k^2 + 12k + 1 = 0$, $k = \dfrac{-12 \pm \sqrt{12^2 - 4(4)(1)}}{2(4)}$ $= \dfrac{-12 \pm \sqrt{128}}{8}$ $= \dfrac{-3 \pm 2\sqrt{2}}{2} = -\dfrac{3}{2} \pm \sqrt{2}$	This question required an algebraic approach so the GC cannot be used to obtain the graph. It is very tedious to sketch the graph without using a GC as the stationary points would have to be found by differentiation. Students would also need to show the nature of the stationary points before they can sketch the graph on their own. Moreover, it would take some time to find the exact y coordinates of the stationary points in order to obtain the range of h. Thus, students are strongly encouraged to use the discriminant instead (see solution)

	$4k^2+12k+1\geq 0$ $\therefore\ k\leq -\frac{3}{2}-\sqrt{2}\ OR\ k\geq -\frac{3}{2}+\sqrt{2}$  Hence, the range of h is $\left(-\infty,-\frac{3}{2}-\sqrt{2}\right]\cup\left[-\frac{3}{2}+\sqrt{2},\infty\right)$	
10(i)	Concentration of the brine entering the tank is $\frac{C}{1000}$ kg/L Solution leaving the tank is $\frac{x}{2000}$ kg/L. The rate at which salt enters the tank is $\frac{5C}{1000}=\frac{C}{200}$ kg/min The rate at which leaves the tank is $\frac{5x}{2000}=\frac{x}{400}$ kg/min. $\frac{\mathrm{d}x}{\mathrm{d}t}=\text{inflow rate }-\text{outflow rate}$ $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{C}{200}-\frac{x}{400}=\frac{1}{400}(2C-x)$ (Shown) When $\frac{\mathrm{d}x}{\mathrm{d}t}=0$, $x=120$ kg and $2C-120=0\Rightarrow C=60$ kg	
(ii)	$\int\frac{1}{120-x}\,\mathrm{d}x=\int\frac{1}{400}\,\mathrm{d}t$ $-\ln\lvert 120-x\rvert=\frac{1}{400}t+B$ where $B=\text{const}$ $120-x=Ae^{-\frac{1}{400}t}$ where $A=\text{const}$ When $t=0$, $x=20$ kg, $Ae^0=A=120-20=100$ $120-x=100e^{-\frac{1}{400}t}$ $x=20\left(6-5e^{-\frac{1}{400}t}\right)$	Remember to include a negative sign and modulus sign after integrating $\frac{1}{120-x}$.

(iii)	 Using GC, $t = 204.33 = 204$ min.	
(iv)	It is assumed that there is no evaporation in the system so that the concentrations of the solution remain as stated/unaffected.	
11(i)	AP : first term = 1000, common difference = 250 $7500 = 1000 + 250(N-1)$ $N = 27$	It is an AP with first term 1000 and common difference 250. $u_N = 7500$. Solve for N.
(ii)	Total number of power banks produced in 60 weeks $= S_{27} + 33(7500)$ $= \frac{27}{2}\left(1000 + 7500\right) + 33(7500)$ =362250	For 1st to 27th week, it is an AP with first term 1000 and common difference 250. For 28th to 60th week, the production is at 7500 per week, so $(60-27)\times 7500$.
(iii)	Number of power banks on demand on week 1= 50 Number of power banks on demand on week 2 = a+50b Number of power banks on demand on week 3 $= a + b(a + 50b) = a + ba + 50b^2$	For 2nd week, the demand is $a + b(50) = a + 50b$ For 3rd week, demand is $a + b(\text{demand of 2nd week})$ $= a + b(a + 50b)$
(iv)	Number of power banks on demand on week 4 $= a + b(a + ba + 50b^2)$ $= a + ba + b^2a + 50b^3$ Number of power banks on demand on week n $= a + ba + b^2a + b^3a + ... + b^{n-2}a + 50b^{n-1}$ $= \frac{a(b^{n-1}-1)}{b-1} + 50b^{n-1}$	Continue working out for week 4 and deduce the demand of the nth week. Note that the last term of the expression is $b^{n-2}a$ for the GP. The first part of the expression is a summation of GP with first term a, common ratio b and number of terms $n-1$, which can be written as $\frac{a(b^{n-1}-1)}{b-1}$.
(v)	Total number of power banks produced in 60 weeks $= \frac{60}{2}\left(2(1000) + 59L\right)$ = 30(2000+59L)	Remember to take summation of the terms from the 1st to 60th week. The total demand can be found using GC (MATH, 0: Summation).

	Total number of power banks on demand in 60 weeks $=\sum_{r=1}^{60}\left(\frac{300(1-1.05^{r-1})}{1-1.05}\right)+\sum_{r=1}^{60}\left(1.05^{r-1}(50)\right)$ = 1779181.493 For $30\left(2000+59L\right)\geq 1779181.493$ $L\geq 971.29$ Least L = 972	For total demand to be met, total production $\geq$ total demand for the first 60 weeks, solve for L.

YISHUN INNOVA JUNIOR COLLEGE
JC 2 PRELIMINARY EXAMINATION
Higher 2

CANDIDATE NAME	

CG		INDEX NO	

MATHEMATICS 9758/02

Paper 2 **19 SEPT 2019**

3 hours

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your CG and name on the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiners' Use

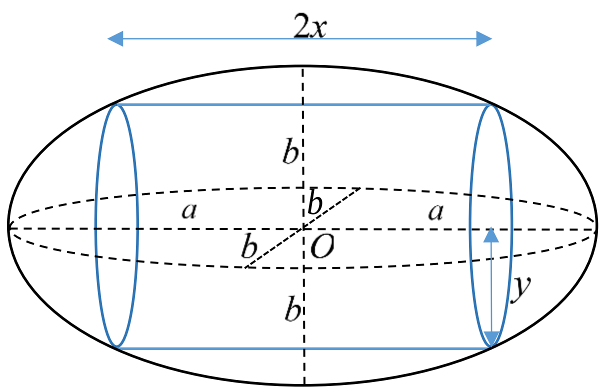
Question	1	2	3	4	5	6
Marks						

Question	7	8	9	10	**Total marks**	
Marks						

This document consists of 22 printed pages and 2 blank pages.

Section A: Pure Mathematics [40 marks]

1



The above diagram shows a hollow ellipsoid with centre O, enclosing a fixed volume of $\frac{4}{3}\pi ab^2$. A solid cylinder of length $2x$ and base radius y is inscribed in the ellipsoid. It is given that $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a and b are positive constants with $a > b$.

Use differentiation to find, in terms of a and b, the maximum volume of the cylinder, proving that it is a maximum. Hence determine the ratio of the maximum volume of the cylinder to that of the volume enclosed by the ellipsoid. [6]

2 **(i)** Find $\int 2\sin(k+1)x \sin kx \, \mathrm{d}x$. [2]

(ii) Hence, determine in terms of k, the value of $\int_0^{\frac{\pi}{2}} \left(\sin(k+1)x - \sin kx\right)^2 \, \mathrm{d}x$, where k is an even integer. [5]

3 The plane p contains the point A with coordinates $(5, -1,\ 2)$ and the line l_1 with equation $\dfrac{x-3}{2} = y$, $z = 1$.

(i) The point B has coordinates $(c,\ 2,\ 2)$. Given that the shortest distance from B to l_1 is $\dfrac{\sqrt{205}}{5}$, find the possible values of c. [3]

(ii) Find a cartesian equation of p. [3]

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$, $\mu \in \mathbb{R}$.

(iii) Find the coordinates of the point at which l_2 intersects p. [3]

The line l_3 has equation $\mathbf{r} = \begin{pmatrix} a \\ 2a \\ a \end{pmatrix} + t\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$, $t \in \mathbb{R}$, where a is a constant.

(iv) Show that p is parallel to l_3. [1]

(v) Given that l_3 and p have no point in common, what can be said about the value of a? [1]

(vi) It is given instead that $a = 1$, find the distance between l_3 and *p*, leaving your answer in exact form. [2]

4 **Do not use a calculator in answering this question.**

(a) The equation $2z^3 - 3z^2 + kz + 26 = 0$, where k is a real constant, has a root $z = 1 + a\text{i}$, where *a* is a positive real constant. Find the other roots of the equation and the values of *a* and *k*. [6]

(b) **(i)** Given that $(x+\mathrm{i}y)^2=15+8\mathrm{i}$, determine the possible values of the real numbers x and y. [3]

(ii) The roots of the equation $z^2-(2+7\mathrm{i})z=15-5\mathrm{i}$ are z_1 and z_2, with $\arg(z_1)<\arg(z_2)$. Find an exact expression for z_2, giving your answer in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r>0$ and $-\pi<\theta\le\pi$. [3]

(iii) Find the argument of $z_1^{\,2} z_2^{\,*}$ in exact form. [2]

Section B: Probability and Statistics [60 marks]

5 An investigation was carried out to determine the effect of rainfall on crop yield. The table below shows the average monthly rainfall, x mm, and the crop yield, y kg. The data is recorded during different months of a certain year.

x	150	163	172	175	180	187	196
y	48	70	87	92	95	89	80

(i) Draw a scatter diagram for these values. State with a reason, which of the following equations, where a and b are constants, provides the most accurate model of the relationship between x and y.

(A) $y = a\ln(x-100)+b$

(B) $y = a(x-180)^2+b$

(C) $y = \dfrac{a}{x-130}+b$ [3]

(ii) Using the model you chose in part **(i)**, write down the equation for the relationship between x and y, giving the numerical values of the coefficients. State the product moment correlation coefficient for this model. [2]

(iii) Calculate an estimate of the crop yield when the average monthly rainfall is 185 mm. Comment on the reliability of your estimate. [2]

6 A factory produces a large number of packets of cornflakes. On average, two in 7 packets contain a toy. The packets of cornflakes are sold in cartons of 12.

(a) A carton is randomly chosen.

(i) Find the probability that there are fewer than 2 toys. [1]

(ii) Find the probability that there are more than 1 but at most 6 toys. [2]

(b) Find the probability that in five randomly selected cartons, two of them contain exactly 4 toys and three of them contain exactly 2 toys. [2]

(c) A mini-mart ordered 40 cartons of cornflakes from the factory. Find the probability that none of the cartons contains fewer than 2 toys. [2]

The factory also produces a large number of packets of oats. A random sample of n packets of oats is chosen. The number of packets of oats containing a toy in the sample is denoted by A. Assume that A has the distribution $\mathrm{B}(n, p)$, where $p > 0.1$.

Given that $n = 25$ and $\mathrm{P}(A = 2 \text{ or } 3) = 0.25$, write down an equation in terms of p and find p numerically. [2]

7 A box contains five balls numbered 1, 3, 5, 6, 8. Three balls are drawn at random from the box.

(a) Find the probability that the sum of the three numbers drawn is an even number. [2]

(b) The random variable S denotes the smallest of the three numbers drawn.

(i) Determine the probability distribution of S. [2]

(ii) Find $\mathrm{E}(S)$ and $\mathrm{Var}(S)$. [2]

(iii) The mean of a random sample of 55 observations of S is denoted by $\overline{S}$. Find the probability that $\overline{S}$ is within 0.5 of $\mathrm{E}(S)$. [3]

8 **(a)** Events A and B are such that $\mathrm{P}(A \cup B) = 0.6$ and $\mathrm{P}(A \mid B) = 0.4$. Given that A and B are independent, find

(i) $\mathrm{P}(B)$, [2]

(ii) $\mathrm{P}(A' \mid B')$. [2]

(b) 12 people are to be seated at 3 different coloured round tables.

(i) Find the number of ways that there are at least 3 people at each table. [4]

(ii) Find the probability that there are 4 people at each table given that there are at least 3 people at each table. [2]

9 A company produces car batteries. The life, in months, of a car battery of the regular type has the distribution $\mathrm{N}\left(\mu,\ \sigma^2\right)$. The mean life of 4 randomly selected car batteries of the regular type is denoted by $\overline{X}$. It is given that $\mathrm{P}\left(\overline{X} < 36.1\right) = \mathrm{P}\left(\overline{X} > 49.1\right) = 0.03355$.

(i) State the value of μ and show that $\sigma \approx 7.10$, correct to 2 decimal places. [4]

(ii) Find the smallest integer value of *k* such that more than 90% of the car batteries of the regular type have a life less than *k* months. [2]

(iii) Past experience shows that 25% of the car batteries of the regular type with lives less than 36 months are due to bad driving habits. A random sample of 100 car batteries of the regular type is selected. Find the expected number of these car batteries which will have lives each less than 36 months due to bad driving habits. [2]

(iv) After research and experimentation, the company produces a premium type of car battery using an improved manufacturing process which is able to increase the life of each car battery by 10%. Find the probability that the total life of 5 randomly chosen car batteries of the premium type is more than the total life of 6 randomly chosen car batteries of the regular type. [4]

10 A previous study revealed that the average time taken to assemble a certain type of electrical component is at least 15 minutes. The manager wants to investigate if the results of the study is valid. A random sample of 40 components is taken and the times taken to assemble the components are summarised in the following table:

Time to assemble a component (min)	9	10	12	13	15	16	17	18
Number of components	1	6	3	8	5	7	8	2

(i) Find unbiased estimates of the population mean and variance. [2]

(ii) Test at the 5% level of significance whether the results of the study is valid. You should state your hypotheses and define any symbols you use. [5]

(iii) Explain why the manager is able to conduct the test without knowing anything about the distribution of the times taken to assemble the electrical components. [1]

(iv) Explain what is meant by the phrase "5% level of significance" in this context. [1]

The manager claims that the average time taken to assemble another type of electrical component is 30 minutes. A random sample of 50 components of this type is chosen and the time taken to assemble each component is recorded. The mean and standard deviation of the sample are 29.7 minutes and k minutes respectively. Find the range of possible values of k if a test at the 8% significance level shows that there is sufficient evidence that the manager's claim is valid. [4]

~ END OF PAPER ~