NATIONAL JUNIOR COLLEGE
SENIOR HIGH 2 PRELIMINARY EXAMINATION
Higher 2

MATHEMATICS 9758/01

Paper 1 **24 August 2018**

3 hours

Additional Materials: Answer Paper
List of Formulae (MF26)
Cover Sheet

READ THESE INSTRUCTIONS FIRST

Write your name, registration number, subject tutorial group, on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in the brackets [ ] at the end of each question or part question.

This document consists of **8** printed pages.

1 The n^{th} term of a sequence is given by $u_n = n!(n-2)$, for all positive integers n where $n \geq 2$.

Show that

$$u_n - u_{n-1} = (n-1)!\left(an^2 + bn + c\right)$$

for some real constants a, b and c to be determined. [2]

Hence find

$$\sum_{n=3}^{N+1}\left[(n-1)!\left(2n^2 - 6n + 6\right)\right].$$ [3]

2 Referred to the origin O, the points A and B are such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The point C is on AB produced such that $AB : BC = 1 : k$, where k is a real constant.

Given that $OA = 2OC$, $\angle AOB = \dfrac{\pi}{2}$, and $\angle AOC = \dfrac{3\pi}{4}$, find the exact value of k, showing your working clearly. [5]

3 By means of the substitution $u = \sin\theta$, find

$$\int \frac{2\cos\theta - 3\sin 2\theta}{1 + \sin^2\theta}\,\mathrm{d}\theta.$$ [5]

4 The curve C has equation $9x^2 - y^2 + 3xy - 5 = 0$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. Hence, explain why C has no turning points. [4]

(ii) Find the equations of the tangent and the normal to C at the point $P(1, -1)$. [2]

(iii) The tangent and normal to C at the point $P(1, -1)$ meets the y-axis at the points Q and R respectively. State the area of triangle PQR. [1]

5 Given that a is a positive constant. A curve C_1 has parametric equations

$$x = \frac{a}{t}, \quad y = 1 + t.$$

Sketch C_1, labelling the coordinates of the point(s) where the curve crosses the x- and y-axes, and the equations of the asymptote(s) in terms of a, if any. [2]

Another curve C_2 has equation $y = \sqrt{1 + \frac{x^2}{a^2}}$.

(i) Show algebraically that the y-coordinates of the point(s) of intersection of C_1 and C_2 satisfies the equation $(y-1)^2\left(y^2-1\right)-1=0$. [2]

(ii) Sketch C_2 on the same diagram as C_1, labelling the coordinates of the point(s) where the curve crosses the x- and y-axes, and the equations of the asymptotes in terms of a, if any.

Find the coordinates of point(s) of intersections of C_1 and C_2 and label the coordinates in this diagram, leaving the answers correct to 3 significant figures, in terms of a. [4]

6 **(a)** The function f is defined by

$$\mathrm{f} : x \mapsto 2x^2 - \lambda x - 3, \text{ where } x \in \mathbb{R},\ \frac{7}{4} < x < 5,$$

where λ is a real constant. Find the set of possible values of λ such that f^{-1} exists. [2]

(b) The function g is defined by

$$\mathrm{g}(x) = \begin{cases} (x-1)^2, & \text{for } 0 \le x \le 1, \\ 2\log_2 x, & \text{for } 1 < x \le 2. \end{cases}$$

(i) Sketch the graph of $y = \mathrm{g}(x)$, labelling clearly the coordinates of the end-points and the points where the curve crosses the x-axis, if any. [2]

(ii) Hence solve the inequality $1 < \mathrm{g}(x) \le 2$ exactly. [2]

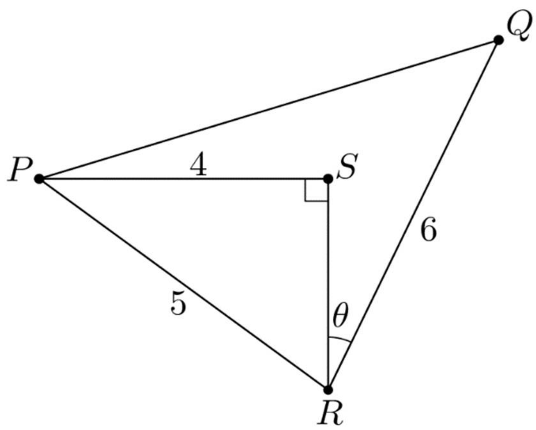
(iii) Given that g^2 exists, define g^2 in a similar form as g. [3]

7 **(a)** Given that $y+2=(x+1)^{\ln(x+1)}$, where $x>-1$, show that

$$(x+1)\frac{\mathrm{d}y}{\mathrm{d}x}=2(y+2)\ln(x+1).$$ [2]

By repeated differentiation of the above result, find the Maclaurin series of y up to and including the term in x^2. [3]

(b)



In the diagram above, $QR=6$, $PS=4$, $PR=5$, $\angle PSR=\frac{\pi}{2}$ and $\angle QRS=\theta$ radians.

(i) Show that $PQ=(61-36\cos\theta+48\sin\theta)^{\frac{1}{2}}$. [2]

(ii) Given that θ is a sufficiently small angle, show that

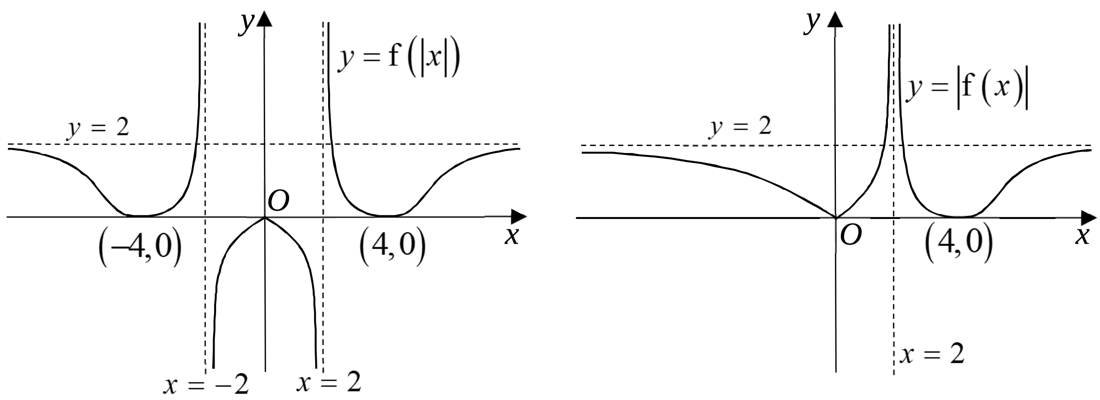
$$PQ\approx 5+p\theta+q\theta^2$$

for some rational constants p and q to be determined exactly. [4]

8 **(a)** Describe a sequence of exactly three transformations that will transform the curve with equation $y=\dfrac{1}{x-3}$ onto the curve with equation $y=\dfrac{2x+a}{3-x}$, where a is a positive constant. [3]

Given instead that $a<-6$, use a non-graphical method to determine the range of values of x where the graph of $y=\dfrac{2x+a}{3-x}$ is concave upwards. [2]

(b) The diagrams below show the graphs of $y=\mathrm{f}\left(|x|\right)$ and $y=\left|\mathrm{f}\left(x\right)\right|$, where the equations of the asymptotes and the coordinates of the turning points are given. The gradient of the graph with equation $y=\mathrm{f}\left(x\right)$ at the origin is $-\dfrac{3}{4}$.



On separate diagrams, sketch the graphs of

(i) $y=\mathrm{f}(x)$, [2]

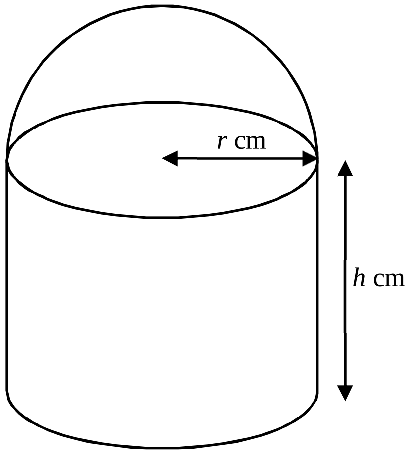
(ii) $y=\dfrac{1}{\mathrm{f}(x)}$, [3]

(iii) $y=\mathrm{f}'(x)$, [3]

including the coordinates of the points where the graphs cross the x- and y- axes and the equations of the asymptotes, if any.

9 [It is given that a sphere of radius r has surface area $4\pi r^2$ and volume $\frac{4}{3}\pi r^3$.]

A manufacturer produces closed hollow cans of fixed volume k cm^3 as shown in the diagram below. The top part is a hemisphere made of tin. The bottom part is a cylinder made of aluminium of cross-sectional radius r cm and height h cm. There is no material between the cylinder and the hemisphere so that any fluid can move freely within the container.



(i) If tin costs 4 cents per cm^2 and aluminium costs 6 cents per cm^2, use differentiation to find the values of r and h such that the total cost of producing the cans is minimised, giving your answers in terms of k. Simplify your answers. [7]

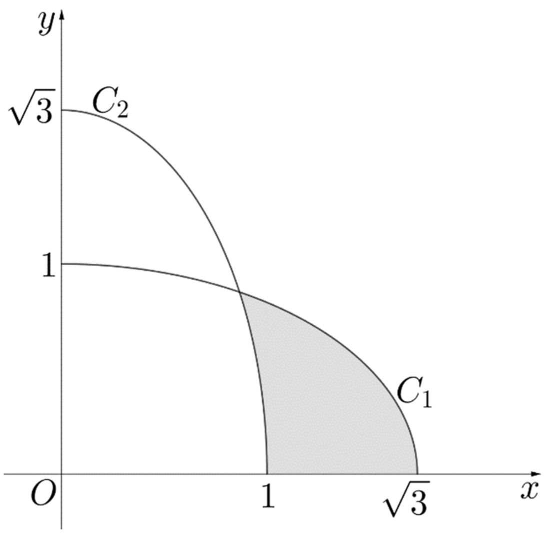
(ii) At the beginning of an experiment, a similar-shaped can of dimensions $r = 4$ and $h = 10$, is filled to its capacity with water. Due to a hole at its base, water is leaking at a constant rate of 2 cm^3 s^{-1} when the can is standing upright. Find the exact rate at which the height of the water is decreasing 80 seconds after the start of the experiment. [5]

10 **(i)** Show that for any real constant p, $\dfrac{p^2x^2}{\sqrt{1-p^2x^2}} = \dfrac{1}{\sqrt{1-p^2x^2}} - \sqrt{1-p^2x^2}$.

Hence, or otherwise, prove that for any constant n such that $0 < n < \dfrac{1}{p}$,

$$\int_n^{\frac{1}{p}} \sqrt{1-p^2x^2}\,\mathrm{d}x = \frac{\pi}{4p} - \frac{1}{2p}\sin^{-1}(pn) - \frac{1}{2}n\sqrt{1-p^2n^2}.$$ [6]

(ii) The curves C_1 and C_2 are ellipses with the origin O as their common centre. It is given that the points $(0,1)$ and $\left(\sqrt{3},0\right)$ are vertices of C_1 and the points $\left(0,\sqrt{3}\right)$ and $(1,0)$ are vertices of C_2. The diagram below shows the parts of the two curves in the first quadrant.



Use the result in part **(i)** to find the area of the shaded region exactly, giving your answer in the form $\dfrac{\pi\sqrt{3}}{m}$, where m is an integer constant to be determined. [6]

[**Question 11 is printed on the next page.**]

11 A swimming pool contains 375000 litres of pure water. Water containing s milligrams of free chlorine per litre flows into the pool at a rate of 10 litres per minute. The pool is also draining at a rate of 10 litres per minute, such that the volume of the water in the pool remains constant.

(i) Show that the rate at which the mass of the free chlorine, x grams, is changing in the pool over time, t minutes, can be modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{375s - x}{37500},$$

State a necessary assumption for the above model to be valid. [3]

(ii) Given that no free chlorine is present in the pool initially, find x in terms of t and s. [4]

(iii) Find, in terms of s, the mass of free chlorine present in the pool after one hour. [1]

(iv) Sketch the graph of x against t which is relevant to the context, labelling the point(s) where the curve crosses the axes and the equation(s) of the asymptote(s). Hence determine the mass of free chlorine present in the pool after a long period in terms of s. [3]

To prevent health complications, the recommended safe level of free chlorine to be used is between 375 grams and 1125 grams in a pool which contains 375 000 litres of water. Find the range of values of s that should be used. [2]

– END OF PAPER –

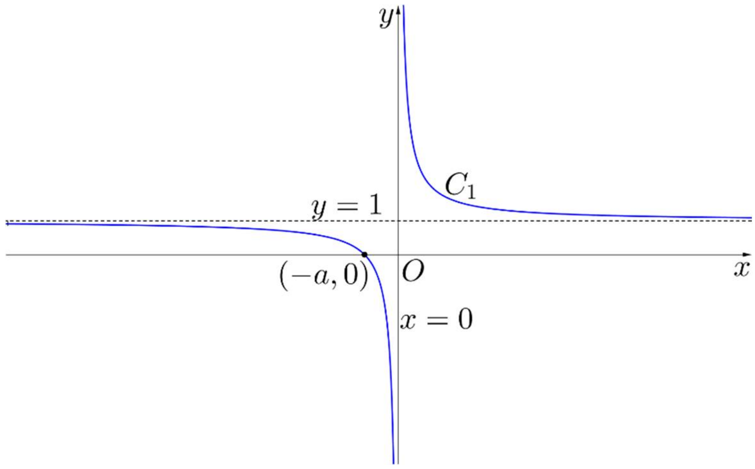
2018 NJC H2 Math Prelim Paper 1 Solution

1	$u_n - u_{n-1}$ $= n!(n-2)-(n-1)!(n-3)$ $=(n-1)!n(n-2)-(n-1)!(n-3)$ $=(n-1)!(n^2-2n-n+3)$ $=(n-1)!(n^2-3n+3)$ $\sum_{n=3}^{N+1}\left[(n-1)!\left(2n^2-6n+6\right)\right].$ $2\sum_{n=3}^{N+1}\left((n-1)!\left(n^2-3n+3\right)\right)$ $=2\sum_{n=3}^{N+1}\left(u_n-u_{n-1}\right)$ $=2\ [\cancel{u_3}-u_2$ $+\cancel{u_4}-\cancel{u_3}$ $+\vdots$ $+\cancel{u_N}-\cancel{u_{N-1}}$ $+u_{N+1}-\cancel{u_N}]$ $=2\left(u_{N+1}-u_2\right)$ $=2\left[(N+1)!(N-1)-2!(0)\right]$ $=2(N+1)!(N-1)$

2	Let $OC = \mathbf{c}$. $\mathbf{b} = \dfrac{k\mathbf{a}+\mathbf{c}}{1+k}$ $(1+k)\mathbf{b} = k\mathbf{a}+\mathbf{c}$ $\mathbf{a}\cdot\mathbf{b}(1+k) = \mathbf{a}\cdot(k\mathbf{a}+\mathbf{c})$ $0 = k\lvert\mathbf{a}\rvert^2 + \mathbf{a}\cdot\mathbf{c}$ $k\lvert\mathbf{a}\rvert^2 = -\lvert\mathbf{a}\rvert\lvert\mathbf{c}\rvert\cos\dfrac{3\pi}{4}$ $k\lvert\mathbf{c}\rvert^2 = -\lvert\mathbf{a}\rvert\cdot\dfrac{1}{2}\lvert\mathbf{a}\rvert\cos\dfrac{3\pi}{4}$ since $\lvert\mathbf{a}\rvert = 2\lvert\mathbf{c}\rvert$ $k = -\dfrac{1}{2}\cos\dfrac{3\pi}{4}$ $k = \dfrac{\sqrt{2}}{4}$

3	$u=\sin\theta \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}\theta}=\cos\theta$ $\displaystyle\int \frac{2\cos\theta-3\sin 2\theta}{1+\sin^2\theta}\,\mathrm{d}\theta$ $\displaystyle=\int \frac{2\cos\theta-6\sin\theta\cos\theta}{1+\sin^2\theta}\,\mathrm{d}\theta$ $\displaystyle=\int \left(\frac{2-6\sin\theta}{1+\sin^2\theta}\right)\cos\theta\,\mathrm{d}\theta$ $\displaystyle=\int \frac{2-6u}{1+u^2}\,\mathrm{d}u$ $\displaystyle=\int \frac{2}{1+u^2}\,\mathrm{d}u-3\int \frac{2u}{1+u^2}\,\mathrm{d}u$ $=2\tan^{-1}u-3\ln\left(1+u^2\right)+c$ $=2\tan^{-1}\left(\sin\theta\right)-3\ln\left(1+\sin^2\theta\right)+c$ $=2\tan^{-1}\left(\sin\theta\right)-3\ln\left(1+\sin^2\theta\right)+c$

4 (i)	Differentiating $9x^2 - y^2 + 3xy - 5 = 0$ w.r.t. x, $18x - 2y\frac{dy}{dx} + 3x\frac{dy}{dx} + 3y = 0$ $2y\frac{dy}{dx} - 3x\frac{dy}{dx} = 18x + 3y$ $\frac{dy}{dx} = \frac{18x+3y}{2y-3x}$ Suppose (x, y) is a turning point of C. Then $$\frac{dy}{dx} = \frac{18x+3y}{2y-3x} = 0 \Rightarrow 18x + 3y = 0 \Rightarrow y = -6x$$ Substituting $y = -6x$ into $9x^2 - y^2 + 3xy - 5 = 0$, we get $9x^2 - (-6x)^2 + 3x(-6x) - 5 = 0$ $9x^2 - 36x^2 - 18x^2 - 5 = 0$ $x^2 = \frac{5}{-45} = -\frac{1}{9}$ which is impossible since $x^2 \geq 0$. Therefore C has no turning points.
4 (ii)	At $P(1, -1)$, $\frac{dy}{dx} = \frac{18-3}{-2-3} = -3$ Then equation of tangent is $\frac{y+1}{x-1} = -3 \Rightarrow y = -3x + 2$ Then equation of normal is $\frac{y+1}{x-1} = \frac{1}{3} \Rightarrow 3y + 3 = x - 1 \Rightarrow y = \frac{1}{3}x - \frac{4}{3}$
4 (iii)	Area of triangle PQR $= \frac{1}{2}\left(2 + \frac{4}{3}\right)(1)$ $= \frac{5}{3}$

<table>
<tr><td>5 (1st part)</td><td>
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<tr><td>5 (i)</td><td>

$x=\frac{a}{t}$, $y=1+t \Rightarrow x=\frac{a}{y-1}$

Substitute $x=\frac{a}{y-1}$ into $y=\sqrt{1+\frac{x^2}{a^2}}$:

$$y=\sqrt{1+\frac{\left(\frac{a}{y-1}\right)^2}{a^2}}$$

$$y^2=1+\frac{1}{(y-1)^2}$$

$$y^2(y-1)^2=(y-1)^2+1$$

$$y^2(y-1)^2-(y-1)^2-1=0$$

$$(y-1)^2(y^2-1)-1=0$$

Alternatively,

$$y=\sqrt{1+\frac{\left(\frac{a}{t}\right)^2}{a^2}}=\sqrt{1+\frac{1}{t^2}}$$

Squaring both sides,

$$y^2=1+\frac{1}{t^2}$$

$$y^2=1+\frac{1}{(y-1)^2}$$

$$y^2-1=\frac{1}{(y-1)^2}$$

$$(y^2-1)(y-1)^2=1$$

$$(y^2-1)(y-1)^2-1=0 \text{ (shown)}$$

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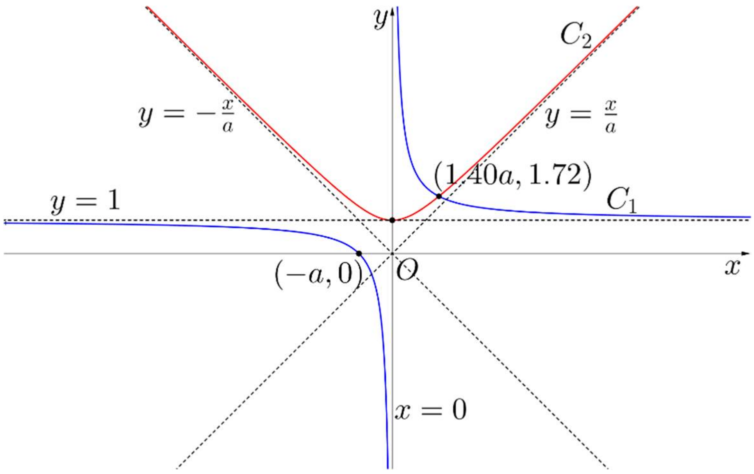
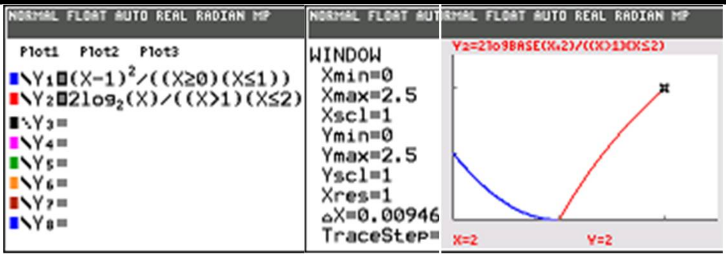
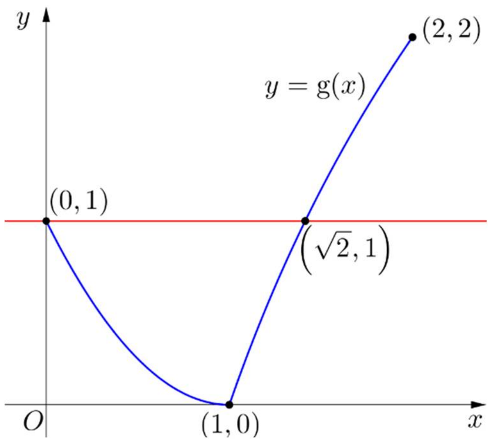
</table>

5 (ii)	 Using GC to solve, $(y-1)^2(y^2-1)-1=0$, $y=-1.106919$ (rejected) or 1.716673 When $y = 1.716673$, $x = 1.39534a$ $\therefore$ The point of intersection is $(1.40a, 1.72)$.

6 (a)	$2x^2-\lambda x-3=2\left(x^2-\frac{\lambda}{2}x\right)-3$ $=2\left(x^2+2\left(-\frac{\lambda}{4}\right)x+\left(-\frac{\lambda}{4}\right)^2\right)-3-2\left(-\frac{\lambda}{4}\right)^2$ $=2\left(x-\frac{\lambda}{4}\right)^2-\frac{\lambda^2+24}{8}$ OR $\frac{\mathrm{d}}{\mathrm{d}x}\left(2x^2-\lambda x-3\right)=0$ $4x-\lambda=0$ $x=\frac{\lambda}{4}$ For f^{-1} to exist, the turning point of $y=2x^2-\lambda x-3$ **cannot** lie in the interval $\frac{7}{4}<x<5$. Therefore, $\frac{\lambda}{4}\le\frac{7}{4}$ or $\frac{\lambda}{4}\ge 5\Rightarrow\lambda\le 7$ or $\lambda\ge 20$

<table>
<tr><td>6 (b)(i)</td><td>




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<tr><td>6 (b)(ii)</td><td>
From the sketch in (b)(i),

Point of intersection between $y = \mathrm{g}(x)$ and $y = 1$ occurs at the points (0, 1) and where

$$2\log_2 x = 1 \Rightarrow \log_2 x = \frac{1}{2}$$
$$\Rightarrow x = 2^{\frac{1}{2}} = \sqrt{2}$$

Therefore, $1 < \mathrm{g}(x) \le 2 \Rightarrow \sqrt{2} < x \le 2$
</td></tr>
<tr><td>6 (b) (iii)</td><td>
For $0 \le x \le 1$, $\mathrm{g}^2(x) = \mathrm{g}\left((x-1)^2\right) = \left((x-1)^2 - 1\right)^2 = \left(x^2 - 2x\right)^2$

Considering the effect of part (b)(ii),

For $1 < x \le \sqrt{2}$, $\mathrm{g}^2(x) = \mathrm{g}\left(2\log_2 x\right) = \left(2\log_2 x - 1\right)^2$

For $\sqrt{2} < x \le 2$, $\mathrm{g}^2(x) = \mathrm{g}\left(2\log_2 x\right) = 2\log_2\left(2\log_2 x\right)$

$$\text{Therefore, } \mathrm{g}^2(x) = \begin{cases} \left(x^2 - 2x\right)^2 & \text{for } 0 \le x \le 1, \\ \left(2\log_2 x - 1\right)^2 & \text{for } 1 < x \le \sqrt{2}, \\ 2\log_2\left(2\log_2 x\right) & \text{for } \sqrt{2} < x \le 2 \end{cases}$$
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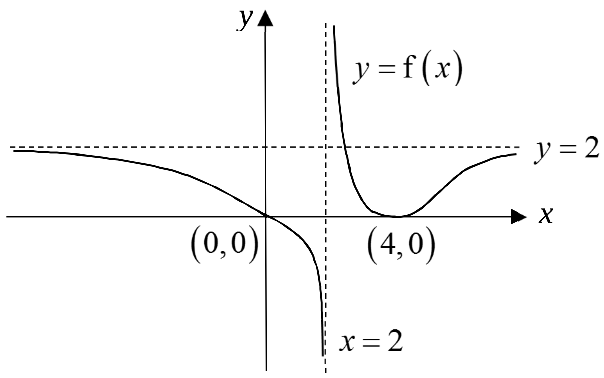
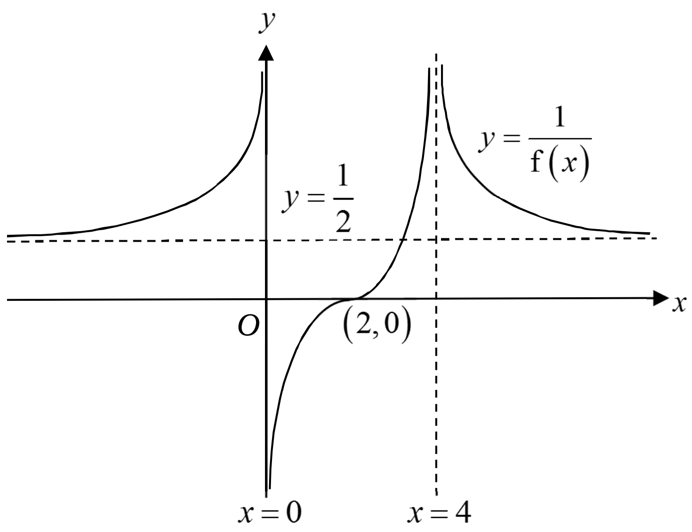
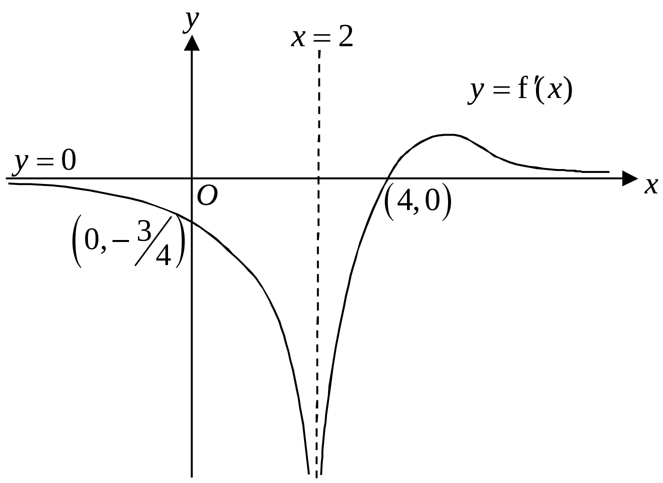
</table>

7 (a)	$y+2=(x+1)^{\ln(x+1)}$ $\ln(y+2)=\ln(x+1)^{\ln(x+1)}$ $\ln(y+2)=(\ln(x+1))^2$ $\frac{1}{y+2}\frac{\mathrm{d}y}{\mathrm{d}x}=2(\ln(x+1))\left(\frac{1}{x+1}\right)$ $(x+1)\frac{\mathrm{d}y}{\mathrm{d}x}=2(y+2)\ln(x+1)$ (shown) $(x+1)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2(y+2)}{x+1}+2\frac{\mathrm{d}y}{\mathrm{d}x}\ln(x+1)$ $(x+1)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\frac{\mathrm{d}y}{\mathrm{d}x}\left[1-2\ln(x+1)\right]=\frac{2(y+2)}{x+1}$ When $x=0$, $y+2=1$ $\therefore y=-1$ $\frac{1}{-1+2}\frac{\mathrm{d}y}{\mathrm{d}x}=0$ $\therefore \frac{\mathrm{d}y}{\mathrm{d}x}=0$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{2(-1+2)}{0+1}$ $\therefore \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2$ $y\approx -1+\frac{x^2}{2!}(2)$ $\therefore y=-1+x^2$

7 (b)(i)	$PQ^2 = PR^2 + QR^2 - 2PR \cdot QR \cos\angle PRQ$ $= 5^2 + 6^2 - 2(5)(6)\cos\left(\angle PRS + \theta\right)$ $= 25 + 36 - 60\left[\cos\left(\angle PRS\right)\cos\theta - \sin\left(\angle PRS\right)\sin\theta\right]$ $= 61 - 60\left(\frac{3}{5}\cos\theta - \frac{4}{5}\sin\theta\right)$ $= 61 - 36\cos\theta + 48\sin\theta$ $\therefore PQ = \left(61 - 36\cos\theta + 48\sin\theta\right)^{\frac{1}{2}}$ (shown)
7 (b) (ii)	$PQ = \left(61 - 36\cos\theta + 48\sin\theta\right)^{\frac{1}{2}}$ $\approx \left(61 - 36\left(1 - \frac{\theta^2}{2}\right) + 48\theta\right)^{\frac{1}{2}}$ $= \left(25 + 48\theta + 18\theta^2\right)^{\frac{1}{2}}$ $= 5\left(1 + \frac{48}{25}\theta + \frac{18}{25}\theta^2\right)^{\frac{1}{2}}$ $\approx 5\left(1 + \frac{1}{2}\left(\frac{48}{25}\theta + \frac{18}{25}\theta^2\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{48}{25}\theta\right)^2\right)$ $= 5\left(1 + \frac{24}{25}\theta + \frac{9}{25}\theta^2 - \frac{288}{625}\theta^2\right)$ $= 5\left(1 + \frac{24}{25}\theta - \frac{63}{625}\theta^2\right)$ $= 5 + \frac{24}{5}\theta - \frac{63}{125}\theta^2,\ p = \frac{24}{5},\ q = -\frac{63}{125}$

8 (a) (i)	1. Reflect graph about the x-axis 2. Scale graph parallel to the y-axis by factor of $(6 + a)$ 3. Translate graph along/in the negative y-direction by 2 units
8 (a) (ii)	$y = \frac{2x+a}{3-x}$ $= -\frac{2x+a}{x-3}$ $= -2 - \frac{6+a}{x-3}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{6+a}{(x-3)^2}(-1)$ $= \frac{6+a}{(x-3)^2}$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{6+a}{(x-3)^3}(-2)$ $= -\frac{2(6+a)}{(x-3)^3}$ Where the graph is concave upwards, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} > 0$. Since $a < -6$, $6+a < 0$. Therefore, $(x-3)^3 > 0 \Rightarrow x > 3$

8 (b) (i)	
8 (b) (ii)	
8 (b) (iii)	

9 (i)	Consider the volume of a can $\frac{2}{3}\pi r^3+\pi r^2 h=k$ $\pi rh=\frac{k}{r}-\frac{2}{3}\pi r^2$ $h=\frac{k}{\pi r^2}-\frac{2}{3}r$ --- (1) Cost of a can, $C=4\left(2\pi r^2\right)+6\left(2\pi rh\right)+6\left(\pi r^2\right)$ $=14\left(\pi r^2\right)+12\left(\pi rh\right)$ $=14\left(\pi r^2\right)+\frac{12k}{r}-8\pi r^2$ (from (1)) $=6\pi r^2+\frac{12k}{r}$ $\frac{\mathrm{d}C}{\mathrm{d}r}=12\pi r-\frac{12k}{r^2}=0$ $r^3=\frac{k}{\pi}$ $\therefore r=\sqrt[3]{\frac{k}{\pi}}$ $\frac{\mathrm{d}^2C}{\mathrm{d}r^2}=12\pi+\frac{24k}{r^3}=36\pi>0$ or $\frac{\mathrm{d}^2C}{\mathrm{d}r^2}=12\pi+\frac{24k}{r^3}>0$ since $r, k>0$ When $r=\sqrt[3]{\frac{k}{\pi}}$, $h=\frac{k}{\pi\left(\sqrt[3]{\frac{k}{\pi}}\right)^2}-\frac{2}{3}\left(\sqrt[3]{\frac{k}{\pi}}\right)$ $=\sqrt[3]{\frac{k}{\pi}}-\frac{2}{3}\left(\sqrt[3]{\frac{k}{\pi}}\right)=\frac{1}{3}\left(\sqrt[3]{\frac{k}{\pi}}\right)$ The cheapest can is manufactured when $r=\sqrt[3]{\frac{k}{\pi}}$ and $h=\frac{1}{3}\left(\sqrt[3]{\frac{k}{\pi}}\right)$.

9 (ii)	Volume of hemisphere $=\frac{2}{3}\pi r^3=\frac{128}{3}\pi$ After 80 seconds, volume of water leaked $=160>\frac{128}{3}\pi$ Hence we only consider the rate of change of the height of water in the **cylinder** after 80 seconds Let H be the height in cm of the water in the can. $V=\pi r^2 H$ $\quad=16\pi H$ $\frac{\mathrm{d}V}{\mathrm{d}t}=16\pi\frac{\mathrm{d}H}{\mathrm{d}t}$ $\frac{\mathrm{d}H}{\mathrm{d}t}=\frac{1}{16\pi}\frac{\mathrm{d}V}{\mathrm{d}t}$ $\quad=\frac{-2}{16\pi}$ $\quad=-\frac{1}{8\pi}$ The height of the liquid is decreasing at $\frac{1}{8\pi}$ cm s^{-1} , 80 seconds after the start of the experiment.

10 (i)	$\text{RHS}=\dfrac{1}{\sqrt{1-p^2x^2}}-\sqrt{1-p^2x^2}$ $=\dfrac{1-\left(1-p^2x^2\right)}{\sqrt{1-p^2x^2}}=\dfrac{p^2x^2}{\sqrt{1-p^2x^2}}=\text{LHS (shown)}$
	<u>Method 1</u> $\int_n^{\frac{1}{p}}\sqrt{1-p^2x^2}\ \mathrm{d}x=\int_n^{\frac{1}{p}}\dfrac{1}{\sqrt{1-p^2x^2}}-\dfrac{p^2x^2}{\sqrt{1-p^2x^2}}\ \mathrm{d}x$ $=\int_n^{\frac{1}{p}}\dfrac{1}{\sqrt{1-p^2x^2}}\ \mathrm{d}x+\dfrac{1}{2}\int_n^{\frac{1}{p}}x\cdot\dfrac{-2p^2x}{\sqrt{1-p^2x^2}}\ \mathrm{d}x$ $=\dfrac{1}{p}\int_n^{\frac{1}{p}}\dfrac{p}{\sqrt{1-(px)^2}}\ \mathrm{d}x$ $+\left[\dfrac{1}{2}x\cdot\dfrac{\sqrt{1-p^2x^2}}{\frac{1}{2}}\right]_n^{\frac{1}{p}}-\dfrac{1}{2}\int_n^{\frac{1}{p}}\dfrac{\sqrt{1-p^2x^2}}{\frac{1}{2}}\ \mathrm{d}x$ $=\dfrac{1}{p}\left[\sin^{-1}(px)\right]_n^{\frac{1}{p}}+\left[0-n\sqrt{1-p^2n^2}\right]-\int_n^{\frac{1}{p}}\sqrt{1-p^2x^2}\ \mathrm{d}x$ $=\dfrac{1}{p}\left[\sin^{-1}(1)-\sin^{-1}(pn)\right]-n\sqrt{1-p^2n^2}-\int_n^{\frac{1}{p}}\sqrt{1-p^2x^2}\ \mathrm{d}x$ $=\dfrac{1}{p}\left[\dfrac{\pi}{2}-\sin^{-1}(pn)\right]-n\sqrt{1-p^2n^2}-\int_n^{\frac{1}{p}}\sqrt{1-p^2x^2}\ \mathrm{d}x$ $=\dfrac{\pi}{2p}-\dfrac{1}{p}\sin^{-1}\left(pn\right)-n\sqrt{1-p^2n^2}-\int_n^{\frac{1}{p}}\sqrt{1-p^2x^2}\ \mathrm{d}x$ Hence, $2\int_n^{\frac{1}{p}}\sqrt{1-p^2x^2}\ \mathrm{d}x=\dfrac{\pi}{2p}-\dfrac{1}{p}\sin^{-1}\left(pn\right)-n\sqrt{1-p^2n^2}$ $\int_n^{\frac{1}{p}}\sqrt{1-p^2x^2}\ \mathrm{d}x=\dfrac{\pi}{4p}-\dfrac{1}{2p}\sin^{-1}\left(pn\right)-\dfrac{1}{2}n\sqrt{1-p^2n^2}$

10 (i)	**Method 2**

$$\int_n^{\frac{1}{p}} \sqrt{1-p^2x^2}\ \mathrm{d}x = \int_n^{\frac{1}{p}} 1\cdot\sqrt{1-p^2x^2}\ \mathrm{d}x$$

$$=\left[x\sqrt{1-p^2x^2}\right]_n^{\frac{1}{p}} - \int_n^{\frac{1}{p}} x\cdot\frac{1}{2\sqrt{1-p^2x^2}}\cdot\left(-2p^2x\right)\mathrm{d}x$$

$$=\left(0-n\sqrt{1-p^2n^2}\right)+\int_n^{\frac{1}{p}}\frac{p^2x^2}{\sqrt{1-p^2x^2}}\ \mathrm{d}x$$

$$=-n\sqrt{1-p^2n^2}+\int_n^{\frac{1}{p}}\frac{1}{\sqrt{1-p^2x^2}}-\sqrt{1-p^2x^2}\ \mathrm{d}x$$

$$=-n\sqrt{1-p^2n^2}+\int_n^{\frac{1}{p}}\frac{1}{\sqrt{1-p^2x^2}}\ \mathrm{d}x-\int_n^{\frac{1}{p}}\sqrt{1-p^2x^2}\ \mathrm{d}x$$

Hence,

$$2\int_n^{\frac{1}{p}}\sqrt{1-p^2x^2}\ \mathrm{d}x=-n\sqrt{1-p^2n^2}+\int_n^{\frac{1}{p}}\frac{1}{\sqrt{1-p^2x^2}}\ \mathrm{d}x$$

$$\Rightarrow\int_n^{\frac{1}{p}}\sqrt{1-p^2x^2}\ \mathrm{d}x$$

$$=\frac{1}{2}\left[-n\sqrt{1-p^2n^2}+\frac{1}{p}\int_n^{\frac{1}{p}}\frac{p}{\sqrt{1-(px)^2}}\ \mathrm{d}x\right]$$

$$=-\frac{1}{2}n\sqrt{1-p^2n^2}+\frac{1}{2p}\left[\sin^{-1}(px)\right]_n^{\frac{1}{p}}$$

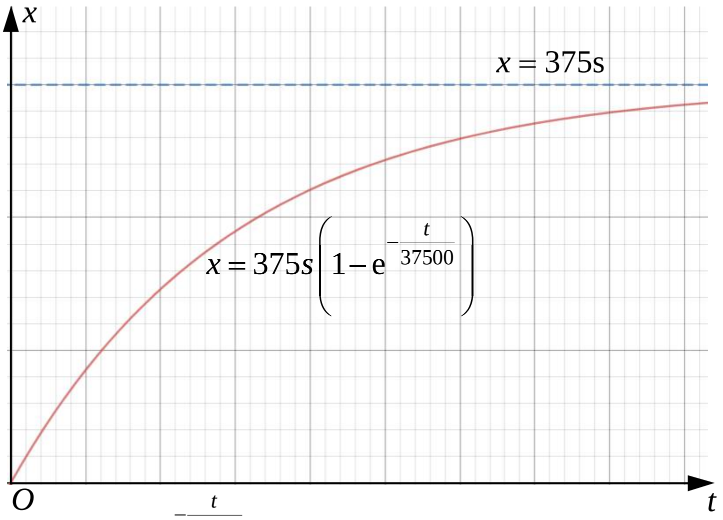
$$=\frac{1}{2p}\left[\sin^{-1}1-\sin^{-1}(pn)\right]-\frac{1}{2}n\sqrt{1-p^2n^2}$$

$$=\frac{1}{2p}\left[\frac{\pi}{2}-\sin^{-1}(pn)\right]-\frac{1}{2}n\sqrt{1-p^2n^2}$$

$$=\frac{\pi}{4p}-\frac{1}{2p}\sin^{-1}(pn)-\frac{1}{2}n\sqrt{1-p^2n^2}\ \text{(shown)}$$

10 (ii)	$C_1 : \frac{x^2}{3} + y^2 = 1 \Rightarrow y^2 = 1 - \frac{1}{3}x^2$ $C_2 : x^2 + \frac{y^2}{3} = 1 \Rightarrow y^2 = 3 - 3x^2$ At the point of intersection, $1 - \frac{1}{3}x^2 = 3 - 3x^2 \Rightarrow \left(3 - \frac{1}{3}\right)x^2 = 3 - 1$ $\Rightarrow \frac{8}{3}x^2 = 2$ $\Rightarrow x^2 = \frac{3}{4} \Rightarrow x = \frac{\sqrt{3}}{2}$ $(\because x > 0)$ Therefore, area of shaded region is given by $\int_{\frac{\sqrt{3}}{2}}^{\sqrt{3}} \sqrt{1 - \frac{1}{3}x^2}\ \mathrm{d}x - \int_{\frac{\sqrt{3}}{2}}^{1} \sqrt{3 - 3x^2}\ \mathrm{d}x$ $= \int_{\frac{\sqrt{3}}{2}}^{\sqrt{3}} \sqrt{1 - \left(\frac{1}{\sqrt{3}}\right)^2 x^2}\ \mathrm{d}x - \sqrt{3}\int_{\frac{\sqrt{3}}{2}}^{1} \sqrt{1 - x^2}\ \mathrm{d}x$ $= \frac{\pi}{4\left(\frac{1}{\sqrt{3}}\right)} - \frac{1}{2\left(\frac{1}{\sqrt{3}}\right)}\sin^{-1}\left(\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2}\right) - \frac{1}{2} \cdot \frac{\sqrt{3}}{2}\sqrt{1 - \left(\frac{1}{\sqrt{3}}\right)^2\left(\frac{\sqrt{3}}{2}\right)^2}$ $-\sqrt{3}\left[\frac{\pi}{4 \cdot 1} - \frac{1}{2 \cdot 1}\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{2} \cdot \frac{\sqrt{3}}{2}\sqrt{1 - \left(\frac{\sqrt{3}}{2}\right)^2}\right]$ $= \frac{\pi\sqrt{3}}{4} - \frac{\sqrt{3}}{2}\sin^{-1}\left(\frac{1}{2}\right) - \frac{\sqrt{3}}{4}\sqrt{\frac{3}{4}}$ $-\sqrt{3}\left[\frac{\pi}{4} - \frac{1}{2}\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \frac{\sqrt{3}}{4}\sqrt{\frac{1}{4}}\right]$ $= -\frac{\sqrt{3}}{2} \cdot \frac{\pi}{6} - \frac{3}{8} + \frac{\sqrt{3}}{2} \cdot \frac{\pi}{3} + \frac{3}{8} = \frac{\pi\sqrt{3}}{12}$

11 (i)	Mass of free chlorine entering per minute $=\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)_{\text{in}}=10s\text{ mg}=\frac{10s}{1000}=\frac{s}{100}\text{ g}$ Mass of free chlorine leaving per minute $=\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)_{\text{out}}=\frac{x}{375000}(10)\text{ g}=\frac{x}{37500}\text{ g}$ $\frac{\mathrm{d}x}{\mathrm{d}t}=\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)_{\text{in}}-\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)_{\text{out}}$ $=\frac{10s}{1000}-\frac{10x}{375000}$ $\Rightarrow\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{375s-x}{37500}$ (shown)
	Assume that the mixture/concentration of free chlorine and water is uniform/homogeneous/well-mixed in the pool.
11 (ii)	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{375s-x}{37500}$ $\int\frac{1}{375s-x}\,\mathrm{d}x=\int\frac{1}{37500}\,\mathrm{d}t$ $\Rightarrow-\ln\lvert 375s-x\rvert=\frac{t}{37500}+C$ $\Rightarrow-\ln(375s-x)=\frac{t}{37500}+C$ (since $375s-x>0$) $\Rightarrow\ln(375s-x)=-\frac{t}{37500}-C$ $\Rightarrow 375s-x=\mathrm{e}^{-\frac{t}{37500}}\cdot\mathrm{e}^{-C}$ $\Rightarrow x=375s-A\mathrm{e}^{-\frac{t}{37500}}$, where $A=\mathrm{e}^{-C}$ When $t=0$, $x=0$. $0=375s-A\Rightarrow A=375s$ $\therefore x=375s\left(1-\mathrm{e}^{-\frac{t}{37500}}\right)$
11 (iii)	In 60 minutes, $x=375s\left(1-\mathrm{e}^{-\frac{60}{37500}}\right)=0.599520s$ Amount of free chlorine in the pool after one hour is $0.600s$ or $375s\left(1-\mathrm{e}^{-\frac{1}{625}}\right)$ grams (or $600s$ milligrams).

<table>
<tr><td>11 (iv)</td><td>

As $t \to \infty$, $e^{-\frac{t}{37500}} \to 0$, $x \to 375s$.</td></tr>
<tr><td>11 (last part)</td><td>$375 < 375s < 1125$
$\Rightarrow 1 < s < 3$</td></tr>
</table>

NATIONAL JUNIOR COLLEGE
SENIOR HIGH 2 PRELIMINARY EXAMINATION
Higher 2

MATHEMATICS 9758/02

Paper 2 **11 September 2018**

3 hours

Additional Materials: Answer Paper
List of Formulae (MF26)
Cover Sheet

READ THESE INSTRUCTIONS FIRST

Write your name, registration number, subject tutorial group, on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in the brackets [ ] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

1 **(a)** In a city, a traveller may use a mobile phone application to choose a variety of modes of transport to travel from the international airport *A*, to a specific hotel *Z*: taking a High Speed Rail, taking a Subway train or walking.

Along the way, he will need to make transfer via various interchange stations or subway stations *B*, *C*, *D*, *E*, *F* or *G* and walk within stations to travel in another mode of transport.

When the traveller uses the travelling application, he is given 3 choices to travel from the international airport to the hotel:

<table>
<tr><td></td><td colspan="4">International Airport ⟶ Hotel</td></tr>
<tr><td>Choice I</td><td rowspan="2">Take the High Speed Rail at A for 60km to Interchange Station B</td><td>Walk 0.3 km to Subway Station within B</td><td>Take the Subway train within B to C that travels 10 km.</td><td>Walks 0.5 km to Z.</td></tr>
<tr><td>Choice II</td><td>Within B, cross over to another platform at a negligible distance away to take the Subway train and travel 5 km to D.</td><td>Walks 1.2 km to Z.</td><td></td></tr>
<tr><td>Choice III</td><td>Takes the Subway train at A for 20 km to Interchange Station E.</td><td>Within E, walks 0.2 km to F.</td><td>Take the Subway train at F to G for 40 km</td><td>Walks 0.2 km to Z.</td></tr>
</table>

Given that the travelling time taken for Choices I, II and III are 51.7 minutes, 56.3 minutes and 69.6 minutes and assuming that the waiting time for the high speed rail or subway is negligible, find the average speed of the high speed rail, subway train and the walking pace of the traveller. Leave your answers in km/h. [4]

(b) **(i)** Solve the inequality $\dfrac{2x-1}{2x^2-1} \le 1$ exactly. [3]

(ii) Hence, solve the inequality $\dfrac{2(\cos x)-1}{\cos 2x} \le 1$ exactly for $0 \le x \le \pi$. [3]

[Turn over

2 The planes p_1 and p_2, have equations $2x+3y+6z=0$ and $\mathbf{r}\cdot\begin{pmatrix}1\\2\\2\end{pmatrix}=6$ respectively.

(i) Find a vector equation of the line of intersection, *l*, between p_1 and p_2. [2]

The line *m* passes through the points $A(2,1,1)$ and $B(5,4,2)$.

(ii) Verify that *A* lies on p_2. [1]

(iii) Find the coordinates of the points on *m* that are equidistant from planes p_1 and p_2. [5]

3 **(a)** The sum, S_n, of the first *n* terms of a sequence is given by

$$S_n = \pi\left(n-\frac{1}{2}\right)^2 - \frac{\pi}{4} + n\pi^2.$$

Show that the sequence follows an arithmetic progression and state the value of the common difference. [5]

(b) **(i)** A kangaroo spotted her runaway joey when she was 20 metres away from her and jumped straight towards her joey. The kangaroo's 1st jump was 2 metres and each successive jump was 1.1 times that of her previous jump. At the same time, the joey jumped away from her mother along the same imaginary straight line, such that its first jump was 1 metre and each successive jump was 0.1 metres more than its previous jump.

Assuming that the time taken for the kangaroo and her joey for each jump is the same, set up an inequality for the least number of jumps, *k*, the kangaroo had to take in order to catch up with her joey. Hence, find the value of *k*. [4]

(ii) Find the distance between the kangaroo and her joey just after the *k*th jump. [1]

4 **(a)** It is given that $z^* = \dfrac{(-1-\mathrm{i})^3}{1-\mathrm{i}\sqrt{3}}$, where z^* is the conjugate of a complex number z.

(i) Find the exact values of the modulus and argument of $\dfrac{1}{z}$. [5]

(ii) Hence determine the exact values of a and b (where $-\pi < b \leq \pi$) in the equation

$$\mathrm{e}^{2a+\mathrm{i}b} = \frac{1}{z^4}.$$ [3]

(b) The complex variables u and v satisfy the equations

$$\mathrm{i}u - v = 3 \quad \text{and} \quad u^* + (1-\mathrm{i})v = 7 + 4\mathrm{i}.$$

Find the values of u and v, giving your answers in the form $x + \mathrm{i}y$. [4]

Section B: Probability and Statistics [60 marks]

5 In this question you should state clearly the values of the parameters of any normal distribution you use.

The masses in grams of dragon fruits and mangoes are independent random variables with the distributions $\mathrm{N}(350, 196)$ and $\mathrm{N}(250, 98)$ respectively. Dragon fruits and mangoes are priced at \$2.40 per kilogram and \$12 per kilogram respectively.

Find the probability that the total cost of 5 randomly chosen dragon fruits and 3 randomly chosen mangoes is less than \$13.50. [4]

6 A school's concert band comprises 24 woodwind players, n brass players and 10 percussion players. $\frac{1}{3}$ of all woodwind players, $\frac{2}{5}$ of all brass players and $\frac{4}{5}$ of all percussion players are Senior High students, while the rest are Junior High students. No student in the band plays more than one type of instrument.

One student from the concert band is selected at random. Find, in terms of n, the probability that the student is neither a percussion player nor a Senior High student. [3]

Suppose instead that two students from the concert band are randomly selected. It is given that the probability that one of them is a Senior High student and the other is a Junior High student is $\frac{1}{2}$. Show that

$$n^2 + pn + q = 0$$

for some integer constants p and q to be determined, and hence find the value of n. [3]

7 Ten circular stickers for temperature taking purposes, each of them indistinguishable apart from their colours, are placed in an opaque box.

(a) It is given that four of the stickers are purple, two are blue and the remaining stickers are pink, orange, yellow and green. Suppose four stickers are given to 4 people, such that each person receives exactly one sticker. Find the number of ways this can be done if

(i) all four stickers are of different colours, [1]

(ii) there are no restrictions on the colours of the stickers. [3]

(b) The 10 stickers labelled with distinct alphabets "A" to "J" are to be packed into zip-lock bags, which may come in different sizes. Bags of the same size are considered to be indistinguishable.

(i) Suppose five zip-lock bags of different sizes are used to contain 2 stickers each. How many ways can this be done? [1]

(ii) Suppose instead that a large-sized bag is used to contain 4 stickers, two medium-sized bags are used to contain 2 stickers each and a small-sized bag is used to contain the remaining 2 stickers. How many ways can this be done? [2]

8 Mary and Jerry play a game using two six-sided dice. One of the dice is fair with '1' to '6' on each face. The other die is weighted such that the score, denoted by Y, has a probability distribution given as follows.

$$\mathrm{P}(Y = y) = \begin{cases} \dfrac{1}{6} & \text{for } y = 1, \\ \dfrac{1}{6}(y-1) & \text{for } y = 2, 3, \\ \dfrac{1}{36}(y-1) & \text{for } y = 4, 5, 6. \end{cases}$$

Mary throws the dice. Jerry pays Mary \$3 if the total score from the two dice is at least "9" and Mary pays Jerry \$4 if the total score from the two dice is at most "4". Otherwise, there is no monetary transaction between both parties. Let \$$X$ be the amount of money Mary gains after one game.

Show that $\mathrm{P}(X = 3) = \dfrac{25}{108}$. Tabulate the probability distribution of X. [4]

The game is played n times, where n is large. Find the least number of games that Mary needs to play so that there is a probability of more than 0.99 for her average winnings in the n games to differ from her expected winnings by at most \$1. State clearly the parameters used in your calculations. [5]

9 Factory Nayma produces balls used by the World Ball Association with diameters that follow a normal distribution. A random sample of 30 balls is chosen and their diameters in centimetres, x, are summarised by

$$\sum(x-22)=-27 \text{ and } \sum\left(x-22\right)^2=321.26.$$

(i) Calculate unbiased estimates of the population mean and population variance. [2]

(ii) Test, at the 10% significance level, whether the population mean diameter of a ball produced by Factory Nayma is 22 cm. [5]

(iii) Another random sample of 30 balls is selected, with mean diameter $\bar{x}$. Use an algebraic method to calculate the set of values of $\bar{x}$ for which there is insufficient evidence to conclude that the population mean diameter of all balls is greater than 22 cm at the 10% level of significance. (Answers obtained by trial and improvement from a calculator will obtain no marks.) [3]

10 5.2% of all insurance agents from a large insurance company, Prodential, have an advanced diploma in insurance (ADI). A random sample of 30 agents from Prodential is obtained.

(i) State, in context, two assumptions for the number of insurance agents with ADI in this sample to be well-modelled by a binomial distribution. [2]

Assume now that these assumptions do in fact hold.

(ii) Find the probability that at least three of the insurance agents in this sample have an ADI each. [2]

$100p$% of all insurance agents from another large insurance company, Avila, have an advanced diploma in insurance (ADI), where $p < 0.5$. A sample of 10 agents from Avila is obtained. It is given that the number of insurance agents with ADI in this sample can be modelled by a binomial distribution.

(iii) It is given that the probability that 5 of the agents in this sample have an ADI each is 0.12294, correct to 5 decimal places. Show that p satisfies an equation of the form $p(1-p)=k$ for some real constant k to be determined, and hence find the value of p correct to 2 decimal places. [3]

(iv) Suppose instead that p = 0.24 and forty samples of 10 Avila insurance agents each are obtained. Find the probability that the average number of insurance agents with ADI of the forty samples is between 2.3 and 2.5. [3]

(v) Explain, stating a reason, how increasing the number of samples of 10 Avila insurance agents each will affect your answer in part **(iv)**. [2]

11 **(i)** Sketch a scatter diagram, that might be expected when x and y are related approximately, for each of the cases (A) and (B) below. In each case your diagram should include 6 points, approximately equally spaced with respect to x, and with all x- and y- values positive.

(A) $y = a + bx^2$, where $a, b \in \mathbb{R}^+$

(B) $y = c + \dfrac{d}{x}$, where $c \in \mathbb{R}^+$ and $d \in \mathbb{R}^-$. [2]

Obesity is becoming increasingly prevalent across the globe. To investigate the effects of obesity on one's health, a study was conducted to determine if the blood pressure of adults aged between 40 and 50 years old is dependent on their Body Mass Index (BMI). Data from six patients in this age-group from a hospital are collected. Their BMI, m kg/m^2, and systolic blood pressure, s, in mmHg, are as follows.

m	22	27	31	36	40	44
s	120	150	168	172	179	183

(ii) Sketch the scatter diagram for these values, labelling the axes clearly. [2]

(iii) With reference to your scatter diagram in part **(ii)**, explain why model (B) in part **(i)** is more appropriate than model (A) for modelling these values and calculate the product moment correlation coefficient for model (B). [2]

(iv) Find the equation of the least-squares regression line of s on $\dfrac{1}{m}$ and use it to estimate the BMI of another patient (of a similar age profile) whose systolic blood pressure is 110 mmHg. Comment on the reliability of your estimate. [3]

(v) Explain why the regression line of $\dfrac{1}{m}$ on s should not be used for your calculations in part **(iv)**. [1]

(vi) State, in context, a limitation of using the regression equation in part **(iv)** to estimate the systolic blood pressure of *other* people with BMI within the range $22 < m < 44$. [1]

(vii) Suppose a new data pair $(\bar{m}, \bar{s})$ is added to the table above, where $\bar{m}$ and $\bar{s}$ are the mean BMI (in kg/m^2) and the mean systolic blood pressure (in mmHg) of the adults in the study respectively, based on the data above. Without any calculations, explain whether the equation of the regression line you have obtained in part **(iv)** would change. [1]

– END OF PAPER –

2018 NJC H2 Math Prelim Paper 2 Solution

Section A : Pure Mathematics [40 marks]

1 (a)	Let a, b and c be the average speed of the high speed rail, subway train and walking pace of a person. For Choice 1, $\frac{60}{a}+\frac{0.3}{c}+\frac{10}{b}+\frac{0.5}{c}=\frac{51.7}{60} \quad \Rightarrow \quad \frac{60}{a}+\frac{10}{b}+\frac{0.8}{c}=\frac{51.7}{60}$ …(1) For Choice 2, $\frac{60}{a}+\frac{5}{b}+\frac{1.2}{c}=\frac{56.3}{60}$ …(2) For Choice 3, $\frac{20}{b}+\frac{0.2}{c}+\frac{40}{b}+\frac{0.2}{c}=\frac{69.6}{60} \quad \Rightarrow \quad \frac{60}{b}+\frac{0.4}{c}=\frac{69.6}{60}$ …(3) Solving (1), (2) and (3) simultaneously, $\frac{1}{a}=\frac{1}{160}$, $\frac{1}{b}=\frac{1}{60}$ and $\frac{1}{c}=\frac{2}{5}$. Thus, the average speed, in km/h, of the high speed rail, subway train and person are 160, 60 and 2.5 respectively.

1 (b) (i)

$$\frac{2x-1}{2x^2-1}\le 1 \quad , \quad x\ne \pm\frac{1}{\sqrt{2}}$$

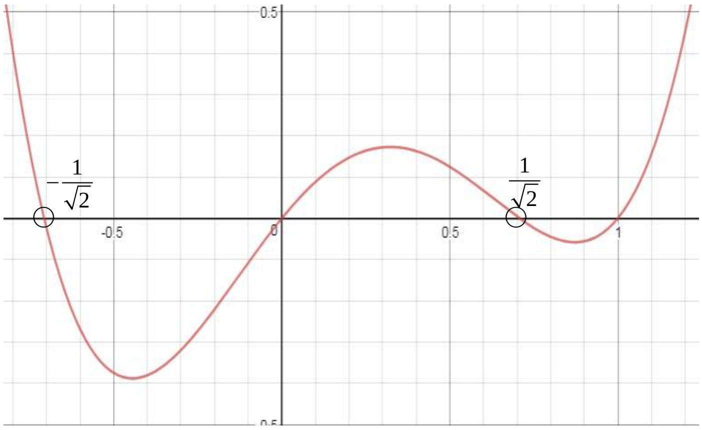
$$\frac{2x-1-2x^2+1}{2x^2-1}\le 0$$

$$2\left(x-x^2\right)\left(2x^2-1\right)\le 0$$

$$x\left(1-x\right)\left(2x^2-1\right)\le 0$$

$$x\left(1-x\right)\left(\sqrt{2}x-1\right)\left(\sqrt{2}x+1\right)\le 0$$

$$\left(\sqrt{2}x+1\right)\left(x\right)\left(x-1\right)\left(\sqrt{2}x-1\right)\ge 0$$



$x<-\frac{1}{\sqrt{2}}$ or $0\le x<\frac{1}{\sqrt{2}}$ or $x\ge 1$. (since $x\ne \pm\frac{1}{\sqrt{2}}$)

1 (b) (ii)

$\frac{2\left(\cos x\right)-1}{\cos 2x}\le 1$ for $0\le x\le \pi$

$$\frac{2\left(\cos x\right)-1}{2\left(\cos x\right)^2-1}\le 1$$

$\cos x<-\frac{1}{\sqrt{2}}$ or $0\le \cos x<\frac{1}{\sqrt{2}}$ or $\cos x\ge 1$

For $\cos x\ge 1$, this is only possible when $x=0$.

For $\cos x<-\frac{1}{\sqrt{2}}$, we have $-1\le \cos x<-\frac{1}{\sqrt{2}}$,

thus $\frac{3\pi}{4}<x\le \pi$.

For $0\le \cos x<\frac{1}{\sqrt{2}}$, we have $\frac{\pi}{4}<x\le\frac{\pi}{2}$.

Hence, the solution is $x=0$ or $\frac{\pi}{4}<x\le\frac{\pi}{2}$ or $\frac{3\pi}{4}<x\le\pi$.

2 (i)	$p_2: \ x+2y+2z=6$ Using GC, $$\mathbf{r}=\begin{pmatrix}-18\\12\\0\end{pmatrix}+\lambda\begin{pmatrix}-6\\2\\1\end{pmatrix}, \lambda\in\mathbb{R}$$
2 (ii)	Since $2+2(1)+2(1)=6$, or $\begin{pmatrix}2\\1\\1\end{pmatrix}\bullet\begin{pmatrix}1\\2\\2\end{pmatrix}=2+2+2=6$, the point A lies on p_2.
2 (iii)	Let the point that is equidistant from both planes be C. $\begin{pmatrix}5\\4\\2\end{pmatrix}-\begin{pmatrix}2\\1\\1\end{pmatrix}=\begin{pmatrix}3\\3\\1\end{pmatrix}$ $\overrightarrow{OC}=\begin{pmatrix}2\\1\\1\end{pmatrix}+t\begin{pmatrix}3\\3\\1\end{pmatrix}$ for some $t\in\mathbb{R}$ Distance of C from p_1 = Distance of C from p_2 $\dfrac{\left\lvert\left[\begin{pmatrix}2+3t\\1+3t\\1+t\end{pmatrix}-\begin{pmatrix}2\\1\\1\end{pmatrix}\right]\cdot\begin{pmatrix}1\\2\\2\end{pmatrix}\right\rvert}{\sqrt{1^2+2^2+2^2}}=\dfrac{\left\lvert\left[\begin{pmatrix}2+3t\\1+3t\\1+t\end{pmatrix}-\begin{pmatrix}0\\0\\0\end{pmatrix}\right]\cdot\begin{pmatrix}2\\3\\6\end{pmatrix}\right\rvert}{\sqrt{2^2+3^2+6^2}}$ $\dfrac{\lvert 3t+6t+2t\rvert}{3}=\dfrac{\lvert 4+6t+3+9t+6+6t\rvert}{7}$ $\dfrac{11\lvert t\rvert}{3}=\dfrac{\lvert 13+21t\rvert}{7}$ $77\lvert t\rvert=\lvert 39+63t\rvert$ $77t=-39-63t$ or $77t=39+63t$ $140t=-39$ or $14t=39$ $t=-\dfrac{39}{140}$ or $t=\dfrac{39}{14}$ $\overrightarrow{OC}=\begin{pmatrix}2\\1\\1\end{pmatrix}+\left(-\dfrac{39}{140}\right)\begin{pmatrix}3\\3\\1\end{pmatrix}=\dfrac{1}{140}\begin{pmatrix}163\\23\\101\end{pmatrix}$ or $\overrightarrow{OC}=\begin{pmatrix}2\\1\\1\end{pmatrix}+\left(\dfrac{39}{14}\right)\begin{pmatrix}3\\3\\1\end{pmatrix}=\dfrac{1}{14}\begin{pmatrix}145\\131\\53\end{pmatrix}$ The two points are $\left(\dfrac{163}{140},\dfrac{23}{140},\dfrac{101}{140}\right)$ and $\left(\dfrac{145}{14},\dfrac{131}{14},\dfrac{53}{14}\right)$

3 (a)	$u_n = S_n - S_{n-1}, \qquad n \geq 2$ $= \pi\left(n-\frac{1}{2}\right)^2 - \frac{\pi}{4} + n\pi^2 - \pi\left((n-1)-\frac{1}{2}\right)^2 + \frac{\pi}{4} - (n-1)\pi^2$ $= \pi\left(n^2 - n + \frac{1}{4}\right) - \frac{\pi}{4} + n\pi^2 - \pi\left(n^2 - 3n + \frac{9}{4}\right) + \frac{\pi}{4} - n\pi^2 + \pi^2$ $= -n\pi + \frac{1}{4}\pi + 3n\pi - \frac{9}{4}\pi + \pi^2$ $= 2n\pi - 2\pi + \pi^2, \quad n \geq 2$ $\because u_1 = S_1 = \pi\left(1-\frac{1}{2}\right)^2 - \frac{\pi}{4} + \pi^2 = \pi^2 = 2(1)\pi - 2\pi + \pi^2$ $\therefore u_n = 2n\pi - 2\pi + \pi^2, \quad n \geq 1$ Method 1: Comparing *n*th term of AP $\because u_n = 2n\pi - 2\pi + \pi^2, \quad n \geq 1$ $= \pi^2 + 2\pi(n-1), \quad n \geq 1$ Method 2: Calculating common difference $u_n - u_{n-1} = 2n\pi - 2\pi + \pi^2 - \left(2(n-1)\pi - 2\pi + \pi^2\right)$ $= 2\pi$ Sequence is follows a arithmetic progression with common difference 2π .
3 (b) (i)	$\frac{2\left((1.1)^k - 1\right)}{1.1-1} \geq \frac{k}{2}\left(2(1) + (k-1)(0.1)\right) + 20$ $20\left((1.1)^k - 1\right) \geq \frac{k}{2}(1.9 + 0.1k) + 20$ $20(1.1)^k - 20 \geq 0.05k^2 + 0.95k + 20$ $20(1.1)^k - 0.05k^2 - 0.95k \geq 40$ By G.C., $k = 11$.
3 (b) (ii)	Required distance $= \frac{2\left((1.1)^{11} - 1\right)}{1.1-1} - \frac{11}{2}\left(2(1) + (11-1)(0.1)\right) - 20$ $= 0.56233$ $= 0.562\text{m}$ (3 s.f.)

<table>
<tr><td>4 (a) (i)</td><td>

$$|z^*| = \left|\frac{(-1-\mathrm{i})^3}{1-\mathrm{i}\sqrt{3}}\right| = \frac{|-1-\mathrm{i}|^3}{|1-\mathrm{i}\sqrt{3}|} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

$$\begin{aligned}\arg\left(z^*\right) &= \arg\left(\frac{(-1-\mathrm{i})^3}{1-\sqrt{3}\mathrm{i}}\right) \\ &= 3\arg(-1-\mathrm{i}) - \arg\left(1-\sqrt{3}\mathrm{i}\right) \\ &= 3\left(-\frac{3}{4}\pi\right) - \left(-\frac{1}{3}\pi\right) \\ &= -\frac{23}{12}\pi \equiv \frac{1}{12}\pi\end{aligned}$$

$$\left|\frac{1}{z}\right| = \frac{1}{|z|} = \frac{1}{|z^*|} = \frac{1}{\sqrt{2}}$$

$$\arg\left(\frac{1}{z}\right) = -\arg(z) = \arg\left(z^*\right) = -\frac{23}{12}\pi \text{ or } \frac{1}{12}\pi$$

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$$-1-\mathrm{i} = \sqrt{2}\mathrm{e}^{-\mathrm{i}\frac{3\pi}{4}},\ 1-\mathrm{i}\sqrt{3} = 2\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{3}\right)}$$

$$z^* = \frac{\left(\sqrt{2}\mathrm{e}^{-\mathrm{i}\frac{3\pi}{4}}\right)^3}{2\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{3}\right)}} = \frac{2\sqrt{2}\mathrm{e}^{-\mathrm{i}\frac{9\pi}{4}}}{2\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{3}\right)}} = \sqrt{2}\mathrm{e}^{-\mathrm{i}\frac{9\pi}{4}-\mathrm{i}\left(-\frac{\pi}{3}\right)} = \sqrt{2}\mathrm{e}^{-\mathrm{i}\frac{23\pi}{12}}$$

$\frac{1}{z} = \frac{1}{\sqrt{2}\mathrm{e}^{\mathrm{i}\frac{23\pi}{12}}} = \frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\frac{23\pi}{12}}$. Therefore,

$$\left|\frac{1}{z}\right| = \frac{1}{\sqrt{2}},\ \arg\left(\frac{1}{z}\right) = -\frac{23}{12}\pi$$

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<tr><td>4 (a) (ii)</td><td>

$$\frac{1}{z^4} = \left(\frac{1}{z}\right)^4 = \left(\frac{1}{\sqrt{2}}\mathrm{e}^{-\frac{23}{12}\pi\mathrm{i}}\right)^4 = \frac{1}{4}\mathrm{e}^{-\frac{23}{3}\pi\mathrm{i}} = \frac{1}{4}\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}$$

$$\mathrm{e}^{2a+\mathrm{i}b} = \frac{1}{z^4} \Rightarrow \mathrm{e}^{2a}\cdot\mathrm{e}^{\mathrm{i}b} = \frac{1}{4}\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}$$

Therefore we have

$$\mathrm{e}^{2a} = \frac{1}{4} \Rightarrow 2a = \ln\frac{1}{4} \Rightarrow a = \ln\frac{1}{2} \text{ or } -\ln 2$$

$$\mathrm{e}^{\mathrm{i}b} = \mathrm{e}^{\frac{1}{3}\pi\mathrm{i}} \Rightarrow b = \frac{1}{3}\pi$$

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4 (b)	$\mathrm{i}u - v = 3 \Rightarrow v = \mathrm{i}u - 3$ Then substituting $w = \mathrm{i}z - 3$ into the other equation, $u^* + (1-\mathrm{i})(\mathrm{i}u - 3) = 7 + 4\mathrm{i}$ $u^* + \mathrm{i}u - 3 - \mathrm{i}^2 u + 3\mathrm{i} = 7 + 4\mathrm{i}$ $u^* + \mathrm{i}u + u = 10 + \mathrm{i}$ $2a + \mathrm{i}(a + \mathrm{i}b) = 10 + \mathrm{i}$ $2a - b + \mathrm{i}a = 10 + \mathrm{i}$ Comparing the real and imaginary parts, we get $a = 1$ and $2a - b = 10 \Rightarrow 2 - b = 10 \Rightarrow b = -8$. Therefore $u = 1 - 8\mathrm{i}$ and $v = \mathrm{i}(1 - 8\mathrm{i}) - 3 = 5 + \mathrm{i}$
	Alternatively, $\mathrm{i}u - v = 3 \quad (1)$ $u^* + (1-\mathrm{i})v = 7 + 4\mathrm{i}$ Let $u = a + b\mathrm{i}$ and $v = c + d\mathrm{i}$ where $a, b, c, d \in \mathbb{R}$. Equation (1) becomes $\mathrm{i}(a + b\mathrm{i}) - (c + d\mathrm{i}) = 3$ $(-b - c) + \mathrm{i}(a - d) = 3 + 0\mathrm{i}$ Comparing the real and imaginary parts, $-b - c = 3 \quad (3)$ $a - d = 0 \quad (4)$ Equation (2) becomes $(a - b\mathrm{i}) + (1-\mathrm{i})(c + d\mathrm{i}) = 7 + 4\mathrm{i}$ $(a + c + d) + \mathrm{i}(-b + d - c) = 7 + 4\mathrm{i}$ Comparing the real and imaginary parts, $a + c + d = 7 \quad (5)$ $-b - c + d = 4 \quad (6)$ Using GC to solve (3), (4), (5) and (6) simultaneously, $a = 1,\ b = -8,\ c = 5$ and $d = 1$ Therefore, $u = 1 - 8\mathrm{i}$ and $v = 5 + \mathrm{i}$.

Section B: Probability and Statistics [60 marks]

5	Let X and Y denote the mass in grams of a randomly chosen dragonfruit and mango respectively, and C be the total cost of 5 randomly chosen dragon fruits and 3 randomly chosen mangoes. Then $$C = \frac{2.4}{1000}\sum_{i=1}^{5} X_i + \frac{12}{1000}\sum_{j=1}^{3} Y_j$$ Therefore, $\mathrm{E}(C) = \frac{2.4}{1000}\times 5\times 350 + \frac{12}{1000}\times 3\times 250 = 13.2$ and $\mathrm{Var}(C) = \left(\frac{2.4}{1000}\right)^2 \times 5\times 196 + \left(\frac{12}{1000}\right)^2 \times 3\times 98 = 0.0479808$ $C \sim \mathrm{N}(13.2, 0.04798)$ $\mathrm{P}\left(C < 13.5\right) = 0.915$ (to 3 s.f.)

6

	Senior High	Junior High	**Total**
Woodwind	8	16	**24**
Brass	$\frac{2}{5}n$	$\frac{3}{5}n$	$\boldsymbol{n}$
Percussion	8	2	**10**
Total	$\frac{2}{5}n+16$	$\frac{3}{5}n+18$	$\boldsymbol{n+34}$

6 (1st part)

P(neither SH student nor percussion player)

$= \text{P(either JH woodwind player or JH brass player)}$

$$= \frac{\text{No. of JH woodwind players} + \text{No. of JH brass players}}{\text{Total no. of concert band students}}$$

$$= \frac{16+\frac{3}{5}n}{24+n+10}$$

$$= \frac{\frac{3}{5}n+16}{n+34}$$

$$= \frac{3n+80}{5n+170} \text{ or } \frac{3n+80}{5(n+34)} \text{ or } 1-\frac{2n+90}{5n+170}$$

Alternatively,

P(neither SH student nor percussion player)

$= \text{P(either JH woodwind player or JH brass player)}$

$$= \frac{24}{24+n+10}\times\frac{2}{3}+\frac{n}{24+n+10}\times\frac{3}{5}$$

$$= \frac{16}{n+34}+\frac{3n}{5(n+34)}$$

$$= \frac{3n+80}{5n+170}$$

6 (2nd part)	P(one is a SH playe and the other is a JH player) $= \frac{1}{2}$ $\dfrac{\frac{2}{5}n+16}{n+34} \times \dfrac{\frac{3}{5}n+18}{n+33} \times 2! = \dfrac{1}{2}$ $\dfrac{2n+80}{5n+170} \times \dfrac{3n+90}{5n+165} = \dfrac{1}{4}$ $4 \cdot 2(n+40) \cdot 3(n+30) = (5n+170)(5n+165)$ $24(n^2+70n+1200) = 25n^2+1675n+28050$ $n^2-5n-750=0$ $n=-25$ (rej) or 30
	Alternatively, $\dfrac{\binom{0.4n+16}{1}\binom{0.6n+18}{1}}{\binom{n+34}{2}} = \dfrac{1}{2}$ $\dfrac{(0.4n+16)(0.6n+18)}{\frac{(n+34)(n+33)}{2!}} = \dfrac{1}{2}$

7 (a) (i)	Number of ways if all 4 stickers are of different colours $=\binom{6}{4}4!=360$
7 (a) (ii)	Case 1 : All 4 different colours are used No. of ways $=360$ Case 2 : 2 are of the same colour, the remaining 2 are different colours No. of ways $=\binom{2}{1}\binom{5}{2}\frac{4!}{2!}=240$ Case 3 : 3 of them have the same colour and the last one has a different colour No. of ways $=\binom{1}{1}\binom{5}{1}\frac{4!}{3!}=20$ Case 4 : 2 are of the same colour, the other 2 have the same colour (only two purple and 2 blue) No. of ways $=\binom{2}{2}\frac{4!}{2!2!}=6$ Case 5 : All 4 stickers are of the same colour. No. of ways $=1$ Total number of ways $=360+240+20+6+1$ $=627$
7 (b) (i)	Number of ways $=\binom{10}{2}\binom{8}{2}\binom{6}{2}\binom{4}{2}\binom{2}{2}=113400$
7 (b) (ii)	No. of ways $=\frac{\binom{10}{2}\binom{8}{2}\binom{6}{2}\binom{4}{4}}{2!}=9450$

8 (1st part)

Presenting all the possible outcomes on the total score of the dice :

Fair Die $\left(\frac{1}{6}\right)$

Biased Die		**1**	**2**	**3**	**4**	**5**	**6**
	(1/6) **1**	2	3	4	5	6	7
	(1/6) **2**	3	4	5	6	7	8
	(2/6) **3**	4	5	6	7	8	9
	(3/36) **4**	5	6	7	8	9	10
	(4/36) **5**	6	7	8	9	10	11
	(5/36) **6**	7	8	9	10	11	12

$$\mathrm{P}(X=3)=\left(\frac{2}{6}\times\frac{1}{6}\times 1\right)+\left(\frac{3}{36}\times\frac{1}{6}\times 2\right)+\left(\frac{4}{36}\times\frac{1}{6}\times 3\right)+\left(\frac{5}{36}\times\frac{1}{6}\times 4\right)$$

$$=\frac{25}{108}$$

$$\mathrm{P}(X=-4)=\left(\frac{1}{6}\times\frac{1}{6}\times 5\right)+\left(\frac{2}{6}\times\frac{1}{6}\times 1\right)$$

$$=\frac{7}{36}$$

$$\mathrm{P}(X=0)=1-\frac{25}{108}-\frac{7}{36}=\frac{31}{54}$$

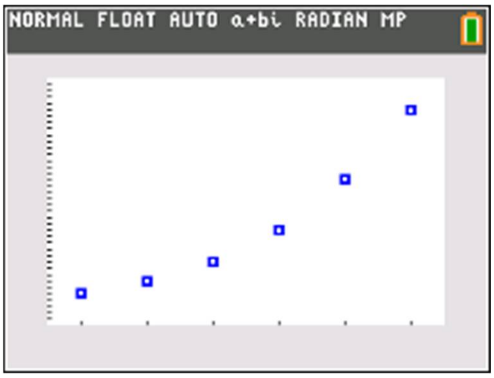
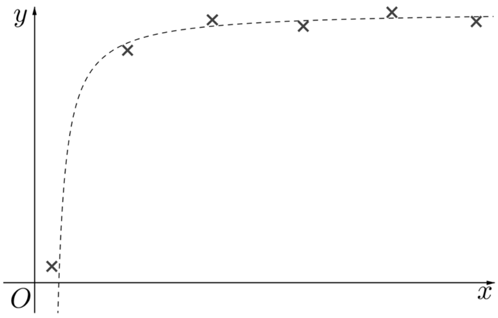
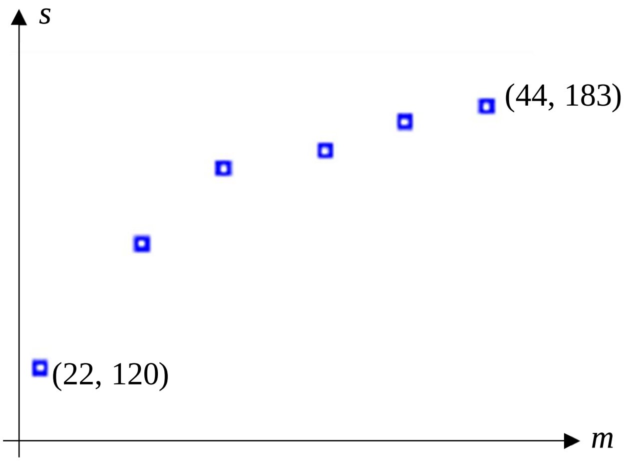
Distribution of X are as follows :

x	3	0	–4
$\mathrm{P}(X=x)$	$\frac{25}{108}$	$\frac{31}{54}$	$\frac{7}{36}$

8 (2nd part)	$\mathrm{E}(X)=3\left(\frac{25}{108}\right)+0\left(\frac{31}{54}\right)-4\left(\frac{7}{36}\right)=-\frac{1}{12}$ $\mathrm{E}(X^2)=3^2\left(\frac{25}{108}\right)+0^2\left(\frac{31}{54}\right)+(-4)^2\left(\frac{7}{36}\right)=\frac{187}{36}$ $\mathrm{Var}(X)=\frac{187}{36}-\left(\frac{1}{12}\right)^2=\frac{83}{16}$ Since ***n* is large, by Central Limit Theorem**, $\bar{X} \sim \mathrm{N}\left(-\frac{1}{12},\frac{83}{16n}\right)$ **approximately**. $\mathrm{P}\left(\left\|\bar{X}-\left(-\frac{1}{12}\right)\right\|\le 1\right)>0.99$ Standardising, $\mathrm{P}\left(\|Z\|\le\frac{1}{\sqrt{83/16n}}\right)>0.99$ $\mathrm{P}\left(\|Z\|\le\frac{4\sqrt{n}}{\sqrt{83}}\right)>0.99$ $\mathrm{P}\left(Z\le-\frac{4\sqrt{n}}{\sqrt{83}}\right)<0.005$ $-\frac{4\sqrt{n}}{\sqrt{83}}<-2.575829$ $n>34.41$ Smallest possible number of games required is 35.

9 (i)	$\bar{x} = \frac{-27}{30} + 22 = 21.1$ $s^2 = \frac{1}{29}\left(321.26 - \frac{(-27)^2}{30}\right)$ $= 10.24$
9 (ii)	Test $H_0 : \mu = 22$ against $H_1 : \mu \neq 22$ Level of significance = 10% (two-tailed) Under H_0, $\bar{X} \sim N\left(22, \frac{10.24}{30}\right)$ approximately. Hence $Z = \frac{\bar{X} - 22}{\sqrt{\frac{10.24}{30}}} \sim N(0,1)$ approximately. Critical region: $\lvert z \rvert > 1.64485$ (6 s.f.) Observed test statistic, $z = \frac{21.1 - 22}{\sqrt{\frac{10.24}{30}}} = -1.54046 \text{ (6 s.f.)} > -1.64485$ (hence do not reject H_0) OR p-value = 0.123446 (6 s.f.) > 0.1 (hence do not reject H_0) ***Step 5:*** We conclude that there is <u>insufficient evidence</u> at the <u>10% significance level</u> to claim that the <u>mean diameter of the balls is not 22 cm</u>.
9 (iii)	To not reject H_0, the observed test statistic, $z \leq 1.28155$ (6 s.f.) $\therefore \frac{\bar{x} - 22}{\sqrt{\frac{10.24}{30}}} \leq 1.28155$ $0 < \bar{x} \leq 22.7$ (3 s.f.)

10 (i)	The event that each agent has an ADI occurs independently for all agents in the sample. The probability that each agent has an ADI is constant.
10 (ii)	Let Y denote the number of insurance agents with ADI out of 30 randomly chosen Prodential insurance agents. Then $$Y \sim \text{B}(30,\ 0.052).$$ $\text{P}(Y \geq 3) = 1 - \text{P}(Y \leq 2)$ $= 0.2032366271$ $= 0.203$ (3sf)
10 (iii)	Let W denote the number of insurance agents with ADI out of 10 randomly chosen Avila insurance agents. Then $$W \sim \text{B}(10,\ p).$$ $\text{P}(W = 5) = 0.12294$ $\binom{10}{5} p^5 (1-p)^5 = 0.12294$ $p(1-p) = \left(\frac{0.12294}{252}\right)^{\frac{1}{5}} = 0.21760$, i.e., $k = 0.21760$ $p^2 - p + 0.21760 = 0$ $p = 0.68$ or 0.32 Since $p < 0.5$, $p = 0.32$
10 (iv)	$\text{E}(W) = 10 \times 0.24 = 2.4$ $\text{Var}(W) = 10 \times 0.24 \times (1 - 0.24) = 1.824$ Since sample size, 40, is large, by Central Limit Theorem, $$\overline{W} \sim \text{N}\left(2.4, \frac{1.824}{40}\right) \text{ approximately.}$$ Therefore, $\text{P}\left(2.3 < \overline{W} < 2.5\right) = 0.36042 = 0.360$ (to 3 s.f.).
10 (v)	With a larger sample size, the variance of $\overline{W}$ will *decrease*. This means that the distribution of $\overline{W}$ will have a higher concentration about its mean, 2.4, and therefore the value of $\text{P}\left(2.3 < \overline{W} < 2.5\right)$ will increase.

11 (i)	$y = a + bx^2$  $y = c + \frac{d}{x}$ 
11 (ii)	
11 (iii)	The equation $y = c + \frac{d}{x}$, is more appropriate because from the scatter diagram, as m increases, <u>s increases at a decreasing rate.</u> $r = -0.982$

11 (iv)	Regression line of s on $\frac{1}{m}$: $s = 248.61 - 2731.4\left(\frac{1}{m}\right)$ $\approx 249 - \frac{2730}{m}$ Since systolic blood pressure depends on weight, we should use the regression line of s on $\frac{1}{m}$: $110 = 248.61 - 2731.4\left(\frac{1}{m}\right)$ $m \approx 19.706$ Since $s = 110$ lies outside the range of values of s, **extrapolation** is required which gives an **unreliable** estimate.
11 (v)	Since s depends on m, m is the controlled variable. Therefore, the regression line of $\frac{1}{m}$ on s should **not** be used.
11 (vi)	• Too few patients are selected for the equation of the regression line to be reliable in estimation. • The data is only valid for estimating the blood pressure for people of a similar age profile. • The data is only valid for estimating the blood pressure of patients with similar medical conditions. • A person's blood pressure is not fixed and is influenced by other factors at time of measurement, such as physical activity and/or varying emotional states like anxiety.
11 (vii)	Since the data point $(\bar{m}, \bar{s})$ does not lie on the regression line, the answer in part **(iii)** will change as the equation of the regression line is affected.